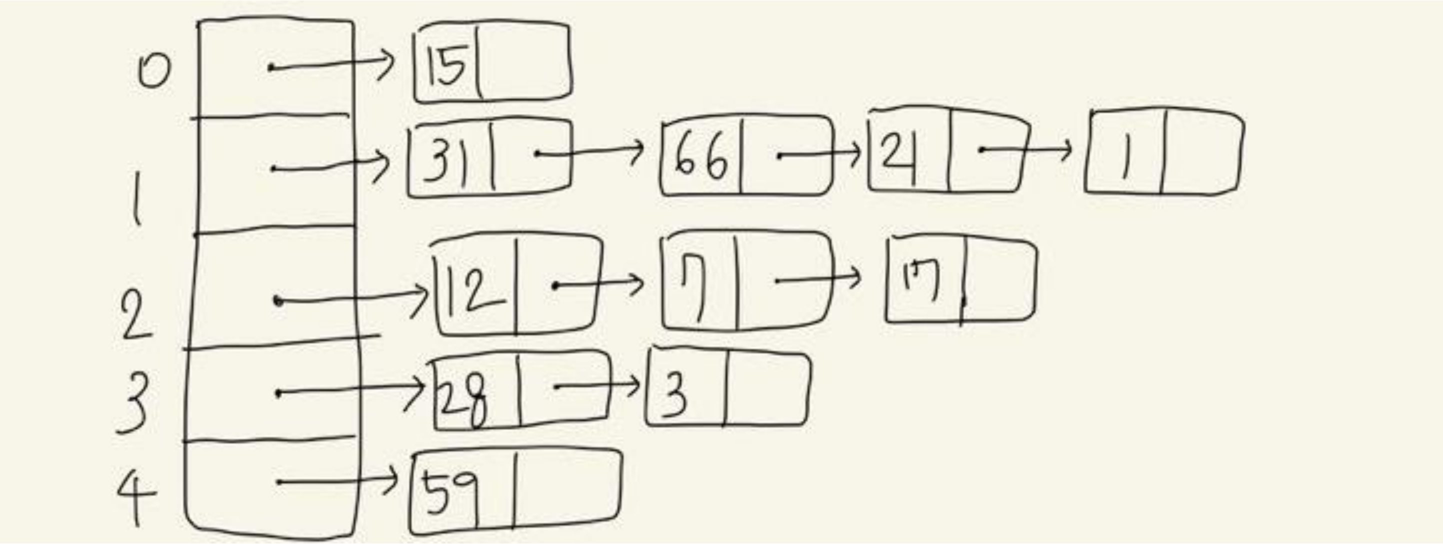
1. Consider inserting the keys 12, 28, 31, 7, 15, 17, 66, 59, 21, 3, 1 into a hash table of length m = 5 using separate chaining where $h(k) = k \bmod m$. Illustrate the result of inserting these keys.



2. Consider inserting the keys 2, 18, 9, 7, 12, 10, 21, 11, 8 into a hash table of length m = 11 using open addressing with the auxiliary hash function $h'(k) = k \; mod \; m$. Draw the hash tables after inserting these keys

a)

$h(k,i) = (h'(k) + i) \bmod m$, $h'(k) = k \bmod n$

0	21
1	12
2	2
3	11
4	8
5	
6	
7	18
8	7
9	9
10	10

$h'(2) = 2 \bmod 11 = 2$

$h(2,0) = (2+0) \bmod 11$, $2 \bmod 11 = 2$

$h'(18) = 18 \bmod 11 = 7$

$h(18,0) = (7+0) \bmod 11$, $7 \bmod 11 = 7$

$h'(9) = 9 \bmod 11 = 9$

$h(9,0) = (9+0) \bmod 11$, $9 \bmod 11 = 9$

$h'(7) = 7 \bmod 11 = 7$ / $h(7,0) = (7+0) \bmod 11 = 7 \bmod 11 = 7$

$h(7,1) = (7+1) \bmod 11 = 8 \bmod 11 = 8$

$h'(12) = 12 \bmod 11 = 1$

$h(12,0) = (12+0) \bmod 11 = 12 \bmod 11 = 1$

$h'(10) = 10 \bmod 11 = 10$

$h(10,0) = (10+0) \bmod 11 = 10 \bmod 11 = 10$

$h'(21) = 21 \bmod 11 = 10$

$h(21,0) = (21+0) \bmod 11 = 21 \bmod 11 = 10$

$h(21,1) = (21+1) \bmod 11 = 22 \bmod 11 = 0$

$h'(11) = 11 \bmod 11 = 0$

$h(11,0) = (11+0) \bmod 11 = 0$

$h(11,1) = (11+1) \bmod 11 = 12 \bmod 11 = 1$

$h(11,2) = (11+2) \bmod 11 = 13 \bmod 11 = 2$

$h(11,3) = (11+3) \bmod 11 = 14 \bmod 11 = 3$

$h'(8) = 8 \bmod 11 = 8$

$h(8,0) = (8+0) \bmod 11$
$= 8$

$h(8,1) = (8+1) \bmod 11$
$= 9$

$h(8,2) = (8+2) \bmod 11$
$= 10$

$h(8,3) = (8+3) \bmod 11$
$= 0$

$\cdots$

$h(8,7) = (8+7) \bmod 11$
$= 4$

2. Consider inserting the keys 2, 18, 9, 7, 12, 10, 21, 11, 8 into a hash table of length m = 11 using open addressing with the auxiliary hash function $h'(k) = k \bmod m$. Draw the hash tables after inserting these keys

b)

0	7
1	10
2	12
3	2
4	11
5	21
6	
7	
8	18
9	8
10	9

$h(2,0) = 2+1+(3\times 0) \bmod 11 = 3 \bmod 11 = 3$

$h(18,0) = 7+1+(3\times 0) \bmod 11 = 8 \bmod 11 = 8$

$h(9,0) = 9+1+(3\times 0) \bmod 11 = 10 \bmod 11 = 10$

$h(7,0) = 7+1+(3\times 0) \bmod 11 = 8 \bmod 11 = 8$

$h(7,1) = 7+1+(3\times 1) \bmod 11 = 11 \bmod 11 = 0$

$h(12,0) = 1+1+(3\times 0) \bmod 11 = 2 \bmod 11 = 2$

$h(10,0) = 10+1+(3\times 0) \bmod 11 = 11 \bmod 11 = 0$

$h(10,1) = 10+1+(3\times 1) \bmod 11 = 14 \bmod 11 = 3$

$h(10,2) = 10+1+(3\times 4) \bmod 11 = 23 \bmod 11 = 1$

$h(21,0) = 10+1+(3\times 0) \bmod 11 = 11 \bmod 11 = 0$

$h(21,1) = 10+1+(3\times 1) \bmod 11 = 14 \bmod 11 = 3$

$h(21,2) = 10+1+(3\times 4) \bmod 11 = 23 \bmod 11 = 1$

$h(21,3) = 10+1+(3\times 9) \bmod 11 = 38 \bmod 11 = 5$

$h(11,0) = 0+1+(3\times 0) \bmod 11 = 1 \bmod 11 = 1$

$h(11,1) = 0+1+(3\times 1) \bmod 11 = 4 \bmod 11 = 4$

$h(8,0) = 8+1+(3\times 0) \bmod 11 = 9 \bmod 11 = 9$

2. Consider inserting the keys 2, 18, 9, 7, 12, 10, 21, 11, 8 into a hash table of length m = 11 using open addressing with the auxiliary hash function $h'(k) = k \ mod \ m$. Draw the hash tables after inserting these keys

c)

0	11
1	12
2	2
3	21
4	7
5	
6	
7	18
8	8
9	9
10	10

$$h(k,i) = (h'(k) + i h_2(k)) \bmod n$$

$$h_2(2) = 1 + (2 \bmod 10) = 1 + 2 = 3$$

$$h(2,0) = (2 + (0 \times 3)) \bmod 11 = 2 \bmod 11 = 2$$

$$h_2(18) = 1 + (18 \bmod 10) = 1 + 8 = 9$$

$$h(18,0) = (7 + (0 \times 9)) \bmod 11 = 7 \bmod 11 = 7$$

$$h_2(9) = 1 + (9 \bmod 10) = 1 + 9 = 10$$

$$h(9,0) = (9 + 0 \times 10) \bmod 11 = 9 \bmod 11 = 9$$

$$h_2(7) = 1 + (7 \bmod 10) = 1 + 7 = 8$$

$$h(7,0) = (7 + (0 \times 8)) \bmod 11 = 7 \bmod 11 = 7$$

$$h(7,1) = (7 + (1 \times 8)) \bmod 11 = 15 \bmod 11 = 4$$

$$h_2(12) = 1 + (12 \bmod 10) = 1 + 2 = 3$$

3. Using Figure 7.1 as a model, illustrate the operation of PARTITION on the array A = <11, 8, 13, 9, 12, 5, 16, 1, 15>. Pivot is the first element of the array A.

i p j r

a. | 11 8 13 9 12 5 16 1 15

p i j r

b. | 11 | 8 13 9 12 5 16 1 15

p i j r

c. 11 | 8 | 13 9 12 5 16 1 15

p i j r

d. 11 | 8 | 13 | 9 12 5 16 1 15

p i j r

e. 11 | 8 9 | 13 | 12 5 16 1 15

p i j r

f. 11 | 8 9 | 13 12 | 5 16 1 15

p i j r

g. 11 | 8 9 5 | 12 13 | 16 1 15

p i j r

h. 11 | 8 9 5 | 12 13 16 | 1 15

p i j r

i. 11 | 8 9 5 1 | 13 16 12 | 15

p i r

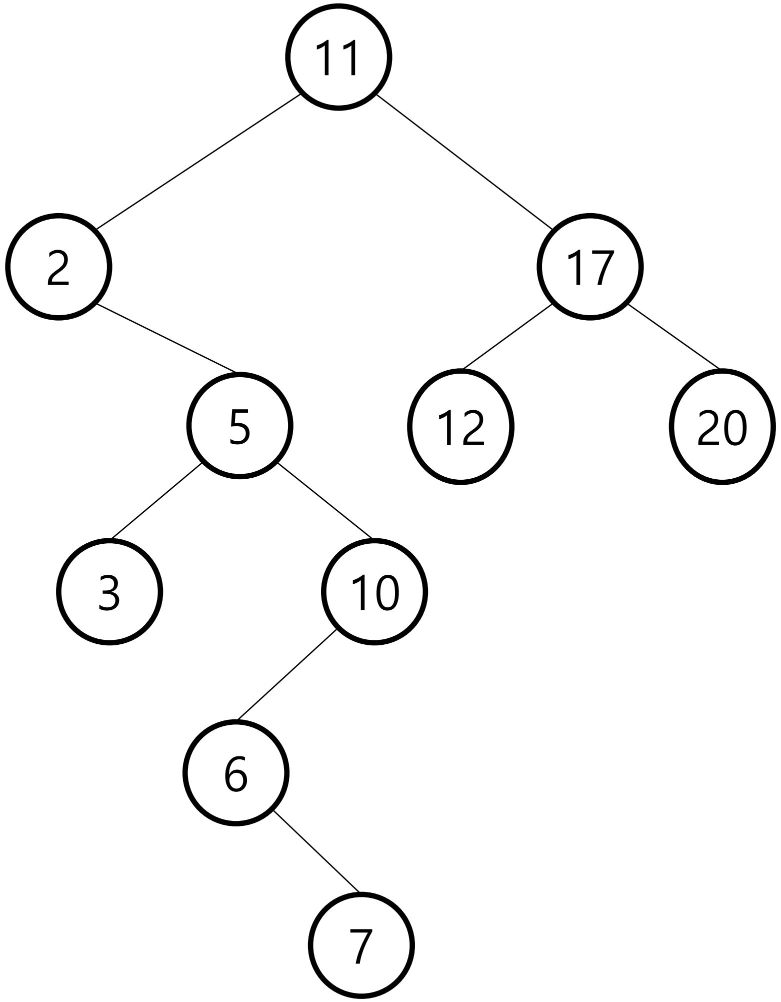
j. 11 | 8 9 5 1 | 13 16 12 15 |

p i r

k. 1 8 9 5 | 11 | 13 16 12 15 |

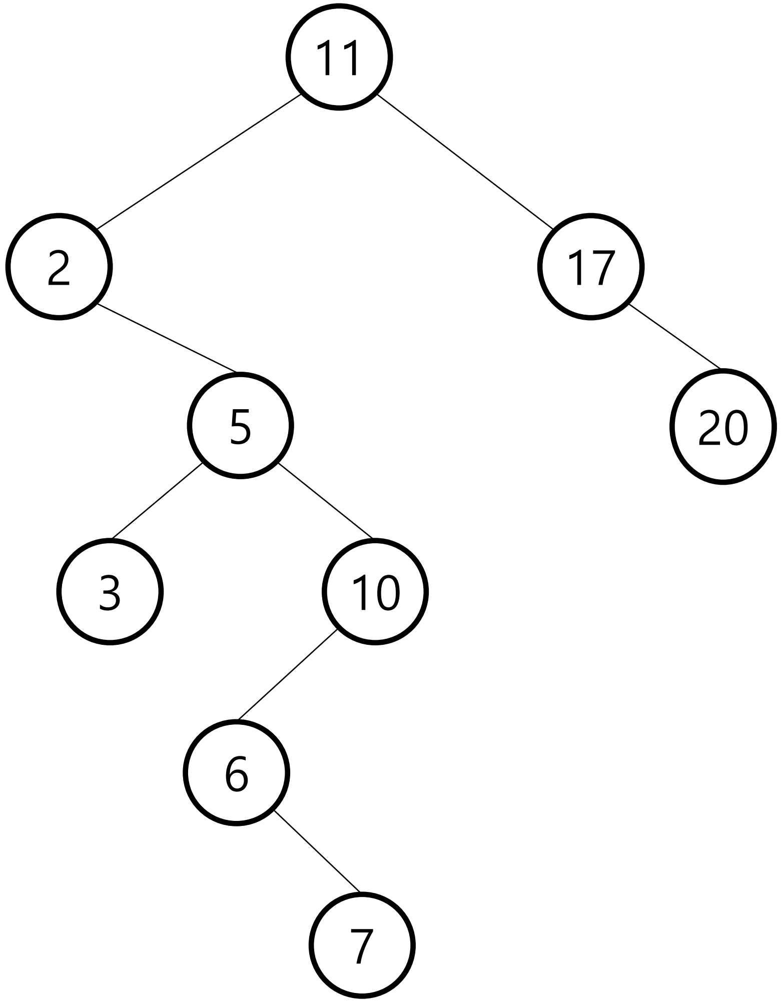
4. Answer the following questions for the keys 11, 2, 17, 5, 3, 10, 6, 7, 12, 20

a)



b-1)

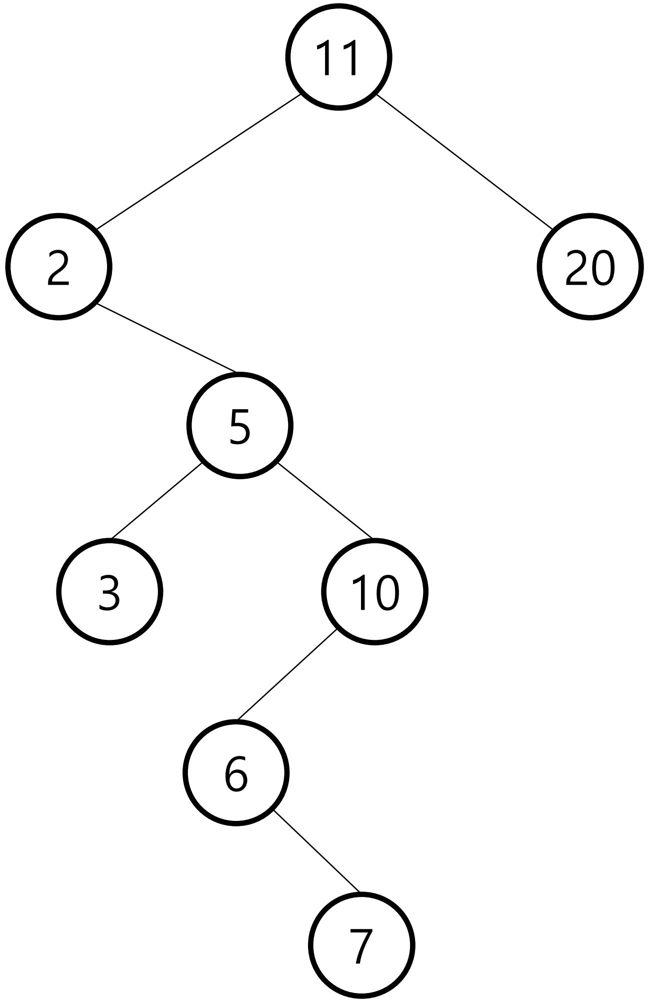
Delete 12



4. Answer the following questions for the keys 11, 2, 17, 5, 3, 10, 6, 7, 12, 20

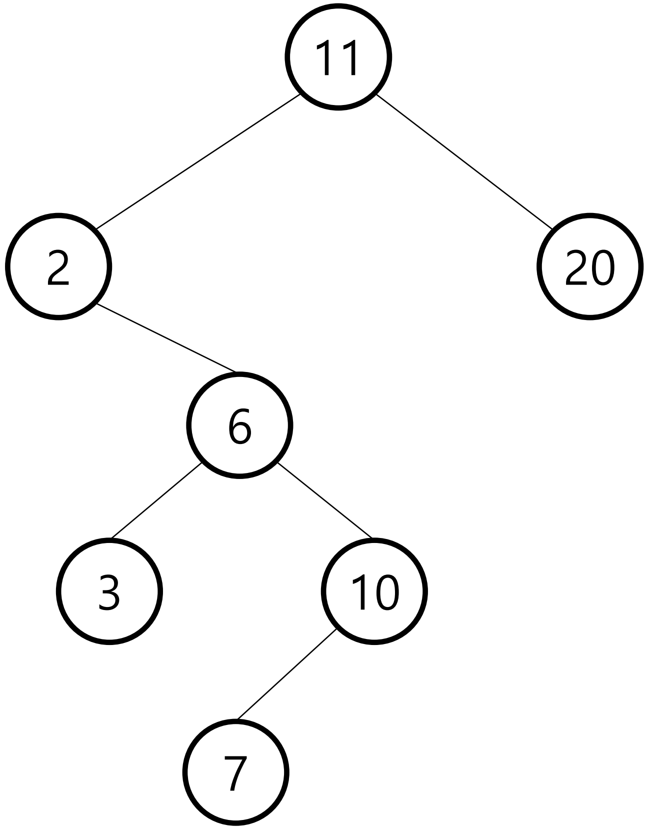
b-2)

Delete 17



b-3)

Delete 5



5. Write the pseudo for MAX(T) in a tree. The MAX(T) finds a node with the maximum key value in T.

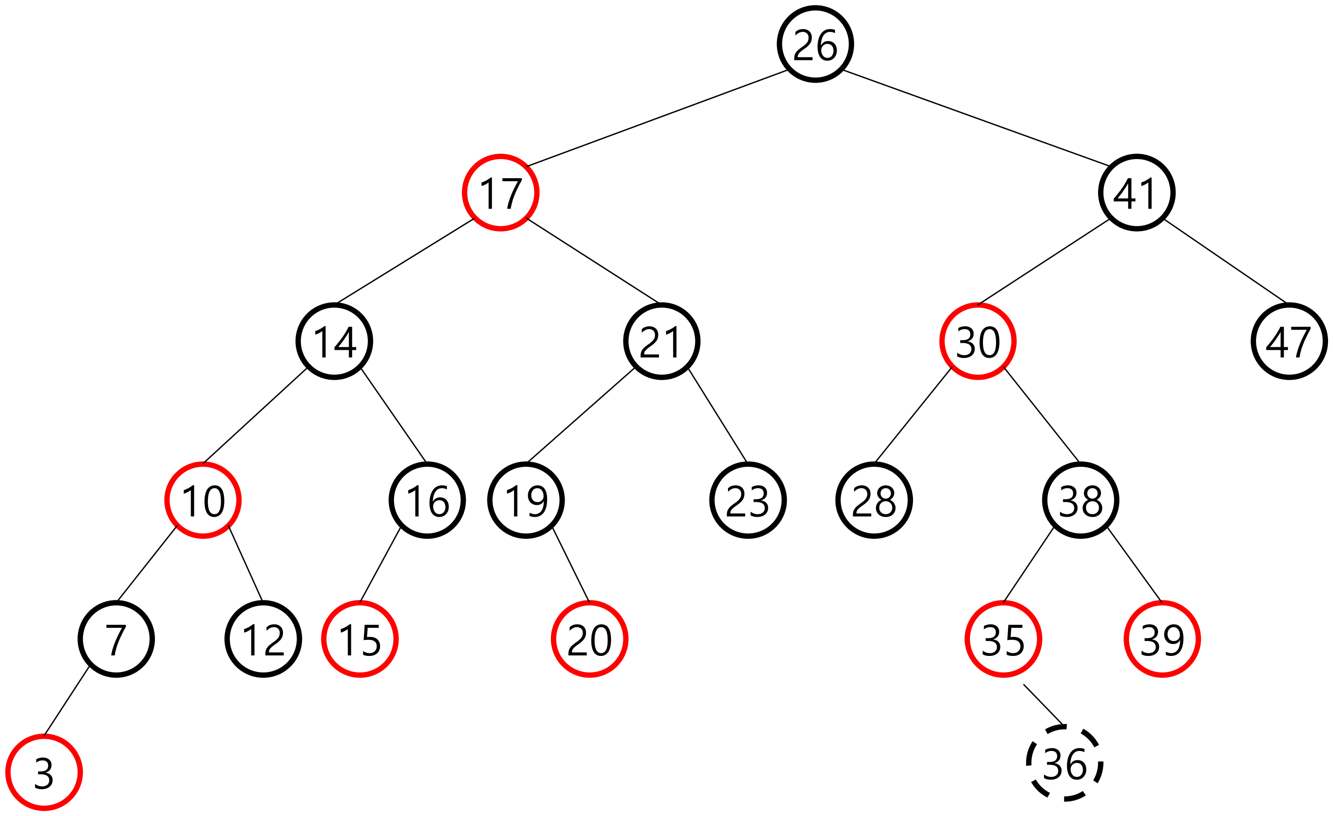
$$Tree - MaxiMum\ (x)$$

$$\boldsymbol{while}\ x.right\ \neq\ NIL$$

$$x = x.right$$

$$\boldsymbol{return}\ x$$

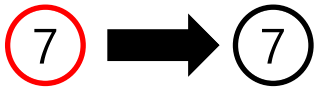
6. Draw the red-black tree that results after TREE-INSERT is called on the tree in Figure 13.1(c) with key 36. If the inserted node is colored red, is the resulting tree a red-black tree? What if it is colored black? Answer without TREE-INSERT-FIXUP execution.



- 노드 36이 Red인 경우 : Double red 문제가 발생한다.
- 노드 36이 Black인 경우 : leaf node까지 가는 블랙 노드의 수가 맞지 않다.

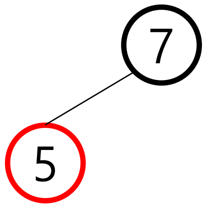
7. Draw the red-black trees that result after successively inserting the keys in the order 7, 5, 9, 2, 8, 3, 13 into an initially empty red-black tree.

1) Insert 7

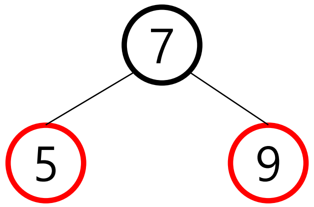


Color change

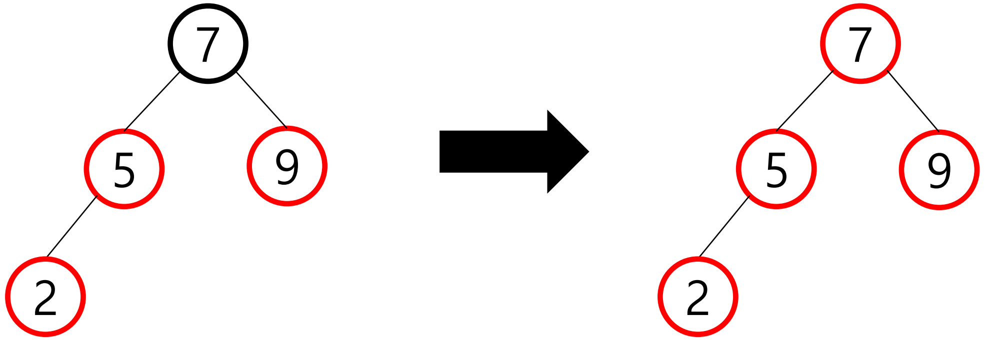
2) Insert 5



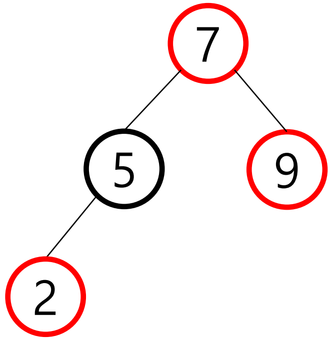
3) Insert 9



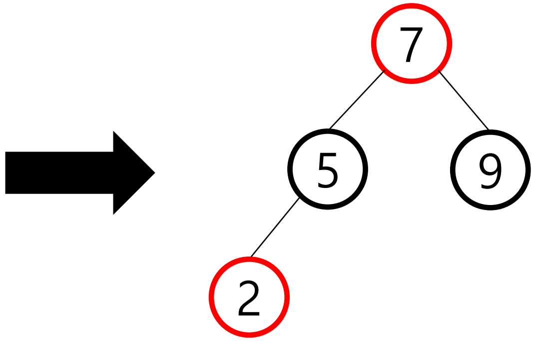
4) Insert 2



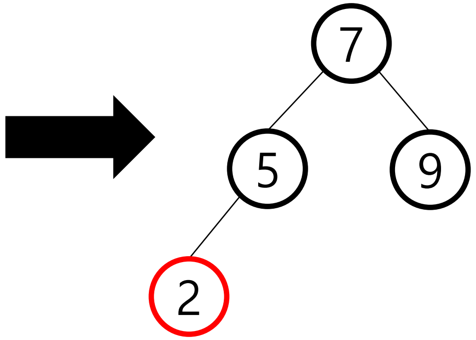
Color change



Color change

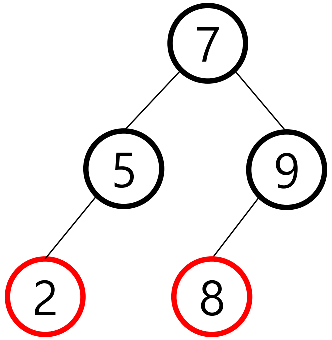


Color change



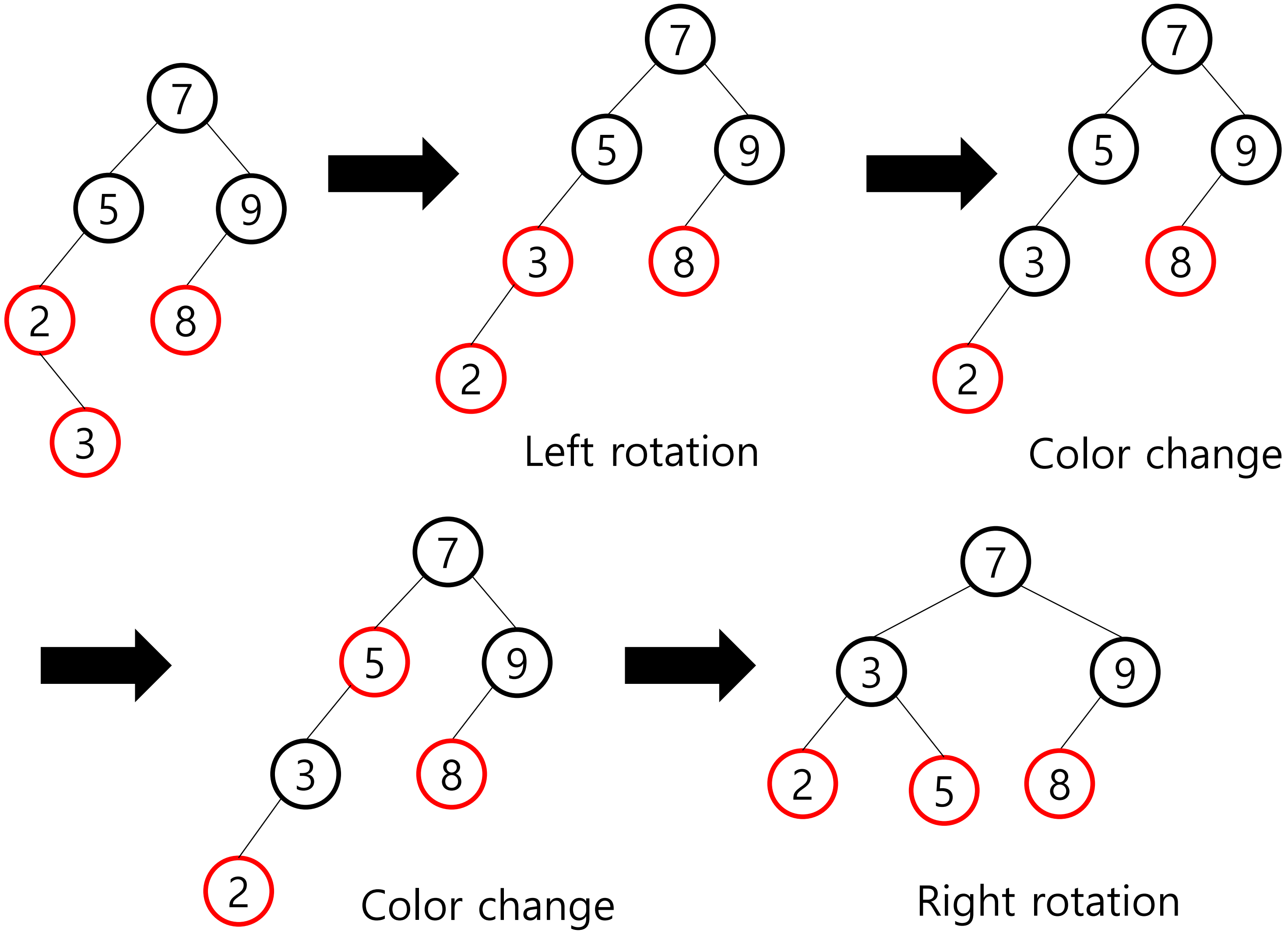
Color change

5) Insert 8

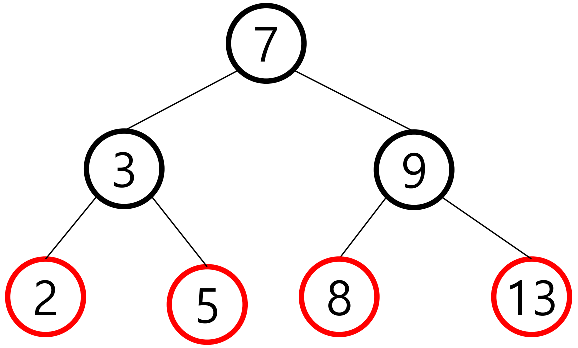


7. Draw the red-black trees that result after successively inserting the keys in the order 7, 5, 9, 2, 8, 3, 13 into an initially empty red-black tree.

6) Insert 3



7) Insert 13

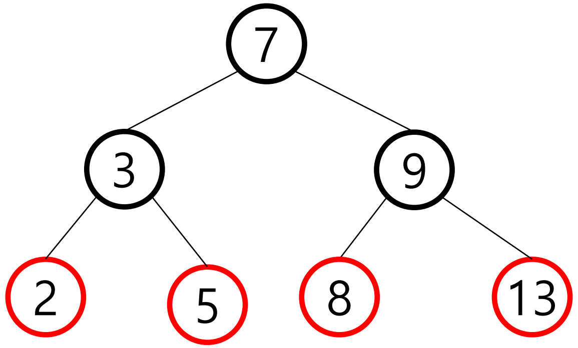


Color-change : 7
Left rotation : 1
Right rotation : 1

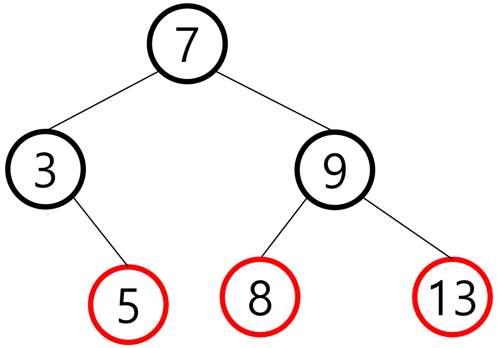
8. Draw the red-black trees that result from the successive deletion of the keys in the order 2, 7, 9, 5, 3, 13, 8 on the tree generated in exercise 7.

Case 1

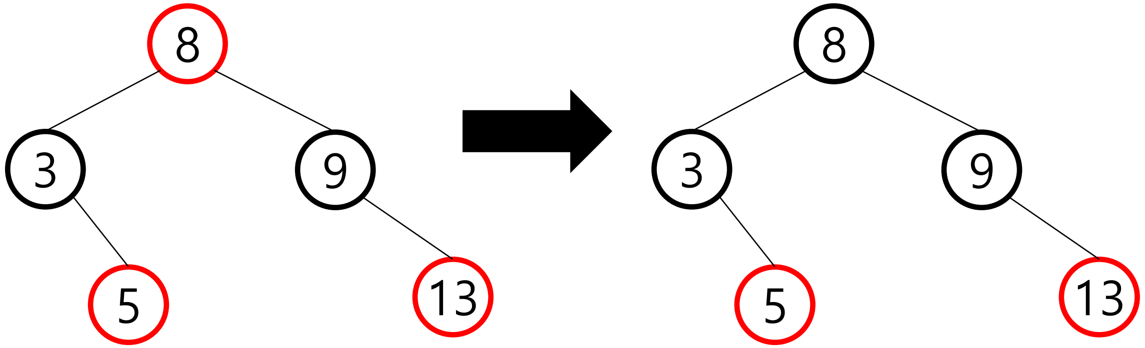
0) init



1) delete 2



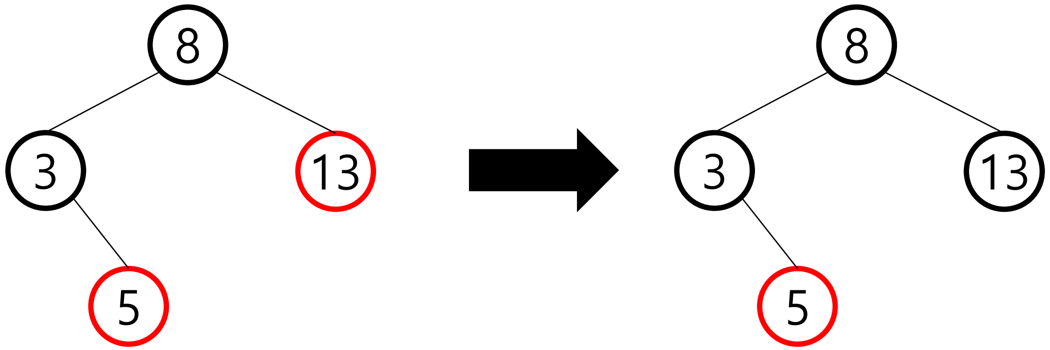
2) delete 7



Color change

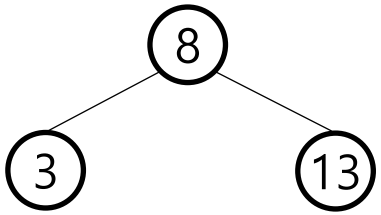
8. Draw the red-black trees that result from the successive deletion of the keys in the order 2, 7, 9, 5, 3, 13, 8 on the tree generated in exercise 7.

3) delete 9

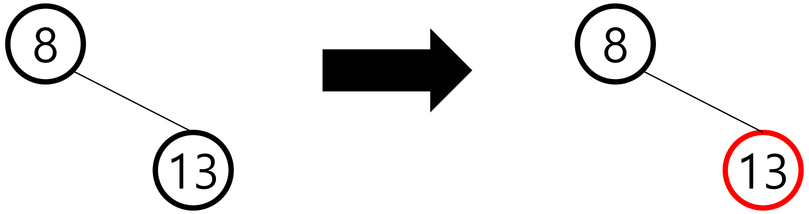


Color change

4) delete 5



5) delete 3



Color change

6) delete 13

7) delete 8

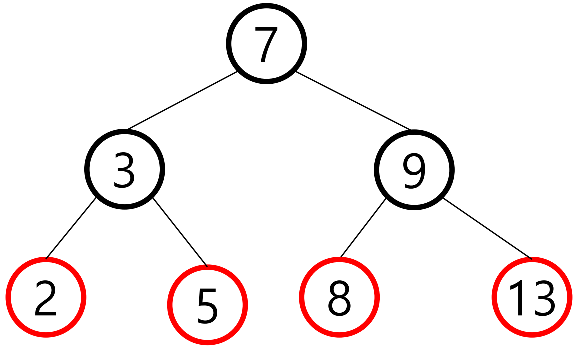
NULL

Color-change : 3
Left rotation : 0
Right rotation : 0

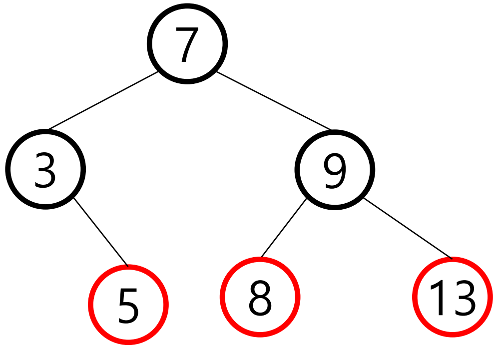
8. Draw the red-black trees that result from the successive deletion of the keys in the order 2, 7, 9, 5, 3, 13, 8 on the tree generated in exercise 7.

Case 2

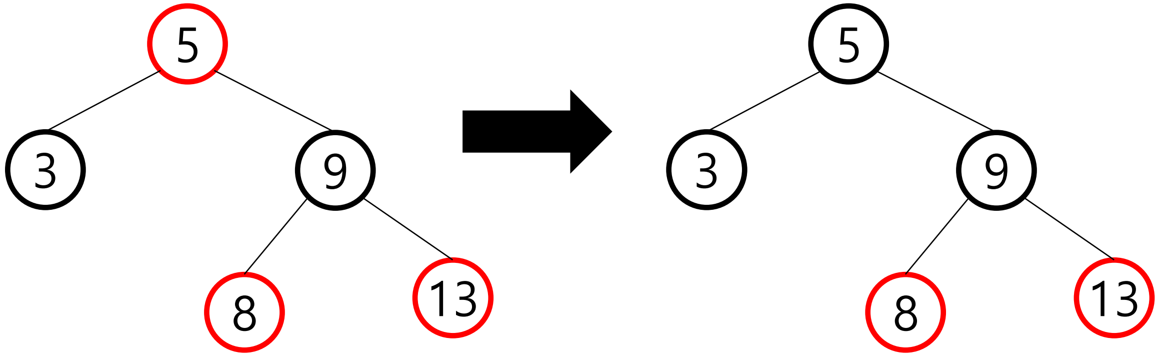
0) init



1) delete 2



2) delete 7



Color change

8. Draw the red-black trees that result from the successive deletion of the keys in the order 2, 7, 9, 5, 3, 13, 8 on the tree generated in exercise 7.