Ex-Post Incentive Compatible Mechanism Design*

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Abstract

We characterize ex post incentive compatible public decision rules, and apply this characterization to (*i*) bilateral trade and (*ii*) public good provision.

1 Introduction

When a planner faces the problem of designing a mechanism to elicit agents' private information and to make, on the basis of this information, some public decision which will affect these agents, the planner must confront the issue of incentives. The mechanism defines the rules of a game played by agents

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and the planner hopes that a solution of this game correpsponds to truthful revelation of information. The theory of mechanism design explores the types of public decision rules for which there exists such an “incentive compatible” mechanism.

The possibilities and impossibilities that result from this theory depend in turn on the theory of games the planner uses to determine what constitutes a solution to his mechanism. The solution concept most widely used in mechanism design is Bayesian Nash equilibrium. In environments with independent private values (IPV), the theory of Bayesian incentive compatible mechanism design has formalized many economically intuitive concepts, such as information rent, constrained efficiency, second-best optimality. On the other hand, when information is correlated, these natural limits on the possibility of Bayesian incentive compatibile mechanism design disappear (see Cremer and McLean (1985), Cremer and McLean (1988), McAfee and Reny (1992)), and the theory loses much of its appeal.

A stronger solution concept is dominant-strategy equilibrium. When the planner insists on the requirement that participation and truthful revelation of information is a dominant strategy for agents, the set of admissible mechanisms is restricted, and appealing features of the IPV literature are restored even when information is correlated. The rich and elegant theory of dominant strategy incentive compatible mechanism design and its connection with IPV Bayesian incentive compatibility is laid out in papers such as Mookherjee and Reichelstein (1992), Williams (1999), and Krishna and Perry (2000).

In recent years there has been a renewed focus on mechanism design in environments with “interdependent valuations.” In these settings, agents preferences depend not only on their own private information but also the information of others. In these environments, Bayesian Nash equilibrium remains very weak (see, for example, Cremer and McLean (1985)), but now dominant strategy equilibrium is too strong: very little can be achieved in dominant strategies. The literature has therefore focused on ex post equilibrium, an intermediate solution concept which entails economically interpretable restrictions on mechanism design possibilities, and which preserves many appealing features of the IPV literature. Ex post equilibrium is equivalent to dominant strategy equilibrium in environments with private values. Ex post incentive compatibility can therefore be viewed as a generalization of dominant-strategy incentive compatibility to interdependent value environments.

This paper attempts to organize and extend the theory of ex post incentive compatible mechanism design. We contribute to the existing literature (e.g., Maskin (1992), Dasgupta and Maskin (2000), Esö and Maskin (2000), Bergemann and Välimäki (2002), Krishna (2003), Perry and Reny (2002)) in a number of ways. First, this literature has focused exclusively on the design of efficient mechanisms. We provide a methodology which can be applied to general design problems, not just those with the goal of efficiency. Second, with the exception of Bergemann and Välimäki (2002), this literature has focused on specific problems (e.g. auctions), presented efficient mechanisms for those environments, and then showed that these mechanisms were ex post incentive compatible. In this paper we approach the design problem from the other direction: we characterize incentive compatibility in general and show how this characterization can be applied to study different applications. This work extends that of Bergemann and Välimäki (2002) who provide a characterization of efficient ex post incentive compatible mechanisms in some specific environments.

A second motivation comes from a recent paper by Bergemann and Morris (2005). They investigate the robustness of Bayesian incentive compatible mechanism design by enlarging agents' type spaces. Such an exercise captures the idea that when a mechanism designer is not sure about agents' type spaces, he may want to design a mechanism that is Bayesian incentive compatible not only with respect to the original type spaces, but with respect to some enlarged type spaces as well. Of course, as the type spaces being entertained get larger and larger, there are more and more restrictions imposed on the mechanism for it to remain Bayesian incentive compatible. Bergemann and Morris (2005) prove that such restrictions translate into requiring that the mechanism, when restricted to the original type spaces, is incentive compatible with respect to a solution concept that is stronger than Bayesian Nash equilibrium, but yet weaker than dominant strategy equilibrium. This intermediate solution concept turns out to be ex post equilibrium. Given this result, researchers who are interested in, say, whether information rent (á lá Myerson and Satterthwaite (1983)) or full surplus extraction (á lá Cremer and McLean (1988)) is a more robust feature of Bayesian incentive compatible mechanism design can readily answer their questions by studying ex post incentive compatible mechanism design. This paper offers a road map to such a study, and provides two examples to illustrate how to use this road map.

A central idea of this paper is that the problem of designing ex post incentive comaptible mechanisms for a group of agents can be decomposed into a collection of single-agent design problems. We provide two propositions (Propositions 4 and 5) formalizing this connection. These propositions show how well known conditions and results for single-agent mechanism design translate immediately to corresponding conditions and results for multi-agent environments for each of the three solution concepts we have discussed. These immediately yield general characterization results for ex post incentive compatibility and establish that the well-known payoff (or revenue) equivalence results extend to this concept. We illustrate the power of these results in applications to bilateral trade and public good provision.

In Section 2 we lay out the framework for the paper and present some preliminary results. Section 3 contains the main characterization results of the paper. We begin by relating multi-agent to single-agent incentive compatibility. As an application of this relation we present two characterization results for ex post incentive compatibility (Subsections 3.1 and 3.2). In Subsection 3.3 we show how "payoff equivalence" results can be translated from the single-agent framework as well. In Subsection 3.4 we discuss conditions under which *Bayesian* incentive compatibility is equivalent to ex post incentive compatibility. We present two applications in the remaining sections. Section 4 studies the Myerson-Satterthwaite model of bilateral trade with interdependent values. Section 5 explores the possibility of ex post incentive compatible and budget-balanced provision of public goods.

2 Setup

There is a finite set N of agents, and an arbitrary set $\hat{Q}$ of feasible social alternatives. Each agent has a type (or signal) s_i which is an element of an arbitrary set S_i. $S = \times_{i \in N} S_i$ is the set of states of nature. Each agent has a utility function $u_i : \hat{Q} \times S \to \mathbf{R}$ specifying his utility as a function of the social alternative and the type profile. If for each agent i, his utility u_i does not depend on s_{-i} (i.e., $u_i(q, s_{-i}, s_i) = u_i(q, \tilde{s}_{-i}, s_i)$ for all q, s_i, s_{-i}, and $\tilde{s}_{-i}$), we say that the environment is one of private values. Otherwise, values are interdependent. We will also suppose that each agent has a reservation utility level which we normalize to zero.

A social planner designs a mechanism in order to implement a social choice function. A social choice function $\hat{f} : S \to \hat{Q}$ specifies a social alternative for each possible state of nature. A mechanism Γ is a pair (M, g), where $M = (M_1, \ldots, M_N)$ is a collection of message spaces, one for each agent, and

$g : \times_j M_j \rightarrow \hat{Q}$ is the outcome function which specifies the social choice $g(m)$ for each profile, m, of messages.

The mechanism Γ determines a strategic game form among the agents, where first agents simultaneously decide whether to participate and if all participate, each agent i chooses a reporting strategy $\sigma_i : S_i \rightarrow M_i$.

2.1 The Bayesian Framework

Suppose that there exists a common prior μ over the set S. Then, given a mechanism, we could evaluate the payoffs to a type s_i of agent i from a strategy profile σ by calculating the interim expected utility $\mathbf{E}_\mu \left[u_i(\sigma(s), s)|s_i\right]$. Based on this preference relation, a widely used criterion for implementation is Bayesian incentive compatibility.

Definition 1 *A social choice function $\hat{f}$ is Bayesian incentive compatible if there exists a mechanism Γ which admits a Bayesian Nash equilibrium strategy profile σ such that $g(\sigma(s)) = \hat{f}(s)$ for all s.*

The revelation principle for Bayesian incentive compatibility states that a social choice function $\hat{f}$ is Bayesian incentive compatible if and only if the truthtelling strategy profile $\sigma(s) = s$ is a Bayesian Nash equilibrium of the direct relvation mechanism $(S, \hat{f})$; i.e., the interim incentive compatibility constraints

$$\mathbf{E}_\mu \left[u_i(\hat{f}(s), s)|s_i\right] \geq \mathbf{E}_\mu \left[u_i(\hat{f}(\tilde{s}_i, s_{-i}), s)|s_i\right], \qquad \forall\ s_i, \tilde{s}_i \in S_i$$

are satisfied. A mechanism should also provide agents with incentives to participate in the first place. Such a mechanism is called individually rational. As a counterpart to Bayesian incentive compatibility, the natural concept of individual rationality would be interim individual rationality: conditional on his realized type, each agent expects to get at least his reservation utility level.

Definition 2 *A social choice function $\hat{f}$ is interim individually rational if $\forall s_i \in S_i$, $\mathbf{E}_\mu \left[u_i(\hat{f}(s), s)|s_i\right] \geq 0$.*

2.2 Ex Post Incentive Compatibility

When the planner does not know agents' interim beliefs about one anothers' types, when he does not have confidence in his estimate of those beliefs, or when he simply does not want to settle for a mechanism which relies critically on the fine details of these beliefs, the planner may demand a more stringent incentive compatibility concept. A well-studied solution concept which is independent of interim beliefs is dominant strategy equilibrium. A social choice function $\hat{f}$ is dominant strategy incentive compatible if for each agent i, truthtelling (i.e., $\sigma_i(s_i) = s_i$) is a dominant strategy in the direct revelation mechanism $(S, \hat{f})$. That is, $\forall\ s_i, m_i \in S_i$, $\forall s_{-i}, m_{-i} \in S_{-i}$,

$$u_i(\hat{f}(s_i, m_{-i}), s_i, s_{-i}) \geq u_i(\hat{f}(m_i, m_{-i}), s_i, s_{-i}).$$

The natural individual rationality counterpart to dominant strategy incentive compatibility is ex post incentive compatibility.

Definition 3 *A social choice function $\hat{f}$ is ex-post individually rational if $\forall s \in S$, $u_i(\hat{f}(s), s) \geq 0$.*

The following straightforward proposition demonstrates why dominant strategy incentive compatibility, together with ex post individual rationality are the appropriate concepts for a planner who does not want the implementation of his social choice function to rest on tenuous conjectures of agents' higher order beliefs.

Proposition 1 *If $\hat{f}$ is dominant strategy incentive compatible and ex post individually rational, then $\hat{f}$ is Bayesian incentive compatible and interim individually rational for* every *prior distribution μ.*

However, if we are primarily interested in those social choice functions which are Bayesian incentive compatible and interim individually rational for all possible interim beliefs of the agents, we are more interested in the converse of Proposition 1. One special case in which the converse holds is when agents have private values. This was proven by d'Aspremont and Gerard-Varet (1979). It means that, in private-value environments, a mechanism designer interested in mechanisms that are Bayesian incentive compatible regardless of agents' higher order beliefs can restrict his attention to those that are dominant strategy incentive compatible.

Unfortunately this result is not true in general. When values are interdependent, dominant strategy incentive compatibility is too strong to characterize belief-independent implementation. This leads us to the incentive compatibility criterion we study in this paper. Say that a social choice function $\hat{f}$ is ex post incentive compatible if $\forall i$, $\forall s$, $\forall \tilde{s}_i$,

$$u_i(\hat{f}(s), s) \geq u_i(\hat{f}(\tilde{s}_i, s_{-i}), s).$$

We call this collection of constraints the ex post incentive compatibility constraints.

Proposition 2 *A social choice function $\hat{f}$ is Bayesian incentive compatible and interim individually rational for every prior distribution if and only if $\hat{f}$ is ex post incentive compatible and ex post individually rational.*

We conclude this section with a version of the revelation principle for ex post incentive compatibility. Let Γ be an arbitrary game. An *ex post equilibrium* of Γ is a strategy profile σ such that, for every prior distribution μ, σ is a Bayesian Nash equilibrium of Γ with respect to μ.

Proposition 3 *A social choice function $\hat{f}$ is ex post incentive compatible and ex post individually rational if and only if there exists a mechanism Γ, and an ex post equilibrium σ of Γ, such that $\forall s$, each agent participates and $g(\sigma(s)) = \hat{f}(s)$.*

2.3 Quasi-Linear Environments

Up to this point we have made no assumptions about utilities and the structure of the set of social alternatives. For the rest of the paper, we will study environments in which money can be transferred among agents and the planner, and agents' utilities are linear in their payments. Formally, we will assume $\hat{Q} = Q \times \mathbf{R}^N$ where $q \in Q$ is a public decision and $t \in \mathbf{R}^N$ represents the vector of payments from agents to the planner. The utility functions take the form:

$$u_i(q, t, s) = v_i(q, s) - t_i,$$

where the valuation function v_i is assumed to be bounded. A public decision rule $f : S \to Q$ picks out a public decision $f(s)$ for each state s. A payment rule $t : S \to \mathbf{R}^N$ specifies the associated payments. When we say that a public decision rule f is incentive compatible according to some criterion, we mean that there exists a social choice function (f, t) that is correspondingly incentive compatible. In the following section we provide a useful and intuitive characterization of the set of all ex post incentive compatible public decision rules.

3 Characterization

Let's ignore the multi-agent setup in the previous section for a moment To avoid confusion with our original multi-agent setup, let's use A to denote the set of public decisions, and use Θ to denote that single agent's type space. The agent's quasi-linear utility function is $u(a, \theta) = v(a, \theta) - t$, where the valuation function v is assumed to be bounded. Let $g : \Theta \to A$ be a public decsion rule. Since there is only one agent, the different versions of incentive compatibility mentioned in Section 2 are equivalent. We can thus speak of necessary and sufficient conditions C for g to be incentive compatible (with no qualification). Since the quadruple (A, Θ, v, g) will fully describe the environment and the public decision rule in question, any necessary or sufficient condition for incentive compatibility must take the form of restrictions on either A (e.g., finiteness), or Θ (e.g., connectivity), or v (e.g., differentiability), or g (e.g., monotonicity), or any combination of these four. In other words, we can view any condition C as a subset of admissible quadruples (A, Θ, v, g).

Our characterization of ex post incentive compatibility will be based on the observation that each version of incentive compatibility mentioned in Section 2 is simply single-agent incentive compatiblity stated with some order of quantifiers. Thus, we can translate characterization results from the literature on Bayesian and dominant strategy incentive compatibility to corresponding characterizations for ex post incentive compatibility after some changes in the quantifiers.

The above observation is formalized in the following proposition. In part 2 of the proposition, ΔQ denotes the set of probability distributions over Q, $f(S_{-i}, \cdot) : S_i \to \Delta Q$ denotes the random function generated from the public decision rule $f : S \to Q$ and the common prior μ, and "independent-type" refers to the case where the common prior μ is a product measure. In part 3 of the proposition, "private-value" refers to the case where agent i's valuation function v_i does not depend on other agents' signals s_{-i}.[1]

Proposition 4 *For any condition C, the following statements are equivalent:*

1. *In the single-agent special case, the public decision rule $g : \Theta \to A$ is incentive compatible if (resp. only if) the quadruple (A, Θ, v, g) satisfies condition C.*

2. *In the independent-type special case, the public decision rule $f : S \to Q$ is Bayesian incentive compatible with respect to the common prior μ if (resp. only if) $\forall i$, the quadruple $(\Delta Q, S_i, \mathbf{E}_\mu v_i, f(S_{-i}, \cdot))$ satisfies condition C.*

3. *In the private-value special case, the public decision rule $f : S \to Q$ is dominant strategy incentive compatible if (resp. only if) $\forall i$, $\forall s_{-i}$, the quadruple $(Q, S_i, v_i, f(s_{-i}, \cdot))$ satisfies condition C.*

4. *In the interdependent-value general case, the public decision rule $f : S \to Q$ is ex post incentive compatible if (resp. only if) $\forall i$, $\forall s_{-i}$, the quadruple $(Q, S_i, v_i(\cdot, s_{-i}, \cdot), f(s_{-i}, \cdot))$ satisfies condition C.*

¿From the previous literature, we can draw numerous examples of condition C that can be plugged into Propostion 4. Instead of enumerating all these examples, we shall hightlight two which we will employ throughout the rest of this paper.

3.1 Example 1: Pseudo Efficiency

In the single-agent special case, when the valuation function v is bounded (which is a maintained assumption throughout this paper), there exists a very simple necessary and sufficient condition for incentive compatibility.

Definition 4 *In the single-agent special case, a public decision rule g is pseudo-efficient with respect to the valuation function v if there exists a function $w : A \to \mathbf{R}$ such that $\forall \theta$,*

$$g(\theta) \in \mathrm{argmax}_{a \in A} v(a, \theta) + w(a).$$

[1] The qualification "private-value" is actually not needed. But since the main purpose of this proposition is to facilitate cross-fertilization across literatures, we insert that qualification in order to conform to standard assumptions in previous literatures.

Lemma 1 *In the single-agent special case, a public decision rule g is incentive compatible if and only if it is pseudo-efficient with respect to the valuation function v.*

Proof: For each $a \in A$, define

$$\mathcal{A}(a) = \{\theta \in \Theta : g(\theta) = a\}.$$

If g is incentive compatible, then there exists incentive compatible mechanism (g, t). Since v is bounded, t must be bounded as well, otherwise (g, t) wouldn't have been incentive compatible in the beginning. Let $M > \sup_\theta |v(\theta)| + \sup_\theta |t(\theta)|$. Incentive compatibility implies that, for each $\theta, \theta' \in \mathcal{A}(a)$,

$$v(a, \theta) - t(\theta) \geq v(a, \theta) - t(\theta'),$$

and

$$v(a, \theta') - t(\theta') \geq v(a, \theta') - t(\theta),$$

which together imply $t(\theta) = t(\theta') =: t(a)$. We define

$$w(a) = \begin{cases} -t(a) & \text{if } \mathcal{A}(a) \neq \emptyset, \\ -M & \text{otherwise.} \end{cases}$$

Consider any $\theta \in \Theta$. Suppose $g(\theta) = a$. Then for any $a' \in A$ and $\theta' \in \mathcal{A}(a')$, incentive compatibility implies that

$$\begin{aligned} & v(a, \theta) - t(\theta) && \geq v(a', \theta) - t(\theta') \\ \Longrightarrow \quad & v(a, \theta) - t(a) && \geq v(a', \theta) - t(a') \\ \Longrightarrow \quad & v(a, \theta) + w(a) && \geq v(a', \theta) + w(a'). \end{aligned}$$

Moreover, for any $a' \in A$ such that $\mathcal{A}(a') = \emptyset$,

$$v(a, \theta) + w(a) \geq v(a', \theta) - M = v(a', \theta) + w(a')$$

as well. Therefore, we have $a \in \text{argmax}_{a' \in A} v(a', \theta) + w(a')$. This completes the proof that g is pseudo-efficient with respect to v.

To prove the converse, suppose g is pseudo-efficient with respect to v. For any $\theta \in \Theta$, set $t(\theta) = -w(g(\theta))$. Then $\forall \theta, \theta' \in \Theta$,

$$\begin{aligned} v(g(\theta), \theta) - t(\theta) &= v(g(\theta), \theta) + w(g(\theta)) \\ &\geq v(g(\theta'), \theta) + w(g(\theta')) \\ &= v(g(\theta'), \theta) - t(\theta'). \end{aligned}$$

Thus, g is incentive compatible. ■

Using Lemma 1 and Proposition 4, we have the following result.

Theorem 1 *The public decision rule $f : S \to Q$ is ex post incentive compatible if and only if $\forall i$, $\forall s_{-i}$, the function $f(s_{-i}, \cdot)$ is pseudo-efficient with respect to $v_i(s_{-i}, \cdot)$.*

Theorem 1 gives us an intuition of why efficient public decision rules are unlikely to be ex post incentive compatible. Let's write down the definition of efficiency and then compare it with that of pseudo-efficiency.

Definition 5 *A public decision rule f is efficient if $\forall i$, $\forall s_{-i}$,*

$$\forall s_i \in S_i, \qquad f(s) \in \text{argmax}_{q \in Q} v_i(q, s) + \sum_{j \neq i} v_j(q, s). \tag{1}$$

However, in order for $f(s_{-i}, \cdot)$ to be pseudo-efficient with respect to $v(\cdot, s_{-i}, \cdot)$, there must be a function w which does *not* depend on s_i such that

$$\forall s_i \in S_i, \qquad f(s) \in \text{argmax}_{q \in Q} v_i(q, s) + w(q). \tag{2}$$

Comparing (2) and (1), one can see that since the function $\sum_{j \neq i} v_j(q, s_{-i}, \cdot)$ in general cannot be replaced by a constant function $w(q)$. Efficient public decision rules are hence in general not ex post incentive compatible. The well-known fact that efficient public decision rules are always dominant strategy (and hence ex post) incentive compatible turns out to be a hairline case: in the private-value special case, the function $\sum_{j \neq i} v_j(q, s_{-i}, \cdot)$ does *not* depend on s_i by assumption.[2]

[2]The canonical efficient mechanism in the private value environment, the VCG mechanism, is constructed by defining i's payment to be identical to this function.

3.2 Example 2: Separable Environment

For the private-value special case, Mookherjee and Reichelstein (1992) look at a specialized environment and provide the necessary and sufficient condition for dominant strategy incentive compatibility. In particular, they look at the environment where, from every agent i's point of view, Q can be condensed into a single dimension, and agents' valuation functions satisfy a supermodularity condition. More formally, they assume that $\forall i$, $S_i = [0,1]$, and there exist $h_i : Q \to \mathbf{R}$ and twice continuously differentiable functions $d_i(\cdot,\cdot) : \mathbf{R} \times S_i \to \mathbf{R}$ such that $v_i(q, s_i) = d_i(h_i(q), s_i)$ (one-dimension condensation). Moreover, $(\partial^2 d_i / \partial h_i \partial s_i) > 0$ (supermodularity condition). Many classical mechanism design problems belong to this environment. For example, in single-unit auctions, from every bidder i's point of view, the set of different allocations can be condensed into a single dimension, namely the probability p_i that he gets the object. The supermodularity condition also follows from the fact that, when his valuation of the object is v_i, his valuation of probabiliy p_i is simply $p_i v_i$, which has positive cross partial derivatives.

Lemma 2 (Proposition 4 of Mookherjee and Reichelstein (1992)) *In the private-value special case, when one-dimension condensation and the supermodularity condition hold, a public decision rule $f : S \to Q$ is dominant strategy incentive compatible if and only if $\forall i$, $\forall s_{-i}$, $h_i(f(s_{-i}, s_i))$ is non-decreasing in s_i.*

In Sections 4 and 5, we will deal with similar applications in the interdependent valuation framework (with some additional structure.) We will refer to this as the *separable environment*.

Definition 6 *A separable environment is one in which for all i, $S_i = [0,1]$ and there exists function $h_i : Q \to [0,1]$ such that agent i's valuation function takes the form of $h_i(q)v_i(s_{-i}, s_i)$, where v_i is strictly increasing in s_i.*

Using Lemma 2 and Proposition 4, we have the following necessary and sufficient condition, namely *monotonicity*, for ex post incentive compatibility in the separable environment.

Theorem 2 *In the separable environment, a public decision rule $f : S \to Q$ is ex post incentive compatible if and only if it is* monotone*; i.e., $\forall i$, $\forall s_{-i}$, $h_i(f(s_{-i}, s_i))$ is non-decreasing in s_i.*

3.3 Payoff Equivalence

Similar to the case of characterizing incentive compatibility, all the results regarding payoff equivalence in the Bayesian and dominant strategy incentive compatible mechanism design literature can be translated into corresponding results for ex post incentive compatible mechanism design.

In the single-agent special case, we say that an incentive compatible public decision rule g satisfies the payoff equivalence property if for any two incentive compatible mechanisms, $(g(\cdot), t(\cdot))$ and $(g(\cdot), t'(\cdot))$, that differ only in the payment rule, we must have $t(\cdot)$ equal to $t'(\cdot)$ up to a constant. In the independent-type special case, we say that a Bayesian incentive compatible (with respect to common prior μ) public decision rule f satisfies interim payoff equivalence if for any two Bayesian incentive compatible mechanisms, $(f(\cdot), t(\cdot))$ and $(f(\cdot), t'(\cdot))$, that differ only in the payment rule, we must have $\forall i$, $\mathbf{E}_\mu t_i(s_{-i}, \cdot)$ equal to $\mathbf{E}_\mu t'_i(s_{-i}, \cdot)$ up to a constant. In the private-value special case (resp. interdependent-value general case), we say that a dominant strategy (resp. ex post) incentive compatible public decision rule f satisfies ex post payoff equivalence if for any two dominant strategy (resp. ex post) incentive compatible mechanisms, $(f(\cdot), t(\cdot))$ and $(f(\cdot), t'(\cdot))$, that differ only in the payment rule, we must have $\forall i$, $\forall s_{-i}$, $t_i(s_{-i}, \cdot)$ equal to $t'_i(s_{-i}, \cdot)$ up to a constant.

A proposition that parallels Proposition 4 is as follows.

Proposition 5 *For any condition C, the following are equivalent:*

1. *In the single-agent special case, an incentive compatible public decision rule $g : \Theta \to A$ satisfies the payoff equivalence property if (resp. only if) the quadruple (A, Θ, v, g) satisfies condition C.*

2. *In the independent-type special case, a Bayesian incentive compatible (with respect to common prior μ) public decision rule $f : S \to Q$ satisfies the interim payoff equivalence property if (resp. only if) $\forall i$, the quadruple $(\Delta Q, S_i, \mathbf{E}_\mu v_i, f(S_{-i}, \cdot))$ satisfies condition C.*

3. *In the private-value special case, a dominant strategy incentive compatible public decision rule $f : S \to Q$ satisfies the ex post payoff equivalence property if (resp. only if) $\forall i$, $\forall s_{-i}$, the quadruple $(Q, S_i, v_i, f(s_{-i}, \cdot))$ satisfies condition C.*

4. *In the interdependent-value general case, an ex post incentive compatible public decision rule $f : S \to Q$ satisfies the ex post payoff equivalence property if (resp. only if) $\forall i$, $\forall s_{-i}$, the quadruple $(Q, S_i, v_i(\cdot, s_{-i}, \cdot), f(s_{-i}, \cdot))$ satisfies condition C.*

Incidentally, most examples of condition C that can be plugged into the above proposition come from the Bayesian incentive compatible mechanism design literature.[3] Once again, we shall not enumerate all these examples, but instead simply hightlight one which is especially relevant for our applications in Sections 4 and 5.

Lemma 3 *In the single-agent special case, if A is a finite set, Θ is a connected topological space, and v is continuous in θ, then any incentive compatible public decision rule g satisfies the payoff equivalence property.*

Proof: Define $\mathcal{A}(a)$ and $t(a)$ as in the proof of Lemma 1. For any subset $E \subset \Theta$, let $\overline{E}$ denote the closure of E. Let $\mathcal{Q}$denote the image of g; i.e., $\mathcal{Q} = g(\Theta)$. For any subset $R \subset \mathcal{Q}$, we claim that

$$\overline{\bigcup_{a \in R} \mathcal{A}(a)} \cap \overline{\bigcup_{a \notin R} \mathcal{A}(a)} \neq \emptyset, \tag{3}$$

otherwise Θ is the union of disjoint closed sets and hence disconnected, a contradiction.

We will use this fact to construct a tree τ whose vertices are the elements of $\mathcal{Q}$ with the property that for any two adjacent vertices, a and a', $\overline{\mathcal{A}(a)} \cap \overline{(a')} \neq \emptyset$. The construction is inductive. Start with an arbitrary $a \in \mathcal{Q}$. By (3), there exists an $a' \in \mathcal{Q}$ such that $\overline{\mathcal{A}(a)} \cap \overline{\mathcal{A}(a')} \neq \emptyset$. Include an edge between a and a'. Now suppose we have constructed a tree over some subset $R \subset \mathcal{Q}$. Then by (3), there exists an element $a \in R$ and an element $a' \notin R$ such that $\overline{\mathcal{A}(a)} \cap \overline{\mathcal{A}(a')} \neq \emptyset$. If we add an edge between a and a' we obtain a new tree over the set $R \cup \{a'\}$. Obviously the tree τ over $\mathcal{Q}$ we eventually obtain by this procedure will have the desired property.

Now let a and a' be adjacent vertices in τ. By construction there is a type $\bar{\theta} \in \Theta$ and sequences $\theta^k \in \mathcal{A}(a)$ and $\beta^k \in \mathcal{A}(a')$ such that $\theta^k \to \bar{\theta}$ and $\beta^k \to \bar{\theta}$. Suppose (g, t) is incentive compatible. Then

$$v(a, \theta^k) - t(a) \geq v(a', \theta^k) - t(a')$$

[3] See, for example, Krishna and Maenner (2001).

and

$$v(a', \beta^k) - t(a') \geq v(a, \beta^k) - t(a)$$

for each k. Taking limits and using the continuity of v in θ, we conclude

$$t(a) - t(a') = v(a, \bar{\theta}) - v(a', \bar{\theta}). \tag{4}$$

Now let a'' and a''' be an aribitrary pair of vertices. Since τ is a tree, it includes a path $a_0, \ldots, a_n$ with $a_0 = a''$ and $a_n = a'''$. Moreover, for every pair of adjacent vertices, a_j and a_{j+1}, there is a corresponding type $\bar{\theta}_j$ as above. Applying (4) to each adjacent pair along this path, we find

$$t(a'') - t(a''') = \sum_{j=0}^{n-1} [t(a_j) - t(a_{j+1})] = \sum_{j=0}^{n-1} \left[v(a_j, \bar{\theta}_j) - v(a_{j+1}, \bar{\theta}_j)\right] = v(a_0, \bar{\theta}_0) - v(a_n, \bar{\theta}_{n-1}).$$

Suppose (g, t') is also incentive compatible, then t' must satisfy the same relationships, and hence

$$t'(a'') - t'(a''') = t(a'') - t(a''')$$

for every pair $a'', a''' \in \mathcal{Q}$. This proves that t' equal to t up to a constant. ■

Using Lemma 3 and Proposition 5, we have the following result.

Theorem 3 *Suppose Q is a finite set, each S_i is a connected topological space, and each v_i is continuous in s_i. Then any ex post incentive compatible public decision rule satisfies the ex post payoff equivalence property.*

In the single-agent special case, whenever an incentive compatible public decision rule g satisfies the payoff equivalence property there exists a mechanism that pointwise maximizes revenue within the set of all incentive compatible and individually rational mechanisms implementing g. To see this, let (g, t) be any incentive compatible mechanism. By payoff equivalence, the set of all incentive compatible mechanisms implementing g is $\{(g(\cdot), t(\cdot) + z) : z \in \mathbf{R}\}$. Let $Z \subset \mathbf{R}$ be the set of z such that $(g, t + z)$ is also individually rational. Since v is bounded (which is a maintained assumption throughout this paper), Z is nonempty,[4] and takes the form of a

[4]Since v is bounded, t must be bounded as well, otherwise (g, t) wouldn't have been incentive compatible in the beginning. So for small enough z, $(g, t+z)$ will be individually rational.

closed interval, $Z = (-\infty, \bar{z}]$. Now $(g, t + \bar{z})$ will be the pointwise revenue maximizing mechanism we mentioned above.

The same logic extends to the interdependent-value general case: Whenever an ex post incentive compatible public decision rule f satisfies the ex post payoff equivalence property, there exists a mechanism that pointwise maximizes revenue within the set of all ex post incentive compatible and ex post individually rational mechanisms implementing f. If f also happens to be an efficient public decision rule, we shall call that revenue maximizing mechanism the generalized VCG mechanism.

3.4 Connection with Bayesian Incentive Compatible Mechanism Design

In this section we develop a useful connection between incentive-compatibility and individual-rationality constraints imposed at the interim stage versus thos imposed at the ex-post stage. The ideas in this section generalize similar methods for the private-value setting due to Mookherjee and Reichelstein (1992), Williams (1999) and Krishna and Perry (2000).

Suppose f is a Bayesian incentive compatible (with respect to common prior μ) public decision rule that satisfies the interim payoff equivalence property. Suppose we start with one implementing mechanism (f, t), and ask first the question of whether we can construct an ex post incentive compatible mechanism (f, t') which is interim-payoff-equivalent to (f, t) (i.e., every agent gets the same interim expected payoffs). If the answer is affirmative, then for robustness reason we definitely would prefer the equivalent mechanism (f, t') to the original mechanism (f, t).

This first question turns out to have a simple answer. If f also happens to be ex post incentive compatible, then there will be ex post incentive compatible mechanism (f, t'), which by definition will also be Bayesian incentive compatible. By the interim payoff equivalence property, t' will give every agent the same interim expected payoffs as t does (up to some constants which can be readily adjusted to zero). The converse is true as well. If we can find an interim-payoff-equivalent ex post incentive compatible mechanism (f, t'), then of course f must also be ex post incentive compatible. So the answer to our first question will be affirmative if and only if f also happens to be ex post incentive compatible.[5]

[5] Mookherjee and Reichelstein (1992) ask the same question for the private-value special

If our original Bayesian incentive compatible mechanism (f, t) is interim individually rational, then our interim-payoff-equivalent ex post incentive compatible mechanism (f, t') by definition is also interim individually rational, but possibly not ex post individually rational. So a natural second question is the following. Start with any ex post incentive compatible, interim individually rational mechanism (f, t'), can it be further reduced to yet another interim-payoff-equivalent ex post incentive compatible mechanism (f, t'') that is also ex post individually rational?

The following is an easy sufficient condition for an affirmative answer. Suppose f also happens to satisfy the ex post payoff equivalence property. Then for any ex post incentive compatible mechanism (f, t'') implementing f, t'' is equal to t' up to some constants. Assume for simplicity that $\forall i$, $\min_{s_i \in S_i} \mathbf{E}_\mu [v_i(f(s_{-i}, s_i), s_{-i}, s_i) - t'_i(s_{-i}, s_i)]$ is attained. Then $\operatorname{argmin}_{s_i \in S_i} \mathbf{E}_\mu [v_i(f(s_{-i}, s_i), s_{-i}, s_i) - t'_i(s_{-i}, s_i)]$ will be the same regardless of whether we use payment rule t' or t'' (for they are the same up to some constants). Consider any $\underline{s}_i$ that minimizes $\mathbf{E}_\mu [v_i(f(s_{-i}, s_i), s_{-i}, s_i) - t'_i(s_{-i}, s_i)]$. If $\underline{s}_i$ also minimizes $v_i(f(s_{-i}, \cdot), s_{-i}, \cdot) - t'_i(s_{-i}, \cdot)$ for μ-almost all s_{-i}, then we are done. Construct a payment rule t'' from t' by adjusting all those constants such that $\forall i$, $t''_i(\cdot, \underline{s}_i) \equiv \mathbf{E}_\mu t'_i(s_{-i}, \underline{s}_i)$, and the resulting mechanism (f, t'') will be ex post incentive compatible, ex post individually rational, and gives every agent the same interim expected payoffs.

The "converse" is also true in the following sense. Suppose $\underline{s}_i$ does not minimize $v_i(f(s_{-i}, \cdot), s_{-i}, \cdot) - t'_i(s_{-i}, \cdot)$ for μ-almost all s_{-i}. Suppose, say, $\mathbf{E}_\mu [v_i(f(s_{-i}, \underline{s}_i), s_{-i}, \underline{s}_i) - t'_i(s_{-i}, \underline{s}_i)] = 0$ (the interim individual rationality constraint is binding). Then for any interim-payoff-equivalent ex post incentive compatible mechanism (f, t'') to be ex post individually rational, t'' must be such that $v_i(f(\cdot, \underline{s}_i), \cdot, \underline{s}_i) - t''_i(\cdot, \underline{s}_i) \equiv 0$ μ-almost everywhere. But then there will be some s_{-i} and $s_i \neq \underline{s}_i$ such that $v_i(f(s), s) - t''_i(s) < 0$, and hence f'' cannot be ex post individually rational.[6]

The sufficient condition above may look complicated. But it actually amounts to nothing more than saying that every agent has an unambiguous

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case, with ex post incentive compatibility replaced by dominant strategy incentive compatibility. Since the specific environment they look at guarantees interim payoff equivalence, their paper can be read as characterizing dominant strategy incentive compatibility.

[6]For example, the Cremer and McLean (1985) surplus extraction mechanism, while ex post incentive compatible, is not ex post individually rational. In fact surplus extraction would not be consistent with this additional constraint due to the same reason outlined in the text.

"lowest type" (which necessarily depends on the public decision rule f in question). Since many classical mechanism design problems belong to the separable environment, and since unambiguous "lowest types" trivially exist in the separable environment,[7] these observations often find useful application.

We conclude this section with a sample theorem illustrating how one can capitalize on the above informal discussion. The theorem identifies a sufficient condition for Bayesian incentive compatible mechanism design to be "equivalent" to ex post incentive compatible mechanism design. Such an "equivalence" was previously developed in the private-value special case by Mookherjee and Reichelstein (1992) and Williams (1999). To state the theorem, we need the following definition.

Definition 7 *In the separable environment, the valuations satisfy the strong single crossing condition if each $v_i(\cdot)$ is differentiable, and whenever $q, q' \in \operatorname{argmax}_q \sum_j v_j(s) h_j(q)$ and $h_i(q) > h_i(q')$ we have $\sum_j \frac{\partial v_j(s)}{\partial s_i}(h_j(q) - h_j(q')) > 0$.*

Theorem 4 *Suppose the environment is separable, the valuations satisfy the strong single crossing condition, Q is a finite set,[8] and signals are independently distributed. Then for any efficient, Bayesian incentive compatible, and interim individually rational mechanism (f, t), there exists an interim-payoff-equivalent ex post incentive compatible and ex post individually rational mechanism (f, t'').*

In the theorem above, the strong single crossing condition[9] ensure that efficient public decision rules are monotone (and hence ex post incentive compatible): when there is a "tie" for the efficient alternative, an increase in i's signal breaks the tie in favor of i's preferred alternative and hence $h_i(f(s_{-i}, \cdot))$ is locally increasing. The finiteness of Q is to ensure that $f(s_{-i}, \cdot)$ is locally constant at all other points.

Proof: By the strong single crossing condition and the finiteness of Q, f is efficient implies f is monotone. By Theorem 2, f is monotone implies

[7] The "lowest type" is $s_i = 0$.

[8] A version of this result holds for a continuum of alternatives at the expense of some additional notations.

[9] This is a generalized version of the (strong) single crossing condition in, for example, Dasgupta and Maskin (2000) and Perry and Reny (2002).

f is ex post incentive compatible. By Theorem 3, f also satisfies the ex post (and hence interim) payoff equivalence property. So there exists ex post incentive compatible and interim individually rational mechanism (f, t') that is interim-payoff-equivalent to (f, t). Since unambiguous "lowest types" exist in the separable environment, there exists ex post incentive compatible and ex post individually rational mechanism (f, t'') that is interim-payoff-equivalent to (f, t'). ■

4 Bilateral Trade

The bilateral trading environment is described as follows. There are two agents, the seller σ and the buyer β. The seller has one indivisible unit of a good. Each observes a private signal from the unit interval $s_\sigma, s_\beta \in [0, 1]$. Each $i = \beta, \sigma$ has a valuation function $v_i : [0, 1]^2 \to \mathbf{R}$ which we assume to be continuous. For the seller, we can interpret $v_\sigma(s)$ as the opportunity cost to the seller of transferring the object to the buyer. This could be a production cost, or simply the seller's own value for an object already produced. Under either interpretation, the seller's ex post individually rational utility level is $v_\sigma(s)$. The buyer's is normalized to zero. We assume that each agent's valuation function is weakly increasing and strictly increasing in his own signal, and that there is at least one s such that $v_\beta(s) > v_\sigma(s)$ and one $\tilde{s}$ such that $v_\beta(\tilde{s}) < v_\sigma(\tilde{s})$ so that the problem is not trivial. The latter can be restated as the assumption that there is neither common knowledge of gains from trade, nor common knowledge of no gains from trade.

This problem was originally studied by Myerson and Satterthwaite (1983) who showed the impossibility of efficient interim incentive-compatible trading mechanisms with independently distributed private values. Adapting the ideas of Cremer and McLean (1985), subsequently, McAfee and Reny (1992) showed that, in the private value setting, when there is correlation between the cost of the seller and value of the buyer, an efficient, budget-balanced mechanism exists in general. Finally, Fieseler, Kittsteiner, and Moldovanu (2003)study the bilateral-trading problem with interdependent valuations under interim incentive-compatibility with independently distributed private signals. They extend the Myerson-Satterthwaite impossibility result to this setting provided values are increasing in all signals. We focus on ex-post incentive compatibility and thus make no assumptions about the distribution of signals. On the other hand, we give a version of the result for interim incentive compatibility as a corollary (see Corollary 1.)

A public decision rule in this context is simply a mapping $f : S \to [0,1]$, where $f(s)$ is the probability that the object will be transferred from the seller to the buyer when the reported type profile is s. We shall call a public decision rule in this bilateral-trade setting a trading rule. If f takes values in $\{0,1\}$, we call f a deterministic trading rule. If (f,t) is an ex post incentive compatible trading mechanism, then the net payoff to the agents in state s are as follows:

$$V_\beta(s) := f(s)v_\beta(s) - t_\beta(s),$$
$$V_\sigma(s) := -f(s)v_\sigma(s) - t_\sigma(s).$$

Definition 8 *A trading rule is monotone if it is monotone non-decreasing in ϕ_β and monotone non-increasing in ϕ_σ.*

By Theorem 2, a trading rule is ex post incentive compatible if and only if it is monotone.

An efficient trading rule in this bilateral-trade setting is a trading rule f such that $f(s) = 1$ whenever $v_\beta(s) > v_\sigma(s)$, and $f(s) = 0$ whenever $v_\beta(s) < v_\sigma(s)$.

There can be multiple efficient trading rules, but they differ only in their ways of breaking ties. Among these efficient trading rules, we will be able to select one that is monotone (and hence ex post incentive compatible) if and only if the following single crossing condition holds.

Definition 9 *The valuation functions satisfy the single crossing property if for any $s, s' \in S$ such that $s'_\beta \geq s_\beta$ and $s'_\sigma \leq s_\sigma$,*

$$[v_\beta(s) > v_\sigma(s)] \implies [v_\beta(s') \geq v_\sigma(s')].$$

If the valuations further satisfy the strong single crossing condition (see Subsection 3.4), then all efficient trading rules will be monotone (and hence ex post incentive compatible). These results are standard and hence we shall omit the proofs

4.1 Budget-Balanced Trading Mechanisms

Suppose the strong single crossing condition holds and hence efficient trading rules are ex post incentive compatible. The next question is whether efficient trading rules are ex post implementable with budget-balanced trading mechanisms.

Consider an efficient, ex post incentive compatible trading mechanism (f, t). The mechanism is *budget-balanced* if $t_\beta(s) + t_\sigma(s) \geq 0$ for all s. If the latter is satisfied with equality for all s, then we say that the mechanism is *exactly budget-balanced.* If on the other hand $t_\beta(s) + t_\sigma(s) \leq 0$ for all s (with at least one strict inequality), then we say that the mechanism runs a deficit. A budget surplus is defined analogously (i.e., the mechanism runs a budget surplus when it is budget-balanced but not exactly budget-balanced). If f is a deterministic trading rule, then by Theorem 3 f satisfies the ex post payoff equivalence property, and the generalized VCG mechanism is well defined (see Subsection 3.3). The generalized VCG payment rule t can be constructed as follows: $t_\sigma(s) = t_\beta(s) = 0$ for all $s \notin \mathcal{T} := \{s : f(s) = 1\}$, and for all $s \in \mathcal{T}$,

$$t_\beta(s) = v_\beta(g_\beta(s_\sigma), s_\sigma), \tag{5}$$

$$t_\sigma(s) = -v_\sigma(s_\beta, g_\sigma(s_\beta)), \tag{6}$$

where $g_\beta(s_\sigma) = \inf\{\tilde{s}_\beta : (\tilde{s}_\beta, s_\sigma) \in \mathcal{T}\}$ and $g_\sigma(s_\beta) = \sup\{\tilde{s}_\sigma : (s_\beta, \tilde{s}_\sigma) \in \mathcal{T}\}$. Note for future reference that $f(0, s_\sigma)v_\beta(0, s_\sigma) - t_\beta(0, s_\sigma) = 0$ for all s_σ, and $-f(s_\beta, 1)v_\sigma(s_\beta, 1) - t_\sigma(s_\beta, 1) = 0$ for all s_β.

For the private-value independent-type special case, Myerson and Satterthwaite (1983) prove that efficiency together with Bayesian incentive compatibility and interim individual rationality is inconsistent with budget balance. On the other hand, when types are correlated, McAfee and Reny (1992) show that budget balance can be achieved. The following result explains why the former result is more robust (in the sense of Bergemann and Morris (2005)) than the latter.

Theorem 5 *There does not exist an efficient, ex post incentive compatible, ex post individually rational, and budget-balanced trading mechanism.*

We prove this theorem in two steps. First in Lemma 4 we show the impossibility result for the special case of deterministic mechanisms. Then, in Lemma 5 we show that the restriction to deterministic mechanisms is without loss of generality.

Lemma 4 *Every deterministic, efficient, ex post incentive compatible, and ex post individually rational trading mechanism runs a deficit.*

Proof: Let f be deterministic, efficient, ex post incentive compatible, and ex post individually rational. By Theorem 3, it suffices to show that (f, t) runs a deficit when t is the generalized VCG payment rule. Let $s \in \mathcal{T}$. Consider the profile $\gamma = (g_\beta(s_\sigma), g_\sigma(s_\beta))$. We claim that $v_\sigma(\gamma) \geq v_\beta(\gamma)$. To show this we consider two cases.

First, suppose $\gamma = (0, 1)$. If $v_\beta(\gamma) > v_\sigma(\gamma)$, then by the efficiency of f, we must have $f(\gamma) = 1$. Because f is ex post incentive compatible, it is monotone (Theorem 2), and hence $f(\tilde{s}_\beta, g_\sigma(s_\beta)) = f(g_\beta(s_\sigma), \tilde{s}_\sigma) = 1$ for any $\tilde{s}_\beta$ and $\tilde{s}_\sigma$. Applying monotonicity again, we conclude that $f(\tilde{s}_\beta, \tilde{s}_\sigma) = 1$ for any $\tilde{s}_\beta$ and $\tilde{s}_\sigma$. But this is a contradiction because there is at least one $\tilde{s}$ such that $v_\sigma(\tilde{s}) > v_\beta(\tilde{s})$ and hence by efficiency, $f(\tilde{s}) = 0$.

Second, suppose (say) $g_\beta(s_\sigma) > 0$ (the case of $g_\sigma(s_\beta) < 1$ is treated in an analogous fashion.) If $v_\beta(\gamma) > v_\sigma(\gamma)$, then by continuity of the valuation functions there exists $\varepsilon > 0$ such that $g_\beta(s_\sigma) - \varepsilon > 0$ and $v_\beta(g_\beta(s_\sigma) - \varepsilon, g_\sigma(s_\beta)) > v_\sigma(g_\beta(s_\sigma) - \varepsilon, g_\sigma(s_\beta))$. By efficiency of f, $f(g_\beta(s_\sigma) - \varepsilon, g_\sigma(s_\beta)) = 1$, and by monotonicity of f, $f(g_\beta(s_\sigma) - \varepsilon, s_\sigma) = 1$. But this contradicts the definition of $g_\beta(s_\sigma)$.

Thus we have

$$t_\beta(s) = v_\beta(g_\beta(s_\sigma), s_\sigma) \leq v_\beta(\gamma) \leq v_\sigma(\gamma) \leq v_\sigma(s_\beta, g_\sigma(s_\beta)) = -t_\sigma(s)$$

so that the deficit is non-negative in every state in which there is trade. We now use the assumption that valuations are strictly increasing in own signal to show that the deficit is strictly positive in some states in which the gains from trade are strictly positive.

Let $\Delta v(s)$ denote $v_\beta(s) - v_\sigma(s)$. By assumption, there exist $s, \tilde{s}$ such that $\Delta v(s) > 0$ and $\Delta v(\tilde{s}) < 0$. Without loss of generality we can pick s such that $s_\sigma < 1$ (by continuity such an s must exist). Since f is efficient, $\Delta v(\tilde{s}) < 0$ implies that $f(\tilde{s}) = 0$. Since f is monotone, $f(\tilde{s}_\beta, 1) = 0$, and finally applying efficiency again, we have $\Delta v(\tilde{s}_\beta, 1) \leq 0$. Write $s' = (\tilde{s}_\beta, 1)$. By the continuity of the valuation functions, the set $R = \{r : \Delta v(rs + (1-r)s') = 0\}$ is not empty. Let $\bar{r} = \sup R$, and $\tau = \bar{r}s + (1-\bar{r})s'$. Note that $\tau_\sigma \geq s_\sigma$. Continuity implies $v_\beta(\tau) = v_\sigma(\tau)$.

Consider any sequence $r_k \downarrow \bar{r}$. Then, writing $s(r_k) = (r_k s + (1-r_k)s')$, we have $s(r_k) \to \tau$ and $s(r_k) \in \mathcal{T}$ for all k. By monotonicity, $(s_\beta(r_k), s_\sigma) \in \mathcal{T}$

for all k and thus

$$\tau_\beta = \lim_{k\to\infty} s_\beta(r_k) \geq \inf_k s_\beta(r_k) \geq g_\beta(s_\sigma)$$

Suppose that the inequality is strict. Then

$$t_\beta(\tau_\beta, s_\sigma) = v_\beta(g_\beta(s_\sigma), s_\sigma) < v_\beta(\tau_\beta, s_\sigma) \leq v_\beta(\tau) = v_\sigma(\tau) \leq v_\sigma(\tau_\beta, g_\beta(\tau_\beta)) = -t_\sigma(\tau_\beta, s_\sigma)$$

yielding a strict deficit. On the other hand, if $g_\beta(s_\sigma) = \tau_\beta$, then we must have $v_\beta(\tau_\beta, s_\sigma) = v_\sigma(\tau_\beta, s_\sigma)$ otherwise by continuity, there would be a $\varepsilon > 0$ such that $\Delta v(\tau_\beta - \varepsilon, s_\sigma) > 0$ and hence $(\tau_\beta - \varepsilon, s_\sigma) \in \mathcal{T}$, contradicting $\tau_\beta = g_\beta(s_\sigma)$. Thus

$$t_\beta(s) = v_\beta(g_\beta(s_\sigma), s_\sigma) = v_\beta(\tau_\beta, s_\sigma) = v_\sigma(\tau_\beta, s_\sigma) \leq v_\sigma(s) < v_\sigma(s_\beta, g_\sigma(s_\beta))$$

where the final inequality follows because $\Delta v(s) > 0$ and continuity imply $s_\sigma < g_\sigma(s_\beta)$. ■

Lemma 5 *For every efficient, ex post incentive compatible, and ex post individually rational trading mechanism, there is a deterministic, efficient, ex post incentive compatible, and ex post individually rational trading mechanism which yields a smaller deficit in every state.*

Proof: Let (f, t) be an efficient trading mechanism. Construct a deterministic rule $\tilde{f}$ by setting $\mathcal{T} = f^{-1}(1)$. The rule $\tilde{f}$ is efficient and monotone because f is. Let $\tilde{t}$ be the generalized VCG payment rule for $\tilde{f}$. We will show that the deficit under $(\tilde{f}, \tilde{t})$ is smaller than under (f, t) in every state s.

First suppose $f(s) = 1$. Then by ex post individual rationality, $v_\beta(s) \geq t_\beta(s)$. Furthermore, since $t_\beta(s)$ is the payment paid by every type of buyer in the set $S^* = \{\tilde{s}_\beta : f(\tilde{s}_\beta, s_\sigma) = 1\}$, these individual rationality constraints together imply

$$t_\beta(s) \leq \inf_{\tilde{s}_\beta \in S^*} v_\beta(\tilde{s}_\beta, s_\sigma) = g_\beta(s_\sigma)$$

Similarly we can argue $-t_\sigma(s) \geq g_\sigma(s_\beta)$. Thus, $t_\beta(s) + t_\sigma(s) \leq g_\beta(s_\sigma) - g_\sigma(s_\beta)$ and the former is the deficit under (f, t), while the latter is the deficit under $(\tilde{f}, \tilde{t})$.

Now suppose $f(s) = p < 1$. By ex post individual rationality, $pv_\beta(s) \geq t_\beta(s)$ and $-t_\sigma(s) \geq pv_\sigma(s)$. Since f is efficient and $p < 1$, it must be that

$v_\sigma(s) \geq v_\beta(s)$ and hence $-t_\sigma(s) \geq pv_\sigma(s) \geq pv_\beta(s) \geq t_\beta(s)$. This shows that under (f, t) there is a deficit in every such state s. Since $f(s) < 1$ implies $\tilde{f}(s) = 0$ and hence $\tilde{t}(s) = (0, 0)$ there is budget balance at each such s under $(\tilde{f}, \tilde{t})$. ■

As a corollary, we have the following result which was obtained via independent means by Fieseler, Kittsteiner, and Moldovanu (2003). We say that a mechanism satisfies ex ante budget balance if the expected value of the deficit is zero.

Corollary 1 *When signals are distributed independently, there does not exist any deterministic,*[10] *efficient, Bayesian incentive compatible, and interim individually rational trading mechanism which satisfies ex ante budget balance.*

Proof: By Theorem 4 such a mechanism could be reduced to an interim-payoff-equivalent efficient, ex post incentive compatible, and ex post individually rational trading mechanism. Since total welfare is fixed by efficiency, interim-payoff-equivalence implies equal expected surplus. Thus, the new mechanism would satisfy ex ante budget balance. But this would contradict Lemma 4. ■

4.2 Deficit-Minimizing Efficient Mechanism

Given the above results, it is natural to investigate the existence of an efficient mechanism that minimizes the deficit among all efficient, ex post incentive compatible, and ex post individually rational mechanisms. Such a mechanism exists, and in fact the minimization is pointwise. Note that this is a different result than the existence of the generalized VCG mechanism (Subsection 3.3). There we fixed f and found the deficit-minimizing payments, whereas here we look for the pair (f, t) which minimizes the deficit among all efficient $(\tilde{f}, \tilde{t})$.

The optimal efficient trading rule f^* is defined as follows. Any efficient, deterministic (which is without loss of generality in light of Lemma 5), and

[10]This is to keep the set of alternatives finite so that Theorem 4 can be applied directly. The result extends to the random case with some additional work. See Fieseler, Kittsteiner, and Moldovanu (2003).

ex post incentive compatible can be described by a monotone cutoff function $g : [0, 1] \to [0, 1]$ such that $f(s) = 1$ if $s_\beta > g(s_\sigma)$, and $f(s) = 0$ if $s_\beta < g(s_\sigma)$. Since g is monotone, it has an "inverse" g^{-1} in the following sense: $g^{-1}(s_\beta) = \sup\{s_\sigma : s_\beta > g(s_\sigma)\}$. Let $\mathcal{G}$ be the set of all monotone cutoff functions obtained in this way from efficient, deterministic, and ex post incentive compatible mechanisms, and let g_* be the pointwise supremum of $\mathcal{G}$. We claim that $g_* \in \mathcal{G}$. Because the pointwise supremum of monotone functions is monotone, g_* is monotone as well. Hence, if we set $f^*(s) = 1$ iff $s_\beta > g_*(s_\sigma)$ (and otherwise $f^*(s) = 0$), we obtain a monotone and hence ex post incentive compatible trading rule. To show that it is efficient, note that $v_\beta(s) >$ (resp: $<$) $v_\sigma(s)$ implies $s_\beta >$ (resp: $<$) $g(s_\sigma)$ for all $g \in \mathcal{G}$ and hence $s_\beta >$ (resp: $<$) $g_*(s_\sigma)$.

Thus f^* is an efficient, deterministic, and ex post incentive compatible trading rule. Let t^* be the associated generalized VCG payment rule. We now show that (f^*, t^*) achieves the smallest deficit at each state among all mechanisms (f, t) "in" $\mathcal{G}$. Clearly the deficit is no greater under (f^*, t^*) at states s where $f^*(s) = 0$ since there the deficit is exactly zero under (f^*, t^*) and by Theorem 5, the deficit is non-negative under (f, t). Consider s at which there is trade in f^*. The payment from the buyer is $v_\beta(g_*(s_\sigma), s_\sigma)$ which by definition of g_* is no smaller than $v_\beta(g(s_\sigma), s_\sigma)$, the payment from the buyer in (f, t). The payment to the seller is $v_\sigma(s_\beta, g_*^{-1}(s_\beta))$ which is no greater than $v_\sigma(s_\beta, g(s_\beta))$ because

$$g_*^{-1}(s_\beta) = \sup\{s_\sigma : s_\beta > g_*(s_\sigma)\} \leq \sup\{s_\sigma : s_\beta > g(s_\sigma)\}$$

because the first set is a subset of the second. We have now established the following theorem.

Theorem 6 *Whenver there exists an efficient and ex post incentive compatible trading rule, there exists a deficit minimizing, efficient, ex post incentive compatible, and ex post individually rational trading rule. Moreover, this trading rule is deterministic.*

4.3 Detail-Free Implementation

It is worth pointing out that, when the valuation functions satisfy the strong single crossing property, not only that all efficient trading rules are ex post incentive compatible, one can pick an efficient mechanism that is ex post dominance solvable as well (so a stronger solution concept comes for free).

Since this result is analogous to our paper on interdependent-value auctions (Chung and Ely (2000)), we shall provide only a sketch here.

Consider the following trading game: the buyer announces a price offer and the seller a price demand. These announcemnts are simulataneous. If (and only if) the buyer's price offer exceeds the seller's demand, the object will be transferred to the buyer, the buyer will pay the seller's demand and the seller will receive the buyer's offer. It can be proved that this game can be solved by iterative elimination of ex post weakly dominated strategies, and the ex post dominance solution (which by definition is also an ex post equilibrium) implements efficient outcomes (see Chung and Ely (2000)).

Following Dasgupta and Maskin (2000), one can also call the above trading game a detail-free mechanism, in the sense that it, instead of using abstract message spaces, uses message spaces that are simpler and have an intrinsic meaning, more like bids in an auction. Moreover, it does not require its designer to know the valuation functions.

4.4 An Example With a Budget Surplus

It was important for Theorem 5 that the valuation functions were weakly increasing. In this section we provide an example in which balanced budget is possible when this assumption is lifted. For some parameter values the budget is balanced in every state, and for other parameter values the mechanism actually runs a budget surplus.

Suppose $v_i(s) = s_i + as_{-i}$ for $i = \beta, \sigma$, where the parameter $a \in \mathbf{R}$ measures the sign and extent of interdependency. Whenever $a < 1$ there is a (essentially) unique efficient and ex post incentive compatible trading rule f in which $f(s) = 1$ if $s_\beta > s_\sigma$. The VCG payments in the event of trade are $t_\beta(s) = s_\sigma + as_\sigma$ and $t_\sigma(s) = -(s_\beta + as_\beta)$. Notice that when $a = -1$, we have $t_\beta(s) = t_\sigma(s) \equiv 0$ so that there is a balanced budget in every state. And when $a < -1$, we have $t_\beta(s) + t_\sigma(s) > 0$ in every state s where there is trade. Thus, the generalized VCG mechanism runs a budget surplus.

5 Provision of Public Goods

In this section we study ex post incentive compatible mechanisms for the provision of a public good when agents' values are possibly interdependent. For simplicity we study the case of two agents who must decide whether or not to proceed with a public project of fixed size and given cost. They will design a mechanism in order to elicit the preferences of each agent, and on the basis of the revealed preferences decide whether or not to proceed with the project, and if so, how to distribute the costs.

Each agent $i = 1, 2$ has a signal $s_i \in S_i = [0, 1]$. Agent i's value for the public project is determined by his valuation function $v_i : S \to \mathbf{R}$, which we assume is weakly increasing, and strictly increasing in own signal. We normalize the value from not producing the public good to zero. The cost of producing the public good is c. The restriction to two agents is for illustrative purposes, the generalization to an arbitrary number is straightforward.

We will study deterministic public decision rules $f : S \to \{0, 1\}$, where $f(s) = 1$ iff the public good is to be produced. We shall call a public decision rule in this public goods setting a provision rule. A provision rule f is efficient if $f(s) = 1$ whenever $V(s) := v_1(s) + v_2(s) > c$, and $f(s) = 0$ whenever $V(s) < c$. A provision mechanism (f, t) satisfies budget balance when $f(s) = 1$ implies $t_1(s) + t_2(s) \geq c$. It satisfies exact budget balance if the latter is always satisfied with equality.

Definition 10 *A provision rule f is monotone if $f(s) = 1$ implies $f(\tilde{s}) = 1$ for every $\tilde{s} \geq s$ where the inequality is weak, and co-ordinate-wise.*

By Theorem 2, a provision rule is ex post incentive compatible if and only if it is monotone.

5.1 Efficient Provision Rules

Our first observation is that efficient provision rules are always ex post incentive compatible because they are always monotone. For any $\tilde{s} \geq s$,

$$V(s) \geq c \implies V(s) \geq c.$$

Theorem 7 *Every efficient provision rule is ex post incentive compatible.*

5.2 Budget-Balanced Provision Mechanisms

Unfortunately, ex post individually rational provision mechanisms are not typically self-financing and require subsidies from outside. In the remainder of this subsection, we study the constraints implied by the restriction to ex post individually rational, budget-balanced mechanisms. In the literature on provision of public goods when agents have private values, a well-known theme is that budget-balanced mechanisms suffer from the free-rider problem. This problem becomes more and more severe in larger populations, asymptotically leading to complete free-riding and no public good provision. The free-rider problem arises in our context with interdependent valuations, but becomes less and less severe as values become "more" common. In the extreme case in which agents have pure common values, $v_1(s) = v_2(s)$ for every s, the efficient mechanism is also exact budget-balancing.

We will distinguish between three classes of enviornments, those with increasing, constant, and decreasing differences. Informally, increasing differences means that agent i's value is more sensitive to changes in his own information than to changes in the information of agent $-i$. Decreasing differences is the opposite case. Constant differences implies that agents have pure common values, $v_i(s) = v_{-i}(s)$ for each s. We think of increasing differences as the typical case. Note that private values is a special case of incresing differences.

Definition 11 *Say that the environment is one of increasing differences, decreasing differences, or pure common values according to whether the difference*

$$v_i(s_i, s_{-i}) - v_{-i}(s_i, s_{-i})$$

is strictly increasing, strictly decreasing, or constant in s_i, respectively.

The proof of the following theorem is straightforward and hence is omitted.

Theorem 8 *Possibilities for efficient, ex post incentive compatible, ex post individually rational and budget-balanced public goods provision are classified as follows.*

1. *With increasing differences, every ex post incentive compatible, ex post individually rational, and budget-balanced provision rule is inefficient.*

2. *With decreasing differences, efficient provision rules are budget-balanced.*

3. *With pure common values, efficient provision rules are exactly budget-balanced.*

Given the above result, we would want to characterization of second-best budget-balanced mechanisms when there are increasing differences. Say that a (deterministic) ex post incentive compatible provision rule f is budget-balanced if its associated generalized VCG mechanism (which by Theorem 3 is well defined) is budget-balanced. Say a budget-balanced f is *dominated* if there exists another budget-balanced $\tilde{f}$ such that

1. $f(s) = 1 \implies \tilde{f}(s) = 1$.
2. If $\tilde{f}(s) = 1$ but $f(s) = 0$ then $V(s) \geq c$.

When searching for the second-best mechanism, which amounts to to searching for the f^* that maximizes $\int f(s)\,[v(s) - c]\,d\mu$ among all budget-balanced f for some prior belief μ over S, a planner can always restrict attention to budget-balanced provision rules that are undominated.

A special role will be played by the following class of provision rules which we call threshold contribution rules.

Definition 12 *f is a threshold contribution rule if there exists a pair of valuations $\bar{v}_1$, $\bar{v}_2$ such that $f(s) = 1$ iff $v_1(s) \geq \bar{v}_1$ and $v_2(s) \geq \bar{v}_2$.*

We can think of $\bar{v}_i$ as the "pivotal" valuation for agent i. Only if each agent's valuation exceeds his pivotal level will the good be provided.

Theorem 9 *Assume increasing differences. Then f is an ex post incentive compatible, budget-balanced, and undominated provision rule if and only if it is a threshold contribution rule with pivotal valuations $\bar{v}_1$, $\bar{v}_2$ such that*

1. *$\bar{v}_1 + \bar{v}_2 = c$, and*
2. *there exists $\bar{s}$ such that $v_i(\bar{s}) = \bar{v}_i$ for $i = 1, 2$.*

Proof: We begin by showing that such a threshold contribution rule is ex post incentive compatible and budget-balanced. Ex post incentive compatibility follows from Theorem 2. The associated generalized VCG payment rule is:

$$t_i(s) = \begin{cases} 0 & \text{if } f(s) = 0 \\ \inf\{v_i(\tilde{s}_i, s_{-i}) : f(\tilde{s}_i, s_{-i}) = 1\} & \text{otherwise} \end{cases}.$$

Because $f(s) = 1$ only if $v_i(s) \geq \bar{v}_i$ and $\bar{v}_1 + \bar{v}_2 = c$, the mechanism (f, t) is budget-balanced.

Now we show that f is undominated. Consider a candidate dominating $\tilde{f}$; i.e. suppose $f(s) = 1 \implies \tilde{f}(s) = 1$ and $\tilde{f}(s) = 1$ for some s such that $v_i(s) < \bar{v}_i$ for some i.

Since f is a threshold contribution rule, $v_j(\bar{s}) = \bar{v}_j$ for each j for some $\bar{s}$ where $\bar{v}_j$ is the pivotal value for agent j. Note that $f(\tilde{s}) = 1$ for every $\tilde{s}$ such that $\tilde{s}_{-i} > \bar{s}_{-i}$, since f is monotone, and hence $\tilde{f}(\tilde{s}) = 1$ as well.

We claim that $s_i \leq \bar{s}_i$. For if $s_i > \bar{s}_i$, then we must have $s_{-i} < \bar{s}_{-i}$ so that $v_i(s) < \bar{v}_i = v_i(\bar{s})$. Then by increasing differences, $v_i(s) - \bar{v}_i > v_{-i}(s) - \bar{v}_{-i}$ which in turn implies that $v_{-i}(s) < \bar{v}_{-i}$. But then $\tilde{f}$ cannot be budget-balanced because $t_i(s) \leq v_i(s)$ (by ex post individual rationality) and hence $t_1(s) + t_2(s) \leq V(s) < \bar{v}_i + \bar{v}_{-i}$ which by assumption is equal to c.

Now because $\tilde{f}$ is ex post incentive compatible, it is monotone and hence $\tilde{f}(\bar{s}_i, s_{-i}) = 1$. According to the associated generalized VCG payment rule, $\tilde{t}_i(\bar{s}_i, s_{-i}) \leq v_i(s) < \bar{v}_i$ and $\tilde{t}_{-i}(\bar{s}_i, s_{-i}) \leq v_{-i}(\bar{s}) = \bar{v}_{-i}$, and hence $\tilde{t}_i(\bar{s}_i, s_{-i}) + \tilde{t}_{-i}(\bar{s}_i, s_{-i}) < \bar{v}_i + \bar{v}_{-i} = c$, hence $\tilde{f}$ is not budget-balanced. This concludes the proof of sufficiency.

Now suppose that f is ex post incentive compatible, ex post individually rational, budget-balanced, and undominated. First we claim that $f(s) = 1$ for some s such that $V(s) = c$. If not, then fix an s such that $V(s) = c$, and let $\hat{f}$ be the threshold contribution rule with pivotal valuations $\bar{v}_i = v_i(s)$. Construct a new provision rule $\tilde{f}$ defined by $\tilde{f}(s) = 1 - (1 - f(s))(1 - \hat{f}(s))$. Since both f and $\hat{f}$ are monotone, so is $\tilde{f}$ and hence $\tilde{f}$ is ex post incentive compatible. Ex post individual rationality and budget balance also are inherited. Thus $\tilde{f}$ dominated f, a contradiction.

Fix an s such that $f(s) = 1$ and $V(s) = c$, and let $\tilde{f}$ be the threshold contribution rule with pivotal valuations $\bar{v}_i = v_i(s)$. Now we claim that f is in fact equal to $\tilde{f}$. If $f(s) = 0$ for some s at which $\tilde{f}(s) = 1$, then the argument of the previous paragraph again yields a dominating provision rule. Finally, if $f(s) = 1$ but $\tilde{f}(s) = 0$, then $v_i(s) < \bar{v}_i$ for some i, and the argument used in the "if" part of the proof will show that f is not budget-balanced, another contradiction. ■

5.3 Detail-Free Implementation

In this subsection we show that when valuations satisfy increasing differences, second-best provision rules have a detail-free indirect implementation in the same sense as in Subsection 4.3. The mechanism can be viewed as a voluntary contingent-contribution scheme in which each agent commits to contribute up to some specified amount provided the pledged contribution of the other agent is at least some level. If each agent's pledged contribution exceeds the minimum specified by the other, then the project is funded and each pays that minimum.

Formally, each agent i announces a pair of numbers, (d_i, p_i). The interpretation is that i is promising to contribute up to p_i provided that agent $-i$ promises to contribute at least d_i. The mechanism works as follows. The public good is produced if and only if $p_i \geq d_{-i}$ for $i = 1, 2$, and if so, agent i is taxed the amount d_{-i} (note that i has promised to pay at least this much). To show that this mechanism implements the second-best, let f be any second-best provision rule. Since we have assumed increasing differences, Theorem 9 implies that f is a threshold contribution rule with associated threshold valuations $\bar{v}_i$. For each s_i, let $F_i(s_i) = \{s_{-i} : v_i(s_i, s_{-i}) > \bar{v}_i, v_{-i}(s_i, s_{-i}) > \bar{v}_{-i}\}$. Define $\phi_{-i}(s_i) = \inf F_i(s_i)$ provided the latter is not empty, otherwise define $\phi_{-i}(s_i) = 1$.

Consider the following strategy profile. First, if $F_i(s_i) = [0, 1]$, then i offers $v_i(s_i, 0)$ contingent upon an offer of $d_i(s_i) = \bar{v}_{-i}$ from agent $-i$. Otherwise, agent i of type s_i offers to contribute $p_i(s_i) = v_i(s_i, \phi_{-i}(s_i))$ contingent upon an offer of at least $d_i(s_i) = v_{-i}(s_i, \phi_{-i}(s_i))$ from agent $-i$.

We now show that this strategy profile is an ex post equilibrium in which the project is approved at state s iff $f(s) = 1$. First, suppose $f(s) = 0$. Then for each i, $\phi_{-i}(s_i) \geq s_{-i}$, and for some i, $\phi_{-i}(s_i) > s_{-i}$. Suppose

$$v_i(s_i, \phi_{-i}(s_i)) = p_i(s_i) \geq d_{-i}(s_{-i}) = v_i(\phi_i(s_{-i}), s_{-i}),$$

then by increasing differences,

$$v_{-i}(\phi_i(s_{-i}), s_{-i}) = p_{-i}(s_{-i}) < d_i(s_i) = v_{-i}(s_i, \phi_{-i}(s_i));$$

i.e., at least one agent's contingency is not met and the project is not approved.

If $d_{-i}(s_{-i}) = \bar{v}_i$, then since $p_i(s_i) = v_i(s_i, \phi_{-i}(s_i)) \geq \bar{v}_i$, we know that i meets the contingency of $-i$. On the other hand, if $d_{-i}(s_{-i}) = v_i(\phi_i(s_{-i}), s_{-i}) > \bar{v}_i$, then we must have $v_{-i}(\phi_i(s_{-i}), s_{-i}) = \bar{v}_{-i}$ (by the definition of ϕ_i), and since $v_{-i}(s_i, \phi_{-i}(s_i)) \geq \bar{v}_{-i}$, increasing differences implies

$$p_i(s_i) = v_i(s_i, \phi_{-i}(s_i)) \geq v_i(\phi_i(s_{-i}), s_{-i}) = d_{-i}(s_{-i})$$

and again i meets the contingency of $-i$. After the identical argument for i's contingency, we have shown that the project is approved if and only if $f(s) = 1$.

To show that the strategies form an ex post equilibrium, note that in any state s, agent i is demanding a contribution from $-i$ which is equal to the minimum of $-i$'s value among all types s_{-i} such that $f(s, s_{-i}) = 1$. Thus, type s_{-i} of agent $-i$ is willing to pay this demand (by meeting i's contingency) if and only if $f(s, s_{-i}) = 1$.

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