The Ideal Transformer

Description and Circuit Symbol

As with all the other circuit elements, there is a physical transformer commonly used in circuits whose behavior can be discussed in great detail. However, in many cases the practical transformer can be adequately approximated by the "ideal transformer," which is much simpler to describe. Physical transformers are often made with two coils of wire, each wound around the same core of magnetic material. Physical transformers have magnetic cores of various configurations and materials, including just air, and they may have more than two coils or windings. This discussion is limited to a basic design which has two windings placed on one core.

The circuit symbol in Fig. 1 is an idealized two-winding transformer. The symbol itself contains reminders of the physical device it is based on. The two curly windings represent the two lengths of wire wound into coils. The two vertical lines represent the presence of a core made of magnetic material, often an iron alloy. The two windings do have a distinct orientation of one with respect to the other: Either coil could be wound clockwise or counterclockwise. Therefore, there are tagged ends on each winding in the symbol to keep track of their relative orientations (the dots tag a given end of each winding). The numbers N_1 and N_2 give the numbers of turns used in each of the two windings.

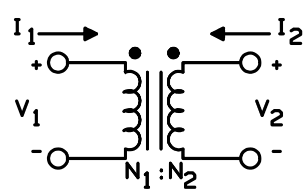

Fig. 1 Circuit symbol for the two-winding transformer.

Circuit Equations

Each winding will be connected into a circuit so that the voltages and currents shown in Fig. 1 are not zero. The rest of the circuitry, which is not shown, determines the relationships between v_1 and i_1, and between v_2 and i_2; however, the transformer enforces a definite relationship between the voltages on the number 1 and the number 2 windings. Likewise, it enforces a relationship between the number 1 and number 2 currents. These relationships are determined by the "turns ratio." The turns ratio n is defined as follows:

$$n = \frac{N_2}{N_1} \qquad (1)$$

The ideal transformer has the following voltage and current relationships:

$$\frac{v_1(t)}{N_1} = \frac{v_2(t)}{N_2}$$
or
$$v_2(t) = \frac{N_2}{N_1} v_1(t) = n\, v_1(t)$$

$$N_1\, i_1(t) = -N_2\, i_2(t) \tag{2}$$
or
$$i_2(t) = -\frac{N_1}{N_2} i_1(t) = -\frac{i_1(t)}{n}$$

Note that passive sign conventions were used for both windings in Fig. 1, and that the currents were signed positive entering the dotted end of the winding, and that the voltages were signed positive at the dotted ends. Changing any of these conventions would change the signs in (2).

The two windings of the transformer were labeled rather generically "1" and "2." It is also quite common to refer to the two windings as "primary" and "secondary." This convention is often used when a generator is connected to a primary winding, and a load is connected to a secondary winding. In that case, the energy flow is into the primary and out of the secondary; however, all transformers are bi-directional, so there is nothing inherently "primary" about either of the two windings.

The transformer turns ratio in Fig. 1 was indicated on the symbol as "N_1:N_2." It could have just as well been labeled "1:n." In honor of the fact that the ideal transformer is a model for a real transformer, the numbers N_1 and N_2 may be the actual physical turns count of a transformer, such as "363:33." For circuit analysis purposes it is equivalent to give the turns ratio as 11:1, or as $n = 1/11 = 0.0909$. It might be noted that the turns ratio of a physical transformer is always a rational number (the ratio of two integers). Therefore, a turns ratio of $\sqrt{3}$:1 is not possible, although 1732 turns and 1000 turns on a physical transformer would do a good job of approximating it.

Equivalent Circuit

An equivalent circuit is often given to explain the operation of a complicated or unfamiliar device by means of an equivalent which contains only familiar devices. If a circuit is truly an "equivalent circuit," the original device can be removed from a system and replaced with its equivalent circuit without changing the behavior or performance of the system. Two equivalent circuits for the ideal transformer are given in Fig. 2. Both of these use a pair of controlled voltage and current sources, where the control factor is either the turns ratio, or its reciprocal. Incidentally, the ideal transformer is not included as a component in the SPICE simulation package; however, controlled sources are available, so one of the equivalents of Fig. 2 is used for SPICE simulation of a circuit containing an ideal transformer.

Fig. 2 Two equivalent circuits for an ideal transformer with a turns ratio of 1:*n*.

AC *vs.* DC Operation of the Ideal Transformer

It is a fact of life that the physical device called the “transformer” cannot operate properly in a dc circuit. The ideal transformer, because it is ideal, is defined to work according to equation (2) equally well for dc or ac voltages and currents, or any combination of the two. Because of the physical limitations of a real transformer, it is rare to see a dc circuit containing a transformer, even if only on paper. However, advances in power electronics over the past 30 years have made an electronic sub circuit which behaves as a “dc transformer” quite commonplace. These power electronic circuits are not usually called “dc transformers,” but that is certainly an acceptable and accurate name. Therefore, a dc circuit which contains a “transformer” is actually a possibility, as long as one understands that the transformer is not the simple device we commonly call a transformer, but rather an electronic switching power converter. These switching power converters normally function with the “turns ratio” as a control input which can be adjusted or varied as the user sees fit.

Equation (2), the defining equation of the ideal transformer, was written showing the voltage and currents of windings 1 and 2 as time-varying. The ideal transformer enforces its turns-ratio relationships between the voltages and currents on an instantaneous basis. Because of this, (2) also applies to each of the attributes of the voltages and currents, such as the peak, average, or rms values. Likewise, the ideal transformer relationships apply to the phasor transforms of the voltages and currents.

$$\begin{aligned} \hat{V}_2(j\omega) &= n\,\hat{V}_1(j\omega) \\ \hat{I}_1(j\omega) &= -n\,\hat{I}_2(j\omega) \end{aligned} \tag{3}$$

Ideal Transformer Properties - Voltage and Current Scaling

The defining equations for the ideal transformer (2) indicate voltage and current scaling. This is illustrated in a sample circuit in Fig. 3. Note the extreme range of voltages and currents in different parts of this circuit. The transformer on the right side is used to scale high voltage down to low voltage, while at the same time scaling low current up to high current. This is a very useful property in a power transmission and distribution system. Fig. 3 is realistic for this application: 25 kV is about the right voltage for a large generator, 138 kV is about the right voltage level for a long transmission line, and

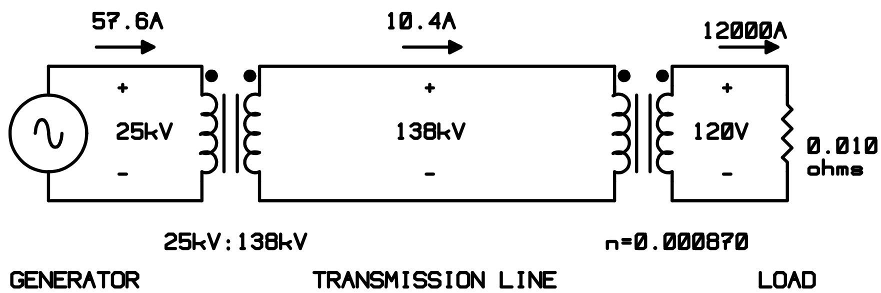

Fig. 3 Transformers used for voltage and current scaling.

120 V may be the voltage required by a certain industrial process (an arc furnace). The two transformers permit each circuit element (the generator, the transmission line, and the load) to operate at its most favorable voltage level. Note that the currents are similarly scaled, and that although the load current seems quite high at 12,000 A, the transmission line current is much more manageable at 10.4 A. It is interesting to note that the product of the voltage and current at every step along the way is the same: 1.44 MW.

Fig. 3 also illustrates two different ways that the transformer turns ratio might be given. The left-hand transformer's turn ratio is stated by the pair of voltages which could simultaneously exist on its two windings. On the right-hand side, the turns ratio is given after reducing the ratio as $n = 0.000\,870$. The actual number of turns is not given in either case, but it is not needed for circuit analysis.

Ideal Transformer Properties - Instantaneous and Complex Power

The ideal transformer does not generate, dissipate, or store energy. Therefore the instantaneous power leaving the transformer is the same as that entering. This could be said in other words by saying that if one were to draw a box around an ideal transformer and sum the power flows into (or out of) the box, the answer is zero at every moment in time. This is illustrated in Fig. 4. In this figure, a source and a load are connected through an ideal transformer. The instantaneous power entering the transformer on its #1 side is denoted by $p_1(t)$; the instantaneous power leaving the transformer on its #2 side is denoted by $p_2(t)$. The power sign convention is indicated in Fig. 4: power is positive if it is flowing in the direction indicated by the arrow. The instantaneous power on the primary side is given by:

$$p_1(t) = i_1(t)\, v_1(t) \tag{4}$$

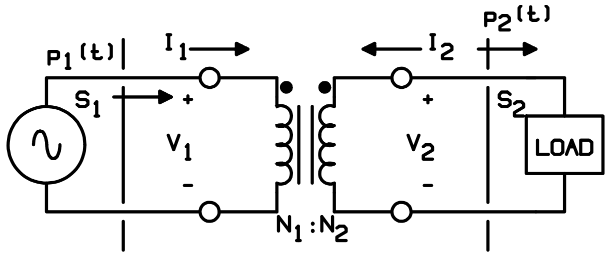

Fig. 4 Instantaneous and complex power flow through an ideal transformer.

The ideal transformer equations (2) can be used to show that the instantaneous power on the secondary side is identical.

$$p_2(t) \;=\; -i_2(t)\,v_2(t) \;=\; +\frac{i_1(t)}{n}\,nv_1(t) \;=\; p_1(t) \tag{5}$$

In an ac system in which phasor analysis has been used on the circuit, it is useful to define complex power. The complex powers flowing into (S_1) and out of (S_2) the transformer are also indicated in Fig. 4. These also have equal real and imaginary components.

$$\begin{gathered} S_2 \;=\; \hat{V}_2\left(-\hat{I}_2\right)^* \;=\; n\hat{V}_1\left(\frac{\hat{I}_1}{n}\right)^* \;=\; S_1 \\ \text{meaning} \\ P_2 \;=\; \mathrm{Re}\left(S_2\right) \;=\; P_1 \;=\; \mathrm{Re}\left(S_1\right) \\ \text{and} \\ Q_2 \;=\; \mathrm{Im}\left(S_2\right) \;=\; Q_1 \;=\; \mathrm{Im}\left(S_1\right) \end{gathered} \tag{6}$$

The ideal transformer does not generate or consume either real or reactive power.

Ideal Transformer Properties - Impedance Scaling

Fig. 5 illustrates the impedance scaling property of the transformer. Consider a source on the #1 (primary) winding, and a complex impedance load on the #2 (secondary) winding. The secondary current will be related to the secondary voltage by the load impedance:

$$\hat{I}_2 \;=\; -\frac{\hat{V}_2}{Z_2} \tag{7}$$

The transformer turns-ratio relationship (2) gives the corresponding primary voltage and current. To the generator, the apparent value of the load impedance is the ratio of the voltage to the current on the primary side.

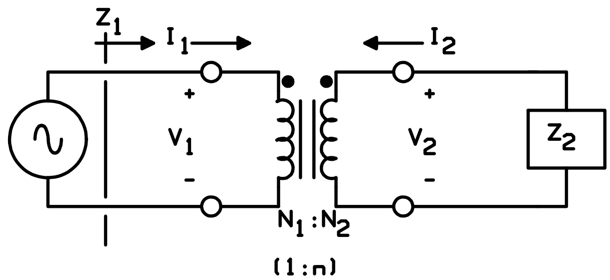

Fig. 5 Impedance scaling property of the transformer.

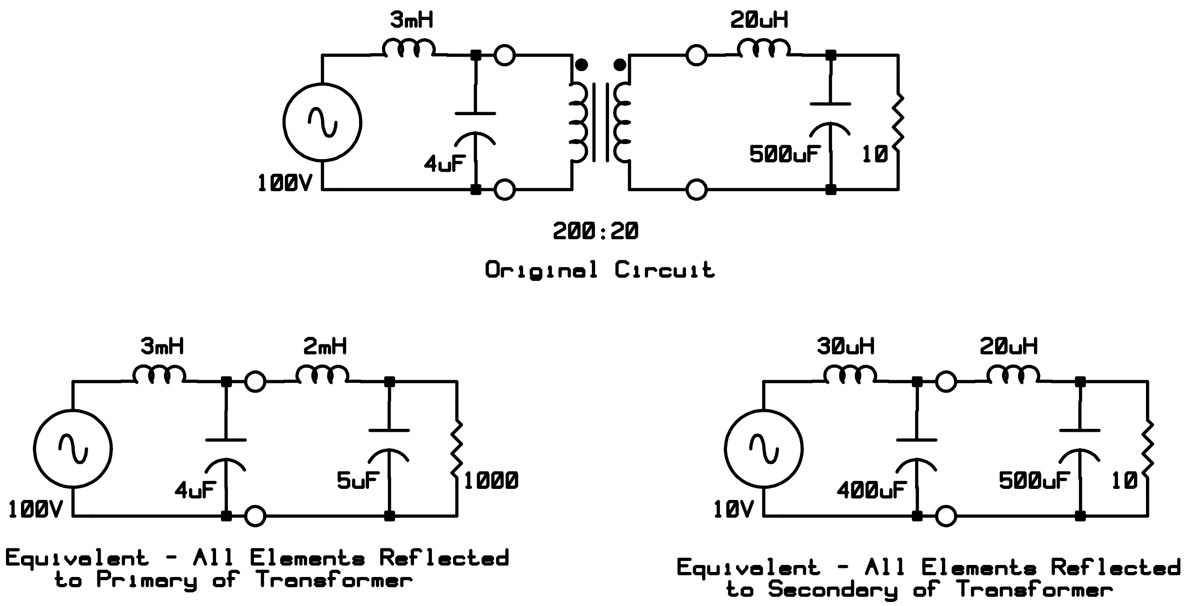

Fig. 6 Illustration of impedance reflection.

$$Z_1 = \frac{\hat{V}_1}{\hat{I}_1} = -\frac{\hat{V}_2}{n^2\hat{I}_2} = \frac{Z_2}{n^2} \tag{8}$$

Equation (8) may be interpreted in words by saying that if a given impedance is connected to one winding of an ideal transformer, it will appear the same at the other winding, except that it is scaled in magnitude by the turns-ratio squared. The impedance appears greater at the winding having the greater number of turns, and smaller at the winding having the fewer.

The impedance scaling property of an ideal transformer allows circuit elements to be moved from one winding to another by scaling their impedances according to the square of the transformer turns ratio. This is illustrated in Fig. 6. Here note that the transformer has a turns ratio of 200:20, or 0.10. Two equivalent circuits are also shown in Fig. 6. On the lower left, all of the circuit elements on the secondary side of the transformer have been “reflected” to the primary side. In each case, the element is the same type as before, but its complex impedance increased by a factor of 100 (10 squared). (Remember that the impedance of an inductor is directly proportional to its inductance value, while the impedance of a capacitor is inversely proportional to its capacitance value.) The lower right side of Fig. 6 shows the result of reflecting all circuit elements to the secondary side of the transformer. Here note that while impedances scale according to the square of the turns ratio, voltages and currents are scaled proportionate to the turns ratio only.

Galvanic Isolation

Galvanic isolation is a reference to the fact that a transformer normally has no direct electrical connection between its primary and secondary windings. This is a very useful property, and sometimes a transformer with a 1:1 turns ratio is added to a circuit solely to take advantage of the galvanic isolation it offers. What this is good for is best explained with an example.

Consider a 50-kW A.M. broadcasting transmitter. The transmitting antenna is a metal tower about 75 m tall, insulated from the earth. The transmitter must deliver approximately 1300 V at the broadcasting frequency, with one connection to the tower and the other to a solid earth ground connection. (How the 50 kW leaves the electrical circuit and is launched as a traveling radio wave is outside the scope of this circuits course, and nowhere explained in our book!) This situation is sketched in Fig. 7. The problem arises that the aviation authorities would like to see a red light at the top of the tower to warn aircraft. The light can be supplied from the 120-V power mains, but one terminal of the power mains is earth grounded. This is required for safety purposes should there be an insulation failure or a lightning strike in the power system.

One could attempt to run wires up the tower to the light, carefully insulating both wires and the light from the tower structure. This will not work, however, because this 75-m of wiring for the light becomes part of the antenna system and interferes with it. At the frequency range used in the A.M. broadcasting band, the capacitance between the wires and the tower is very significant. In effect, there is a unwanted capacitor connected between the transmitter output terminal and the power system. This ties the power system into the transmitting antenna, putting a strong radio signal into the power system, and interfering with the antenna's purpose of launching a radio wave.

The solution is connect one of the lighting wires to the tower, making the light and its wiring clearly part of the metallic mass of the transmitting antenna. In order to get the 120-V supply to run the light, an isolating transformer is used. This is shown in Fig. 7. Study of the circuit will show that the transformer primary and secondary windings are each operating with 120 V at 60 Hz appearing across them. However, there is 1300 V at radio frequency appearing between the bottom terminal of the primary and the bottom terminal of the secondary. The transformer must simply have good enough insulation to handle this voltage level. The transformer is said to supply "galvanic isolation" for the 1300-V radio signal.

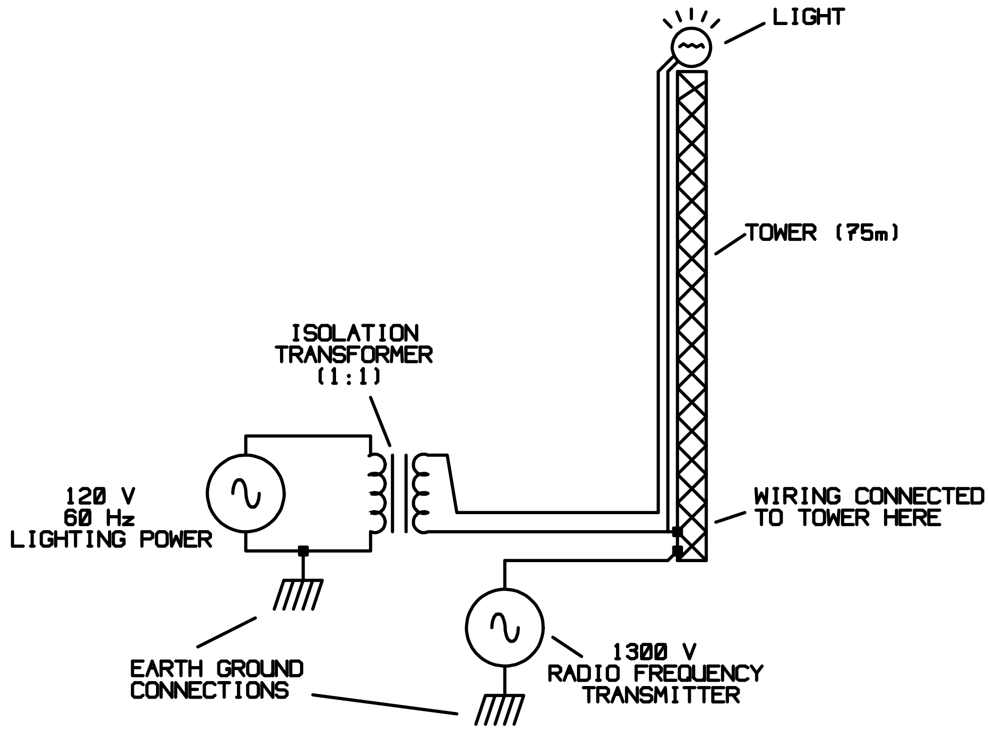

Fig. 7 Galvanic isolation properties of the transformer allow the connection of the 120-V lighting power to the transmitting antenna tower without harm to the power system or the transmitter.

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