Predicting Parking Lot Occupancy in Vehicular Ad Hoc Networks

Murat Caliskan*, Andreas Barthels*, Björn Scheuermann‡, and Martin Mauve‡

*Volkswagen AG, Wolfsburg, Germany
{murat.caliskan, andreas.barthels}@volkswagen.de

‡Institute of Computer Science, Heinrich Heine University, Düsseldorf, Germany
{scheuermann, mauve}@cs.uni-duesseldorf.de

Abstract— **The search for free parking places is a promising application for vehicular ad hoc networks (VANETs). In order to guide drivers to a free parking place at their destination, it is necessary to estimate the occupancy state of the parking lots within the destination area at time of arrival. In this paper, we present a model to predict parking lot occupancy based on information exchanged among vehicles. In particular, our model takes the age of received parking lot information and the time needed to arrive at a certain parking lot into account and estimates the future parking situation at time of arrival. It is based on queueing theory and uses a continuous-time homogeneous Markov model. We have evaluated the model in a simulation study based on a detailed model of the city of Brunswick, Germany.**

I. Introduction

Searching for free parking places in urban traffic conditions is a serious mobility problem. A study in [1] provides results regarding the parking situation in Schwabing, a district of Munich, Germany. For this area, an annual total economical damage of 20 million Euros (about 25 million US dollars) has been estimated, caused only by the traffic searching for free parking lots. It would thus be a great benefit for the driver of a vehicle to have up-to-date knowledge on the traffic situation, particularly information on free parking places near the destination area.

Vehicular ad hoc networks present a promising way to build up a decentralized parking guidance system. Designing such an application can be decomposed into three major issues: (1) Which information on a parking place needs to be known by the vehicles and thus has to be distributed in the vehicular ad hoc network? (2) How should the protocol for the dissemination of parking place information look like? And finally, (3) how can this information be used to maximize the benefit for the driver? The second question is relatively disjoint from the others, while number one and three need to be considered conjointly. A bandwidth efficient protocol for disseminating parking place information in VANETs has already been proposed in [2]. Here we concentrate on the other two parts, with a focus on the third aspect: how should the information received through the VANET be interpreted?

We use the model depicted in Figure 1 for a decentralized parking guidance system: the occupancy information is collected at the respective parking lot, e. g. by parking meters or parking fee payment terminals. This information is broadcasted, received by vehicles within communication range, and then disseminated within the vehicular ad hoc network. Vehicles on their way to some destination area can then use it to make their decision amongst several possible parking opportunities.

There is significant latency in the network, mostly because of temporary partitioning. This is particularly serious during the initial rollout phase of VANETs, when the number of equipped vehicles is small. Also, the time between receiving the information and arriving at a particular parking place must be considered. Thus, simply distributing the occupancy of parking lots and hoping that this information is still valid at the time the driver arrives at the parking lot is not an optimal solution. Instead, we propose to estimate the future parking lot occupancy from the information that is available in the VANET.

Our contributions in this paper are (1) a mathematical model for parking lot occupancy prediction in a vehicular ad hoc network, (2) a concept how this model can be applied in practice, and (3) a simulative evaluation of the proposed approach using a detailed simulation model.

The remainder of this paper is organized as follows. The next section discusses work related to decentralized parking place search. Section III introduces our prediction algorithm in detail. Section IV describes our evaluation methodology comprising the utilized simulation environment and the model used to verify our algorithm. The results are presented and discussed in Section V. Finally, Section VI concludes this paper.

II. Related Work

There are a number of projects and proposals that use wireless communication to alleviate the parking place problem. Here, we discuss these approaches. However, none of them considers the problem of outdated occupancy information. Thus, our contributions here are complementary to these approaches.

An approach for distributing parking place information is planned in the project SmartPark [3]. The project focus—besides distribution of information—is on using wireless sensor and actor networks in order to allow convenient parking

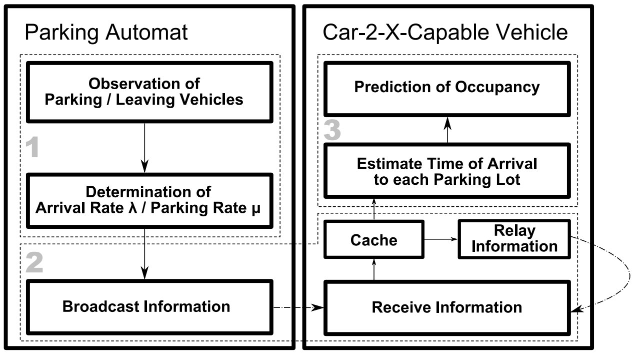

Fig. 1. Information flow in predicting parking lot occupancy.

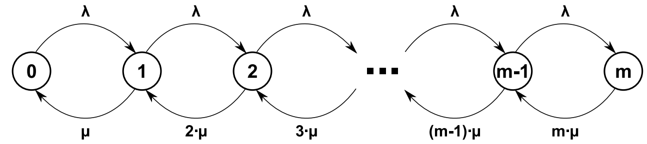

Fig. 2. Markov chain corresponding to our model.

for drivers. This comprises hardware solutions for detecting free parking lots [4] and information distribution.

The authors in [5] propose also a scenario of wireless ad hoc networks for finding free parking places. They use multi-hop dissemination of information only among interlinked parking meters, and not among vehicles. Requests from vehicles for parking places are received and handled by such a parking meter via single-hop communication.

ParkSens [6] presents an approach to sense the occupancy status of a parking place by using magnetic sensors that can detect small fluctuations in the earth's magnetic field. Sensors send their information to a central database, or they distribute them via wireless communications. According to their web page, their sensors are close to being brought into market.

In [2] we concentrate on the dissemination of parking place occupancy information via multi-hop vehicular ad hoc communication, i. e., the second of the three issues mentioned in the introduction. We have used the dissemination protocol presented in that paper for the evaluation of our prediction model.

III. The Algorithm

In this section, we introduce our algorithm for the prediction of future parking place occupancy, and we show how it can be implemented in a vehicular ad hoc network. Our approach is based on results from queueing theory and applied stochastics. We model a parking lot as a queue and use a Markov chain to describe it. Since vehicles can park or leave at arbitrary times, we use a continuous-time model [7], [8].

A. Dissemination of Parking Lot Information

In our approach, five values for each parking lot are distributed in the network, namely timestamp, total capacity of the parking lot, number of parking places that are currently occupied, and finally, two rates: *the arrival rate* of vehicles, and the *parking rate*. The parking rate is the inverse of the average time for which a vehicle stays on its parking lot before it leaves again. These rates are measured at the parking lots.

B. Mathematical Foundation of the Prediction Model

For the queue representing the parking lot we use a homogeneous Markov model with exponentially distributed inter-arrival and parking times. Thus, the flow of incoming vehicles is a Poisson process. Since the space on each parking lot is finite, the queue has the same capacity as the number of parking places on the parking lot. Each state of the Markov model represents the respective number of vehicles currently parking on the parking lot. If a vehicle arrives at a fully occupied parking lot, it is rejected. In Kendall notation, we deal with an $M/M/m/m$ queue, where m is the number of parking places on the parking lot. This special type of queue is also called a loss system.

Because of the homogeneity we can use the following notation. For all times $t, \tau \in \mathbb{R}_0^+$,

$$p_{ij}(t) := P(X_{\tau+t} = j \mid X_\tau = i) = P(X_t = j \mid X_0 = i).$$

$p_{ij}(t)$ denotes the probability that the Markov chain is in state j at t time units in the future, given that its present state is i. For ease of notation the state set is from now on assumed to be $\{0, 1, 2, \ldots, m\}$.

In the theory of continuous-time Markov chains, the concept of the Q-matrix is used in order to be able to calculate the transition probabilities for all $t \in \mathbb{R}_0^+$. The transition rate q_{ij} from state i to state j is defined as the right-hand derivative of $p_{ij}(t)$ at $t = 0$:

$$\forall i \neq j : q_{ij} := \lim_{t \searrow 0} \frac{p_{ij}(t)}{t}.$$

By conservation of probability, the probability of staying in a certain state i decreases with rate

$$q_{ii} = -\sum_{j \neq i} q_{ij}.$$

The Q-matrix is then defined by $Q = (q_{ij})$; its dimension is $(m+1) \times (m+1)$.

If the parameter of the arrival Poisson process is denoted by λ, and μ is the parking rate, this results in a Q-matrix with the following tri-diagonal pattern:

$$Q = \begin{pmatrix} -\lambda & \lambda & & & \\ \mu & -(\lambda+\mu) & \lambda & & \\ & 2\mu & -(\lambda+2\mu) & \lambda & \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ & & (m-1)\mu & -(\lambda+(m-1)\mu) & \lambda \\ & & & m\mu & -m\mu \end{pmatrix}$$

The corresponding Markov chain is depicted in Figure 2.

C. Application of the Prediction Model in a Vehicle

When information on a parking lot's λ (arrival rate), μ (parking rate), occupancy status and total capacity is received from the vehicular ad hoc network, a vehicle knows the corresponding Q-matrix. Utilizing this knowledge it can calculate the probabilities for the Markov model being in some state at any point in time in the future. For a driver in search of a parking place, one probability is of particular interest: *the probability that at least one parking place is free upon arrival*. We denote the timestamp of the occupation data of the parking lot by T_0, the number of occupied parking places at this point in time by n, the total capacity of the parking lot by m and the estimated arrival time by T_a. Then, in terms of our model, we are interested in calculating

$$P(X_{T_a} < m \mid X_{T_0} = n) = 1 - p_{nm}(T_a - T_0).$$

We can expect that the higher this probability is, the better it is to choose the respective parking lot. In combination with information from the navigation system on the distance between the respective parking lot and the destination, user preferences and, optionally, other factors like the parking fee etc., the vehicle can provide recommendations to the driver.

The question that now arises is whether the calculation of the probability is computationally feasible in a vehicle's on-board unit. In order to evaluate the model and to obtain the probability of being in some state at $t = T_a - T_0$ time units in the future, an initial value problem of the form $\dot{\pi}(t) = \pi(t) \cdot Q$, $\pi(0) = \pi_0$ needs to be solved, where $\pi : \mathbb{R}_0^+ \rightarrow [0,1]^{m+1}$ is a function mapping the time t to the vector of predicted probabilities, and π_0 is the vector representing the model's state probabilities at time T_0. Since we know for sure that the model is in state n at time T_0, π_0 is simply the vector with one entry with value one at position n, and zeros at all other positions. The solution of this initial value problem is $\pi(t) = \pi_0 e^{tQ}$. Like evaluating the matrix exponential operator in general, calculating $\pi_0 e^{tQ}$ is non-trivial.

In [9] a survey on different techniques for approximating the matrix exponential is provided. Two approaches seem particularly well-suited for our specific problem structure. Scaling and squaring with rational Padé approximation in combination with Krylov subspace approximations [10] seems very promising for the specific structure of our tri-diagonal Q-matrix. There exist ready-to-use libraries for this. A more common alternative are general ordinary differential equation (ODE) solver algorithms, which are also widely available.

We have made comparative performance measurements with these approaches. It turned out, that the combination of Padé approximation and Krylov techniques provides superior performance. For a parking lot with a capacity of 100 vehicles, the necessary computation time is in the order of 10^{-2} seconds on a 2 GHz x86 CPU. Due to the computational power that can be expected for on-board computers by the time of the rollout of VANETs, the computational effort seems feasible.

IV. Evaluation Methodology

A. Simulation Environment

We have evaluated our prediction model in an extensive simulation study. This study has been carried out using a simulation environment consisting of several runtime-coupled simulators, which allows for a realistic simulation of VANETs. This simulation environment has been presented in [11].

The vehicular movements are generated by the microscopic traffic simulator VISSIM [12]. It includes, e. g., multi-lane traffic, traffic lights, many types of vehicles, and takes driver psychologies into account. For the evaluation we have used a model of the city of Brunswick, Germany, with a size of approximately 16×19 km, 522 road kilometers, and up to 10 000 vehicles. The vehicular traffic pattern is based on measurements taken by the city of Brunswick for the purpose of traffic planning and models the time from 06:00 am to 10:00 am. The model includes the locations of 129 real parking lots. In our simulations, each one generates its occupancy and broadcasts its data into the VANET.

VISSIM is coupled with the ns-2 [13] network simulator in order to simulate the network traffic. MATLAB libraries are employed for the necessary numerical calculations, and a navigation suite estimates realistic arrival times in each vehicle. All components continuously exchange data and adapt their behavior at runtime, according to the events occurring in the integrated simulation environment. This environment allows us to evaluate our algorithm in a realistic setting.

For the purposes of this study, we took the specifications from the ORiNOCO 11b [14] client PC card which complies with the IEEE 802.11b specifications and adopted them to the network simulator ns-2. The data transmission rate used for the simulations is 11 Mbps with a transmission range of 300 m. The two-ray ground propagation model has been used in conjunction with the modelling of radio obstacles in the Brunswick scenario. Obstacle modelling allows to discard transmissions at the physical layer, if an obstacle prevents two cars from communicating. To analyze worst case connectivity and long prediction times, every object except for streets and junctions is considered as an obstacle.

At first, only a small fraction of vehicles will be able to participate in a VANET. Thus, in our simulations, only five percent of the vehicles are equipped with Wireless-LAN. Each vehicle caches and disseminates the parking lot data. The prediction model in a vehicle is re-evaluated whenever it receives new parking lot information.

B. Simulation of Parking Lot Occupancy

Our data on Brunswick's parking lots is taken from a study carried out by the city's transportation officials [15]. However, it is not detailed enough to provide us with exact occupation data of the parking lots. Thus, based on this data we have developed a simulation model for parking lot occupancy. We have intentionally designed this simulation model very different from the modelling assumptions made in our prediction algorithm: we do not want to evaluate the algorithm using a simulation model that is the same as in our prediction.

The occupancy is modelled as an autoregressive Gaussian process. We define a parameter N_{ref} that determines the expected occupation level per vehicle in the scenario. The ratio between the current number of vehicles in the simulation N and N_{ref} yields the expected percentage of occupied parking places. Furthermore, we introduce a parameter σ_{rel} that determines the "noisiness" of the occupation.

For a parking lot with capacity m, the initial occupancy is a Gaussian random sample with mean $m \cdot N_{\text{ref}}/N$ and standard deviation $m \cdot \sigma_{\text{rel}}$. Every 60 seconds a new sample from this distribution is drawn, and a new occupancy $\mathcal{O}'$ is calculated by an exponential sliding average with smoothing parameter γ from the random sample x and the old occupation $\mathcal{O}$:

$$\mathcal{O}' = \gamma \cdot \mathcal{O} + (1 - \gamma) \cdot x$$

Initially as well as after each iteration the new occupancy is rounded to the next integer. In case the new value is below zero or greater than the parking lot's capacity, it is set to the respective border value.

This process exhibits the desired properties. It is of increasing order, has varying coefficients and an embedded stochastic process as its mean. Adjusting the parameters allows to fine-tune the model. In our experience, $N_{\text{ref}} = 10\,000$, $\sigma_{\text{rel}} = 0.1$, and $\gamma = 0.9$ yield sensible occupation values. Therefore we will use these settings to evaluate our approach.

C. Estimation of Arrival and Parking Rates

To complete our simulation model, we need an estimation algorithm for the arrival and parking rates. The occupancy simulation model provides us only with the parking lot's occupation. From this information, the simulated parking automats need to derive the arrival and parking rates. The information available from the municipality of Brunswick says that the expected parking time μ^{-1} is 51 minutes. Therefore, in our simulations, we have fixed μ at this value.

We then use a relatively simple estimation scheme for the arrival rate. Just like a real parking automat could do, the calculation of λ is done by analyzing the past occupancy over some time period, it is fixed to five minutes in our simulations. First, the number of vehicles that have left the parking lot during the simulation interval is estimated. This includes vehicles that have been replaced by newly arriving ones. For the estimation, the process of leaving vehicles is approximated by a Poisson process. Then the number of leaving vehicles can be estimated by taking the expected value. From the number of departing vehicles and the new occupancy the number of arrivals can be determined. We consider the new occupancy prior to limiting it to the total capacity of the parking lot in this calculation. Therefore we also account for vehicles that have been rejected because of the parking lot being full. From the departure and arrival counts a maximum likelihood estimation of λ is performed.

V. Evaluation Results

All equipped vehicles perform predictions based on the mathematical model presented in Section III-B. Each such prediction is a probability vector. It denotes the estimated probability of all possible occupancy states at the time of arrival. Figures 3, and 4 depict probability vectors as estimated by our model, for one of the parking lots with a capacity of 70 vehicles. The prediction time, which lies between 0 and 300

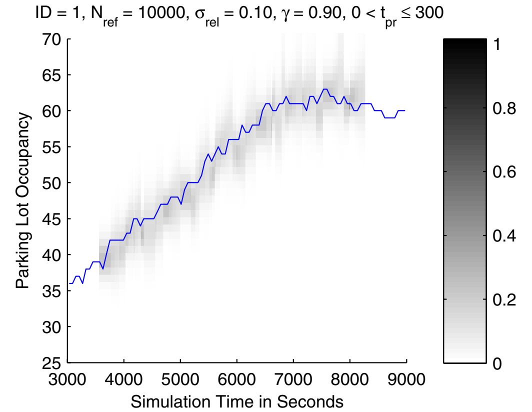

Fig. 3. Probability density of predictions with $0 < t_{\text{pr}} \leq 300$.

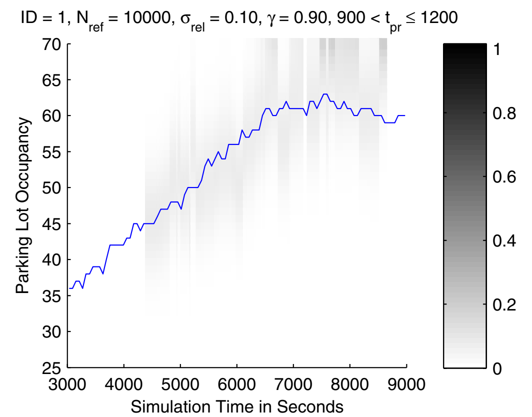

Fig. 4. Probability density of predictions with $900 < t_{\text{pr}} \leq 1200$.

seconds in Figure 3 and between 900 and 1200 seconds in Figure 4, consists of the age of the parking place information in the vehicles cache, plus the arrival time for each vehicle at this particular parking lot and is denoted by t_{pr} .

The x-axis denotes the simulation time and the y-axis the occupancy states of a parking lot. The occupancy curve represents the simulated parking lot real time occupancy. The grey shaded area in the background represents the predictions of the model. The darker the color at a particular point in the diagram, the higher is the predicted probability of the occupancy state at the respective time. This means that for one specific time, the vector of predicted probabilities is plotted vertically using different shades of grey.

Since each vehicle has its own prediction of the parking lot, depending on the information that it has, we had to choose one for each value on the x-axis. At each time value on the x-axis, the drawn probability vector belongs to the vehicle with the lowest prediction time t_{pr} within the prediction time interval considered in the respective figure.

Our prediction algorithm is most effective for prediction times $t_{\text{pr}} \leq 15$ minutes, even though the simulated occupancy can vary quite significantly over this timespan. With increasing prediction time, the uncertainty of the model—visible by a more blurred grey area—increases.

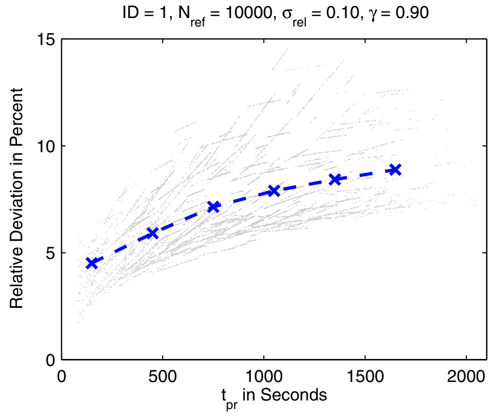

Fig. 5. Relative deviation of predictions.

The model fits very well for the most important prediction time interval: parking lots, at least in Europe, are densely located in city centres (many hundreds in an area of less than 10 square kilometers), and each parking lot can be reached within a few hundred seconds. Therefore, the vast majority of predictions is for prediction times less than fifteen minutes.

In Figure 5, we show another evaluation of our simulation data. It shows the relative error of the model's estimate, depending on the prediction time t_{pr}. Recall that the prediction model's output is not a single occupancy status, but a probability for each possible occupancy. Therefore we calculate the mean deviation from the true value relative to the capacity of the parking lot, weighted with the respective probability. Let π_i denote the probability assigned for occupancy state i, m the capacity of the parking lot, and $\mathcal{O}$ the actual occupancy at the predicted time. The mean relative deviation is then given by

$$\sum_{i=0}^{m} \pi_i \cdot \frac{|i-\mathcal{O}|}{m}.$$

As an example, this means that if the model predicted some state i to occur almost surely (i. e., with probability 1), then this sum would yield the relative deviation of this state from the correct occupancy.

There is one gray dot in the background of the figure for each prediction. The bold, dashed line shows the average errors of each 300 s interval. The results show that the deviation of the predicted occupancy from the real value grows slowly with increasing prediction time. So, although the model's uncertainty increases, the predictions remain quite close to what happens at the parking lot. Note that here the prediction relies on a rather simple rate estimation algorithm in the simulated parking automates. Without modifying the algorithm employed in the vehicle, it could be increased further with more accurate rate estimates, e. g., if the parking lot takes past experience or additional knowledge into account. Thus, our algorithm can assist the driver to select the best suitable parking lot out of many options received through a VANET.

VI. Conclusion

In this paper, we have considered an application for vehicular ad hoc networks. We have proposed an algorithm that uses parking lot data disseminated in a VANET to estimate the future occupancy of parking lots. This enables each vehicle to choose an appropriate parking lot. We have introduced a model based on queueing theory and continuous-time Markov chains, and evaluated it using an integrated simulation environment with realistic vehicular movements. Moreover, we have presented mathematical tools to make the necessary calculations possible on resource-constrained on-board computers.

The results of our evaluation prove that our approach is well suited to estimate the parking place situation, and can serve well to minimize the effort of searching for a free parking lot in a real world VANET application.

From a macro-perspective, an initial orientation of vehicles requires only coarse information about the parking place situation in different areas around the destination. Hence, our future work will focus on aggregating information about individual parking lots within the network, in order to build area information. Since for such aggregates, the need for predicting the situation at the time of arrival, is of same importance as for predicting single parking lot information, we will extend our proposed prediction model and show its applicability for aggregates.

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