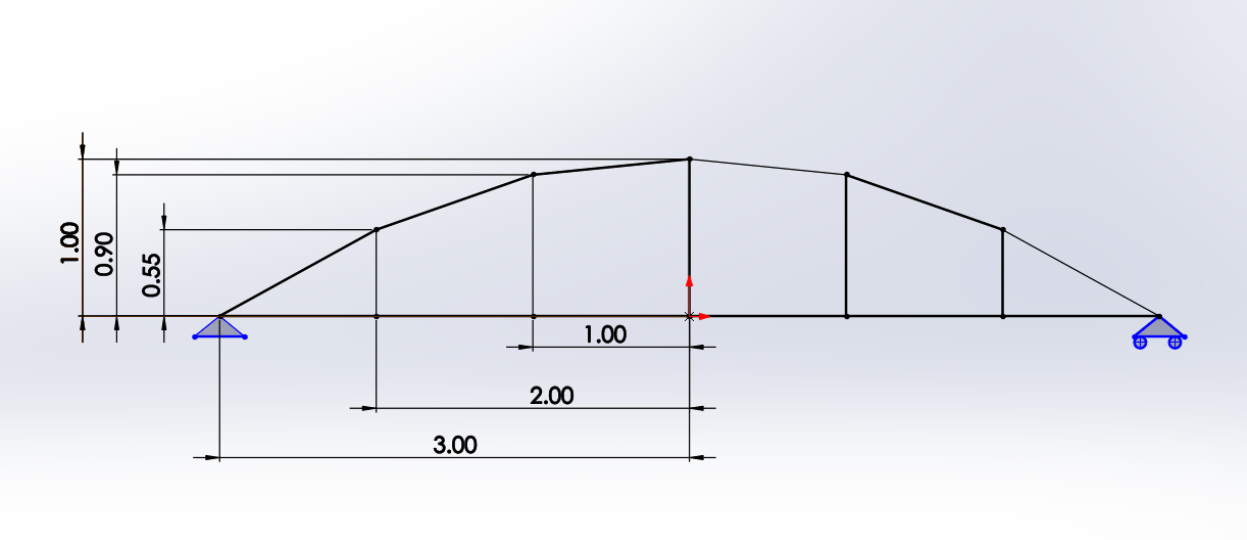
João Vitor Sanches 9833704

Em conjunto com Victor Chacon Codesseira 9833711

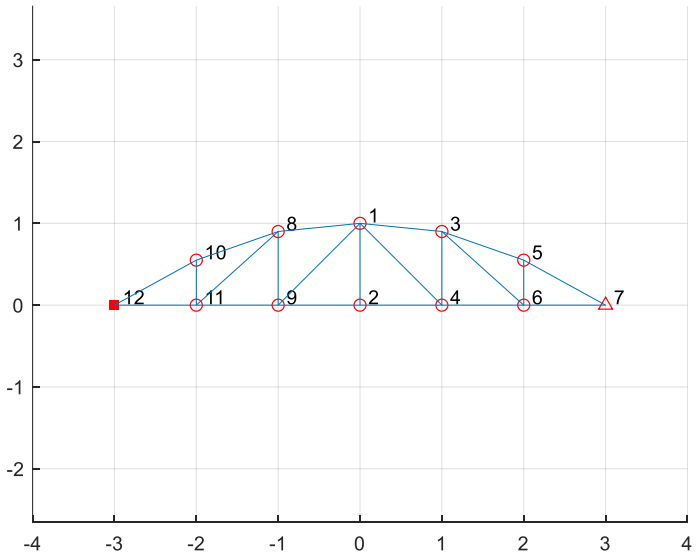
Aula 3 – Dinâmica de Treliças

Com o desenvolvimento anterior do código em matlab, é possível fazer a análise estática da ponte de pedestres, cujas dimensões são:



Adotou-se área da seção transversal 0.0015 m^2, módulo de elasticidade 206 GPa e massa específica 7800 Kg/m^3.

Aplicando 50KN no ponto central superior, obtém-se os resultados apresentados abaixo



Estrutura como modelada

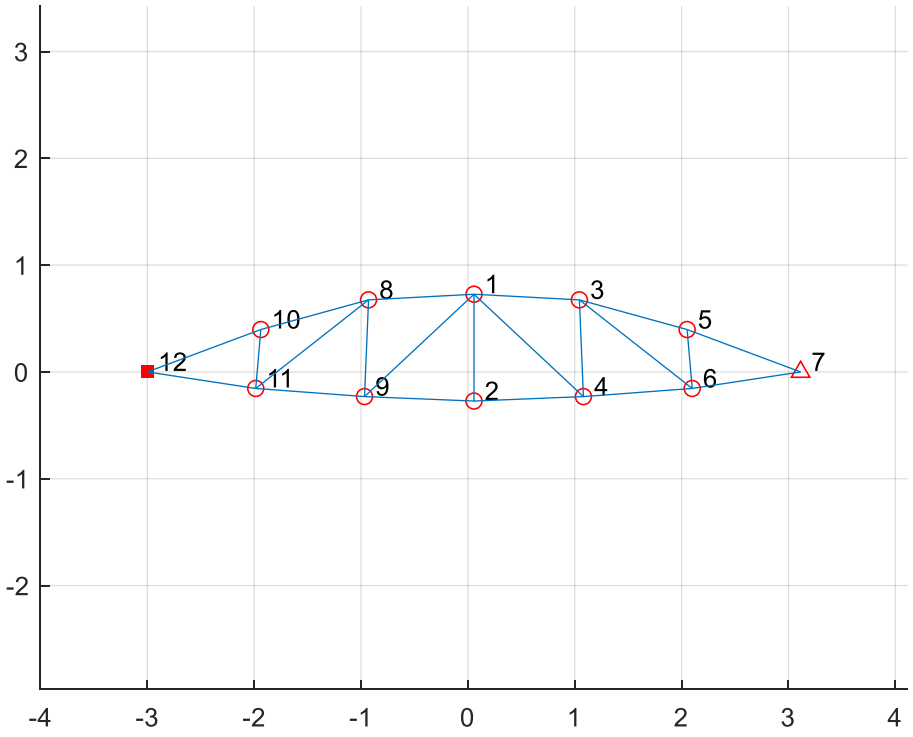
Arquivo de entrada utilizado:

```
#HEADER
Análise estática - ponte de pedestres

#DYNAMIC
0

#NODES
0 1
0 0
1 0.9
1 0
2 0.55
2 0
3 0
-1 0.9
-1 0
-2 0.55
-2 0
-3 0

#ELEMENTS
1 2 0.0015 206843000000 7800
1 4 0.0015 206843000000 7800
3 4 0.0015 206843000000 7800
3 6 0.0015 206843000000 7800
5 6 0.0015 206843000000 7800
1 9 0.0015 206843000000 7800
8 9 0.0015 206843000000 7800
8 11 0.0015 206843000000 7800
10 11 0.0015 206843000000 7800
1 3 0.0015 206843000000 7800
3 5 0.0015 206843000000 7800
5 7 0.0015 206843000000 7800
7 6 0.0015 206843000000 7800
6 4 0.0015 206843000000 7800
2 4 0.0015 206843000000 7800
9 2 0.0015 206843000000 7800
9 11 0.0015 206843000000 7800
11 12 0.0015 206843000000 7800
12 10 0.0015 206843000000 7800
10 8 0.0015 206843000000 7800
8 1 0.0015 206843000000 7800

#LOADS
@1
0 -50000


#CONSTRAINTS
@12
0 0
@7
u 0
```

Saída do programa:

```
Displacement on node 1 is 0.00056729 in x and -0.0027257 in y
Displacement on node 2 is 0.00056729 in x and -0.0027257 in y
Displacement on node 3 is 0.00043308 in x and -0.0022503 in y
Displacement on node 4 is 0.00080902 in x and -0.0023067 in y
Displacement on node 5 is 0.00051115 in x and -0.0015295 in y
Displacement on node 6 is 0.00098808 in x and -0.0015456 in y
Displacement on node 7 is 0.0011346 in x and 0 in y
Displacement on node 8 is 0.0007015 in x and -0.0022503 in y
Displacement on node 9 is 0.00032556 in x and -0.0023067 in y
Displacement on node 10 is 0.00062343 in x and -0.0015295 in y
Displacement on node 11 is 0.0001465 in x and -0.0015456 in y
Displacement on node 12 is 0 in x and 0 in y
Stress on element 1 is 0 MPa
Stress on element 2 is 18.3211 MPa
Stress on element 3 is -12.981 MPa
Stress on element 4 is 9.0139 MPa
Stress on element 5 is -6.1384 MPa
Stress on element 6 is 18.3211 MPa
Stress on element 7 is -12.981 MPa
Stress on element 8 is 9.0139 MPa
Stress on element 9 is -6.1384 MPa
Stress on element 10 is 37.2001 MPa
Stress on element 11 is 32.0595 MPa
Stress on element 12 is 34.3702 MPa
Stress on element 13 is -30.55 MPa
Stress on element 14 is -37.0969 MPa
Stress on element 15 is -50.0182 MPa
Stress on element 16 is -50.0182 MPa
Stress on element 17 is -37.0969 MPa
Stress on element 18 is -30.55 MPa
Stress on element 19 is 34.3702 MPa
Stress on element 20 is 32.0595 MPa
Stress on element 21 is 37.2001 Mpa
```

Ilustração da estrutura deformada:



(Escala de deformação 100X)

Análise dinâmica

Exercício 1

O programa foi alterado para lidar com problemas dinâmicos. Para o exercício 1, o arquivo de entrada definido foi:

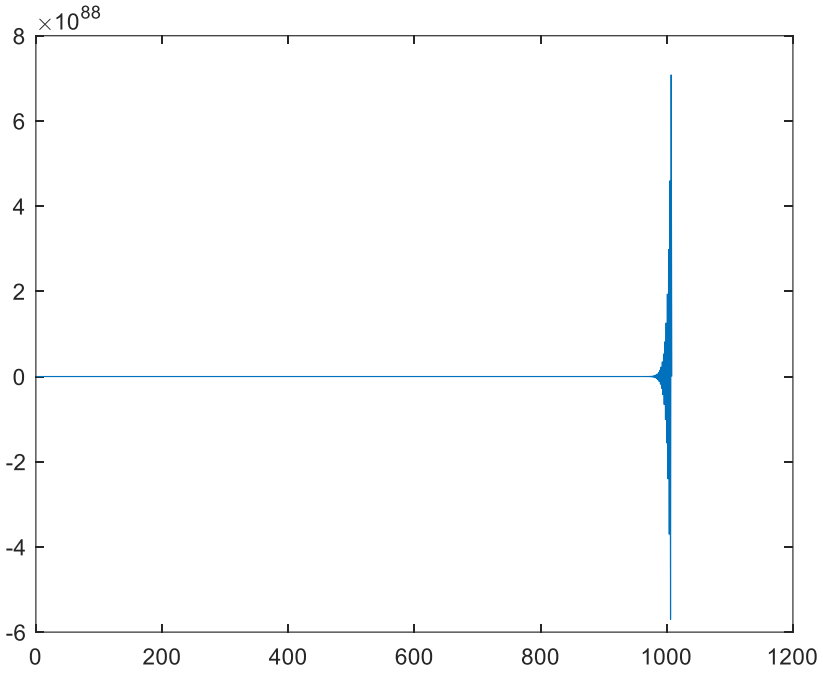
```
#HEADER
Exercicio 1 - dinamico
Dynamic values are zero, unless defined

#DYNAMIC
1

#TIMESTEP
0.0001

#SIMTIME
1

#NODES
0 0
-0.508 0

#ELEMENTS
1 2 0.000625 206843000000 7800


#LOADS
@2
0 0
450 0


#CONSTRAINTS
@1
0 0
@2
u 0


#INITIALDISP

#INITIALVEL

#INITIALACCEL
```

Como as condições iniciais são nulas, as seções foram deixadas vazias. O degrau na força é definido descrevendo a força inicial e a do primeiro passo (que é mantido no decorrer da simulação).

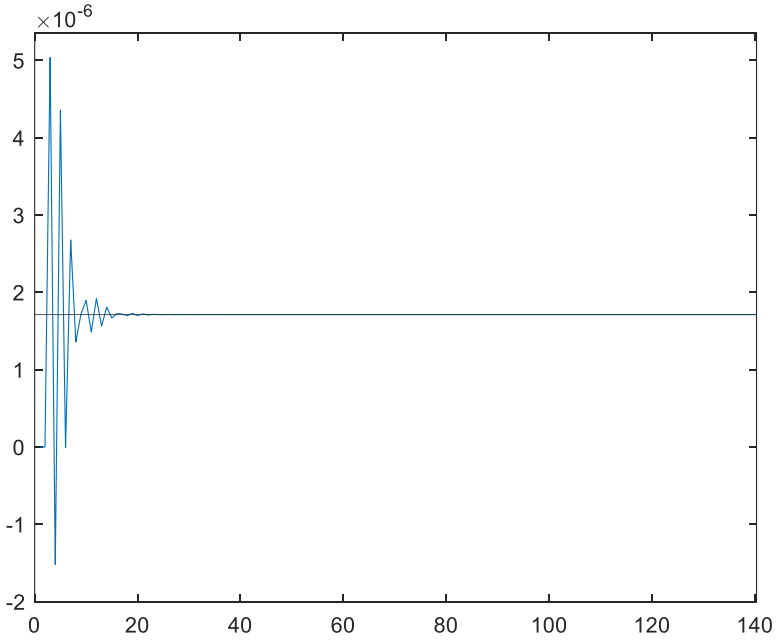
Calcula-se o timestep crítico com

$$\Delta t_{cr} = \frac{L_{mesh}}{\sqrt{E/\rho}} = \frac{0.5}{\sqrt{206843000000/7800}} = 9.87e^{-5}$$

Com $t_1 = 1.007 * t_{cr} = 9.94e^{-5}$, a posição do nó é mal calculada e o gráfico aponta a não convergência da resposta:



Já com $t_2 = 1.007 * t_{cr} = 9.7662e^{-5}$, a posição converge exatamente para o valor do equilíbrio estático:



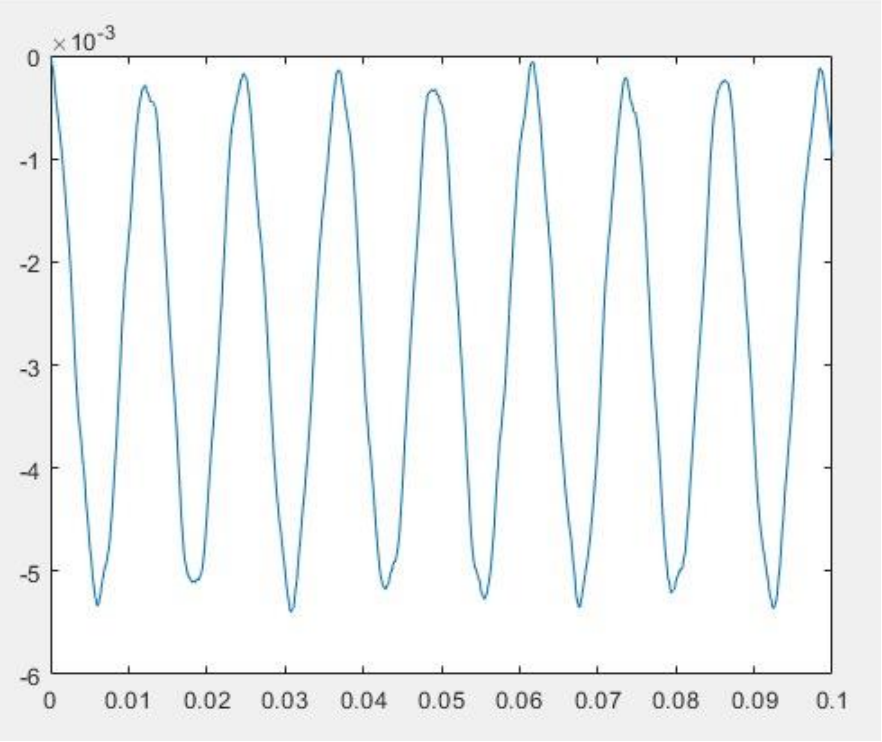
Exercício 2

Para o exercício 2, foi criado um arquivo de entrada para a ponte, definido abaixo.

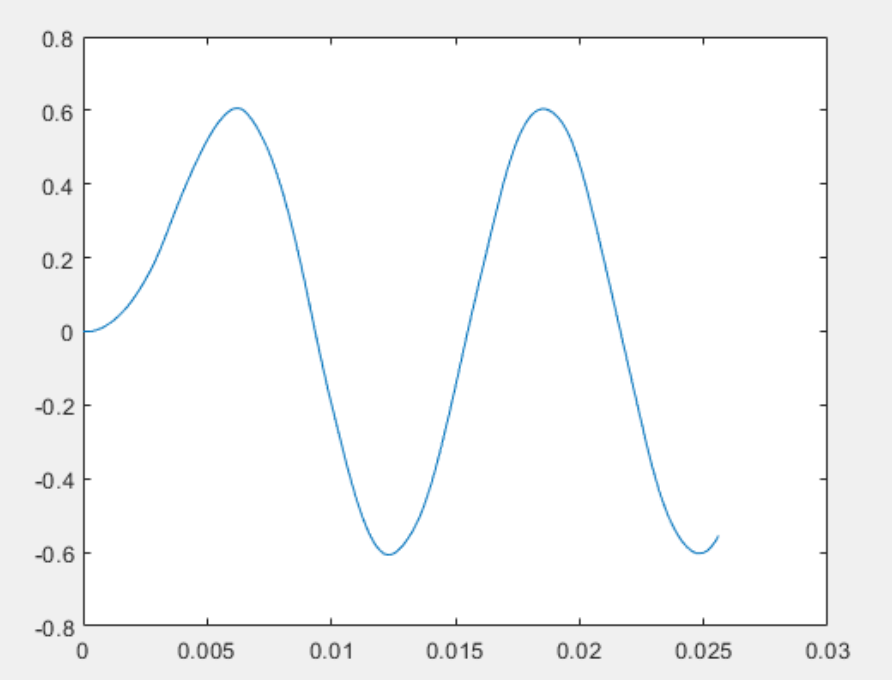
```
#HEADER
Exercicio 2 - Ponte
Dynamic values are zero, unless defined

#DYNAMIC
1

#TIMESTEP
0.00001

#SIMTIME
1

#NODES
0 1
0 0
1 0.9
1 0
2 0.55
2 0
3 0
-1 0.9
-1 0
-2 0.55
-2 0
-3 0

#ELEMENTS
1 2 0.0015 206843000000 7800
1 4 0.0015 206843000000 7800
3 4 0.0015 206843000000 7800
3 6 0.0015 206843000000 7800
5 6 0.0015 206843000000 7800
1 9 0.0015 206843000000 7800
8 9 0.0015 206843000000 7800
8 11 0.0015 206843000000 7800
10 11 0.0015 206843000000 7800
1 3 0.0015 206843000000 7800
3 5 0.0015 206843000000 7800
5 7 0.0015 206843000000 7800
7 6 0.0015 206843000000 7800
6 4 0.0015 206843000000 7800
2 4 0.0015 206843000000 7800
9 2 0.0015 206843000000 7800
9 11 0.0015 206843000000 7800
11 12 0.0015 206843000000 7800
12 10 0.0015 206843000000 7800
10 8 0.0015 206843000000 7800
8 1 0.0015 206843000000 7800

#LOADS
@1
```

```
0    50000

#CONSTRAINTS
@12
0 0
@7
u 0

#INITIALDISP

#INITIALVEL

#INITIALACCEL
```

Com essa configuração, aconteceu um impulso no nó 1, que gerou a seguinte resposta:



A partir desse gráfico, foi possível obter a primeira frequência natural do sistema, que ficou em torno de 78 Hz(ou um período de 0.0128 s). Então, foi aplicada uma força no formato pedido pelo enunciado, de forma a fazer o deslocamento do nó 2 ser 0.1*L, ou seja, 0.6 m. Após ajustar esse valor da força, chegamos num valor de 7.5 MN para uma deformação tão alta. A resposta do sistema, então, foi a seguinte, na qual podemos perceber o sistema oscilando na frequência da força.



Em https://github.com/jvSanches/PMR5026 é possível ter acesso ao programa completo...
O código utilizado nos exercícios é listado a seguir:

DynamicreProcessor.m

```
steps = round(simtime/timestep);
dof = 2*length(nodes);
D = zeros(steps+1, dof);
Ddot = zeros(steps+1, dof);
Dddot = zeros(steps+1, dof);
F = zeros(steps+1, dof);

D(1,1:dof) = initial_disp(1:dof);
Ddot(1,1:dof) = initial_vel(1:dof);
```

```
Dddot(1,1:dof) = initial_accel(1:dof);

for i=1:length(nodes)
    for j=1:steps+1
        if j > length(nodes(i).fx)
            F(j,2*i-1) = F(j-1,2*i-1);
        else
            F(j,2*i-1) = nodes(i).fx(j);
        end
        if j > length(nodes(i).fy)
            F(j,2*i) = F(j-1,2*i);
        else
            F(j,2*i) = nodes(i).fy(j);
        end
    end
end

%% Builds system matrices
disp('Building global stiffness matrix...');
Kglobal = zeros(2*length(nodes));
for i=1:length(elements)
    Kdist = zeros(2*length(nodes));
    [k11, k12, k22, index1, index2] = elements(i).decomposeStiffnes();
    index1 = 2 * index1 -1;
    index2 = 2 * index2 -1;
    Kdist(index1,index1) = k11(1,1);
    Kdist(index1,index1+1) = k11(1,2);
    Kdist(index1+1,index1) = k11(2,1);
    Kdist(index1+1,index1+1) = k11(2,2);

    Kdist(index1,index2) = k12(1,1);
    Kdist(index1,index2+1) = k12(1,2);
    Kdist(index1+1,index2) = k12(2,1);
    Kdist(index1+1,index2+1) = k12(2,2);

    Kdist(index2,index1) = k12(1,1);
    Kdist(index2,index1+1) = k12(1,2);
    Kdist(index2+1,index1) = k12(2,1);
    Kdist(index2+1,index1+1) = k12(2,2);

    Kdist(index2,index2) = k22(1,1);
    Kdist(index2,index2+1) = k22(1,2);
    Kdist(index2+1,index2) = k22(2,1);
    Kdist(index2+1,index2+1) = k22(2,2);

    Kglobal = Kglobal + Kdist;
end
disp('Done')
disp('Building global mass matrix...');
Mglobal = zeros(2*length(nodes));
for i=1:length(elements)
    Mdist = zeros(2*length(nodes));
    [m11, m12, m22, index1, index2] = elements(i).decomposeMass();
```

```
    index1 = 2 * index1 -1;
    index2 = 2 * index2 -1;
    Mdist(index1,index1) = m11(1,1);
    Mdist(index1,index1+1) = m11(1,2);
    Mdist(index1+1,index1) = m11(2,1);
    Mdist(index1+1,index1+1) = m11(2,2);

    Mdist(index1,index2) = m12(1,1);
    Mdist(index1,index2+1) = m12(1,2);
    Mdist(index1+1,index2) = m12(2,1);
    Mdist(index1+1,index2+1) = m12(2,2);

    Mdist(index2,index1) = m12(1,1);
    Mdist(index2,index1+1) = m12(1,2);
    Mdist(index2+1,index1) = m12(2,1);
    Mdist(index2+1,index1+1) = m12(2,2);

    Mdist(index2,index2) = m22(1,1);
    Mdist(index2,index2+1) = m22(1,2);
    Mdist(index2+1,index2) = m22(2,1);
    Mdist(index2+1,index2+1) = m22(2,2);

    Mglobal = Mglobal + Mdist;
end
disp('Done')

Cglobal = 0.0004 * (0.3*Mglobal + 0.03*Kglobal);
```

DunamicSolver.m

```
disp('Starting Solver')

%% Reduces system with given constraits

for i=1:length(nodes)
    if nodes(i).xconstrained

        for j = 1:length(Kglobal)
            F(:, j) = F(:, j) - Kglobal(j,2*i-1) * nodes(i).dx;
            Kglobal(2*i-1,j) = 0;
            Kglobal(j,2*i-1) = 0;
            Mglobal(2*i-1,j) = 0;
            Mglobal(j,2*i-1) = 0;
        end
        F(:,2*i-1) = nodes(i).dx;
        Kglobal(2*i-1, 2*i-1) = 1;
        Mglobal(2*i-1, 2*i-1) = 1;
    end
    if nodes(i).yconstrained
        for j = 1:length(Kglobal)
            F(:, j) = F(:, j) - Kglobal(j,2*i) * nodes(i).dy;
            Kglobal(2*i,j) = 0;
```

```
            Kglobal(j,2*i) = 0;
            Mglobal(2*i,j) = 0;
            Mglobal(j,2*i) = 0;
        end
        F(:,2*i) = nodes(i).dy;
        Kglobal(2*i, 2*i) = 1;
        Mglobal(2*i, 2*i) = 1;
    end
end

%Cglobal = 1*Mglobal;

M_inv = (Mglobal./timestep^2)^-1;
MC1 = (2/timestep^2)*Mglobal - (1/timestep)*Cglobal;
MC2 = (1/timestep^2)*Mglobal - (1/timestep)*Cglobal;

D0 = D(1,:) - timestep*Ddot(1,:) + (timestep^2/2)*Dddot(1,:);
D(2, :) = M_inv*(F(1,:)' - Kglobal*D(1,:)' + MC1*D(1,:)' - MC2*D0');
for i=2:steps-1
    Dn = D(i,:)';
    Dn_1 = D(i-1,:)';
    R_int = Kglobal*Dn;
    Fn = F(i,:)';

    Dn_new = M_inv*(Fn- R_int + MC1*Dn - MC2*Dn_1);
    D(i+1,:) = Dn_new;
end

disp('Done')

%% Plot dos valores desejados
% limit = 10000; figure; plot(0:timestep:(limit-1)*timestep,D(1:limit,2))
T2 = 0.0128;
t = 0:timestep:2*T2;
figure; plot(t,D(1:length(t),4))
```

truss.m

```
classdef truss
    %UNTITLED truss element
    %   Detailed explanation goes here

    properties
        n1,
        n2,
        A,
        E,
        L,
        m,
        l,
        K,
```

```
    M,
    ro,


end

methods
    function obj = truss(node1, node2, area, e_modulus, density)
        %UNTITLED Construct an instance of this class
        %   Detailed explanation goes here
        obj.n1 = node1;
        obj.n2 = node2;
        obj.A = area;
        obj.E = e_modulus;
        obj.ro = density;
        obj.L = sqrt((node2.x - node1.x)^2 + (node2.y - node1.y)^2);
        l = (node2.x-node1.x) / obj.L;
        m = (node2.y-node1.y) / obj.L;
        obj.K = (obj.E*obj.A/obj.L)*[l*l l*m 0 -l*l -l*m 0;
                                     l*m m*m 0 -l*m -m*m 0;
                                     0 0 1 0 0 0;
                                    -l*l -l*m 0 l*l l*m 0;
                                     -l*m -m*m 0 l*m m*m 0;
                                     0 0 0 0 0 1];
        obj.M = (obj.A * obj.ro * obj.L/6)*[2*l*l 2*l*m l*l l*m;
                                            2*l*m 2*m*m l*m m*m;
                                            l*l l*m 2*l*l 2*l*m;
                                            l*m m*m 2*l*m 2*m*m];

    end

    function [k11, k12, k22, index1, index2] = decomposeStiffnes(obj)
        %METHOD1 Summary of this method goes here
        %   Detailed explanation goes here
        k11 = obj.K(1:3,1:3);
        k12 = obj.K(1:3,4:6);
        k22 = obj.K(4:6,4:6);
        index1 = obj.n1.index;
        index2 = obj.n2.index;
    end
    function [m11, m12, m22, index1, index2] = decomposeMass(obj)
        %METHOD1 Summary of this method goes here
        %   Detailed explanation goes here
        m11 = obj.M(1:2,1:2);
        m12 = obj.M(1:2,3:4);
        m22 = obj.M(3:4,3:4);
        index1 = obj.n1.index;
        index2 = obj.n2.index;
    end
    function tension = getTension(obj)
        nx1 = obj.n1.x+obj.n1.dx;
        ny1 = obj.n1.y+obj.n1.dy;
        nx2 = obj.n2.x+obj.n2.dx;
        ny2 = obj.n2.y+obj.n2.dy;
        L_strain = sqrt((nx2 - nx1)^2 + (ny2 - ny1)^2);
        tension = (obj.L-L_strain)/obj.L * obj.E;
    end
```

```
    end
end
```

node.m

```
classdef node < handle
    %NODE Node for mef
    %   Detailed explanation goes here

    properties
        index,
        x,
        y,
        theta,
        fx,
        fy,
        mo,
        dx,
        dy,
        dtheta,
        xconstrained,
        yconstrained,
        thetaconstrained,


    end

    methods
        function obj = node(n_x,n_y)
            %NODE Construct an instance of this class
            %   Detailed explanation goes here
            obj.x = n_x;
            obj.y = n_y;
            obj.theta = 0;
            obj.fx = 0;
            obj.fy = 0;
            obj.mo = 0;
            obj.dx = 0;
            obj.dy = 0;
            obj.xconstrained = 0;
            obj.yconstrained = 0;
            obj.thetaconstrained = 0;
        end
        function setLoad(obj, nfx, nfy, nmo)
            obj.fx = obj.fx + nfx;
            obj.fy = obj.fy + nfy;
            obj.mo = obj.mo + nmo;
        end
        function setIndex(obj, ind)
            obj.index = ind;
        end

        function constrain(obj, c_x, c_y, c_theta)

            if c_x ~= 'u'
```

```
                c_x = str2num(c_x);
                obj.xconstrained = 1;
                obj.dx = c_x;
            end
            if c_y ~= 'u'
                c_y = str2num(c_y);
                obj.yconstrained = 1;
                obj.dy = c_y;
            end
            if c_theta ~= 'u'
                c_theta = str2num(c_theta);
                obj.thetaconstrained = 1;
                obj.dtheta = c_theta;
            end
        end
        function setDeltas(obj, ndx, ndy)
            obj.dx = ndx;
            obj.dy = ndy;
        end
    end
end
```

plotter.m

```
figure
hold on;
grid on;
scale = 100;

%%Display nodes
max_x = -inf;
min_x = inf;
max_y = -inf;
min_y = inf;
for i=1:length(nodes)
    nx = nodes(i).x+scale*nodes(i).dx;
    ny = nodes(i).y+scale*nodes(i).dy;
    max_x = max(max_x, nx);
    max_y = max(max_y, ny);
    min_x = min(min_x, nx);
    min_y = min(min_y, ny);
    if nodes(i).xconstrained
        if nodes(i).yconstrained
            scatter(nx, ny, 's', 'filled', 'red');
        else
            scatter(nx, ny, '>', 'filled', 'red');
        end
    else
        if nodes(i).yconstrained
            scatter(nx, ny,'^' ,'red');
        else
            scatter(nx, ny, 'red');
        end
    end
    text(nx+0.1, ny+0.1, string(i));
end
offset = 1;
```

```
axis([min_x-offset max_x+offset min_y-offset max_y+offset])

%display trusses
for i=1:length(elements)
    line_x = [elements(i).n1.x+scale*elements(i).n1.dx,
elements(i).n2.x+scale*elements(i).n2.dx];
    line_y = [elements(i).n1.y+scale*elements(i).n1.dy,
elements(i).n2.y+scale*elements(i).n2.dy];
    line(line_x,line_y)
end
```