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Intelligent parking systems

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“The right to move a car is superior to the right to store cars on the public way” (*American City* magazine, 1920)

Abstract

The basic concepts of the parking reservation system and parking revenue management system are discussed in this paper. The proposed ‘‘intelligent’’ parking space inventory control system that is based on a combination of fuzzy logic and integer programming techniques makes ‘‘on line’’ decisions whether to accept or reject a new driver’s request for parking. In the first step of the proposed model, the best parking strategies are developed for many different patterns of vehicle arrivals. These parking strategies are developed using integer programming approach. In the second step, learning from the best strategies, specific rules are defined. The uniqueness of the proposed approach is that the rules are derived from the set of chosen examples assuming that the future traffic arrival patterns are known. The results were found to be close to the best solution assuming that the future arrival pattern is known.

Keywords: Traffic; Uncertainty modeling; Control; Parking; Fuzzy logic

1. Introduction

Every day a significant percentage of drivers in single-occupancy vehicles search for a parking space. Additionally, less experienced drivers or out-of-towners further contribute to the increase of traffic congestion. Search for a vacant parking space is a typical example of a search process. Every parking search strategy is composed of a set of vague rules. It is usually difficult to describe these rules explicitly. The type of the planned activity, time of a day, day of the week, current congestion on particular routes, knowledge of city streets, and potentially available parking places have significant influence on a chosen parking search

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strategy. During the last four decades numerous parking search models have been developed (Van der Goot, 1982; Axhausen and Polak, 1991; Polak and Axhausen, 1990; Young et al., 1991a,b; Saltzman, 1997; Shoup, 1997; Steiner, 1998; Thompson and Richardson, 1998; Arnott and Rowse, 1999; Tam and Lam, 2000; Wong et al., 2000; Waterson et al., 2001). In many decision-making situations in transportation (modal split, choice of air carrier, choice of airport, etc.) the competitive alternatives and their characteristics are reasonably well known in advance to the decision maker (passenger, driver). On the other hand, the drivers usually discover different parking alternatives one by one in a temporal sequence. Clearly, this temporal sequence has a very strong influence on the driver's final decision about the parking place.

During the past two decades, traffic authorities in many cities (Helsinki, Cologne, Mainz, Stuttgart, Wiesbaden, Aalborg, Hague) have started to inform and guide drivers to parking facilities with real-time variable message signs [directional arrows, names of the parking facilities, status (full, not full, number of available parking spaces, etc.)]. Information about the number of available parking spaces could be displayed on the major roads, streets and intersections, or it could be distributed through the Internet.

It is logical to ask the question about the benefits of the parking guidance systems. Current practice shows that parking guidance systems usually do not change the occupancy rate or average parking duration. Drivers easily become familiar with the parking guidance systems, and majority of them use, thrust and appreciate the help of the systems. Guidance systems significantly increase the probability of finding vacant parking space, mitigate frustration of the drivers–visitors unfamiliar with the city center, decrease the queues in front of parking garages, decrease the total amount of vehicle-miles traveled (particularly in the city centers), decrease the average trip time, energy consumption, and air pollution. Parking guidance system is a part of comprehensive parking policy and traffic management system, whose other elements are street parking control (including sanctions for the illegally parked vehicles), parking fare structure, and parking revenue management system.

Parking guidance systems help drivers to find vacant parking spaces when they are already on the network, and approaching their final destination. Throughout this research the concepts of the *parking reservation system* and *parking revenue management system* are proposed. Such systems would help drivers to find a vacant parking space even *before* beginning their trip. The proposed "intelligent" parking space inventory control system that is based on the combination of *simulation*, *optimization techniques*, and *fuzzy logic* makes "real-time" decisions as to whether to reject or accept a new request for parking. The proposed methodology could be applied for parking lots and parking garages in cities and at the big international airports.

The paper is organized as follows: Parking-pricing problems are presented in Section 2. Analogies between parking problems and some other industries are presented in Section 3. The parking revenue management system is introduced in Section 4, and the Intelligent parking space inventory control system is introduced in Section 5. The algorithm to create intelligent parking spaces inventory control system is presented in Section 6. Results obtained with the "intelligent" parking system are given in Section 7, and Section 8 presents the concluding remarks and further research orientations.

2. Parking pricing

In majority of cities throughout the world drivers pay for using different parking facilities. In some instances, traffic congestion can be significantly reduced as a result of parking price. The parking revenue is usually used to cover parking facility costs (access gates, ticket printers, parking meters, parking signs, attendants), or to improve some other traffic and transportation activities. Different parking pricing strategies should be a part of the comprehensive solution approach to the complex traffic congestion problems. There is no doubt that parking pricing represents one of the important demand management strategies. For example, traffic authorities, local governments and private sector could introduce higher parking tariffs for solo drivers or for long-term parkers in congested city areas. They could provide special parking discounts to vanpoolers. Obviously parking pricing should be carefully studied in the context of the considered city area (down-town, residential, commercial, retail use areas).

In some cities (Madison, Wisconsin) there are already time dependent parking fees that force commuters to switch to different alternatives of public transportation (http://danenet.wicip.org/bcp/Parking.html). Trying to promote public transit San Francisco traffic authorities increased parking tariffs at public and commercial garages. The Chicago authorities raised parking rates few times (www.fta.dot.gov/fta/library/planning/tdmstatus/ftaprkng.htm). As a consequence, the total number of cars parked significantly decreased, as well as parking duration time. The greatest decrease was in the number of all day parkers. Authorities in Seattle significantly reduced parking tariffs for car-pool at two Seattle parking facilities in downtown (ntl.bts.gov/DOCS/mtp17b.html). Active role in parking pricing strategies could also have employers paying for employees' parking. Employers who remove parking subsidies for the employees could significantly decrease the total number of solo drivers. The main role of any parking pricing strategy should be reducing the total number of vehicle trips during certain time periods, shifting commuters to alternative transportation modes, and to different parking locations. At the same time, when trying to implement any parking strategy, it is very important to provide enough parking space for shoppers, to provide preferential parking for residents in considered city area, to provide preferential parking for different parking locations, to consider low income families, and to protect streets in the neighborhood from illegal parking.

The basic economic concepts of supply and demand should be more utilized when solving complex traffic congestion and parking problems (Vickrey, 1969, 1994; Verhoef et al., 1995). So-called value pricing is also known as congestion pricing, or variable tolling. The basic idea behind the concept of congestion pricing is to force drivers to travel and use transportation facilities more during off-peak hours and less during peak hours. The idea of congestion pricing is primarily connected with the road (drivers pay for using private, faster roads, drivers with lower vehicle occupancy pay for using High Occupancy Vehicle lanes, drivers pay more to enter city's downtown on weekdays) or airport operators (more expensive landing fees during peak hours). In the context of parking problems, this means: (a) that different parking tariffs should exist for different users; (b) that the parking fees should increase and/or decrease few times during a day.

3. Parking problems and revenue management systems: Analogies with some other industries

Airline industry, hotels, car rental, rail, cruise, healthcare, broadcast industry, energy industry, golf, equipment rental, restaurant, and other industries are utilizing revenue management concepts when selling their products (Cross, 1997). Revenue management could be described as a group of different scientific techniques of managing the company revenue when trying to deliver the right product to the right client at the right price at the right time. The roots of the revenue management are in the airline industry. The basic characteristics of the industries to which different revenue management concepts were successfully applied are: (a) variable demand over time; (b) variable asset utilization; (c) perishable assets; (d) limited resources; (e) market segmentation; (f) adding new capacity is expensive, difficult or impossible; (g) direct cost per client is negligible part of the total cost of making service available; (h) selling products in advance. The main characteristics of the parking space inventory control problems are the following:

- Parking demand is variable over time.
- Like hotel rooms, or restaurant chairs, parking spaces also have daily opportunity to be "sold" (used by clients).
- Any parking lot or garage has limited number of parking spaces that can be used by drivers.

- Market segmentation means that different customers are willing to pay different prices for the same asset (hotel room, airline seat, seat in a rented car). Businessman wanting to park a car near a meeting point 15 minutes before the meeting would be ready to pay much higher parking fee than a pensioner planning to walk with his wife through the downtown, who made parking reservation four day in advance.
- Building new garages and parking lots could be very expensive and sometimes very difficult.
- Parking places can be easily reserved in advance.

Introducing and developing parking reservation system (created in an Internet and cell phone environment) would present further improvement in modern parking technologies. Drivers would be advised and guided before beginning of the trip, as well as during the trip. Parking reservation system should be coupled with the parking revenue management system. In this way, parking operators and traffic authorities would be able to implement different parking strategies. Once the driver is allowed to park, it is possible to implement internal garage guidance system that guides the driver to an empty parking place.

4. Introducing parking revenue management system

Let us assume that we have parking reservation system. Drivers make their requests for parking at random moments of time (by phone from home, by cell phone while driving, through the Internet, etc.).

A certain number of drivers would maybe cancel their reservations before beginning of the parking. These cancellations would also be made at random moments of time. Like in some other industries, a certain number of drivers would not appear in parking garage for which they have a confirmed reservation and purchased ticket. Would these drivers be penalized for their behavior? Depending on ration between parking demand and parking supply, the answer could be "Yes" or "No".

Reservation system should be flexible enough allowing some drivers to appear right before wished beginning of parking, looking for an empty space in a garage, even though they do not have a confirmed reservation. Would it be good to have few different parking tariffs? The answer is obviously "Yes". Drivers paying lower parking tariffs could be disabled and senior citizens, people who reserve parking space few days in advance, or HOV drivers. Drivers paying higher tariffs could be solo drivers, long term parking drivers, or drivers showing up and asking for parking without making reservation in advance. Obviously, there is a lot of possible parking pricing strategies.

The stochastic nature of reservation generation and cancellation, the stochastic nature of driver show-up during reserved time slot, variety of parking tariffs, and the need to respond to drivers' requests in real time, indicate that the management of parking garage revenues represents a complex problem.

In the past 30 years a relatively large number of papers have been devoted to different aspects of the airline seat inventory control problem (Littlewood, 1972; Belobaba, 1987; Brumelle and McGill, 1993; Teodorović et al., 2002). The model proposed in this paper is highly inspired by the developed airline yield management stochastic and/or deterministic models.

Let us assume that we have few different parking tariffs. The simplest reservation system (similar to some airline reservation systems in the past) could be "distinct tariff class parking space inventories" (Fig. 1(a)), indicating separate parking spaces in the garage for each tariff class. In this case, once the parking space is assigned to a tariff class, it may be booked only in that tariff class or else remains unsold. There are certain advantages, as well as certain disadvantages in the case of distinct parking space inventories. In this case users paying lower tariffs would be relatively well "protected". In other words, this system would pay a lot of attention to the disabled person, senior citizens, people who reserve parking space few days in advance, and HOV drivers. Obvious disadvantage of the distinct parking space inventories is the fact that very often some parking spaces assigned to lower tariff users would be empty even the higher tariff users demand is very high. In other words, it would be possible to reject some drivers even all parking spaces in garage are not occupied.

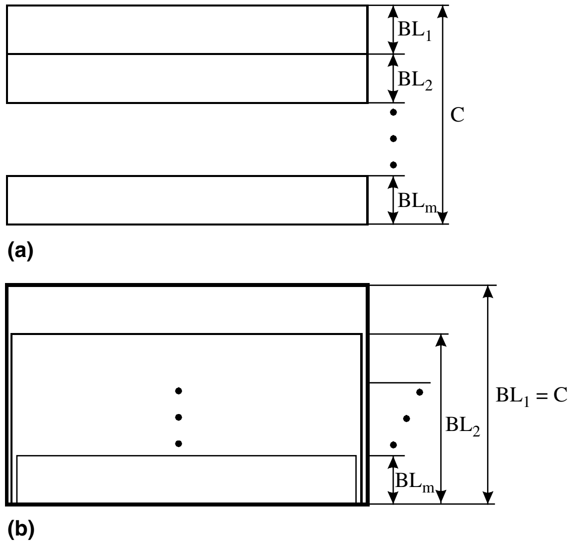

Fig. 1. Distinct (a) and nested (b) tariff class parking spaces inventories.

In case of a "nested reservation system", the high tariff request will not be rejected as long as any parking spaces are available in lower tariff classes. For example, if we have four tariff classes, then there is no booking limit for class 1, but there are booking limits (BL_i, $i = 2, 3, \ldots, m$) for each of the remaining three classes (Fig. 1(b)). As we can see from Fig. 1(b), all parking spaces are always available to class 1. There are always a certain number of parking spaces protected for class 1, certain number of parking spaces protected for classes 1 and 2, and certain number of parking spaces protected for classes 1, 2 and 3. If we make a request-by-request revision of booking limits, there is no longer a difference between distinct and nested reservation system.

In this research (like in the paper of Teodorović et al., 2002) an attempt was made to make reservation decisions on the "request-by-request" basis. In the scenario that we consider, we assume that there are more than two types of tariffs. The basic characteristic of the parking space inventory control model that we propose is "real-time" decision making about each driver request. The developed model is called an "intelligent" parking space inventory control system.

Let us assume that a parking garage offers m tariff classes to the drivers to park their cars. We denote the parking tariffs respectively by $T_1, T_2, \ldots, T_m$, where

$$T_1 > T_2 > \cdots > T_m.$$

By parking tariff T_i $(i = 1, 2, \ldots, m)$ we mean the total number of monetary units that user from the ith class pay for one hour of parking.

In this paper we will introduce the assumption that we have statistical information available regarding drivers' requests on previous days and months. Let us denote by $E_i(t)$ the stochastic process representing cumulative number of driver entered the garage in the ith parking tariff class by time moment t (Fig. 2). Let us also denote by $D_i(t)$ the stochastic process representing cumulative number of driver departures in the ith parking tariff class by time moment t. Time point $t = 0$ represents beginning of the garage working time.

The number of drivers entered is smaller than or equal to the total number of drivers who expressed their desire to park. The number of accepted drivers depends on the ratio between the garage capacity and the

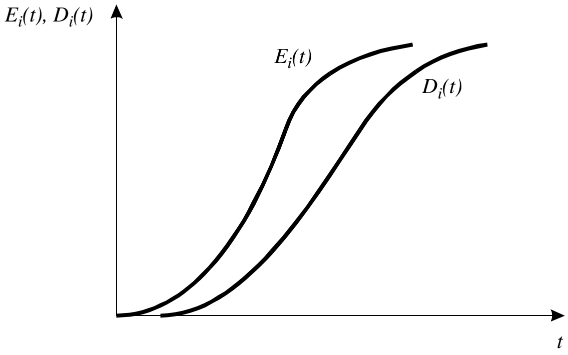

Fig. 2. One realization of the stochastic process $E_i(t)$ representing cumulative number of driver entered the garage and one realization of the stochastic process $D_i(t)$ representing cumulative number of driver departures in the ith parking tariff class by time moment t.

total number of drivers who wish to park. A section of rejected drivers decide to abandon the garage in question. The remaining rejected drivers decide to make subsequent selection of parking possibilities offered by the other parking facilities. In this research we treated the cumulative number of driver entered in the ith driver tariff class by time point t, $E_i(t)$ as the number finally arrived at garage, considering the "original" desires to park in garage in question, certain number of drivers who abandon some other garages, as well as certain number of drivers who choose parking garage in question as their subsequent selection.

The problem considered in this paper can be defined as follows: *For known parking tariffs $T_1, T_2, \ldots, T_m$, based on a large number of realizations of stochastic processes $E_i(t)$ and $D_i(t)$ in different parking tariff classes, develop a parking space inventory control model that will maximize parking operator revenues, while constantly making "on-line" decisions regarding the acceptance or rejection of driver requests.*

5. Intelligent parking space inventory control system

The complexity of the problem, and the uncertainty of different parameters lead us to the conclusion that it is practically impossible to solve the problem analytically. Artificial neural networks and fuzzy systems are "intelligent" systems since they have the ability to "learn from experience" (Teodorović and Vukadinović, 1998). The fuzzy system is not the learning mechanism, per se. On the other hand, an expanding number of fuzzy systems are being generated based on numerical data. In this paper, we have acquired numerical data by previously discovering optimal solutions to various scenarios. In such a way, fuzzy rule base was derived from "the smartest possible decisions". Fuzzy system proposed here embodies a learning mechanism, since it allows for continuous monitoring of parking requests and occasional updating of the fuzzy rule base. The initial assumption in this research is that it is possible to develop an "intelligent" parking space inventory control system that makes *real-time* decisions for each driver request. In other words, the paper assumes that it is possible to develop a system that will recognize a situation characterized by the number of reservations made by individual driver classes and the number of canceled reservations at a certain moment in time. As in other intelligent systems, the "intelligent" parking space inventory control system should be able to generalize, adapt, and learn based on new knowledge and new information.

The developed "intelligent" system is based on fuzzy logic. Theoretical results reached during the past several years (Wang and Mendel, 1992) have indicated that fuzzy logic systems are universal approximators and this explains why fuzzy logic systems are so successful in engineering applications.

5.1. Parking space inventory control model that will maximize parking operator revenue in the case of perfect prediction of future events

We assume that every driver making a request for parking specify entering time point in a garage and leaving time point from the garage. Let us assume, for the moment, that we are able to predict future without a mistake. In context of parking space inventory control problem, this means that we are able to exactly predict random moments of time in which different classes of drivers are making their reservations, the random moments of time in which drivers are making their cancellations, the numbers of drivers from different tariff classes who will not show up to park, the random moments of time in which drivers are leaving the garage, etc. In other words, this indicates that we are able to exactly predict the cumulative number of driver entered the garage $E_i(t)$, as well as the cumulative number $D_i(t)$ of driver departures in the ith parking tariff class by time moment t $(i = 1, 2, \ldots, n)$. *In the case of perfect prediction we must be able to make optimal decisions*. Let us show how we will reach the maximum revenue in the case of a garage for which we know in advance the dynamics of drivers' reservations, cancellations, departures from garage, etc. Let us also introduce the following notation:

m the total number of parking tariff classes,
C garage capacity (the number of parking spaces in the garage),
e_{ij} entering time of the jth car belonging to the ith parking tariff class,
d_{ij} departing time of the jth car belonging to the ith parking tariff class,
T_i ith parking tariff,
R_i the total number of requests for parking in the ith class,
t_b garage opening time,
t_e garage closing time,
(t_b, t_e) time interval when garage is open,
Δt length of the small time interval,
p the total number of small time intervals contained in the time interval (t_b, t_e),
l index of the small time intervals,
$(t_b + (l-1)\Delta t,\ t_b + l\Delta t)$ lth small time interval.

$$x_{ij} = \begin{cases} 1, & \text{if } j\text{th request belonging to the } i\text{th parking tariff class is accepted,} \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

In every moment, garage capacity constraint must be satisfied, i.e.:

$$\sum_{i=1}^{m} [E_i(t) - D_i(t)] \leqslant C, \quad \forall t. \tag{2}$$

In other words, the total number cars parked in a garage cannot exceed garage capacity in any time point.

Let us consider parking operations during time interval (t_b, t_e). The entire period of time can be subdivided into p small intervals. Let Δt be length of the small interval. If l is an index of the small interval, then lth small interval could be given in the following way: $(t_b + (l-1)\Delta t,\ t_b + l\Delta t)$.

Let us introduce the following binary variables:

$$q_{ijl} = \begin{cases} 1, & \text{if } (e_{ij}, d_{ij}) \cap (t_b + (l-1)\Delta t, t_b + l\Delta t) \neq \emptyset, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

When $q_{ijl} = 1$ it means that the jth driver that belongs to the ith parking tariff class would like to be in the garage during lth small time interval (Fig. 3).

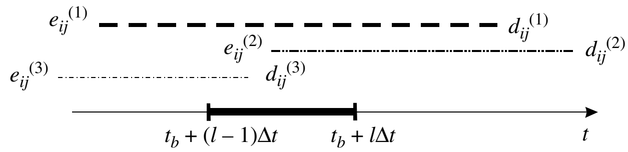

Fig. 3. Examples of parking requests for which $q_{ijl} = 1$.

The garage capacity constraint could be also written in the following way:

$$\sum_{i=1}^{m}\sum_{j=1}^{R_i} q_{ijl}x_{ij} \leqslant C, \quad l = 1,2,\ldots,p. \tag{4}$$

The revenue r_{ij} that the garage operator could earn in the case of acceptance the jth driver's request that belongs to the ith tariff class equals:

$$r_{ij} = T_i(d_{ij} - e_{ij}), \quad i = 1,2,\ldots,m; \;\; j = 1,2,\ldots,R_j. \tag{5}$$

The total garage's revenue R equals:

$$R = \sum_{i=1}^{m}\sum_{j=1}^{R_j} T_i(d_{ij} - e_{ij})x_{ij}. \tag{6}$$

As we already mentioned, we assumed that we know the values of the cumulative representing cumulative number of drivers entered the garage and the cumulative number of driver departures in particular parking tariff classes, as well planned parking duration of all parking activities. We must decide about acceptance of rejection of every particular parking request in order to reach the maximum revenue. Our problem is:

(P):

$$\text{Maximize:} \quad R = \sum_{i=1}^{m}\sum_{j=1}^{R_j} T_i(d_{ij} - e_{ij})x_{ij} \tag{7}$$

$$\text{subject to:} \quad \sum_{i=1}^{m}\sum_{j=1}^{R_i} q_{ijl}x_{ij} \leqslant C, \quad l = 1,2,\ldots,p, \tag{8}$$

$$x_{ij} = \{0,1\}. \tag{9}$$

The problem (P) could be solved using any of the commercially available computer packages (in this paper, we use optimizer ILOG CPLEX). The problem (P) was solved many times for different scenarios. If we can precisely predict the future, we can easily get the optimal solution. Instead of predicting it though, we can simulate future events. In other words, we will simulate realizations of the stochastic processes representing cumulative number of drivers' requests. In the next step, after solving problem (P), we can get the optimal solution. We can repeat the simulation, and again, after solving the problem (P) we can get the optimal solution. After third simulation, we will get the third optimal solution, etc. In this way we can get the optimal solution for every simulated "scenario". This "statistical material" enables the generation of a fuzzy rule base. This procedure will be explained in later sections.

5.2. Real-time decisions about rejecting or accepting driver request

Consider just one of the tariff class. Let 1 denote the situation when the specific request that belong to that tariff class is accepted, 0 the situation when the specific request is not accepted. Each request in considered tariff class may be designated either 0 or 1, and the sequence of the numbers such as the following indicates accepted requests from the considered tariff class:

1 0 1 0 1 1 1 0 0 0 0 1 1 1 1 1 0 0 . . . . 1 1 0 0 0 1 0 1 0 1 1 0 0 0 1 1 1

If we have, for example three tariff classes the optimal solution could be shown in the following way:

Tariff class	Sequence of numbers indicating accepted requests
Tariff class 1	1 0 1 0 1 1 1 0 0 0 0 1 1 1 1 1 0 0 . . . 1 1 0 0 0 1 0
Tariff class 2	1 0 1 1 1 1 1 0 0 1 0 1 1 1 1 1 0 1 . . . 0 1 0 1 0 1 1
. . .	. . .
Tariff class m	1 0 1 0 1 1 1 0 0 0 0 1 1 1 1 1 0 0 . . . 0 1 1 1 1 1 1

After analyzing optimal solution, as well as the cumulative numbers of requests in the particular parking tariff classes, it is possible *at any time* to determine the *available parking capacity*, and to "read" all future requests that will be accepted. It is also possible to calculate in *any time* the new requests appears, the number of future accepted requests that make less revenue than the request in question. In other words, at any time moment t when new request appears, we know the availability of the parking capacity (expressed in %), and the percentage of all future accepted requests (after time t) that make less revenue than the observed request.

The availability of the parking capacity X_1 (expressed in %) equals:

$$X_1 = \left[1 - \frac{\text{Maximum number of vehicles on a lot during the parking time of the current request}}{C}\right](100) \quad [\%]. \tag{10}$$

The relative request's revenue X_2 expressed in % is defined as:

$$X_2 = \frac{\text{Revenue}}{\text{Maximum possible revenue}}(100) \quad [\%]. \tag{11}$$

The percentage of all future accepted requests Y that make less relative revenue than the observed request equals:

$$Y = \frac{\text{The total number of requests accepted after current request with tariff class same or lower than tariff class of current request}}{\text{The total number of requests accepted after current request}} \times (100) \quad [\%]. \tag{12}$$

It is clear that Y depends on the X_1 and X_2. The X_1 and X_2 are antecedents, while Y the consequence. We can establish fuzzy sets for all the antecedents and the consequence. Typical fuzzy rule in the fuzzy rule base could be, for the example, the following one:

If X_1 is A_2 and X_2 is B_4,
Then Y is E_3,

where: A_i, B_i, C_i, D_i and E_i $(i = 1, 2, 3, \ldots)$ are the established fuzzy sets for the antecedents and consequence.

Fuzzy rule base is generated from numerical examples. Detailed description of this procedure will be given in the following section. It is important to underline, that for each tariff class corresponding fuzzy rule base has to be developed separately. The total number of fuzzy rule bases is equal to the total number of parking tariff classes m. Since our fuzzy rule base generates a single output, we have created a separate fuzzy rule base for each tariff class. In this way, the total number of fuzzy rule bases is equal to the total number of parking tariff classes, m. Influx of a parking request triggers the system to "recognize" the type of the request and direct it to an appropriate fuzzy rule base, in which a decision to accept or decline a request is made.

It is possible for every parking request to determine the tariff class it belongs to, and to calculate values of the variables X_1 and X_2. The outcome of the fuzzy rule base Y, is the percentage of all future accepted requests that make the lower revenue than the observed request. We accept considered parking request when produced outcome Y is greater than or equal to $a_{i\ \mathrm{i}} = 1, 2, \ldots, m)$, where a_i are constants subjectively determined by the decision-maker (analyst) for every tariff class. In the case when Y is less than a_i the considered parking request is rejected. The available numbers of parking spaces must be updated every time when some driver is accepted for parking.

5.3. Generating fuzzy rule base from numerical examples

In this research the fuzzy rule base is generated from numerical examples. For doing this we used the procedure proposed by Wang and Mendel (1992). As we already mentioned, availability of parking capacity and relative request's revenue are antecedents, while the percentage of future accepted parking requests that make less revenue than the observed request is the consequence. We also mentioned that we could establish fuzzy sets for all the antecedents and the consequence. We will do it in such a way that, at the very beginning, we will establish the domain intervals for all input and output variables (Fig. 4).

As it was suggested by Wang and Mendel (1992), we can divide each domain interval into a prespecified number of overlapping regions (Fig. 4). We can mention that the number of overlapping regions is not equal for each variable. The lengths of these overlapping regions are usually equal, but not necessarily so. In the next step, each overlapping region is labeled and one membership function is assigned to it. We can also use different types of membership functions for different variables.

Let us assume that we have the following set of the input–output data pairs:

$$(x_1^1, x_2^1, \ldots, x_n^1; y^1), (x_1^2, x_2^2, \ldots, x_n^2; y^2), \ldots,$$

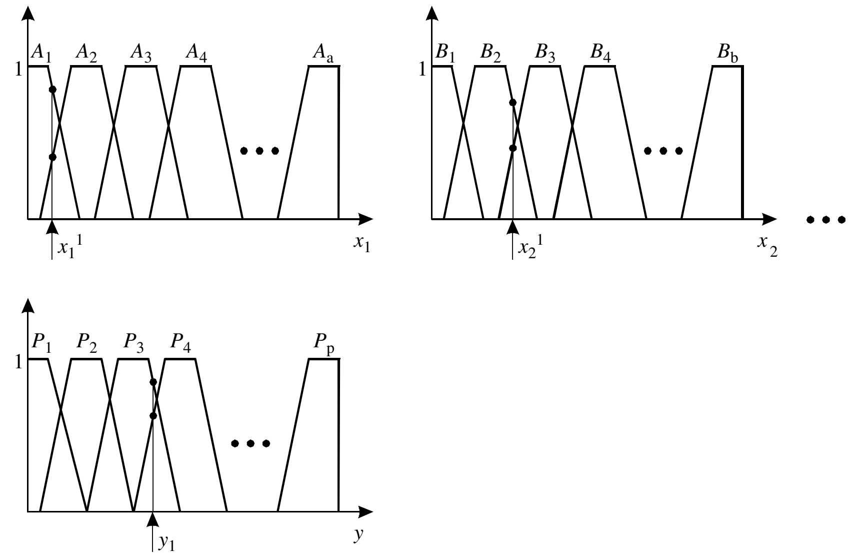

Fig. 4. Division of the domain interval into a prespecified number of overlapping regions.

where $x_1, x_2, \ldots, x_n$ are inputs and y is output. We will generate the fuzzy rule base from this set of input–output data pairs. The values $x_1, x_2, \ldots, x_n$ and y belong to the domain intervals $(x_{1\,\min}, x_{1\,\max})$, $(x_{2\,\min}, x_{2\,\max})$, …, $(x_{n\,\min}, x_{n\,\max})$, $(y_{\min}, y_{\max})$ respectively. From every input–output data pair we can eventually generate one fuzzy rule. Let us explain how we can generate fuzzy rule from the first input–output data pair $(x_1^1, x_2^1, \ldots, x_n^1; y^1)$.

We have to determine the membership function values of the elements. From Fig. 4 we see that x_1^1 has degree 0.35 in $\mathbf{A}_2$, and 0.8 in $\mathbf{A}_1$, x_2^1 has degree 0.4 in $\mathbf{B}_3$, and 0.7 in $\mathbf{B}_2$, … and y^1 has degree 0.6 in $\mathbf{P}_4$, and 0.8 in $\mathbf{P}_3$. In the next step, we will assign each variable to the region with maximum degree. This means that x_1^1 is considered to be $\mathbf{A}_1$, x_2^1 is considered to be $\mathbf{B}_2$, … and y^1 is considered to be $\mathbf{P}_3$. The rule, which we are obtaining from the first pair of the input–output data, is:

Rule 1:

$$\begin{array}{ll} \text{If} & x_1 \text{ is } \mathbf{A}_1 \text{ and } x_2 \text{ is } \mathbf{B}_2 \cdots \text{ and } x_n, \\ \text{Then} & y_1 \text{ is } \mathbf{P}_3. \end{array}$$

This is the way in which we generate the rules from the input–output data pairs. Because we have many input–output data pairs it can happen that we produce some conflicting rules. The conflicting rules are the rules with the same antecedents but different consequents. Wang and Mendel (1992) resolved this "by assigning a degree to each rule and accept only the rule from a conflict group that has maximum degree". Degree of a rule 1, which we got from the first input–output data pair equals:

$$D(\text{Rule 1}) = \mu_{\mathbf{A}_1}(x_1^1) \cdot \mu_{\mathbf{B}_2}(x_2^1) \cdots \mu_{\mathbf{P}_3}(y^1) = 0.8 \cdot 0.7 \cdots 0.8.$$

6. The algorithm to create intelligent parking spaces inventory control system

The fuzzy rules of the "intelligent" reservation system are generated using corresponding numerical data based on the procedures proposed by Wang and Mendel (1992).

The algorithm to create the fuzzy system developed in this paper consists of the following steps:

Step 1: Based on a large number of parking activities in garage in question, create the cumulatives $(E_i(t), D_i(t))$.

Step 2: Formulate a corresponding integer programming problem and find the optimal solution for each generated "scenario".

Step 3: Based on the statistical data resulting from Steps 1 and 2, use the Wang–Mendel's algorithm to generate the fuzzy rule bases. Each fuzzy rule base corresponds to one tariff class.

7. Results obtained using the intelligent parking space inventory control system

The developed model was tested on ten (10) different numerical examples. The examples considered differed in parking size, requests' arrival distribution and service (parking time) distribution. All the examples have in common the following values: $\Delta t = 300$ seconds; $t_e - t_b = 8$ hours; $m = 3$; $T_0 = 3$ \$/hour; $T_1 = 2.5$ \$/hour; $T_2 = 1.5$ \$/hour. Exponential probability density function was used to generate vehicle arrivals and service (parking) time. We first have produced ten examples to generate fuzzy rule bases (training data set) and than we tested developed model on ten new generated examples (control data set).

One of the examples used to generate fuzzy rule bases is shown in Table 1. Last column (x_{ij}) represents integer programming solution of the problem (P). In the next step, we prepare a set of input–output data

Table 1
Parking request characteristics (one randomly generated example)

Request number	Parking tariff class i	e_{ij} [seconds]	d_{ij} [seconds]	$T_i(d_{ij} - e_{ij})$ [\$]	x_{ij}
1	2	14,912.9238	28,800.0000	5.7863	1
2	1	16,374.5718	18,137.2922	1.2241	1
3	2	16,233.1950	24,208.7761	3.3232	1
4	0	1653.2101	14,239.5799	10.4886	1
5	0	17,330.0834	28,800.0000	9.5583	1
6	1	3885.9979	3955.5341	0.0483	1
7	2	22,073.5421	28,800.0000	2.8027	0
8	0	223.9673	13,188.8049	10.8040	1
9	2	25,854.4324	28,800.0000	1.2273	0
10	2	25,102.4678	28,800.0000	1.5406	0
⋮	⋮	⋮	⋮	⋮	⋮
705	1	28,796.1022	28,800.0000	0.0027	0
706	2	9302.0143	28,800.0000	8.1242	1

pairs (statistical data) in order to apply the Wang–Mendel's algorithm for fuzzy rule bases creation. An example of input–output data pairs is given in Table 2.

After building all fuzzy rule bases (in this example $m = 3$) we generate new set of ten parking request patterns and proceed with test phase. The comparison was made between the results obtained using the ‘‘intelligent'' system and those obtained using integer programming. The comparison was also made between the results obtained using the ‘‘intelligent'' system and those obtained using FIFO rule (in our case FIFO rule means that the parking request is accepted immediately if there is vacant parking space). The results obtained are shown in Table 3 and in Fig. 5. In all considered cases FL solution was significantly better than the FIFO solution. The integer programming results are the *maximum* revenue values attainable when the *future is known* (*ideally predicted*). Bearing this fact in mind, as well as the fact that the ‘‘intelligent'' system operates in an on-line regime in conditions of *uncertainty*, it can be concluded that good results would be achieved using the intelligent parking space inventory control system.

Table 2
Input–output data pairs example [related just to one tariff class ($i = 1$)]

Data-pair number	The availability of the parking capacity (X_1) [%]	Relative request's revenue (X_2) [%]	The percentage of all future accepted requests that make less revenue than the observed request (Y) [%]
1	95	13.06	43.90
2	92	17.85	43.60
3	99	1.36	43.56
4	93	13.94	43.41
5	92	1.43	44.10
6	90	4.79	43.94
7	88	13.38	43.68
8	90	6.15	43.64
9	89	10.18	43.15
10	89	1.87	43.90
⋮	⋮	⋮	⋮
1408	1.33	24.35	60.00
1409	2.00	1.62	55.56
1410	1.00	2.89	50.00

Table 3
Comparison of the FL solution with the IP solution and the FIFO solution

Problem	Parking capacity	IP solution [$]	FL solution [$]	FIFO solution [$]	Relative error [%] (IP − FL)/IP
1	100	1787.49	1675.17	1514.63	6.28
2	150	2350.26	2217.68	2059.50	5.64
3	250	3647.42	3367.59	3250.38	7.67
4	350	4385.44	4256.81	4148.09	2.93
5	300	3831.10	3724.33	3556.88	2.79
6	120	2340.11	2125.63	1890.95	9.16
7	200	3125.01	3063.64	3044.88	1.96
8	180	3101.62	2962.03	2709.01	4.50
9	350	4860.19	4461.43	4185.12	8.20
10	100	1115.74	1029.09	974.72	7.77

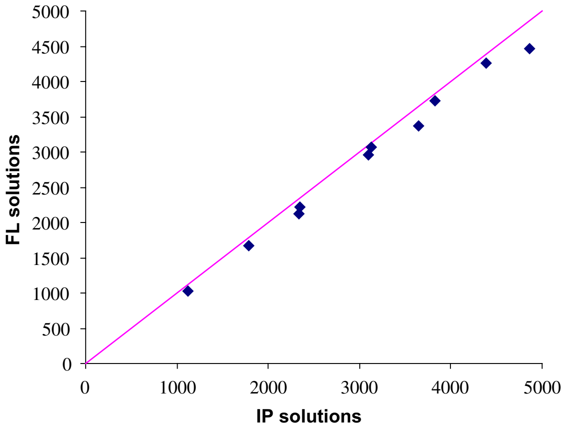

Fig. 5. Comparison of the FL solution with the IP solution.

In all test examples parking capacity ranges from 100 to 350 spots. Additionally, in all examples different PDF parameters were used to generate parking requests.

8. Conclusion

The proposed model belongs to the category of "real-time" reservation systems. The output of the model is a decision (made in real time) to accept/reject requested parking activity. To clarify, the proposed model tells each incoming request, either: "Yes, you can come and park", or: "No, you cannot park at this time". The decision to accept or reject a requested parking activity is contingent upon the current state of the parking system (including cancellations, no shows, space availability, etc.). The proposed fuzzy system allows for continuous monitoring of parking requests and occasional updating of the fuzzy rule base. The numbers of accepted and/or rejected drivers from certain driver types could be used as measures of customer satisfaction.

As we have indicated previously, the main role of any parking pricing strategy should be to reduce the total number of vehicle trips during certain time periods. This is indeed one of the essential public sector objectives. However, in our model, we have focused on the maximization of revenue for the parking management, which is, above all, very much a private sector objective. These two objectives might seemingly appear to be inconsistent with each other and one could question how the latter relates to the former objective of reducing total vehicle trips during certain time periods.

Market segmentation entails a situation in which different drivers agree to pay different prices for the same asset. We have previously illustrated an example of two types of drivers, a businessman and a pensioner—while a businessman who wants to park a car near a meeting place 15 minutes prior to the meeting is prepared to pay a much higher parking fee, a pensioner planning to walk with his wife through the downtown would make parking reservation a day in advance in order to pay a lower parking fee. However, even the wealthiest businessman would not agree to pay a million dollars for short-term parking. Thus, it becomes obvious that the number of parking demands, which depict different groups of drivers *over time*, depend on the manner in which market segmentation has previously been carried out (such as defining two, three, ... or ten different types of drivers and/or types of parking), as well as parking fees that apply for certain types of parking. In extreme cases, depending on the mode of market segmentation, as well as the parking fees that are being offered, theoretically only the wealthiest would be able to drive. In such instances, even though the total number of vehicle trips would decrease in certain time periods, the social injustices would certainly increase to an extent. Hence, maximizing revenue can be achieved with constraints associated with protected number of parking spaces reserved for particular drivers, as well as parking types. By simple introduction of specified restrictions, it is feasible to protect a defined number of parking spaces for elderly and handicapped, as well as some other classes of driver/parking activities.

The intention of this manuscript is not to tackle the analysis of private ownerships and laws associated with parking regulations, since they differ among countries. It is noteworthy though, that in instances when traffic authorities (which supervise public sector objectives) are involved in parking pricing strategy and market segmentation, revenue maximization can represent a surrogate objective function, in an effort to reduce the total number of vehicle trips in certain time periods, and thus spread traffic flows more evenly over time. Maximization of revenue for the parking management represents an attempt to employ basic market principles and to achieve and maintain equilibrium between transportation supply and demand, in the best possible fashion. As a consequence for example, the potential driver who has been denied the right to utilize one of the lower parking fees several times and is still not prepared to pay the higher parking fee will: (a) forgo certain activities in a particular city area, (b) change the time period in a day and/or day of the week in which he/she completes certain activities, (c) abstain from driving a car and chose to exploit public transportation. We believe that the objective function of this paper (maximization of revenue) can greatly influence the change in traffic patterns, as well as the spread of traffic flows more evenly over time.

Future research will be needed to determine the finest form of the objective function(s) which would best depict the need of analysts to spread traffic flows more evenly over time. Possible objective function would, for example, be to maximize the number of accepted requests from one or more request categories, while still providing a defined number of parking spaces for other categories of parking requests.

The intelligent parking system proposed in this paper, could represent further improvement in modern parking technologies. The proposed methodology could be applied for a parking lot, parking garages in cities, as well as for parking lots at big international airports. Cell phones, Personal Computers, and Internet have made a revolution in numerous daily activities. These technologies could be also of great help in our attempts to solve complex parking problems.

The method presented in this paper is characterized by adaptability. The proposed intelligent parking space inventory control system is based on the concept of learning from examples (fuzzy rules are generated by learning from examples). In other words, as the number of various input–output data pairs increases, the fuzzy rule base becomes "fuller", and in some cases, even the certain fuzzy rules of fuzzy rule base are altered. The results obtained are the best confirmation of the accuracy of such an approach.

There are a lot of questions that need to be answered in future research. The problem studied should be expanded and considered within area-wide parking policies. The most important components of the parking revenue management system (parking supply, variable demand, fare structure, variable pricing, market segmentation, occupancy rate, average parking duration, spillover parking problems, and parking violation enforcement) should be studied in more details. It would also be necessary to study future impacts of the parking revenue management systems on modal split (solo driving, public transit, and car-pooling).

Although the proposed system launches a spectrum of new questions, some of which extend beyond our current knowledge, we emphasize several practical benefits resulting from our study. In many countries, local governments and/ or individual businesses are responsible for forming the parking price. Further, off-street parking pricing is often controlled by individual companies. Many drivers consider market segmentation and, even more so parking fees at particular locations at particular time periods, as additional taxes. On the other hand, a successful combination of road pricing and parking pricing techniques will immensely enable to mitigate congestion, distribute traffic flows more evenly through time and space and decrease average travel time and costs. Road pricing/parking pricing is in many countries indeed an important political issue, thus successful implementation of our and similar models, will vastly depend on the extent of the agreement and cooperation between private companies, traffic authorities, local governments and citizen organizations. Various practical questions related to equipment type, equipment costs, operating costs, and user convenience need further in-depth analysis and span beyond the scope of this paper.

It is unknown if drivers would accept the parking reservation and revenue management systems. Further research would be done to check if intelligent parking systems will be able to further decrease the queues in front of parking garages, the total amount of vehicle-miles traveled, the average trip time, energy consumption, and air pollution.

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