

The Twin Rural Postman Problem

Marcos Negreiros

Universidade Estadual do Ceará
Laboratório de Computação Científica (LCC-UECE)
Av. Silas Munguba, 1700 (Campus do Itaperi)
Fortaleza, Ceará, Brazil, CEP: 601740-903
marcos.negreiros@uece.br

Augusto Palhano

Graphvs Consultoria Ltda
R. Vicente Linhares, 521 sl 913, Aldeota,
Fortaleza - Ceará - Brazil, CEP: 60135-270
augustopalhano@gmail.com

Nelson Maculan

Universidade Federal do Rio de Janeiro,
Programa de Eng de Sistemas e Computação (PESC-COPPE)
Ilha do Fundão, Sala H-322B, Rio de Janeiro - RJ - Brazil, CEP: 21.941-972
maculan@cos.ufrj.br

Claudio Barbieri da Cunha

Universidade de São Paulo,
Depto. de Engenharia de Transportes - Escola Politécnica
Av. Prof. Almeida Prato, trav. 2, n. 83 - Edif. Paula Souza
Cidade Universitária, São Paulo, Brazil, CEP: 05508-070
cbcunha@usp.br

ABSTRACT

This paper presents a new combinatorial optimization problem within the class of arc routing problems, referred to as the Twin Rural Postman Problem. In this problem, a mixed and strongly connected graph composed of a set of required and positive weighted links must be fully covered by two servers with minimum total distance. One server covers a subset of required links in a tour that departs and ends from a garage, forming a rural postman tour. Meanwhile, the other server covers the remaining required links in a min-max open rural postman tour. Here we present the problem and its multi-objective set-partitioning formulation. We also discuss different forms and extensions of the problem, and we provide a solution methodology and its application with results to the domiciliary waste collection for the city of Andradina/SP.

KEYWORDS. Arc Routing. Rural Postman. General Routing Problem.

CO, L&T, OA

1. Introduction

Domiciliary waste collection is a common activity in Brazil, where the municipality is the responsible to its management. Also known as organic collection, domiciliary waste collection represents 63-75% of the total operational costs for any worldwide solid waste management system (SWMS). Along years this service has been evolving due to the increasing costs in manpower, transportation and environmental impacts, as pointed out by Hess et al. [2023], Negreiros et al. [2023a], Negreiros et al. [2024a], Olawade et al. [2024].

The basic waste management system in Brazil is planned according to the knowledge of the geographical limits of regions, transportation features and their solid waste generations. In a region, the service is divided into zones, each of which must be covered with an assigned frequency throughout the week. Within these zones, sectors are designated for daily coverage by a vehicle and a crew. Additionally, circuits are established where vehicles and crews complete their maximum tonnage before discharging waste in a dump site. This logistics system is complemented by the obligation for vehicles to perform tours that cover each street within the sectors, providing door-to-door service, where collectors do their work manually removing garbage disposed by owners at their domiciles, while vehicle traverses slowly receiving and compacting the garbage until it reaches full capacity, Negreiros et al. [2022], Negreiros et al. [2023a], Negreiros et al. [2024a], Negreiros et al. [2024b].

The above process has undergone changes over the years in many Brazilian cities, particularly where as major suburban regions expanded, becoming vast and impractical to be covered within reasonable costs and time. The common idea has been to change the process in order to reduce the impact of the vehicle's travels and staff labour costs, combining coverage of a sector through pedestrian (walking collector) and motorized (vehicle) means. This arrangement has led to numerous problems, disappointments and disturbs for municipalities. However, it is currently under investigation in practice by many of the existing concessionaires operating in Brazil.

The extant literature theoretically offers various solutions to the problem of domiciliary waste collection problem, Hess et al. [2023], but most of them consider door-to-door arc routing, or in majority node routing, primarily due to the USA/Canada-based model's use of bins and lift-arm tankers without crews, Negreiros et al. [2024a].

Based on real-world field experience, we are developing a research project for Conservita, a concessionary of domiciliary waste collection operating in Andradina/SP, as well as other cities such as Jales and Santa Cruz do Rio Pardo, also located in the state of São Paulo. In this environment, combined work is a common practice, and the process relies on the driver to control and manage the distribution of the responsibilities between manual and mechanical collection tasks performed by the crew (collectors). Two situations may occur:

1. Asynchronous routes: In this scenario, the servers work independently. Typically, the collector departs early (1 or 2 hours) to cover a circuit within the sector, moving the garbage from their assigned links to the links where the vehicle will transverse along its route .
2. Synchronous routes: Here, the two servers coordinate their coverage efforts in a way that both depart around the same time to complete the circuit of a sector, ensuring that all its street segments are serviced. This allows the corresponding vehicle to traverses part of the sector, passing through a subset of its required links, while a collector pulls garbage from the remaining required links and brings it to specific points (typically a street intersection) along the vehicle's route.

Both cases can be integrated as a new combinatorial optimization problem with a multi-objective function. The routes in this problem are performed with different impacts: for the worker

(collector), it is necessary to minimize walking distance while collecting waste considering the weight/distance he/she must travel to pick-up and deliver to a street corner. In contrast, the vehicle needs to travel less, thus reducing the amount of fuel/energy consumption and costly maintenance. The balance between both routes results in economies of scale to the concessionaire and improves the speed of the collection process. However, the garbage left at the intersection needs to be placed in an adequate container to prevent health problems to the municipality.

Negreiros et al. [2023b] investigated this multi-objective combined coverage and indicated that this alternative model of collection can reduce in about 40-45% , and about 15-25% in average the total distance of the tour travelled in a trip by the vehicles. They mean that there are expressive savings in a full traversal of all service street segments of the sector.

In Figure 1, we can observe the instance (Sub-figure 1a); the general routing (GRP), Corberán et al. [2014], tour and its total distance travelled (Sub-figure 1b); in the subsequent Sub-figures 1c and 1d, the combined coverage with the tours and their total distance traveled, respectively. The reduction in the total distance by the vehicle is 23.08%, but it requires a 12.67Km walking tour for one collector. This effort must be evaluated appropriately, to fit the worker effort to their capacity to healthily complete the work.

The rural postman problem (RPP) is a well-known problem in the arc routing literature, with established methods for both its symmetric and asymmetric versions. The asymmetrical version has been studied by a few authors. Initially, Corberán et al. [2000] developed a Tabu Search procedure wasbased on a constructive method adapted from Nobert and Picard [1996], which was originally designed for the mixed Chinese postman problem. This work was later expanded by Corberán et al. [2003], who developed a mathematical model and provided a polyhedral description of the problem. Further advancements were made by Corberán et al. [2014], who introduced two new formulations for the Mixed RPP and the Mixed General Routing Problem (GRP). Most recently, Corberán et al. revised these formulations for the symmetric case and extended the methodology to address very large-scale instances of the GRP.

This work is organized as follows: in Section 2 we introduce and formulate the problem as a graph optimization problem, and propose a multi-objective set-partitioning formulation to the problem. In section 3 we propose a method to solve the problem based on this formulation using k -shortest paths, and a two phase semi-automatic procedure, while in Section 4 we evaluate the results of the proposed methods for the city of Andradina/SP. Finally we conclude this work by discussing the results and indicating future research directions being investigated by our team.

2. Problem Definition and Some Variations

Given a mixed and strongly connected graph $G(V, L)$, where V are the set of vertices, L is the set of links between any par of adjacent vertices $ij \in L$ formed by a subset of non oriented edges E and a subset of oriented arcs A , $L = E \cup A$. Let R be the required set of vertices and links of this graph, so V_R, E_R and A_R are the subsets of required vertices, required edges and required arcs, respectively or simply L_R . For the purpose of solving any RPP, let the set $W = \{w_{ij} > 0, \forall ij \in L\}$ represent the set of weighted costs of the links.

A General Routing tour is defined by a tour that covers all the required links and vertices, starting from a vertex $v_0 \in V$ with minimum total weighted cost, Negreiros [2024a]. While a Min-Max Open General Routing tour is here defined as the minimal cost tour that traverses all required vertices and links (maximum cover) from-to any vertex of G , Negreiros [2024b].

The General Routing Problem (GRP) can be easily transformed to an equivalent Rural Postman Problem (RPP), just by imposing the expansion of the required vertices into required links in polynomial time, Corberán et al. [2014]. So, it is easy to verify that the min-max open general routing (OGRP) problem can be also converted into an equivalent RPP. Thus, the GRP can be



Figure 1: Detailed savings in domiciliary waste collection from combined coverage.

transformed into an RPP_v (RPP vehicle's tour), and the min-max OGRP can be transformed into an RPP_o (RPP collector's or open tour) through a polynomial transformation.

2.1. The Twin RPP

Let us define the Twin Rural Postman Problem (TRPP) - asynchronous, as the problem of minimizing both RPP_v and RPP_o in such a way that $L_R = L_R^v \cup L_R^o$, and $L_R^v \cap L_R^o = \emptyset$, meaning that L_R^v are the set of links covered by the RPP_v (vehicle) and L_R^o are the set of links covered by the RPP_o (collector). Without loss of generality, a subset of links of RPP_o can be covered by a subset of links of RPP_v , but if one of RPP_o is a required link in the RPP_v route, the other is non-required and vice-versa. In Figure 2, we can observe a partition p of the required links of a GRP instance, (Sub-figure 2a), the vehicle's required links that should be considered (Sub-figure 2b), and the collector's required links (Sub-figure 2c).

A set-partitioning formulation of the asynchronous TRPP can be proposed as follows, after applying the appropriate transformation to the original graph:

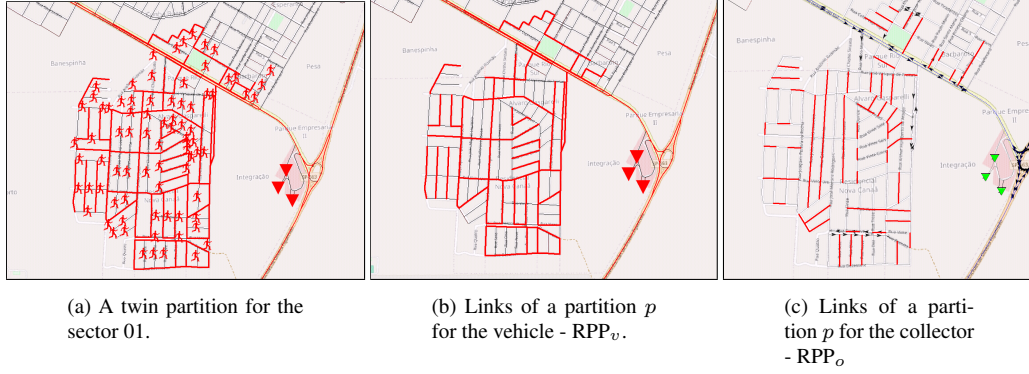


Figure 2: Division of links in a p partition of the TRPP.

Sets:

- V - Set of vertices;
- E - Set of edges;
- $E - E_R$ - Set of non required edges;
- E_R - Set of required edges;
- A - Set of arcs;
- $A - A_R$ - Set of non required arcs;
- A_R - Set of required arcs;
- P - The set of valid partitions of L_R ;
- $L_R^{v,p}$ - A subset p of a partition L_R for v ;
- $L_R^{o,p}$ - A subset p of a partition of L_R for o .

Parameters:

- v_0 - vertex corresponding to the garage;
- α - factor to multiply the cost of RPP_v route;
- w_{ij} - cost of the links;
- $cost(RPP_v^p)$ - cost of the optimal RPP_v route using partition p ;
- β - factor to multiply the cost of RPP_o route;
- $cost(RPP_o^p)$ - cost of the optimal RPP_o route using partition p .

Variables:

$$\begin{aligned}
 z^k &= \begin{cases} 1, & \text{if partition } k \text{ of the required links is used by the route } RPP_v; \\ 0, & \text{otherwise.} \end{cases} \\
 u^s &= \begin{cases} 1, & \text{if partition } s \text{ of the required links is used by the open route } RPP_o; \\ 0, & \text{otherwise} \end{cases} \\
 x_{ij}^p &= \begin{cases} 1, & \text{if the required link } ij \text{ is in the partition } p \text{ by the } RPP_v; \\ 0, & \text{otherwise} \end{cases} \\
 y_{ij}^p &= \begin{cases} 1, & \text{if the required link } ij \text{ is in the partition } p \text{ by the } RPP_o; \\ 0, & \text{otherwise} \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 \text{(TRPP) Minimize } & \sum_{p \in P} (\alpha \text{ cost}(RPP_v^p) z^p + \beta \text{ cost}(RPP_o^p) u^p) \\
 \text{Maximize } & \sum_{ij \in L_R^v} x_{ij}^p + \sum_{ij \in L_R^o} y_{ij}^p
 \end{aligned} \tag{1}$$

Such that:

$$\sum_{p \in P} (x_{ij}^p + y_{ij}^p) = 1, \forall ij \in L_R \tag{2}$$

$$\sum_{ij \in L_R^v} x_{ij}^k \leq |L_R^{k,v}| z^k, \forall L_R^v, k \in P \tag{3}$$

$$\sum_{ij \in L_R^o} y_{ij}^s \leq |L_R^{s,o}| u^s, \forall L_R^o, s \in P \tag{4}$$

$$x_{ij}^p, y_{ij}^p, z^p, u^p \in \{0, 1\}, \forall ij \in L_R, p \in P \tag{5}$$

The TRPP formulation considers the objectives of minimizing the total cost of both RPP tours, while maximizing the coverage of the links in both tours. In constraint (2) we guarantee that the covering of each twin set is unique. Constraint (3) imposes that the covering of the twin set corresponds to a vehicle's tour. Constraint (4) guarantees the covering of the twin set in the collector's (open) tour. The constraint (5) correspond to the domain of the problem's decision variables.

It can be seen that the set P is formed by combinations, in such a way that:

$$|P| = \sum_{i=0}^{|L_R|} \binom{|L_R|}{i} \tag{6}$$

This problem is naturally NP-Hard as its basic problem, RPP, is also NP-Hard.

In Figure 3, it is illustrated more detailed the solution of the problem previously depicted in Figure 1, for a TRPP's instance (Sub-figure 3a) using a p partition of the required links (Sub-figures 3b and 3c).

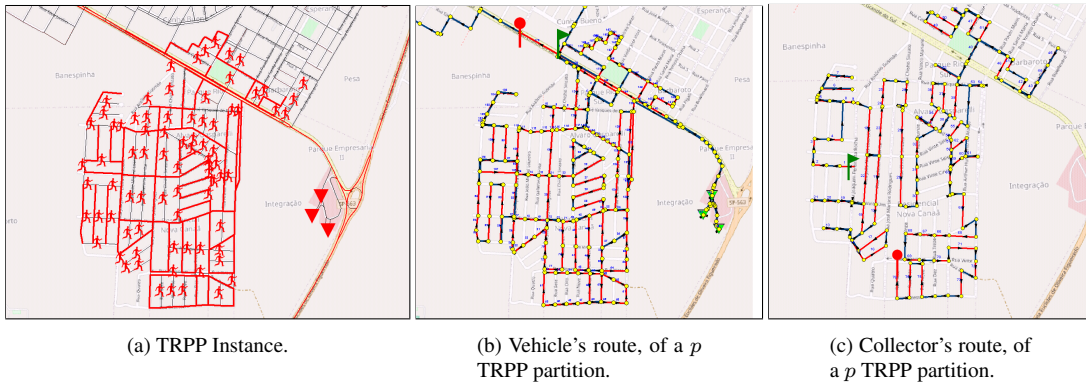


Figure 3: A TRPP feasible solution.

2.2. Some TRPP's Variations

The asynchronous form of the TRPP can be extended to other variations of this problem, as follows:

1. Synchronous with Hierarchy: In this form, the twins must coordinate their efforts so that when one twin traverses links serviced by the other, the latter has already completed their covering task. In other words, this means that it may exist an hierarchy of service such that both twins can work together, helping each other by departing from their respective vertices at the same time;
2. Using mandatory links for vehicle and collector: In this case, for both synchronous and asynchronous TRRP, some required links must be visited by one or both twins;
3. Max covering distance - D_{max} : In this form of the problem, a maximum distance must be defined between pickup links by one twin to the closest link of the route of the other twin in the vehicle.
4. Combination of variations: It can be represented as any form of the original problem that combines previous proposed forms into a new one.

The relevance of this problem is straightforward. It introduces the combinatorial concept of a dynamic waste collection problem observed in the field. And it can be used to solve different arc-routing problems with complementary parts.

3. Methods

In the following subsections we propose two new heuristic methods for evaluating TRPP routes: the general method based on distinct partitions of the local/optimal RPP route over the GRP instance, and the semi-automatic construction over a given required set of links (L_R).

3.1. General method for the Twin RPP

In this method, we first generate a full RPP route over L_R . A full connected graph where the vertices are the required links, and the links are formed by the adjacent list of links of the full RPP Euler tour previously built (Figure 4).

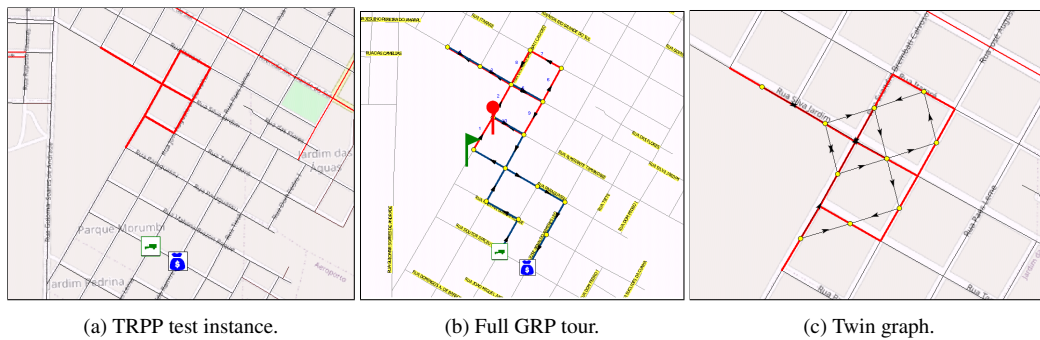


Figure 4: First step of the k -Path method.

From the resulting graph, it selects a starting required link from the set of L_R ordered by the RPP full route, then proceeds to find a k -shortest path (k being a parameter) to the end vertex (the required link corresponding to the last one visited by the full RPP tour). It saves all paths in a list of paths, and for each path the uncovered links of the network is assigned to the collector's partition (RPP_o).

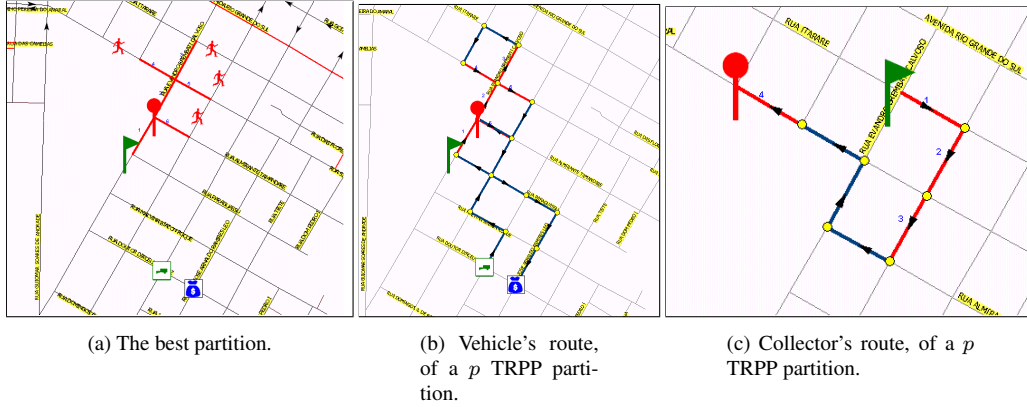


Figure 5: The TRPP optimal solution.

For all paths, which are in fact partitions of L_R , it is calculated the open RPP tour over the (L_R^o) links. The cost of this composition is obtained from the following equation:

$$\alpha \text{ cost}(RPP_v^p) + \beta \text{ cost}(RPP_o^p) \quad (7)$$

If the cost improves the current best known solution, this partition is saved as the best and the actual total best cost is saved. The method proceeds until all k -shortest paths have been evaluated. It returns the best partition found and the corresponding twin RPP tours (Figure 5).

This method is of high computational complexity, $O(2^{L_R})$, but it works properly, very close to/or optimal, for short networks ($|L_R| \leq 20$, and $30 \leq k \leq 40$) as see in Section 4. For higher instances, it is necessary to use higher k , that can grow exponentially.

3.2. A semi-automatic method

In the semi-automatic method, a given partition is considered initially, and only a vehicle tour (RPP_v) and a collector's tour (RPP_o) are calculated, keeping the costs of both and their routes.

As demonstrated in Negreiros et al. [2023b], this method can devise different coverage strategies and obtain the best between them by incorporating additional factors such as knowledge of the region, local collection the features, street segments where there is no collection due to houses facing other parallel street, forbidden streets, and other.

4. Results

In the following subsections we apply both methods to evaluate the computational efficiency of the solutions and their quality. All the tests were performed using a PC-DELL G3, Intel(R) Core(TM) i7-10750H CPU 2.60GHz 2.59 GHz, 16,0 GB (NVMe). In all tests we used one processor. The software environment used was **SisRot® LIX, v5.0**, compiled with EMBAR-CADERO Delphi 32bits under Windows 11. We used basically its GRP and OGRP heuristic solvers (Negreiros [2024a]).

4.1. Applying the General Method

In Table 1, we present the results of the instances created to evaluate the computational behaviour and solutions of the set partitioning method. For our experiments, we have used the street network of Andradina as support network, where $|V|=4915$ and $|L|=6112$ indicate the number of vertices and links, respectively. For all instances, the RPP routes depart and end from different vertices, but they are the same for all instances. We maintain the parameters $\alpha = 1$ and $\beta = 2$ for the tests with 10-20 required links, $\alpha = 1$ and $\beta = 5$ for the tests with 30 required links and

$\alpha = 1$ and $\beta = 10$ for the tests with 40-50 required links. The starting vertex (v_0) of the Euler tour's corresponding graph, was always selected as the middle link considering the GRP tour order.

Table 1 comprises the following information: 'Instance' denotes the label of the instance, *GRP* corresponds to the full route distance (m), k is the number of paths from a selected starting required link to destination, followed by: $|L_R|$, the cardinality of the set of required links; $|L_R^v|$, the cardinality of the set of required links covered by the vehicle; $|L_R^o|$, the cardinality of the set of required links covered by the collector; $C(RPP_v)$ the cost of the route performed by the vehicle; $C(RPP_o)$ the cost of the route performed by the collector; and D_{max} the maximum distance (in meters) from the collector's center links to the vehicle's route.

Instance	<i>GRP</i>	k	$ L_R $	$ L_R^v $	$ L_R^o $	$C(RPP_v)$	$C(RPP_o)$	D_{max}
TEST-TRPP-01	2.518	10	10	6	4	2.077	0.769	57.38
TEST-TRPP-02	4.047	10	10	5	5	3.704	0.574	49.32
TEST-TRPP-03	4.811	20	15	8	7	3.894	0.995	54.69
TEST-TRPP-04	5.426	20	15	11	4	4.824	0.807	48.85
TEST-TRPP-05	7.922	40	20	12	8	7.458	1.256	64.41
TEST-TRPP-06	7.148	40	20	9	11	6.135	1.441	55.24
TEST-TRPP-07	6.747	300	30	16	14	5.833	2.216	65.60
TEST-TRPP-08	8.760	300	30	9	21	6.350	2.591	129.18
TEST-TRPP-09	8.021	1000	40	18	22	6.744	2.494	55.14
TEST-TRPP-10	9.262	1000	40	13	27	6.913	3.481	111.54
TEST-TRPP-11	11.173	3000	50	25	25	9.392	3.154	89.23
TEST-TRPP-12	10.155	3000	50	28	22	8.196	2.532	124.08

Table 1: Results with the test instances.

It should be noted that Table 1 includes the information D_{max} , i.e., the minimum-maximum distance from any link of the collector to any link of the vehicle. This information is a result of our model, not an input parameter. The track of this result is important because the solution will return how far the collector's required link is from the vehicle's tour. The results show that the solutions obtained from TEST-TRPP-08, 10 and 12 are not good enough, requiring substantial effort by the k -shortest path procedure over the Euler Tour Graph.

In the following Sub-figures 6a-6c of Figure 6, the evolution of the calculation for the best $p = k$ partition (evaluated from the last 50 of 300 k -shortest paths) is illustrated using the proposed procedure. The graphics for the instance TEST-TRPP-07, with 30 required links, demonstrated that the objective function value is not necessarily dependent on D_{max} . This behavior is further confirmed in Figures 7 and 8 for the instances TEST-TRPP-09 and TEST-TRPP-11, respectively.

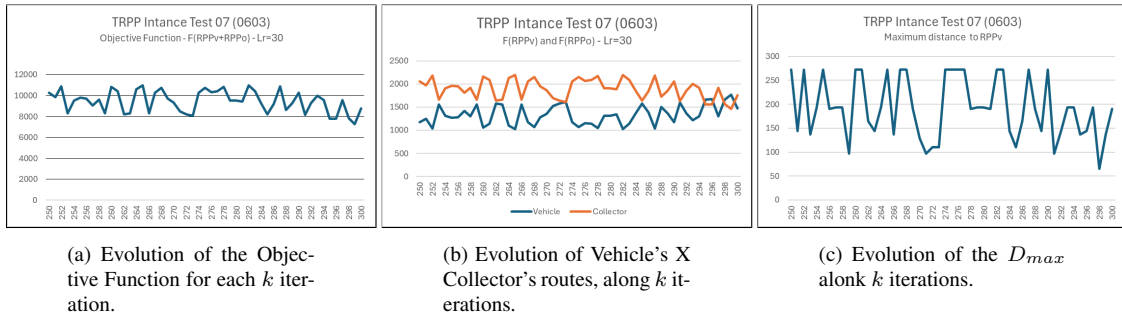


Figure 6: The TRPP k partitions evaluation.

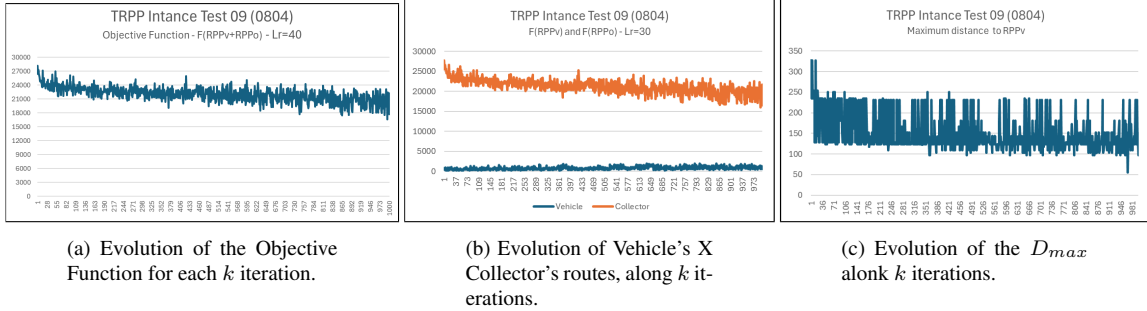


Figure 7: The TRPP k partitions evaluation for instance TEST-TRPP-09.

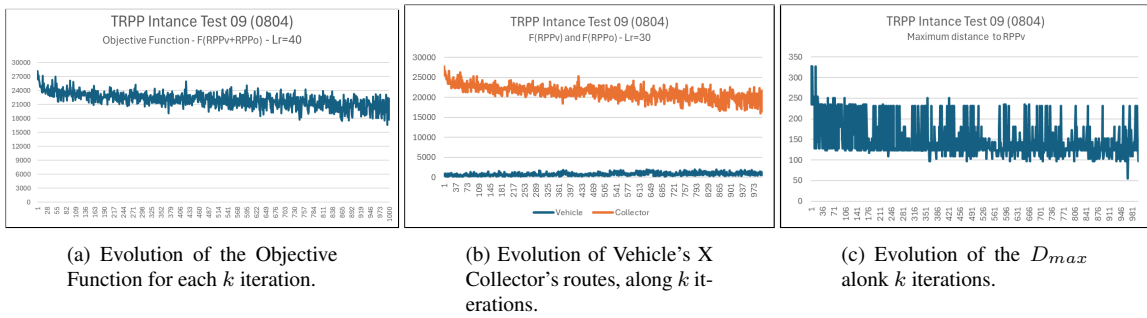


Figure 8: The TRPP k partitions evaluation for instance TEST-TRPP-11.

4.2. Applying the semi-automatic method

For the semi-automatic method, our research project developed and editor for the **SisRot[®] LIX, v5.0** software, to define manually the partition of places covered by vehicles and collectors for each sector. With this on hand, it is given to the user the possibility to generate the separated list of both required links, and proceed the routes with the GRP solver for both type of routes, Negreiros [2024a]. Finally, we obtained the solutions for all planned circuits, in the asynchronous TRRP form.

In Table 2 it is presented the set of instances solved using the semi-automatic method, where: “Instance” is the label of the instance, $|V, L|$ are respectively the number of vertices and links of the support network used to solve the problem, $|L_R|$ is the cardinality of required links set, $|L_R^v|$ is the cardinality of required links set that is covered by the vehicle, $|L_R^o|$ is the cardinality of required links set that is covered by the collector, $C(GRP)$ is the cost (in Km) of the route performed by the vehicle in all L_R , $C(RPP_v)$ is the cost (in Km) of the route performed by the vehicle in combination, $C(RPP_o)$ is the cost (in Km) of the route performed by the collector, T_f is the total time (mm:ss.ds) spent to solve the RPP for all the indicated instances, T_v is the total time (mm:ss.ds) spent to solve the RPP for all the indicated instances for the vehicle and T_o is the total time (mm:ss.ds) spent to solve the ORPP for all the indicated instances for the collector.

All the circuits covered by Andradina’s domiciliary waste collection were generated, returning the solutions as presented in Table 2.

Table 2 returns the service of a manual selection for a twin partition. Circuits from 0101 to 0402 are executed on Mon-Wed-Fri and the remaining circuits (from 0501 to 0802) are executed on Tue-Thu-Sat. The odd circuits starts from a support point (minus 0401 and 0801) and once a vehicle reaches its load capacity, it goes to the city’s landfill, from where all other subsequent circuits start and end their routes.

In Table 3 ‘GRP’ denoted a full route, RPP_v the vehicle’s route, RPP_o the collector’s

Instance	$ V, L $	$ L_R $	$ L_R^v $	$ L_R^o $	$C(GRP)$	$C(RPP_v)$	$C(RPP_o)$	T_f	T_v	T_o
0101	4965 5525	263	183	80	38.23	29.43	13.14	6:52.79	3:16.48	2:30.96
0102		206	139	67	32.08	24.02	10.08			
0201		172	100	72	29.98	24.32	8.97			
0202		287	183	104	36.51	26.12	14.29			
0301		260	212	48	40.02	35.53	8.72			
0302		256	204	52	32.53	28.72	8.20			
0401		137	114	23	28.06	24.23	4.31			
0402		199	132	67	45.40	37.63	9.96			
0501	4986 6553	221	161	60	40.49	33.49	12.49	8:14.76	3:22.83	3:57.43
0502		253	172	81	44.29	34.17	11.50			
0601		279	218	61	62.54	53.72	10.56			
0602		326	242	84	44.02	37.17	11.30			
0701		235	169	66	34.56	24.61	11.94			
0702		292	242	50	54.66	47.91	10.00			
0801		150	133	50	31.23	29.08	6.68			
0802		153	97	56	30.37	24.57	9.36			

Table 2: TRPP semi-automatic solved instances.

route, Gap% meaning the savings in distance between the GRP and RPP_v routes, and the relation RPP_o/RPP_v means the relative portion of the collector's work over the vehicle's work. As can be observed, the imposed system operates with high rates of tonnage for a collector meaning 2.6ton per collector/day in average (working averagely 3.2h per circuit at 3Km/h), or his/her work represents 43.47% of all garbage collected, although the load represents the full service of two heavier days of the week (Mon and Tue). The reduction in the costs are high (17.72%) from the conventional optimized system (already reduced in 15% averagely), and finally the collector's walking paths represents 31.38% of vehicle routes in average.

	GRP	RPP_v	RPP_o	Gap%	RPP_o/RPP_v
Km	625.006	514.281	161.362	-17.72%	31.38%
Kg	79278.29	44,813.63	34464,66	-	43.47%

Table 3: Summary of the service, comparing GRP full tour and Twing tours for all circuits of Andradina

5. Concluding Remarks

In this paper we presented in detail a new combinatorial optimization problem, characterized by the movement of two servers in a network, a direct application to the domiciliary waste collection in Brazilian cities. With the proposed formulation, we developed an automatic procedure to solve the problem using shortest paths. However, this approach is limited by memory capacity of the computer used to solve the problem. As an alternative, we developed a procedure to address real-life instances of the problem, considering its asynchronous form.

The results revealed an impressive contribution of the collectors over the routes, up to 43.47% contribution in removing garbage during an typical day of collection. Although this percentage seems high, the actual system demands even more from them in terms of walking (over 45%) and removing garbage.

Our research in this theme is evaluating heuristics and metaheuristics from the node routing maximum cover tour problem, adapted to arc routing and the TRPP. Our challenge is to include constraints like “must to do links” for both services, hierarchy, max distance to the other or both twin routes. Our first results are encouraging.

Acknowledgment

This research is supported by the project CNPq-407652/2022-1. We thank to CONSERVITA, in the person of Marcio Rossi, Director, for his local support and liberation of the studies we are developing in Andradina, Jales and Santa Cruz do Rio Pardo/SP. The last author acknowledges Brazil's CNPq (National Council for Scientific and Technological Development) financial support [grant number 306485/2023-0].

References

- Corberán, A., Martí, R., and Romero, A. (2000). Heuristics for the mixed rural postman problem. *Computers Operations Research*, 27:183–203.
- Corberán, A., Plana, I., and Sanchis, J. (2014). The rural postman problem on directed, mixed, and windy graphs. In Corberan, L. G., A., editor, *Arc Routing: Problems, methods and applications*, chapter 6, p. 101–127. MOS-SIAM Series on Optimization.
- Corberán, A., Plana, I., Sanchis, J., and Segura, P. Theoretical and computational analysis of a new formulation for the rural postman problem and the general routing problem. *Computers and Operations Research*, 162. URL [10.1016/j.cor.2023.106482](https://doi.org/10.1016/j.cor.2023.106482).
- Corberán, A., Romero, A., and Sanchis, J. (2003). The mixed general routing polyhedron. *Mathematical Programming*, 96:103–137. URL [10.1007/s10107-003-0391-9](https://doi.org/10.1007/s10107-003-0391-9).
- Hess, C., Dragomir, A., Doerner, k., and Vigo, D. (2023). Waste collection routing: a survey on problems and methods. *Central European Journal of Operations Research*. URL [10.1007/s10100-023-00892-y](https://doi.org/10.1007/s10100-023-00892-y).
- Negreiros, M. J., Palhano, A. W. C., and Reis, E. (2022). Sector arc routing-based spatial decision support system for waste collection in brazil. *Waste Management & Research*, 41.
- Negreiros, M. J., Palhano, A. W. C., and Reis, E. (2023a). Optimized planning and management of domiciliary and selective solid waste: Results of application in brazilian cities(sisrot lix). In Zailani, S. and Suriyanarayanan Sarvajakesavalu Ed, I., editors, *Solid Waste and Landfills Management Recent Advances*, chapter 2, p. 25–56. URL DOI:10.5772/intechopen.100756.
- Negreiros, M. (2024a). Generalized travelling salesman based heuristics for the mixed rural postman problem. *Journal of Heuristics*, Submitted.
- Negreiros, M. (2024b). The open rural postman problem. *Computers & Operations Research*, Submitted.
- Negreiros, M., Cunha, C., Reis, E., and Reis, A. (2023b). Uma proposta de solução para o problema de cobertura da coleta domiciliar. *Annals of XXXVII ANPET, Santos/SP*.
- Negreiros, M., Maculan, N., Cunha, C., Viana, F., and Mello, F. (2024a). Waste collection: Sectoring, routing and scheduling for challenging services. volume Springer - To Appear.
- Negreiros, M., Maculan, N., Palhano, A., and Batista, P. The heterogeneous arc sector routing problem. Submitted for publication, 2024b.

- Nobert, Y. and Picard, J.-C. (1996). An optimal algorithm for the mixed chinese postman problem. *Networks*, 27(2):95–108.
- Olawade, D., Fapohunda, O., Wada, O., Usman, S., Ige, A., Ajisafe, O., and Oladapo, B. I. (2024). Smart waste management: A paradigm shift enabled by artificial intelligence. *Waste Management Bulletin*, 2:244–263. URL <https://doi.org/10.1016/j.wmb.2024.05.001>.