

# Deep Graph Clustering with Disentangled Representation Learning

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## Abstract

Deep graph clustering, which aims to uncover the underlying structure within graphs and partition nodes into distinct groups, is a challenging research spot. However, the formation of the cluster in real-world graphs typically governed by the highly complex interaction of many underlying latent factors. Existing methods typically rely on the features and structure associated with the graph, and neglect the entanglement of these factors, resulting in sub-optimal clustering performance. In this paper, we propose a novel deep graph clustering framework named DisenCluster, which learns disentangled representations to simultaneously consider node separation results from diverse perspectives. Specifically, we introduce a disentangled graph encoder that iteratively identifies the latent factors of the input graph by modeling the distribution over different factors for each edge. Subsequently, we utilize a factor-wise contrastive loss to encourage clustering-friendly disentangled representations, allowing us to derive different clustering results based on the corresponding factor. These results are then structured as anchor graphs and seamlessly integrated into a unified graph. Finally, we formulate the framework as a continuous relaxation of the high-order graph cut problem and optimize the objective to obtain effective cluster assignments. Results from experiments on a variety of publicly available datasets further reveal the effectiveness and superiority of our DisenCluster compared with baselines.

## CCS Concepts

- Computing methodologies → Neural networks; Learning latent representations;
- Mathematics of computing → Graph theory.

## Keywords

Graph Neural Network; Deep Graph Clustering; Disentangled Representation Learning

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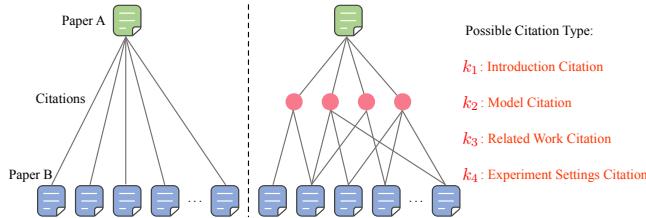
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## 1 Introduction

Graphs are one of the most prevalent data structures to store information about objects with complex relations and have been the subject of investigation for decades. Especially graph learning algorithms, which extract relevant features of graphs by taking advantage of machine learning algorithms, have proven enormously successful in a wide range of domains, e.g., chemical compound classification [11], social network analysis [1, 48, 52] and recommender system [21, 45, 50, 55], etc. Among these various directions in the field of graph learning, one fundamental yet challenging task, namely graph clustering, has recently garnered intensive attention [3, 24–27, 32, 39, 40, 47, 61, 64].

Built upon the superior representation power of deep learning, particularly graph neural networks (GNNs) [16, 28, 42, 57], a variety of deep graph clustering methods are successively proposed, wherein the core idea is to encode nodes through neural networks and assign them into disjoint clusters. According to the learning mechanisms, current approaches can be classified into three main categories, namely generative methods [3, 40], adversarial methods [22, 32, 39] and contrastive methods [24, 25, 61]. Generative and adversarial methods try to capture cluster-oriented node representations by recovering the graph information and distinguishing between real and synthetic samples respectively. Since most of these approaches rely on a clustering-guided loss to minimize the deviation of generated sample embeddings from pre-defined cluster centers, their performance is heavily reliant on the quality of initial cluster centers. Contrastive learning methods, on the other hand, substitute the clustering-guided loss with a more discriminative contrastive loss, offering an effective solution to mitigate the need for manual trial-and-error adjustments.

However, these existing deep graph clustering methods still remain unsatisfactory performance due to the existing limitations as follow: ① Neglect feature representation entanglement. The graph data is frequently sourced from diverse origins or acquired through



**Figure 1: An illustration of diverse citation types in a citation network corresponds to different aspects of semantics between two papers, resulting in diverse clustering result structures.**

various extractors, resulting in inherent variations in their feature representations. For example, the different sections within a paper in the citation network may emphasize various aspects of the paper. **② Unable to capture intricate semantic nuances for clustering.** The cluster formation in a real-world graph typically results from an intricate and diverse process influenced by the interplay of numerous underlying factors. We illustrate a typical example in Figure 1, where a paper in a citation network usually has edges with others for various purposes (e.g., citations of a paper scattered across different sections) [53]. Therefore, it exhibits varying node separations under each distinct aspect, concurrently, the corresponding clustering outcomes possess the potential to complement each other. Therefore, we anticipate an approach capable of investigating various underlying factors and explicitly combining node separation results from their respective diverse perspectives to enhance the performance of the clustering task.

Recently, disentangled representation learning has attracted increasing attention, which aims to acquire factorized representations that can reveal essential or explanatory aspects of the data. Additionally, disentangled representations have been shown to possess greater versatility and robustness when confronted with complex variations. In other words, each underlying factor is therefore expected to be independent and less susceptible to noise stemming from other factors [2]. In the context of graph-structured data, where numerous diverse relations are often intertwined and merged into a single graph, disentangled representation learning offers substantial advantages for downstream tasks. For graph clustering, this approach necessitates the model to generate interpretable features and disentangle the underlying relations by leveraging these features. As a result, the similarities between nodes can manifest in distinct ways under various relations, offering a more comprehensive yet abundant source of information for graph clustering.

Toward this end, in this paper, we propose **DisenCluster**, a novel **Disentangled** graph representation learning framework for deep graph **Clustering**, which is capable of seeking a cluster structure that effectively spans across multiple relationships within a graph. In contrast to existing works that exploit entangled influential factors for graph clustering, our DisenCluster distinguishes various underlying latent factors within the graph data and explicitly fuses the corresponding node separation outcomes to achieve a consensus clustering result. In particular, we first leverage a disentangled graph convolutional layer to discern the factor responsible for the link between a given node and its neighbors, and accordingly

extract pertinent features from the neighbors. Then, a contrastive loss is adopted for each component of the disentangled representation to make the node clustering-friendly. Hence, different clustering results could be individually generated through disentangled representations to investigate the underlying structure of each relation. We represent these clustering results as anchor graphs and seamlessly integrate all of them into a unified graph. Finally, a probabilistic cluster assignment matrix is learned by minimizing relaxed formulations of the graph cut problem, which is founded on the high-order connections within the constructed unified graph. To summarize, we make the following contributions:

- We propose to learn the disentangled representation for deep graph clustering. To the best of our knowledge, this is the first attempt to explicitly consider the underlying factors for graph clustering. The proposed mechanism can be potentially generalized to other unsupervised graph learning tasks.
- We propose a disentangled graph encoder to infer the clustering result for each underlying factor as anchor graphs and explicitly formulate the graph clustering problem as a continuous relaxation of the normalized graph cut problem founded on the high-order connections within the integrated unified graph.
- We perform thorough experiments across various benchmark datasets to evaluate the DisenCluster. The results demonstrate the effectiveness and strong interpretability of our proposed framework for the deep graph clustering task.

## 2 Related Work

### 2.1 Deep Graph Clustering

The fundamental idea of deep graph clustering is to leverage deep learning techniques, such as graph neural networks (GNNs) [16, 42, 57], to perform clustering on graph data. This approach aims to discover hidden community structures or groupings within a given graph, where nodes with similar characteristics are grouped together, facilitating data analysis and uncovering patterns and relationships within the data [3, 8, 23–25, 32, 39, 40, 61, 64]. On the one hand, numerous existing works adopt generative [3, 40] and adversarial [22, 32, 39] architectures to learn effective node representations. For example, DFCN [40] combines representations from AE and GAE, and incorporates target distribution generation and triplet self-supervision for enhanced cross-modality information utilization. Dink-Net [22] optimizes the cluster shrink loss and the cluster dilation loss in an adversarial manner for large scale graph clustering. On the other hand, many studies utilize the remarkable discriminative capabilities of contrastive learning [4, 35] to better serve clustering tasks. For instance, SCGC [24] enhances existing methods by implementing an innovative data augmentation technique and introducing a novel objective function for cross-view structural consistency. CCGC [61] leverages high-confidence clustering results to construct meaningful sample pairs for enhancing their discriminative capability and reliability. Additionally, there are some works on multi-view graph clustering that are highly relevant to our research [6, 9, 10, 14, 20, 31]. However, these approaches often fail to adequately disentangle the various underlying factors behind the graph data, thus ignoring the structure diversity of clustering results from diverse perspectives.

## 2.2 Disentangled Representation Learning

Disentangled representation learning seeks to decompose the representations to isolate the underlying explanatory factors in observed data [2] and has been widely applied across various domains [5, 19, 34, 46, 49, 51, 53, 60]. For example,  $\beta$ -VAE [13] adds the hyper-parameter  $\beta$  as the weight of the KL divergence for the Variational Autoencoders (VAEs) objective to balance the independence constraints and reconstruction accuracy. InfoGAN [5] decomposes the representation into noise and extra class code and estimates the mutual information between the class code and data for controllable image generation. MacridVAE [30] models user behavior data to study hierarchical user intentions via macro and micro representation disentanglement. DisenCite [53] learns the semantics of different sections in the paper via disentangled representation for context-specific citation generation. Recently, disentangled representation learning approaches have been increasingly explored in graph-structured scenarios [18, 29, 54, 63]. DisenGCN [29] introduces a neighborhood routing mechanism that enables the learning of disentangled node representations by partitioning each node's neighborhood into several disjoint subsets. FactorGCN [63] extracts block-wise interpretable features from the entire graph, facilitating graph-level classification tasks. DGCL [18] presents a contrastive learning-based self-supervised framework that learns disentangled representations without reliance on labeled data. Our work focuses on employing disentangled representation for deep graph clustering and the objective is to design a comprehensive framework capable of exploring and combining node separation results from diverse perspectives to effectively enhance the final clustering results.

## 3 METHODOLOGY

This section introduces the notations and problem definition with necessary background. The framework of DisenCluster is illustrated in Figure 2 and we present the details in the following.

### 3.1 Preliminaries

**Notations.** We denote a graph as a tuple  $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$  with node set  $\mathcal{V}$  and edge set  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ , where  $|\mathcal{V}| = N$  and  $|\mathcal{E}| = M$ . We use adjacency matrix  $A \in \mathbb{R}^{N \times N}$  to characterize the structure information of  $\mathcal{G}$ , where entry  $A(u, v) = 1$  if  $(u, v) \in \mathcal{E}$  (otherwise entries of  $A$  are equal to 0). And  $D = \text{diag}(A\mathbf{1}_N)$  is the degree matrix. The node feature matrix is represented as  $X \in \mathbb{R}^{N \times d'}$ , where each row  $x_u \in \mathbb{R}^{d'}$  corresponds to the  $d'$ -dimensional feature vector of node  $u$ . In our work, we seek to develop a disentangled graph encoder, where the output node embedding  $h_u^{(I)}$  can be factorized into  $K$  channels, i.e.,  $h_u^{(I)} = [h_{u,1}^{(I)}, h_{u,2}^{(I)}, \dots, h_{u,K}^{(I)}]$ , to facilitate the downstream deep graph clustering task.

**Problem Definition.** The goal of deep graph clustering is to partition the nodes within a graph into several disjoint groups in an unsupervised manner. We define a discrete cluster assignment matrix  $R \in \{0, 1\}^{N \times C}$ , where  $R(u, j) = 1$  if  $u \in \mathcal{S}_j$  and 0 otherwise. We denote  $\mathcal{S}_j$  as the nodes belonging to the  $j$ -th cluster, which indicates the  $j$ -th column of  $R$ , namely,  $R(j) = [R(1, j), \dots, R(N, j)]^\top$ .

**Graph Cut and Normalised Cut.** Graph clustering involves partitioning a graph into  $C$  disjoint groups, thereby nodes within intra-group are strongly connected than those between groups [43]. Thus,

one of the most explicit and effective ways is to formulate the task as the Normalized Cut problem [38], expressed as:

$$\min_{\mathcal{S}_1, \dots, \mathcal{S}_C} \sum_{j=1}^C \frac{\text{cut}(\mathcal{S}_j, \bar{\mathcal{S}}_j)}{\text{vol}(\mathcal{S}_j)}, \quad (1)$$

where  $\bar{\mathcal{S}}_j$  denotes the node set not belong to cluster  $\mathcal{S}_j$ , namely can be,  $\bar{\mathcal{S}}_j = \mathcal{V} \setminus \mathcal{S}_j$ . Meanwhile, the numerator  $\text{cut}(\mathcal{S}_j, \bar{\mathcal{S}}_j) = \sum_{u \in \mathcal{S}_j, v \in \bar{\mathcal{S}}_j} A(u, v)$  means counting volume of edges that exist between different clusters, and the denominator  $\text{vol}(\mathcal{S}_j) = \sum_{u \in \mathcal{S}_j, v \in \mathcal{V}} A(u, v)$  means counting the edges between the nodes in a cluster and the rest of the graph. The normalized cut can be:

$$\begin{aligned} & \min_{\mathcal{S}_1, \dots, \mathcal{S}_C} \sum_{j=1}^C \frac{\sum_{u \in \mathcal{S}_j, v \in \bar{\mathcal{S}}_j} A(u, v)}{\sum_{u \in \mathcal{S}_j, v \in \mathcal{V}} A(u, v)} \\ &= \max_{\mathcal{S}_1, \dots, \mathcal{S}_C} \sum_{j=1}^C \frac{\sum_{u, v \in \mathcal{S}_j} A(u, v)}{\sum_{u \in \mathcal{S}_j, v \in \mathcal{V}} A(u, v)}, \end{aligned} \quad (2)$$

where the last equivalence follows from:

$$\sum_{u, v \in \mathcal{S}_j} A(u, v) + \sum_{u \in \mathcal{S}_j, v \in \bar{\mathcal{S}}_j} A(u, v) = \sum_{u \in \mathcal{S}_j, v \in \mathcal{V}} A(u, v). \quad (3)$$

### 3.2 Disentangled Graph Encoder

Given the graph  $\mathcal{G}$  as input, we first assume that there are  $K$  underlying latent factors to be disentangled. Therefore, for each node  $u \in \mathcal{V}$ , we can project its feature  $x_u$  into different subspaces:

$$z_{u,k} = \frac{\sigma(W_k^\top x_u + b_k)}{\|\sigma(W_k^\top x_u + b_k)\|_2}, \quad (4)$$

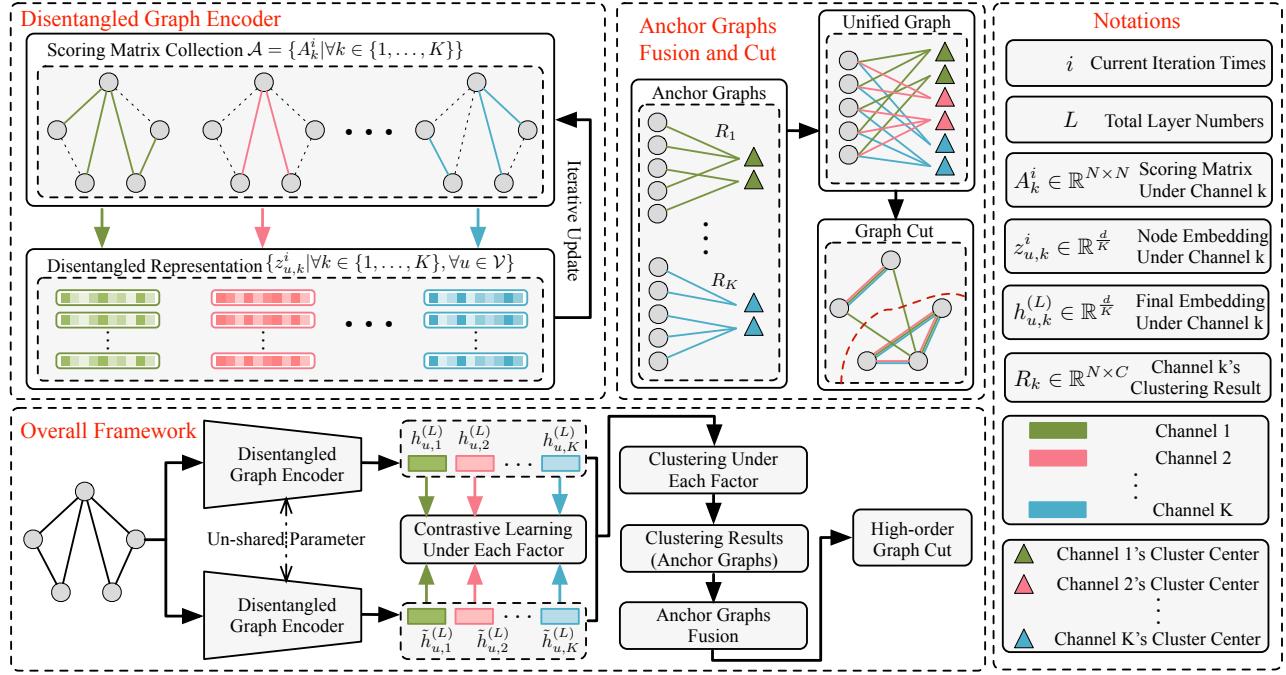
where  $W_k \in \mathbb{R}^{d' \times \frac{d}{K}}$  and  $b_k \in \mathbb{R}^{\frac{d}{K}}$  are the weight of the  $k$ -th channel,  $\sigma(\cdot)$  is a nonlinear activation function. We use the  $l_2$  norm to ensure numerical stability and prevent neighbors with overly rich features.

For different factors of the representation, we define a collection of scoring matrices  $\mathcal{A} = \{A_k | \forall k \in \{1, \dots, K\}\}$  to represent finer-grained relationships between nodes. Within this collection, each entry  $A_k(u, v)$  of the scoring matrix signifies the  $k$ -th relationship between node  $u$  and node  $v$ . We initialize them as  $\mathcal{A}_k^0(u, v) = 1/K$  if  $(u, v) \in \mathcal{E}$ , which assumes that each relationship makes equal contributions at the initial stage and can be iteratively updated.

Within each channel  $k$  of node  $u \in \mathcal{V}$ , we aim to update independent component representation  $z_{u,k}$ . The main intuition is that we only use neighbor  $v$  connects with  $u$  due to factor  $k$  for the updating process. Specifically, at the iteration  $i \in \{1, \dots, I\}$ , we assume the scoring entry  $A_k^i(u, v)$  acting as an attention score, highlighting the primary driving factor behind a particular relationship between node  $u$  and  $v$  under factor  $k$ . We then perform message passing to update the corresponding components of node  $u$ 's representation:

$$z_{u,k}^i = \frac{z_{u,k} + \sum_{v \in N(u)} A_k^{i-1}(u, v) \cdot z_{v,k}}{\|z_{u,k} + \sum_{v \in N(u)} A_k^{i-1}(u, v) \cdot z_{v,k}\|_2}, \quad (5)$$

where  $z_{u,k}^i$  is the refined  $k$ -th component. Intuitively, nodes whose connections are driven by the same underlying factors tend to share similar representations within the corresponding components.



**Figure 2: A schematic view of the DisenCluster.** We first encode the two-view graph via the proposed Siamese disentangled graph encoders with unshared parameters. Then, we perform factor-wise contrastive learning to make each component of representation clustering-friendly. Following that, distinct clustering results are obtained under each factor and formulated as anchor graphs. Lastly, we integrate all anchor graphs into a unified graph and minimize the relaxed formulations of the graph cut problem. The process leverages the high-order connectivity within the unified anchor graph to ensure consistent cluster assignments across multiple factor channels, thereby enhancing the overall clustering performance.

Thus, we iteratively adjust the relationship strengths based on the updated representation, which is defined as follows:

$$A_k^i(u, v) = \frac{\exp(z_{v,k}^i \top z_{u,k}^i / \tau)}{\sum_{k'=1}^K \exp(z_{v,k'} \top z_{u,k'}^i / \tau)}, \quad (6)$$

where  $\tau$  is the temperature parameter. After  $I$  iterations, we ultimately obtain the output of one disentangled graph convolutional layer, i.e.,  $h_{u,k}^{(1)} = z_{u,k}^I$ . Notice that we can stack multiple disentangled graph convolutional layers to get  $h_u^{(L)} = [h_{u,1}^{(L)}, h_{u,2}^{(L)}, \dots, h_{u,K}^{(L)}]$ , where  $L$  is the total stacking layers number. In this way, our model aggregates comprehensive factor-specific information and generates disentangled node representation.

### 3.3 Contrastive Graph Clustering

As effective representations benefit the clustering process, we introduce a factor-wise contrastive learning approach to enhance the discriminative qualities of the generated disentangled node representations. The idea is to create diverse graph sample views and define a self-supervised objective function by pulling together corresponding representations from the same sample while pushing away the different ones. To be specific, we construct different views of the graph with two Siamese disentangled graph encoders, where the parameters are un-shared between two sub-networks. We get node embedding matrix  $H = \{h_1^{(L)}, \dots, h_N^{(L)}\}^\top$  and  $\tilde{H} = \{\tilde{h}_1^{(L)}, \dots, \tilde{h}_N^{(L)}\}^\top$

and calculate the sample similarity matrix cross two views to formulate the contrastive loss,

$$L_{contra}^k(u) = -\log\left(\frac{e^{\text{sim}(h_{u,k}^{(L)}, \tilde{h}_{u,k}^{(L)})}}{e^{\text{sim}(h_{u,k}^{(L)}, \tilde{h}_{u,k}^{(L)})} + \sum_{v \neq u} e^{\text{sim}(h_{u,k}^{(L)}, \tilde{h}_{v,k}^{(L)})}}\right), \quad (7)$$

where  $\text{sim}(\cdot)$  here denotes the similarity function. In practice, we set it as the cosine similarity function. Therefore, the contrastive loss can be calculated between the view pair under  $k$ -th channel. And the overall contrastive loss can be formulated as:

$$L_{contra} = \frac{1}{N} \sum_{u \in \mathcal{V}} \sum_{k=1}^K L_{contra}^k(u). \quad (8)$$

As such, we can minimize the contrastive objective function  $L_{contra}$  to keep the structural consistency across two views and enhance the discriminative capability of the learned disentangled representation, thereby further improving the clustering performance.

### 3.4 Anchor Graphs Generation and Fusion

Since each component of the disentangled representation stems from the different factors of one graph, the clustering result characterizes the inherent groupings or patterns presented within the corresponding semantic context. Thus, we implement a differentiable K-means clustering [56] based on each component of the

disentangled representation. Considering a node  $u$  and  $k$ -th component of its representation  $h_{u,k}^{(L)}$ , we assume  $\mu_k^j$  as the center of cluster  $\mathcal{S}_k^j$  and denote  $R_k \in \mathbb{R}^{N \times C}$  as the cluster assignment matrix derived from the soft K-means. We employ a soft-min mechanism to assign nodes to cluster centers based on their distances.

$$R_k(u, j) = \frac{\exp(-\sigma \|h_{u,k}^{(L)} - \mu_k^j\|)}{\sum_{j'=1}^C \exp(-\sigma \|h_{u,k}^{(L)} - \mu_k^{j'}\|)}, \quad (9)$$

where the distance can also be negative cosine similarity, which shows strong performance. Meanwhile, the cluster centers can be refined iteratively following a process in typical K-means:

$$\mu_k^j = \frac{\sum_u R_k(u, j) h_{u,k}^{(L)}}{\sum_u R_k(u, j)}, \forall j = 1, \dots, C. \quad (10)$$

These iterated processes finally converge to a fixed point when  $\mu_k^j$  remains consistent across consecutive updates. Note that the backward pass for this layer can be efficiently approximated without having to unfold the entire iteration sequence.

For each channel  $k$  of the graph, we formulate the clustering result  $R_k$  as an anchor graph which consists of two types of nodes, i.e., original graph nodes and anchors (cluster centers) respectively, and the edges connecting the anchor-node pairs reflect the weight to which node is assigned to the corresponding cluster. Since anchor sets obtained in different channels are not consistent when K-means clustering performs on each channel separately, we directly concatenate all the anchor sets as well as their connected edges to the graph nodes into a joint anchor graph  $\mathcal{G}_R$  with adjacency matrix  $A_R = R_1 \parallel \dots \parallel R_K$ , where the fused result  $A_R \in \mathbb{R}^{N \times KC}$  and  $\parallel$  means the matrix concatenation operation helping capture the cluster structure diversity of different channels.

### 3.5 High-order Graph Cut

Different from the Normalized Cut builds on first-order connectivity patterns (i.e., edges), we propose to cluster the nodes based on high-order connectivity of  $\mathcal{G}_R$ . The intuition is that when two nodes are consistently assigned to the same cluster across multiple channels, their influence on belonging to the same cluster in the final clustering result is strengthened. Formally, we introduce the high-order adjacency matrix  $\hat{A} = A_R A_R^\top$ , where each entry  $\hat{A}(u, v)$  quantifies the frequency with which nodes  $u$  and  $v$  are assigned to the same cluster across various channels.

For now, we focus on high-order graph cut on the joint anchor graph  $\mathcal{G}_R$ , we rewrite Equation (2) as:

$$\begin{aligned} & \max_{S_1, \dots, S_C} \sum_{j=1}^C \frac{\sum_{u,v \in S_j} \hat{A}(u, v)}{\sum_{u \in S_j, v \in \mathcal{V}} \hat{A}(u, v)} \\ &= \max_{R \in \{0,1\}^{N \times C}} \sum_{j=1}^C \frac{\sum_{u,v \in \mathcal{V}} \hat{A}(u, v) R(u, j) R(v, j)}{\sum_{u \in S_j, v \in \mathcal{V}} R(u, j) \hat{A}(u, v)} \\ &= \max_{R \in \{0,1\}^{N \times C}} \sum_{j=1}^C \frac{R(j)^\top \hat{A} R(j)}{R(j)^\top \hat{D} R(j)} \\ &= \min_{R \in \{0,1\}^{N \times C}} -\text{Tr}\left(\frac{R^\top \hat{A} R}{R^\top \hat{D} R}\right), \end{aligned} \quad (11)$$

where  $\hat{D} = \text{diag}(\hat{A}1_N)$  is the degree matrix of  $\hat{\mathcal{G}}$ . The final cluster assignment matrix  $R$  can also be generated by soft K-means with the whole representation of the node. We approximate the relaxed formulation of the high-order graph cut problem as follows:

$$L_{cut} = -\frac{1}{C} \cdot \text{Tr}\left(\frac{R^\top \hat{A} R}{R^\top \hat{D} R}\right). \quad (12)$$

Furthermore, we introduce an auxiliary loss function to explicitly enforce hard cluster assignments,

$$L_{orth} = \frac{1}{\sqrt{C}-1} \left( \sqrt{C} - \frac{1}{N} \sum_{j=1}^C \|R(j)\|_F \right). \quad (13)$$

where  $\|\cdot\|_F$  here indicates the Frobenius norm. And the orthogonality loss encourages the formation of well-balanced and distinguishable clusters, ensuring that nodes are primarily assigned to one cluster with high probability and less likely to be allocated to other clusters, which discourages further degenerate solutions.

### 3.6 Optimization

The proposed DisenCluster jointly optimizes three objectives, including contrastive loss  $L_{contra}$ , high-order graph cut loss  $L_{cut}$  and corresponding orthogonality loss  $L_{orth}$ . In summary, the objective of DisenCluster is formulated as:

$$L = L_{contra} + \alpha L_{cut} + \beta L_{orth}, \quad (14)$$

where  $\alpha$  and  $\beta$  for the objective denote the trade-off parameters.

## 4 Experiments

### 4.1 Experimental Settings

**Datasets.** To evaluate the performance of our DisenCluster, in our experimental setting, we apply our model to six well-known and public accessible benchmark datasets following the previous works [25, 61], which includes Cora [36], Citeseer [7], Amazon Photo (AMAP) [37], Brazil AirTraffic (BAT) [24], Europe Air-Traffic (EAT) [24], and USA Air-Traffic (UAT) [24].

**Baselines.** To demonstrate the superiority of our DisenCluster, we adopt two groups of baselines for comparison, including (A) classical deep graph clustering methods, i.e., DAEGC [44], ARGA [32], SDCN [3] and DFCN [40], and (B) contrastive deep graph clustering methods, i.e., AGE [7], MVGRL [12], AutoSSL [15], AFGRL [17], GDCL [64], ProGCL [58], CCGC [61] and CDGC [62].

**Evaluation Metrics.** To ensure the comprehensive evaluation of the clustering methods, we adopt four benchmark metrics following [3] for evaluation: Accuracy (Acc), Normalized Mutual Information (NMI), Average Rand Index (ARI) and Macro F1-score (F1). The number of clusters  $C$  in the graph is set equal to the number of ground-truth classes in each dataset and the best map between cluster ID and class ID is constructed by the Kuhn-Munkres algorithm [33]. Larger values imply better clustering results. We perform ten repeated runs for each baseline method and report the mean and standard deviation of four metrics to reduce the impact of the random seed variation. The source code of DisenCluster is available at <https://github.com/jamesyifan/DisenCluster>.

**Table 1: The comparison of clustering performance (%) across six benchmark datasets (mean $\pm$ std). We highlight the best and second-best results among all methods in bold and underline, respectively.**

Dataset	Metric	DAEGC	ARGA	SDCN	DFCN	AGE	MVGRL	AutoSSL	AGC-DRR	AFGRL	GDCL	ProGCL	CCGC	CDGC	DisenCluster
Cora	ACC	70.43 $\pm$ 0.36	71.04 $\pm$ 0.25	35.60 $\pm$ 2.83	36.33 $\pm$ 0.49	73.50 $\pm$ 1.83	70.47 $\pm$ 3.70	63.81 $\pm$ 0.57	40.62 $\pm$ 0.55	26.25 $\pm$ 1.24	70.83 $\pm$ 0.47	57.13 $\pm$ 1.23	73.88 $\pm$ 1.20	74.91 $\pm$ 1.78	<b>78.33<math>\pm</math>0.93</b>
	NMI	52.89 $\pm$ 0.69	51.06 $\pm$ 0.52	14.28 $\pm$ 1.91	19.36 $\pm$ 0.87	57.58 $\pm$ 1.42	55.57 $\pm$ 1.54	47.62 $\pm$ 0.45	18.74 $\pm$ 0.73	12.36 $\pm$ 1.54	56.30 $\pm$ 0.36	41.02 $\pm$ 1.34	56.45 $\pm$ 1.04	<u>58.16<math>\pm</math>0.83</u>	<b>59.86<math>\pm</math>0.76</b>
	ARI	49.63 $\pm$ 0.43	47.71 $\pm$ 0.33	07.78 $\pm$ 3.24	04.67 $\pm$ 2.10	50.10 $\pm$ 2.14	48.70 $\pm$ 3.94	38.92 $\pm$ 0.77	14.80 $\pm$ 1.64	14.32 $\pm$ 1.87	48.05 $\pm$ 0.72	30.71 $\pm$ 2.70	52.51 $\pm$ 1.89	<u>53.82<math>\pm</math>2.25</u>	<b>57.81<math>\pm</math>1.77</b>
	F1	68.27 $\pm$ 0.57	69.27 $\pm$ 0.39	24.37 $\pm$ 1.04	26.16 $\pm$ 0.50	69.28 $\pm$ 1.59	67.15 $\pm$ 1.86	56.42 $\pm$ 0.21	31.23 $\pm$ 0.57	30.20 $\pm$ 1.51	52.88 $\pm$ 0.97	45.68 $\pm$ 1.29	70.98 $\pm$ 2.79	<u>73.33<math>\pm</math>1.86</u>	<b>76.78<math>\pm</math>1.72</b>
Citeseer	ACC	64.54 $\pm$ 1.39	61.07 $\pm$ 0.49	65.96 $\pm$ 0.31	69.50 $\pm$ 0.20	69.73 $\pm$ 0.24	62.83 $\pm$ 1.59	66.76 $\pm$ 0.67	68.32 $\pm$ 1.83	31.45 $\pm$ 0.54	66.39 $\pm$ 0.65	65.92 $\pm$ 0.80	69.84 $\pm$ 0.94	<u>70.12<math>\pm</math>0.36</u>	<b>72.28<math>\pm</math>0.15</b>
	NMI	36.41 $\pm$ 0.86	34.40 $\pm$ 0.71	38.71 $\pm$ 0.32	43.90 $\pm$ 0.20	44.93 $\pm$ 0.53	40.69 $\pm$ 0.93	40.67 $\pm$ 0.84	43.28 $\pm$ 1.41	15.17 $\pm$ 0.47	39.52 $\pm$ 0.38	39.59 $\pm$ 0.39	44.33 $\pm$ 0.79	43.56 $\pm$ 0.35	<b>45.38<math>\pm</math>0.27</b>
	ARI	37.78 $\pm$ 1.24	34.32 $\pm$ 0.70	40.17 $\pm$ 0.43	45.50 $\pm$ 0.30	45.31 $\pm$ 0.41	44.85 $\pm$ 0.69	38.73 $\pm$ 0.55	45.34 $\pm$ 2.33	14.32 $\pm$ 0.78	41.07 $\pm$ 0.96	36.16 $\pm$ 1.11	<u>45.68<math>\pm</math>1.80</u>	44.85 $\pm$ 0.69	<b>48.65<math>\pm</math>0.31</b>
	F1	62.20 $\pm$ 1.32	58.23 $\pm$ 0.31	63.62 $\pm$ 0.24	64.30 $\pm$ 0.20	64.45 $\pm$ 0.27	59.54 $\pm$ 2.17	58.22 $\pm$ 0.68	64.82 $\pm$ 1.60	30.20 $\pm$ 0.71	61.12 $\pm$ 0.70	57.89 $\pm$ 1.98	62.71 $\pm$ 2.06	<u>65.01<math>\pm</math>0.39</u>	<b>66.42<math>\pm</math>0.61</b>
AMAP	ACC	75.96 $\pm$ 0.23	69.28 $\pm$ 2.30	53.44 $\pm$ 0.81	76.82 $\pm$ 0.23	75.98 $\pm$ 0.68	41.07 $\pm$ 3.12	54.55 $\pm$ 0.97	76.81 $\pm$ 1.45	75.51 $\pm$ 0.77	43.75 $\pm$ 0.78	51.53 $\pm$ 0.38	<u>77.25<math>\pm</math>0.41</u>	77.24 $\pm$ 0.87	<b>78.26<math>\pm</math>0.64</b>
	NMI	65.25 $\pm$ 0.45	58.36 $\pm$ 2.76	44.85 $\pm$ 0.83	66.23 $\pm$ 1.21	65.38 $\pm$ 0.61	30.28 $\pm$ 3.94	48.56 $\pm$ 0.71	66.54 $\pm$ 1.24	64.05 $\pm$ 0.55	37.32 $\pm$ 0.28	39.56 $\pm$ 0.39	<u>67.44<math>\pm</math>0.48</u>	67.12 $\pm$ 0.92	<b>67.79<math>\pm</math>0.90</b>
	ARI	58.12 $\pm$ 0.24	44.18 $\pm$ 4.41	31.21 $\pm$ 1.23	58.28 $\pm$ 0.74	55.89 $\pm$ 1.34	18.77 $\pm$ 2.34	26.87 $\pm$ 0.34	<u>60.15<math>\pm</math>1.56</u>	54.45 $\pm$ 0.48	21.57 $\pm$ 0.51	34.18 $\pm$ 0.89	57.99 $\pm$ 0.66	58.14 $\pm$ 0.82	<b>60.48<math>\pm</math>1.22</b>
	F1	69.87 $\pm$ 0.54	64.30 $\pm$ 1.95	50.66 $\pm$ 1.49	71.25 $\pm$ 0.31	71.74 $\pm$ 0.93	32.88 $\pm$ 5.50	54.47 $\pm$ 0.83	71.03 $\pm$ 0.64	69.99 $\pm$ 0.34	38.37 $\pm$ 0.29	31.97 $\pm$ 0.44	72.18 $\pm$ 0.57	<b>73.02<math>\pm</math>2.34</b>	72.22 $\pm$ 0.63
BAT	ACC	52.67 $\pm$ 0.00	67.86 $\pm$ 0.80	53.05 $\pm$ 4.63	55.73 $\pm$ 0.06	56.68 $\pm$ 0.76	37.56 $\pm$ 0.32	42.43 $\pm$ 0.02	47.79 $\pm$ 0.02	50.92 $\pm$ 0.44	45.42 $\pm$ 0.54	55.73 $\pm$ 0.79	75.04 $\pm$ 1.78	75.50 $\pm$ 0.87	<b>78.22<math>\pm</math>0.48</b>
	NMI	21.43 $\pm$ 0.35	49.09 $\pm$ 0.54	25.74 $\pm$ 5.71	48.77 $\pm$ 0.51	36.04 $\pm$ 1.54	29.33 $\pm$ 0.70	19.91 $\pm$ 0.24	27.55 $\pm$ 0.62	31.70 $\pm$ 0.42	28.69 $\pm$ 0.92	50.23 $\pm$ 2.43	<u>50.58<math>\pm</math>0.90</u>	<b>54.51<math>\pm</math>1.07</b>	
	ARI	18.18 $\pm$ 0.29	42.02 $\pm$ 1.21	21.04 $\pm$ 4.97	37.76 $\pm$ 0.23	26.59 $\pm$ 1.83	13.45 $\pm$ 0.03	13.11 $\pm$ 0.81	14.59 $\pm$ 0.74	21.89 $\pm$ 0.74	19.33 $\pm$ 0.57	21.84 $\pm$ 1.34	46.95 $\pm$ 3.09	47.45 $\pm$ 1.53	<b>53.07<math>\pm</math>1.13</b>
	F1	52.23 $\pm$ 0.03	67.02 $\pm$ 1.15	46.45 $\pm$ 5.90	50.90 $\pm$ 0.12	55.07 $\pm$ 0.80	29.64 $\pm$ 0.49	34.84 $\pm$ 0.15	42.33 $\pm$ 0.51	46.53 $\pm$ 0.57	39.94 $\pm$ 0.57	56.08 $\pm$ 0.89	74.90 $\pm$ 1.80	<u>75.40<math>\pm</math>0.88</u>	<b>78.11<math>\pm</math>0.45</b>
EAT	ACC	36.89 $\pm$ 0.15	52.13 $\pm$ 0.00	39.07 $\pm$ 1.51	49.37 $\pm$ 0.19	47.26 $\pm$ 0.32	32.88 $\pm$ 0.71	31.33 $\pm$ 0.52	37.37 $\pm$ 0.11	37.42 $\pm$ 1.24	33.46 $\pm$ 0.18	43.36 $\pm$ 0.87	57.19 $\pm$ 0.66	<u>57.22<math>\pm</math>0.73</u>	<b>58.55<math>\pm</math>0.49</b>
	NMI	05.57 $\pm$ 0.06	22.48 $\pm$ 1.21	08.83 $\pm$ 2.54	32.90 $\pm$ 0.41	23.74 $\pm$ 0.90	11.72 $\pm$ 1.08	07.63 $\pm$ 0.85	07.00 $\pm$ 0.85	11.44 $\pm$ 1.41	13.22 $\pm$ 0.33	23.93 $\pm$ 0.45	33.85 $\pm$ 0.87	33.47 $\pm$ 0.34	<b>33.95<math>\pm</math>0.63</b>
	ARI	05.03 $\pm$ 0.08	17.29 $\pm$ 0.50	06.31 $\pm$ 1.95	23.25 $\pm$ 0.18	16.57 $\pm$ 0.46	04.68 $\pm$ 1.30	02.13 $\pm$ 0.67	04.88 $\pm$ 0.91	06.57 $\pm$ 1.73	04.31 $\pm$ 0.98	27.71 $\pm$ 0.41	26.21 $\pm$ 0.81	<b>27.80<math>\pm</math>0.61</b>	
	F1	34.72 $\pm$ 0.16	52.75 $\pm$ 0.07	33.42 $\pm$ 3.10	42.95 $\pm$ 0.04	45.54 $\pm$ 0.40	25.35 $\pm$ 0.75	21.82 $\pm$ 0.98	35.20 $\pm$ 0.17	30.53 $\pm$ 1.47	25.02 $\pm$ 0.21	42.54 $\pm$ 0.45	57.09 $\pm$ 0.94	57.53 $\pm$ 0.67	<b>57.94<math>\pm</math>0.52</b>
UAT	ACC	52.29 $\pm$ 0.49	49.31 $\pm$ 0.15	52.25 $\pm$ 1.91	33.61 $\pm$ 0.09	52.37 $\pm$ 0.42	44.16 $\pm$ 1.38	42.52 $\pm$ 0.64	42.64 $\pm$ 0.31	41.50 $\pm$ 0.25	48.70 $\pm$ 0.06	45.38 $\pm$ 0.58	<u>56.34<math>\pm</math>1.11</u>	55.31 $\pm$ 2.42	<b>58.51<math>\pm</math>1.35</b>
	NMI	21.33 $\pm$ 0.44	25.44 $\pm$ 0.31	21.61 $\pm$ 1.26	26.49 $\pm$ 0.41	23.64 $\pm$ 0.66	21.53 $\pm$ 0.94	17.86 $\pm$ 0.22	11.15 $\pm$ 0.24	17.33 $\pm$ 0.54	25.10 $\pm$ 0.01	22.04 $\pm$ 2.23	<u>28.15<math>\pm</math>1.92</u>	24.40 $\pm$ 1.69	<b>29.03<math>\pm</math>1.19</b>
	ARI	20.50 $\pm$ 0.51	16.57 $\pm$ 0.31	21.63 $\pm$ 1.49	11.87 $\pm$ 0.23	20.39 $\pm$ 0.70	17.12 $\pm$ 1.46	13.13 $\pm$ 0.71	09.50 $\pm$ 0.25	13.62 $\pm$ 0.57	21.76 $\pm$ 0.01	14.74 $\pm$ 1.99	<u>25.52<math>\pm</math>2.09</u>	22.14 $\pm$ 1.67	<b>28.43<math>\pm</math>1.68</b>
	F1	50.33 $\pm$ 0.64	50.26 $\pm$ 0.16	45.59 $\pm$ 3.54	25.79 $\pm$ 0.29	50.15 $\pm$ 0.73	39.44 $\pm$ 2.19	34.94 $\pm$ 0.87	35.18 $\pm$ 0.32	36.52 $\pm$ 0.89	45.69 $\pm$ 0.08	39.30 $\pm$ 1.82	<u>55.24<math>\pm</math>1.69</u>	52.77 $\pm$ 2.61	<b>56.91<math>\pm</math>1.96</b>

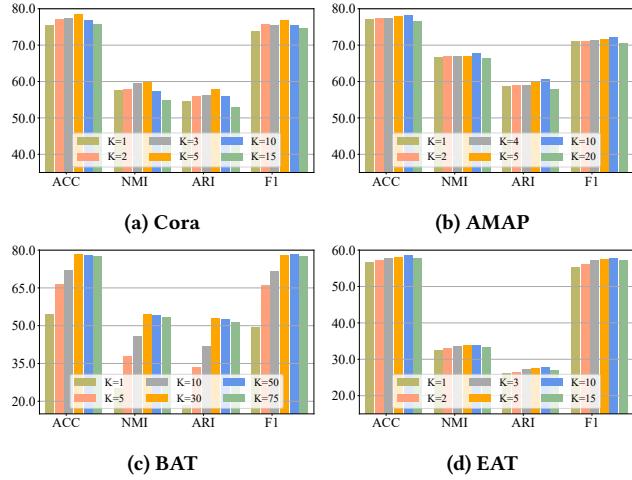
## 4.2 Performance Comparison

We present the quantitative results of our proposed DisenCluster, compared with two groups of competitive deep graph clustering baselines, as reported in Table 1. And from the results, we can derive following key observations:

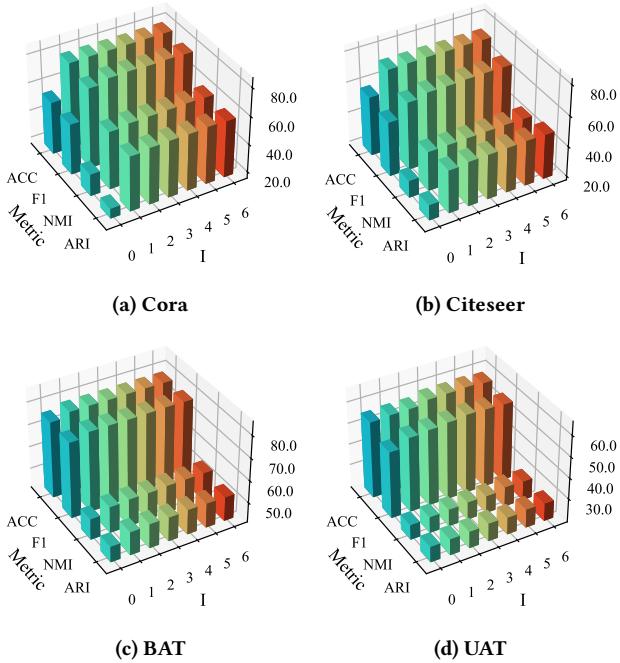
- Classical deep graph clustering methods, which include some reconstructive and adversarial approaches, often perform notably worse than contrastive graph clustering approaches. This indicates that contrastive learning is an effective way to capture the supervision information for graph-structured data. By learning discriminative representations in a principled manner, contrastive learning better serves the clustering task.
- Contrastive graph clustering approaches achieve suboptimal performance compared with ours. This is primarily due to the intricate semantic nuances associated with different factors in real-world graphs. Traditional contrastive learning approaches, when applied to graph-structured data, are less effective in exploiting the intrinsic semantic information of the graph, resulting in consistently distinct clustering outcomes for the graph data.
- Overall, our proposed DisenCluster consistently outperforms other baselines on all six datasets and significantly outperforms the runner-up on many datasets. For instance, on the Cora dataset, we surpass the runner-up by 4.5%, 2.92%, 7.41%, and 4.70% on four evaluation metrics, which significantly demonstrates the effectiveness of our proposed model. We attribute the performance gain to two factors: (i) The effectiveness of capturing intricate semantic nuances for the graph. We encode the graph features into the disentangled representation, which notably captures the latent semantics of the graph. (ii) The distinct clustering results combination framework. We construct anchor graphs from clustering results and integrate them via high-order graph cut,

**Table 2: Ablation studies on six datasets. Our full model achieves the best performance consistently.**

Dataset	Metric	w/o Siam	w/o Contra	w/o Cut	w/o Orth	DisenCluster
Cora	ACC	77.36 $\pm$ 0.68	70.67 $\pm$ 1.77	76.39 $\pm$ 0.45	76.91 $\pm$ 0.48	<b>78.33<math>\pm</math>0.93</b>
	NMI	58.41 $\pm$ 1.11	49.82 $\pm$ 1.79	57.92 $\pm$ 1.10	57.76 $\pm$ 1.26	<b>59.86<math>\pm</math>0.76</b>
	ARI	56.66 $\pm$ 1.07	46.86 $\pm$ 2.19	54.73 $\pm$ 0.69	55.85 $\pm$ 0.87	<b>57.81<math>\pm</math>1.77</b>
	F1	75.93 $\pm$ 1.10	68.19 $\pm$ 2.51	73.05 $\pm$ 1.62	75.83 $\pm$ 1.14	<b>76.78<math>\pm</math>1.72</b>
Citeseer	ACC	72.06 $\pm$ 0.43	66.19 $\pm$ 1.76	71.24 $\pm$ 0.54	71.98 $\pm$ 0.57	<b>72.28<math>\pm</math>0.15</b>
	NMI	44.99 $\pm$ 0.49	39.81 $\pm$ 1.84	44.01 $\pm$ 0.55	45.03 $\pm$ 0.38	<b>45.38<math>\pm</math>0.27</b>
	ARI	48.16 $\pm$ 0.59	39.45 $\pm$ 1.43	47.15 $\pm$ 0.62	48.33 $\pm$ 0.38	<b>48.65<math>\pm</math>0.31</b>
	F1	66.10 $\pm$ 1.05	60.32 $\pm$ 2.06	66.07 $\pm$ 0.49	66.11 $\pm$ 0.69	<b>66.42<math>\pm</math>0.41</b>
AMAP	ACC	75.93 $\pm$ 0.73	74.83 $\pm$ 0.43	75.63 $\pm$ 0.55	76.74 $\pm$ 0.39	<b>78.26<math>\pm</math>0.64</b>
	NMI	63.17 $\pm$ 1.38	64.96 $\pm$ 0.69	64.37 $\pm$ 0.85	65.52 $\pm$ 0.78	<b>67.79<math>\pm</math>0.90</b>
	ARI	56.99 $\pm$ 1.74	55.96 $\pm$ 0.89	55.73 $\pm$ 0.98	58.04 $\pm$ 0.66	<b>60.48<math>\pm</math>1.22</b>
	F1	69.61 $\pm$ 2.16	69.74 $\pm$ 1.01	70.11 $\pm$ 0.53	71.01 $\pm$ 1.02	<b>72.22<math>\pm</math>0.63</b>
BAT	ACC	77.45 $\pm$ 0.49	73.64 $\pm$ 2.49	76.69 $\pm$ 0.70	77.45 $\pm$ 0.34	<b>78.22<math>\pm</math>0.48</b>
	NMI	52.88 $\pm$ 0.79	48.19 $\pm$ 2.46	51.24 $\pm$ 1.17	53.52 $\pm$ 0.52	<b>54.51<math>\pm</math>1.07</b>
	ARI	51.12 $\pm$ 0.95	44.68 $\pm$ 3.59	49.81 $\pm$ 1.38	51.92 $\pm$ 0.60	<b>53.07<math>\pm</math>1.13</b>
	F1	77.53 $\pm$ 0.50	73.82 $\pm$ 2.51	76.66 $\pm$ 0.71	76.50 $\pm$ 0.36	<b>78.11<math>\pm</math>0.45</b>
EAT	ACC	57.39 $\pm$ 0.39	57.23 $\pm$ 0.51	56.87 $\pm$ 0.63	57.14 $\pm$ 0.49	<b>58.55<math>\pm</math>0.49</b>
	NMI	33.49 $\pm$ 0.62	31.47 $\pm</math$			



**Figure 3: Performance (%) w.r.t. different numbers of disentangled factors on four datasets.**



**Figure 4: Performance (%) w.r.t. different iteration times on four datasets.**

- **w/o Siam** (without Parameter Un-shared Siamese Encoders): To simplify the graph augmentation process, we employ the parameter un-shared Simases encoders as two graph views. To verify its effectiveness, we introduce four other widely used graph data augmentations, including edge addition [59], edge dropping [59], graph diffusion [12], and feature masking [64] as variants, where the edge dropping/edge addition/graph teleportation/feature masking rate are all set as 20%. We report the best performance of these data augmentation methods.

- **w/o Contra** (without Contrastive Graph Clustering): We remove the contrastive graph clustering component and solely employ high-order graph cut loss and the corresponding orthogonality loss as the objective function ( $L = L_{cut} + \beta L_{orth}$ ).
- **w/o Cut** (without High-order Graph Cut): We remove the high-order graph cut component and leverage K-means on the learned disentangled graph representation to obtain the final clustering results, and use both contrastive loss and orthogonality loss as the loss function ( $L = L_{contra} + \beta L_{orth}$ ).
- **w/o Orth** (without Orthogonality Loss): We remove the orthogonality regularization loss and use both contrastive loss and high-order graph cut loss as the loss function ( $L = L_{contra} + \alpha L_{cut}$ ).

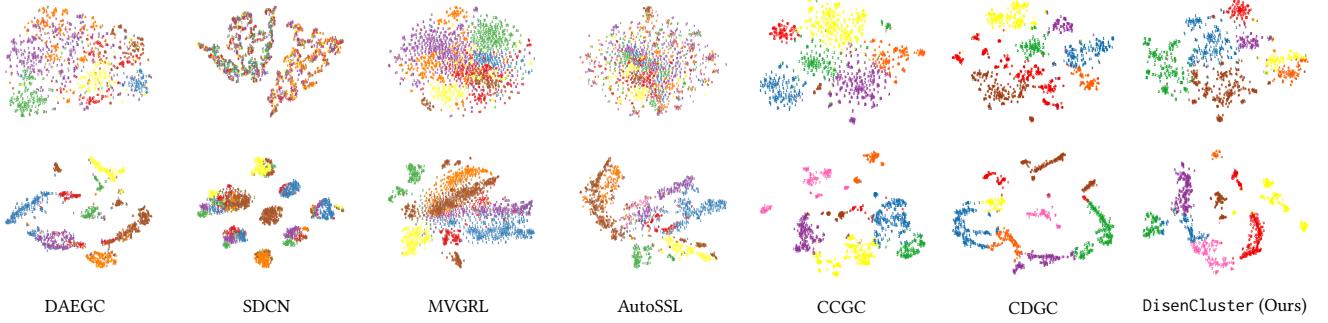
The result of DisenCluster and its variants are shown in Table 2, from which we can draw the following key observations:

- Compared with conducting different node-views by parameter un-shared Simases encoders, the clustering performance is dropped by common graph augmentation strategies (w/o Siam). This indicates that augmentation strategies without careful design may lead to semantic drift [17]. Moreover, it demonstrates the effectiveness of our proposed parameter un-shared encoders.
- Without the contrastive graph clustering component (w/o Contra), the performance fails to achieve satisfactory performance. This indicates that factor-wise graph contrastive learning plays a key role in learning clustering-friendly node representation.
- Removing either the high-order graph cut component (w/o Cut) or orthogonality regularization (w/o Orth) also hinders the model performance, but to a lesser extent compared to removing the contrastive graph clustering component (w/o Contra). This reveals that both components make essential contributions to boosting the clustering performance.

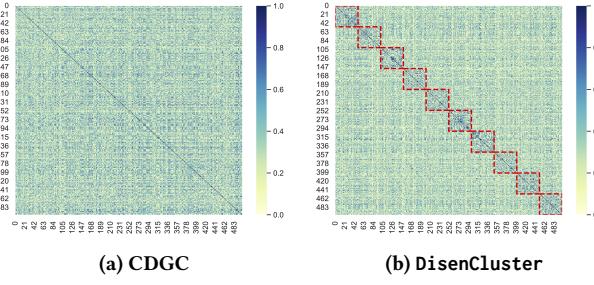
#### 4.4 Parameter Sensitivity

We also examine the sensitivity of the proposed DisenCluster to various hyper-parameters. Specifically, we investigate the effect of varying numbers of latent factors  $K$  in a disentangled graph encoder across four representative datasets. Figure 3 summarizes the experimental results, from which we derive the following key observations:

- In the case where  $K = 1$ , the model degenerates into a graph clustering framework based on entangled representations with poor performance. This observation underscores the importance of modeling various aspects of semantic information to greatly facilitate the clustering performance.
- Increasing the number of disentangled factors  $K$  can substantially enhance the clustering performance. Notably, DisenCluster achieves the best performance at  $K = 5$  for Cora,  $K = 10$  for AMAP/EAT and 30 for BAT, which represents the optimal aspects of semantics for graph clustering.
- However, when the number of disentangled factors is too large (i.e.,  $K \geq 10$  for Cora, AMAP and EAT), the performance will



**Figure 5: The t-SNE visualization on two datasets. The first and second rows represent Cora and AMAP, respectively.**



**Figure 6: Representation correlation analysis. The absolute correlation values are computed between the elements of the 500-dimensional representations obtained through CDGC and our proposed DisenCluster with 10 channels, respectively, using the AMAP dataset.**

degrade slowly. This may be attributed to the use of an overly complex semantic structure on the graph.

**4.4.2 Impact of Iteration Times.** To further examine how the number of iterations affects the model’s performance, we maintain the optimal number of disentangled factors and explore different model variants with varying iteration times on Cora, Citeseer, BAT and UAT datasets. As shown in Figure 4, we set the iteration times for the disentangled graph encoder in the range of [1, 6] and we have the following findings:

- Increasing the number of iterations typically results in improved performance until it stabilizes. For example, once the iteration times reach 3, the performance stabilizes, indicating that a small number of iterations is sufficient to achieve satisfactory results.
- Compared with the disentangled factor number, the model performance is not sensitive to the iteration times, especially on the BAT/UAT dataset. This can be attributed to the fact that training the model for multiple epochs can also drive convergence, even with fewer iteration times.

## 4.5 Visualization Analysis

In this part, we conduct two qualitative assessments to comprehensively investigate how our proposed DisenCluster facilitates graph clustering, including visualizations of the clustering result and the correlations between elements in learned representations.

**4.5.1 Visualizations of Clustering Result.** To unveil the inherent clustering structure, we plot the distributions of the learned representation on Cora and AMAP, compared with six baselines via t-SNE algorithm [41]. As depicted in Figure 5, our DisenCluster significantly improves cluster separability by fostering tighter aggregation of data points within individual clusters and expanding the separation margins between distinct clusters. This underscores the effectiveness of DisenCluster in learning highly discriminative feature representations, thereby facilitating more accurate and robust cluster assignments relative to existing approaches.

**4.5.2 Visualizations of Representation Correlation.** Besides the quantitative evaluation, we visualize the absolute correlation values between the elements of the learned representations. Figure 6 shows the correlation analysis of competitive baseline CDGC and our DisenCluster on the AMAP dataset. Although CDGC performs well on downstream tasks, its latent features are hidden entangled. By contrast, the correlation of DisenCluster reveals ten diagonal blocks, indicating that the highly independent representations across different factors. This demonstrates that disentangled representations make a significant contribution in capturing diverse semantic components and achieve a better performance in the graph clustering task.

## 5 Conclusion

In this paper, we propose a disentangled graph representation approach termed DisenCluster to improve the discriminative capability of nodes for deep graph clustering. Specifically, we propose a disentangled graph encoder to iteratively discerns the disentangled relationships between nodes, which are governed by different latent factors, and accordingly extracts the node representation from the corresponding channel. Then we propose a factor-wise contrastive loss to ensure the learned representations are clustering-friendly. Hence clustering results from different channels are formulated as anchor graphs and fused into one unified graph. Furthermore, we design a high-order graph cut to explore the cluster structure that effectively spans across multiple relationships within a graph. Extensive experiments on six benchmark graph clustering datasets demonstrate the efficacy of our DisenCluster. In future work, we will explore the application of our DisenCluster to a broader range of realistic graph representation-based downstream tasks, including graph classification and anomaly node detection.

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