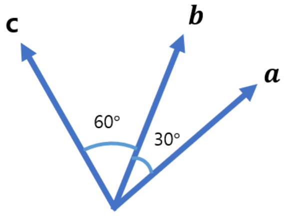
머신 러닝을 위한 수학 복습과제

1. $f(x, y)=x^2y+2x+y^3$를 x, y에 대하여 각각 편미분하시오.

① $f_x(x,y) = 2xy+2$

② $f_y(x,y) = x^2+3y^2$

2. $\mathbf{a}=(\sqrt{3}, 1)$, $\mathbf{b}=(1, \sqrt{3})$, $\mathbf{c}=(-1, \sqrt{3})$라고 할 때 $<\mathbf{a}, \mathbf{b}>$, $<\mathbf{b}, \mathbf{c}>$, $<\mathbf{c}, \mathbf{a}>$를 각각 계산하시오.



① $\langle a, b\rangle = \|a\|\|b\|\cos 30° = \sqrt{10}\cdot\sqrt{10}\cdot\frac{\sqrt{3}}{2} = 5\sqrt{3}$

② $\langle b, c\rangle = \|b\|\|c\|\cos 60° = \sqrt{10}\cdot\sqrt{10}\cdot\frac{1}{2} = 5$

③ $\langle c, a\rangle = \|c\|\|a\|\cos 90° = 0$

3. $A=\begin{pmatrix} 3 & -1 & 0 \\ 2 & 5 & 1 \\ -7 & 1 & 3 \end{pmatrix}$, $B=\begin{pmatrix} 6 & -1 & 0 \\ 0 & 1 & -2 \\ 3 & -8 & 1 \end{pmatrix}$일 때 AB를 계산하시오.

$$AB = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 5 & 1 \\ -7 & 1 & 3 \end{bmatrix}\begin{bmatrix} 6 & -1 & 0 \\ 0 & 1 & -2 \\ 3 & -8 & 1 \end{bmatrix} = \begin{bmatrix} 18 & -3-1 & 2 \\ 12+3 & -2+5-8 & -10+1 \\ -42+9 & 7+1-24 & -2+3 \end{bmatrix} = \begin{bmatrix} 18 & -4 & 2 \\ 15 & -5 & -9 \\ -33 & -16 & 1 \end{bmatrix}$$

4. A=$\begin{pmatrix} 2 & -3 & 1 \\ 2 & 0 & -1 \\ 1 & 4 & 5 \end{pmatrix}$의 행렬식을 계산하시오.

$$|A| = 2\begin{vmatrix} 0 & -1 \\ 4 & 5 \end{vmatrix} + 3\begin{vmatrix} 2 & -1 \\ 1 & 5 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 1 & 4 \end{vmatrix}$$

$$= 2(0+4) + 3(10+1) + (8-0)$$

$$= 8 + 33 + 8 = 49$$

5. A=$\begin{pmatrix} 3 & 4 \\ 6 & 8 \end{pmatrix}$의 역행렬을 구하시오.

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} 8 & -4 \\ -6 & 3 \end{bmatrix}$$

$$|A| = 24 - 24 = 0$$

$|A|$, 즉 행렬식이 0 이므로 역행렬 존재하지 않는다.

6. A=$\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$의 고윳값과 고유벡터를 구하시오.

$$\det(A - \lambda I) = 0$$

$$\Leftrightarrow \det\left(\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = 0$$

$$\Leftrightarrow \begin{vmatrix} 1-\lambda & 4 \\ 3 & 2-\lambda \end{vmatrix} = 0 \quad \Leftrightarrow (1-\lambda)(2-\lambda) - 12 = 0$$

$$\Leftrightarrow 2 - 3\lambda + \lambda^2 - 12 = \lambda^2 - 3\lambda - 10 = (\lambda-5)(\lambda+2) = 0$$

① 고윳값 $\lambda = 5$일 때,

$\left(\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) x = 0$, $x = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$로 놓으면,

$$\Leftrightarrow \begin{bmatrix} -4 & 4 \\ 3 & -3 \end{bmatrix} \cdot \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = 0$$

$\Leftrightarrow -4\alpha + 4\beta = 0$ and $3\alpha - 3\beta = 0$

$\Leftrightarrow \alpha = \beta$

$\therefore$ 고유벡터 $x = t\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

② 고윳값 $\lambda = -2$일 때,

$\left(\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) x = 0$, $x = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$로 놓으면,

$$\Leftrightarrow \begin{bmatrix} 3 & 4 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = 0$$

$\Leftrightarrow 3\alpha + 4\beta = 0$

$\therefore$ 고유벡터 $x = t\begin{bmatrix} 1 \\ -\frac{3}{4} \end{bmatrix} = \frac{1}{4}t\begin{bmatrix} 4 \\ -3 \end{bmatrix}$