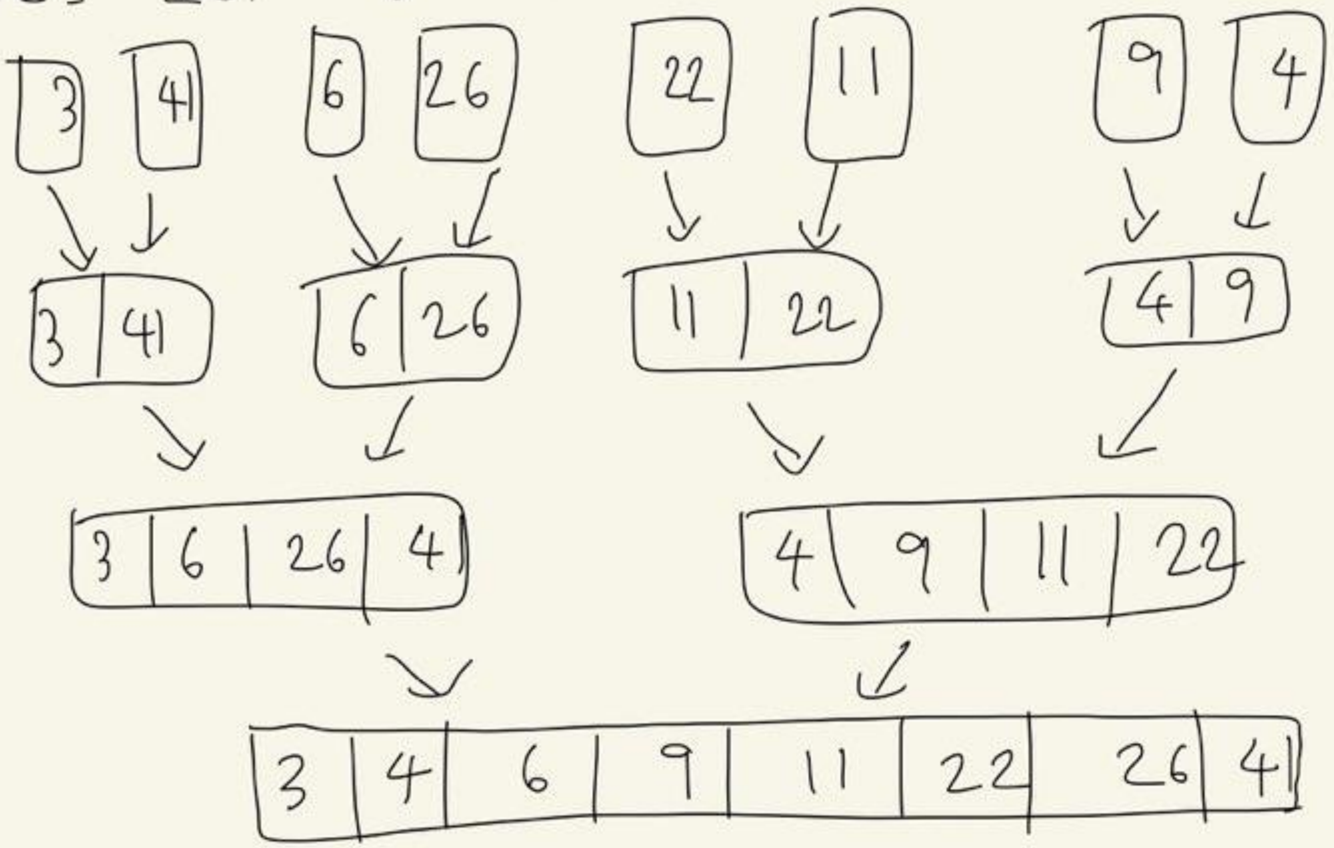
Report 1 1번> merge sort



2. Consider sorting n numbers stored in array A by first finding the largest element of A and exchanging it with the element in A[1]. Then find the second largest element of A, and exchange it with A[2]. Continue in this manner for the first n-1 elements of A.

a. Write pseudocode for this algorithm, which is known as selection sort.

$$selection\ sort(A)$$

$$for\ i \leftarrow 1\ \ to\ \ n-1$$

$$do\ smallest \leftarrow i$$

$$for\ j \leftarrow i+1\ \ to\ \ n$$

$$if\ A[j] < \ \ A[smallest]$$

$$do\ smallest\ \ \leftarrow j$$

$$Exchange\ A[i]\ \ \leftrightarrow A[smallest]$$

b. Why does it need to run for only the first n-1 elements, rather than for all n elements?

Selection sort는 두번째 데이터부터 시작하여 마지막 데이터까지 비교하여 최댓값을 찾고
첫번째 데이터와 교환하는 것을 시작으로 n-1번 실행된다. (or 마지막 n개는 원소의 가장 작은 수)
따라서 n번째의 마지막 데이터는 자동으로 정렬된다.

c. Give the best-case and worst-case running times of selection sort in Θ-notation.

best-case와 worst-case의 실행시간은 모두 $\Theta(n^2)$ 이다.

d. Using Figure 2.2 as a model, illustrate the operation of the selection sort on the array A = <13, 16, 12, 21, 7, 8, 25, 32>.

13	16	12	21	7	8	25	32

32	16	12	21	7	8	25	13

32	25	12	21	7	8	16	13

32	25	21	12	7	8	16	13

32	25	21	16	7	8	12	13

32	25	21	16	13	8	12	7

32	25	21	16	13	12	8	7

32	25	21	16	13	12	8	7

3. Express the following functions in terms of O-notation.

a) $2n^2 + 2\log n$

$since\ 2n^2 + 2\log n \ \leq 2n^2 + 2n^2 = 4n^2\ for\ all\ n \geq 1,$

$we\ may\ take\ C_1 = 4\ \ and\ N_1 = 1\ \ in\ the\ definition\ and\ conclude\ that$

$$2n^2 + 2\log n = \ O(n^2)$$

b) $3n^3 + 5n + 5$

$3n^3 + 5n + 5\ \leq 3n^3 + 5n^3 + 5n^3 = 13n^3\ for\ all\ n \geq 1$

$we\ may\ take\ C_1 = 13\ \ and\ N_1 = 1\ \ in\ the\ definition\ and\ conclude\ that$

$$3n^3 + 5n + 5 = O(n^3)$$

4. Show that the function $3n^5 - n^3 + 2n^2 - 2n + 2 = \theta(n^5)$

$f(n) = 3n^5 - n^3 + 2n^2 - 2n + 2 - 2n + 2 = \theta(n^5)$, $\quad \mathrm{g(n)} = n^5$ 라고 하자.

$f(n) = O(\mathrm{g(n)})$ 임을 보이기 위해 $f(n) = O(\mathrm{g(n)})$ 과 $f(n) = \Omega(\mathrm{g(n)})$ 가 참인지 보여야 한다.

1) For all $n \geq 2$, $\quad 0 \leq 3n^5 - n^3 + 2n^2 - 2n + 2 \leq 5 \cdot n^5$

$$\therefore f(n) = O(\mathrm{g(n)})$$

2) For all $n \geq 2$, $\quad 0 \leq 1 \cdot n^5 \leq 3n^5 - n^3 + 2n^2 - 2n + 2$

$$\therefore f(n) = \Omega(\mathrm{g(n)})$$

따라서 , $3n^5 - n^3 + 2n^2 - 2n + 2 - 2n + 2 = \theta(n^5)$ 이다.

5)

i) $n=0$ $\sum_{i=0}^{0} ar^i = \frac{a(r-1)}{r-1} = a$, $n=0$일 때 참

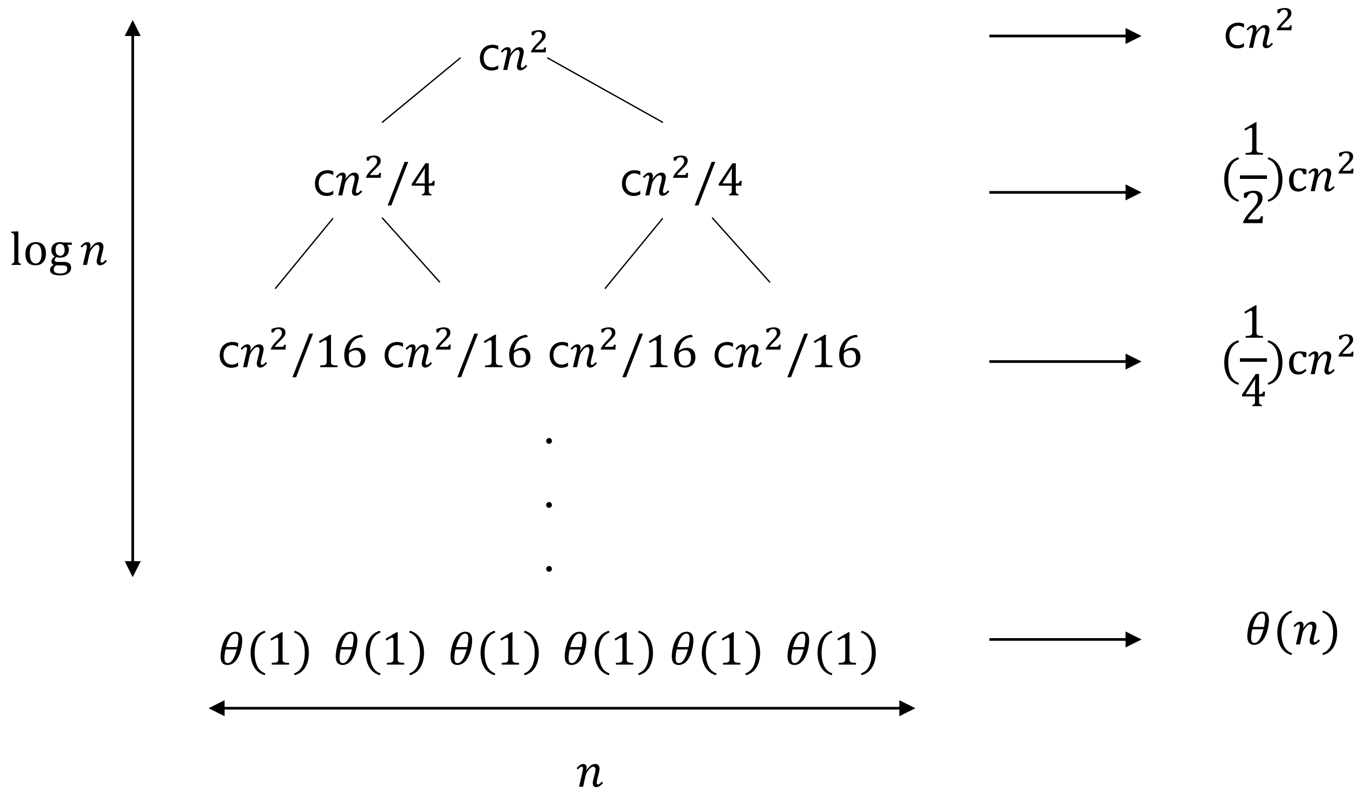
ii) $n=k$일 때 $\sum_{i=0}^{k} ar^i = \frac{a(r^{i+1}-1)}{r-1}$ 라고 가정

$$\sum_{i=0}^{k+1} ar^i = \frac{a(r^{i+1}-1)}{r-1} + ar^{i+1} = \frac{a(r^{i+1}-1)}{r-1} + \frac{a(r^{i+2}-r^{i+1})}{r-1} = \frac{a(r^{i+2}-1)}{r-1}$$

$\therefore$ $n=k+1$일 때 주어진 식은 참

6. Draw the recursion tree for $T(n) = 2T\left(\frac{n}{2}\right) + cn^2$ where, c is constant. Provide a good asymptotic upper bound (O-notation). Also, verify your bound by the substitution method.

Recursion tree



· Geometric series

$x \neq 1$인 실수에 대하여

$$\sum_{k=0}^{n} x^k = 1 + x + x^2 + \cdots + x^n$$

이 식은 geometric 또는 exponential.

1) $$\sum_{k=0}^{n} x^k = \frac{x^{n+1}-1}{x-1}$$

합이 무한, $|x|<1$일 때, 무한 감소 기하 급수

2) $$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$$

1> 1번식을 이용하여 total 계산

$$T(n) = cn^2 + \left(\frac{1}{2}\right)cn^2 + \left(\frac{1}{4}\right)cn^2 + \cdots + \left(\frac{1}{2}\right)^{\log_2 n - 1} cn^2 + \theta(n)$$

$$= \sum_{i=0}^{\log_2 n - 1} \left(\frac{1}{2}\right)^i cn^2 + \theta(n)$$

$$= \left(\frac{\left(\frac{1}{2}\right)^{\log_2 n - 1 + 1} - 1}{\frac{1}{2} - 1}\right) cn^2 + \theta(n) = \left(\frac{\left(\frac{1}{2}\right)^{\log_2 n} - 1}{-\frac{1}{2}}\right) cn^2 + \theta(n) \text{ (by equation 1)}$$

$$= -2\left(\left(\frac{1}{2}\right)^{\log_2 n} - 1\right) cn^2 + \theta(n)$$

$$= -2\left(\frac{1}{n} - 1\right) cn^2 + \theta(n)$$

$$= 2\left(-\frac{1}{n} + 1\right) cn^2 + \theta(n) = 2\left(1 - \frac{1}{n}\right) cn^2 + \theta(n) = O(n^2)$$

· Geometric series

$x \neq 1$ 인 실수에 대하여

$$\sum_{k=0}^{n} x^k = 1 + x + x^2 + \cdots + x^n$$

이 식은 geometric 또는 exponential.

1) $$\sum_{k=0}^{n} x^k = \frac{x^{n+1} - 1}{x - 1}$$

합이 무한, $|x| < 1$ 일 때, 무한 감소 기하 급수

2) $$\sum_{k=0}^{\infty} x^k = \frac{1}{1 - x}$$

2> 2번식을 이용하여 total 계산

$$T(n) = \sum_{i=0}^{\log_2 n - 1} \left(\frac{1}{2}\right)^i cn^2 + \theta(n)$$

$$< \sum_{i=0}^{\infty} \left(\frac{1}{2}\right)^i cn^2 + \theta(n) \text{ (by equation 2)}$$

$$= \left(\frac{1}{1 - \left(\frac{1}{2}\right)}\right) cn^2 + \theta(n) = 2cn^2 + \theta(n) = O(n^2)$$

$\therefore$ total $= O(n^2)$

< substitution method >

$T(n) = 2T\left(\frac{n}{2}\right) + cn^2$

$Guess \quad O(n^2)$

$Assume\ that\ T(k) \le dk^2 \quad for \quad k < n$ $\qquad\qquad \leftarrow \quad Induction\ Hypothesis\ (I.H.)$

$Prove\ T(n) \le dn^2\ by\ induction$

- **Basis step**

$T(n) = \theta\,(1) \quad for\ all \quad n < n_0$

$for\ 1 \le n \le n_0 \qquad\qquad \theta(1) \le dn^2$

- Inductive step

$$T(n) = 2T(n/2) + cn^2$$
$$\leq 2d(n/2)^2 + cn^2$$
$$= (d/2)n^2 + cn^2$$
$$= dn^2 - ((d/2)n^2 - cn^2) \leftarrow \text{desired-residual}$$
$$\leq dn^2 \leftarrow \text{desired}$$

whenever $(d/2)n^2 - cn^2 \geq 0$, for example

$\nwarrow$ residual

if $d \geq 2$ and $n \geq 1$

$$(d/2)n^2 \geq cn^2$$
$$(d/2)n^2 \geq n^2$$
$$d \geq 2$$

7. a) $a=9,\ b=3 \Rightarrow n^{\log_b a} = n^2,\ f(n) = n$

case 1

$$f(n) = n = O(n^{\log_b a - \varepsilon}) = O(n^{2-\varepsilon}) \text{ for } \varepsilon = 1$$

$$T(n) = \Theta(n^2)$$

b)

$a=9,\ b=3 \Rightarrow n^{\log_b a} = n^2,\ f(n) = n^2$

case 2:

$f(n) = n^2 = \Theta(n^2)$

$\therefore T(n) = \Theta(n^2 \log n)$

c)

$a=9,\ b=3 \Rightarrow n^{\log_b a} = n^2,\ f(n)=n^3$

case 3:

$f(n) = \Omega(n^{2+\varepsilon})$ for $\varepsilon = 1$

and $9(n/3)^3 \leq cn^3$ for $c = \frac{1}{3}$

$\therefore T(n) = \Theta(n^3)$

1. Write the BUBBLE-SORT function to sort into ascending order.

Write in pseudo-code (style as shown in the text book).

$Bubblesort\ (A)$

$for\ \ i\ =\ 1\ \ to\ \ A.length\ -1$

$\quad for\ \ j\ =A.length\ downto\ \ i\ +\ 1$

$\quad\quad if\ \ A\,[j]\ <\ \ A\,[j-1]$

$\quad\quad\quad exchange\ \ A\,[j]\ with\ A[j\ -1]$