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Exercício 1

Partindo da eq. governante:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(E\mathrm{A_c}\frac{\mathrm{d}u}{\mathrm{d}x}\right) + q = A\rho\ddot{u}$$

E adotando as condições de contorno e estabilidade do problema:

$$\ddot{u} = 0$$

$$q = 0$$

Temos:

$$E\mathrm{A_c}\frac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{P} \rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{\mathrm{P}}{\mathrm{E\,A_c}}$$

Com Ac, a seção transversal da barra escrita como:

$$\mathrm{A_c(x)} = \mathrm{A} + \left(1\text{-}\frac{\mathrm{x}}{\mathrm{L}}\right)\mathrm{A} = \frac{\mathrm{A}}{\mathrm{L}}\,(2\mathrm{L}\text{-}\mathrm{x})$$

É possível fazer a integração entre x = 0 e x = X:

$$\int_0^{\mathrm{X}} \frac{du}{dx}\,dx = \int_0^{\mathrm{X}} \frac{\mathrm{P}}{\mathrm{E\,A_c(x)}}dx$$

$$\mathrm{u}|_{\mathrm{x=X}} = \int_0^{\mathrm{X}} \frac{\mathrm{P}}{\mathrm{E\,A_c(x)}}dx = \frac{\mathrm{P}}{\mathrm{E}}\int_0^{\mathrm{X}} \frac{\mathrm{L}}{\mathrm{A}}\frac{1}{(2\mathrm{L}\text{ - }\mathrm{x})}dx$$

$$\mathrm{u}|_{\mathrm{x=X}} = \frac{\mathrm{P\,L}}{\mathrm{E\,A}}\int_0^{\mathrm{X}} \frac{1}{(2\mathrm{L}\text{ - }\mathrm{x})}dx = \frac{\mathrm{P\,L}}{\mathrm{E\,A}}\,(\text{- }\ln(\text{-X} + 2\mathrm{L}) + \ln(2\mathrm{L})\,)$$

$$\mathrm{u_X} = \frac{\mathrm{P\,L}}{\mathrm{E\,A}}\,\ln\left(\frac{2\mathrm{L}}{2\mathrm{L}\text{ - }\mathrm{X}}\right)$$

Dado que $\mathrm{A_R} = 2\mathrm{A}$ e $\mathrm{A_r} = \mathrm{A}$, pode-se substituir na equação dada para verificação do resultado acima:

$$u(x) = \frac{P\,L}{E(A_{\mathrm{R}} - A_{\mathrm{r}})}\,ln\left(\frac{A_{\mathrm{R}}}{A_{\mathrm{R}} - \left(\frac{A_{\mathrm{R}}\text{ - }A_r}{L}\right)\mathrm{X}}\right)$$

$$u(x) = \frac{P\,L}{E\,(2\mathrm{A} - \mathrm{A})}\,ln(\frac{2\mathrm{A}}{2\mathrm{A} - \left(\frac{2\mathrm{A}\text{ - }\mathrm{A}}{L}\right)\mathrm{X}})$$

$$u(x) = \frac{P\,L}{E\,\mathrm{A}}\,ln(\frac{2\mathrm{L}}{2\mathrm{L} - \mathrm{X}})$$