

# Solution of the Inverse Jet in a Crossflow Problem by a Predictor-Corrector Technique

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## Abstract

A predictor-corrector method, developed early, was modified to suit the inverse jet flow in a crosswind problem. The methodology was tested against both numerical and experimental data. The jet was generated by heating compressed air with a velocity range of  $0 - 5\text{ m/s}$  and temperatures up to  $425\text{ K}$ . The method attempts to predict the jet velocity, temperature, inlet axial location, and elevation with a self imposed limitation on the number of sample points within the domain. The case where all four of the parameters are unknown led to inaccurate and unacceptable results with 9 sample points. The thermal self-similarity of the problem results in an infinite number of solutions to the problem, with no possibility of narrowing the solution count without more information. Knowing the elevation of the jet results in a maximum error of 9%, but typically much better. Experimental tests indicate the methodology is sensitive to error in the sampling data with a few cases reaching an error over 20%. This technique may be extended to applied areas such as exhaust stacks and fuel injection systems.

*Keywords:* Inverse Problems, Computational Heat Transfer, Convection

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## 1. Introduction

Thermal-fluid systems often create situations in which the engineering problem is an inverse heat transfer problem. These problems often have

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## Nomenclature

|   |  |                     |   |
|---|--|---------------------|---|
| $\mathbf{r}$  | vector location of sampled points  | $\delta$            | vector distance between the actual sampled location and the current test location         |
| $a$   | number of sample locations used in the predictor stage                       | $\lambda$           | thermal conductivity  |
| $b, m, \Gamma_0, \Gamma_1, \Gamma_2$                        | model parameters   | $\mu$               | dynamic viscosity   |
| $C_1, C_2, C_{1\epsilon}, C_\mu, \sigma_k, \sigma_\epsilon$ | $k - \epsilon$ model coefficients  | $\mu_t$             | eddy viscosity  |
| $d$   | number of simulations  | $\phi$              | normalized temperature $\phi = \frac{T-T_\infty}{T_S-T_\infty}$                           |
| $E$   | thermal energy   | $\rho$              | density   |
| $F$   | minimization function for temperature  | $\varepsilon$       | error associated with the inverse convection method at a location with given sampled data |
| $G$   | minimization function for velocity   |                     |   |
| $k, \epsilon$   | turbulence kinetic energy, dissipation rate                                  |                     |   |
| $l, I$  | turbulence length scale and intensity  |                     |   |
| $n$   | number of sample locations   |                     |   |
| $P_{rt}$  | turbulent Prandtl number   |                     |   |
| $T$   | temperature  |                     |   |
| $U$   | free stream velocity   |                     |   |
| $X, Y$  | normalized coordinates   |                     |   |
| $x, y$  | coordinates  |                     |   |
| <b>Greek Symbols</b>  |  |                     |   |
| $\Delta$  | relative difference between the first sampled point and other sampled points |                     |   |
|   |  | <b>Superscripts</b> |   |
|   |  | *                   | predictor stage, alternative heat flux eqn.   |
|   |  | <b>Subscripts</b>   |   |
|   |  | 0, 1, 2             | sample point indexes  |
|   |  | $\infty$            | free stream   |
|   |  | $A, B$              | data set A,B  |
|   |  | $i, j, k$           | index   |
|   |  | $mod$               | modified  |
|   |  | $P$                 | predicted   |
|   |  | $S$                 | source  |

limited physical access, very limited to no boundary condition knowledge, and/or limited domain information.

For example, the temperature distribution at the wall of an optical fiber drawing furnace is difficult to measure directly due to shape, inaccessibility, and high temperatures. The center of the furnace is easily accessible, where the temperature distribution on an inserted rod may be measured. This leads to the inverse heat transfer problem to obtain the wall temperature distribution that gives rise to the measured rod temperature distribution. Issa et al. [1] developed a regularization technique, utilizing this approach, to determine the wall temperature distribution.

The inverse convection problems have been gaining popularity as of late. Prud'homme and Nguyen [2] solved transient inverse convection problems with a single sensor utilizing the conjugate gradient method, but the sensor needed to be moved closer to the boundary layer as the Rayleigh number increased. A partially adiabatic enclosure with heat loss was solved. Liu et al. [3] determined the thermal profiles in a slot vented enclosure, also utilizing the conjugate gradient approach, requiring tens of iterations to achieve less than 1% error. Hong et al. [4] solved the inverse problem of a differentially heated enclosure with constant wall temperatures. They demonstrated that using the conjugate gradient method required at least nine sample points to resolve the heat flux into the enclosure. The conjugate gradient method is a popular method for solving inverse convection problems and is used a number of other works (e.g. [2–6]).

A different approach is that of the artificial neural network to solve the inverse heat transfer problem. Both Ghosh et al. [7], Kumar and Balaji [8] successfully applied the technique. Ghosh et al. [7] solved for the heat conduction in a plate. While Kumar and Balaji [8] solved a similar differentially heated enclosure as [4]. Although requiring more sample points than [4] the technique once trained is non-iterative and thus, typically quicker.

Another example is the inverse plume in a crossflow problem. The problem entails solving for the plume boundary conditions, utilizing limited domain knowledge. A novel predictor-corrector method was developed by VanderVeer and Jaluria [9] to solve such a problem. The method requires a specific pattern of known points to match exactly against a set of simulations to predict the inverse solution. The specific pattern was optimized to require the least number of data points for plume in a crossflow problem [10]. With zero error in the data, a minimum of three known points was possible. However, small amounts of error, as is usually the case, would require the

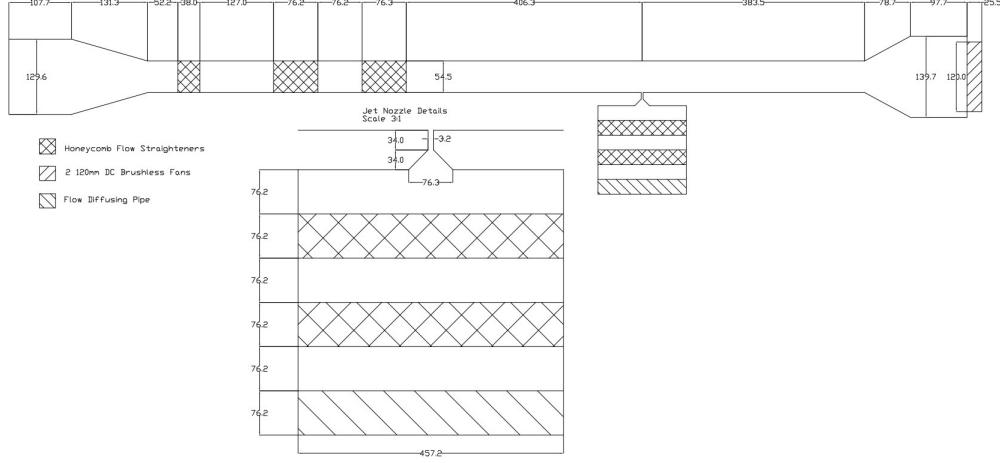


Figure 1: Schematic of a jet in a wind tunnel

known point count to increase to at least five.

The present work is the logical progression of the inverse plume in a crossflow problem, that is the inverse jet in a crossflow problem. The inverse jet in a crossflow problem has many more practical applications, such as exhaust stacks and fuel injection systems. The previous technique will be modified to meet the needs of the new problem.

## 2. Experimental System

The experiment consists of a wind tunnel with a surface level jet located within the test section. The jet uses compressed air flowing through straighteners to achieve a velocity  $U_S$  and is heated to temperature  $T_S$ . The jet is subjected to a perpendicular crossflow velocity  $U_\infty$ . Figure 1 is a diagram of the wind tunnel and the jet, with dimensions in millimeters.

The wind tunnel test section dimensions are  $54.5 \times 305 \times 254 \text{ mm}$ . The maximum velocity of the wind tunnel is  $5.0 \text{ m/s}$ . The jet is heated by electric cartridge heaters (Omega AHP-7561) with a maximum temperature of  $425 \text{ K}$ , due to material limitations of the wind tunnel. The X-direction is directed downstream of the wind tunnel with the zero at the center of the jet. The Y-direction is in the direction of the jet and is zero at the surface of the wind tunnel. Due to the large aspect ratio of the wind tunnel  $\approx (5 : 1)$ , the flow

is assumed to be two-dimensional.

The free stream velocity is determined by a Pitot-Static tube attached to a NIST traceable differential pressure sensor from Omega(PX655-0.1DI). The pressure sensor has a full scale reading of 2.54 *mm* of water and is accurate to 0.05% of full scale. This results in a maximum error of 3% in the calculated velocity. The jet velocity is determined utilizing a rotameter and verified using a Pitot-Static tube attached to the same previously described pressure sensor. This results in the same amount of error in the jet velocity.

The temperature of domain is measured using a K-type thermocouple mounted to an X-Y traversing stage. Sampled data over the course of several days indicate repeatability of the experiment to within 2%.

### 3. Numerical Simulations

The simulations were all performed using Ansys Fluent[11]. The Navier-Stokes equations were solved using a three-dimensional, steady state, realizable  $k - \epsilon$  model with enhanced wall effects. Conjugate heat transfer effects are modeled. The free stream Reynolds number  $Re_{infty}$  is of order  $6 \times 10^3$ , while the jet Reynolds number  $Re_S$  is between  $10^3$  and  $10^4$ . The Rayleigh number  $Ra$  is of order  $10^7$ .

The governing equations are expressed below:

$$u_i = \overline{u_i} + u_i' \quad (1)$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\rho u_i) = 0 \quad (2)$$

$$\begin{aligned} \frac{\partial}{\partial t} (\rho u_i) + \frac{\partial}{\partial x_j} (\rho u_i u_j) = \\ \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \left( 2S_{ij} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right) - \rho \overline{u_i' u_j'} \right] \end{aligned} \quad (3)$$

$$\begin{aligned} \frac{\partial}{\partial t} (\rho E) + \frac{\partial}{\partial x_i} [u_i (\rho E + P)] = \\ \frac{\partial}{\partial x_i} \left[ \left( \lambda + \frac{C_p \mu_t}{Pr_t} \right) \frac{\partial T}{\partial x_i} \right] \end{aligned} \quad (4)$$

$$\begin{aligned}
\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho k u_j) = \\
\frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + \frac{\partial u_j}{\partial x_i} \left( -\rho \overline{u'_i u'_j} \right) \\
- g_i \frac{\mu_t}{\rho P_{rt}} \frac{\partial \rho}{\partial x_i} + \rho \epsilon
\end{aligned} \tag{5}$$

$$\begin{aligned}
\frac{\partial}{\partial t}(\rho \epsilon) + \frac{\partial}{\partial x_j}(\rho \epsilon u_j) = \\
\frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{\partial x_j} \right] + \rho C_1 S \epsilon - \rho C_2 \frac{\epsilon^2}{k + \sqrt{\nu \epsilon}} \\
- C_{1\epsilon} \frac{\epsilon}{k} C_{3\epsilon} g_i \frac{\mu_t}{\rho P_{rt}} \frac{\partial \rho}{\partial x_i}
\end{aligned} \tag{6}$$

$$- \rho \overline{u'_i u'_j} = 2\mu_t S_{ij} - \frac{2}{3} \delta_{ij} \left( \rho k + \mu_t \frac{\partial u_k}{\partial x_k} \right) \tag{7}$$

The constants for the turbulence model are [12, 13] :

$$C_{1\epsilon} = 1.44, C_2 = 1.9, \sigma_k = 1.0, \sigma_\epsilon = 1.2, P_{rt} = 0.85 \tag{8}$$

$$C_1 = \max \left[ 0.43, \frac{Sk/\epsilon}{Sk/\epsilon + 5} \right], S = \sqrt{2S_{ij}S_{ji}}, C_{3\epsilon} = \tanh \left( \frac{u_g}{u_p} \right) \tag{9}$$

$$\mu_t = \frac{\rho C_\mu k^2}{\epsilon} \tag{10a}$$

$$C_\mu = \frac{1}{A_0 + \frac{A_1 k U^*}{\epsilon}} \tag{10b}$$

$$U^* \equiv \sqrt{S_{ij}S_{ji} + \Omega_{ij}\Omega_{ji}} \tag{10c}$$

$$A_0 = 4.04 \tag{10d}$$

$$A_1 = \sqrt{6} \cos \left[ \frac{1}{3} \cos^{-1} \left( \sqrt{6} \frac{S_{ij}S_{jk}S_{ki}}{(S_{ij}S_{ji})^{\frac{3}{2}}} \right) \right] \tag{10e}$$

$$S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \tag{10f}$$

$$\Omega_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right) \tag{10g}$$

where the  $u_g$  and  $u_p$  are the velocity component parallel and perpendicular to gravity respectively. All of the symbols are defined in the nomenclature.

The inflow boundary conditions are:

$$u = U_\infty, v = 0, T = T_\infty, P = P_\infty, l = 4mm, I = 5\% \quad (11a)$$

$$k = \frac{3}{2} (U_\infty I)^2 \quad (11b)$$

$$\epsilon = C_\mu^{3/4} \frac{k^{3/2}}{l} \quad (11c)$$

The jet inflow boundary conditions are:

$$u = 0, v = U_S, T = T_S, P = P_\infty, l = 4mm, I = 5\% \quad (12a)$$

$$k = \frac{3}{2} (U_\infty I)^2 \quad (12b)$$

$$\epsilon = C_\mu^{3/4} \frac{k^{3/2}}{l} \quad (12c)$$

The upper boundary is very far from the jet and therefore should have negligible effect on the numerical result. Thus, the boundary was taken to be symmetric to reduce the possibility of errors by the experimentally accurate no-slip condition. The outflow boundary is at the pressure  $P_\infty$ . The wind tunnel walls are 12 mm thick acrylic, while the test section bottom is modeled as 25.4 mm thick acrylic. The external boundary conditions are isothermal with a temperature of  $T_\infty$ . The test section bottom external temperature is modeled as isothermal  $\frac{1}{2} (T_S + T_\infty)$ . This temperature is used to attempt to model the complicated geometry of the test section floor. Two sheets of acrylic are separated by a small air gap of approximately 0.75 mm and the exterior sheet is exposed to jet temperature air. While not ideal, the experimental-numerical comparisons, shown later, demonstrate this is a reasonable model.

### 3.1. Simulation Validation

A simulation validation study was performed to verify the results of the simulations. Typical verification was performed including flow model, grid independence, and comparison with experimental results. The conditions of the simulation validation are shown in table 1.

The results from Spalart-Allmaras(SA),  $k - \epsilon$ , and  $k - \omega$  were compared and the three models have similar trends. SA tends to be a bit off, but this is

| Parameter          | Value            |
|--------------------|------------------|
| $U_{\infty} (m/s)$ | $2.0 \pm 0.02$   |
| $U_S (m/s)$        | $2.0 \pm 0.2$    |
| $T_{\infty} (K)$   | $305 \pm 0.5$    |
| $P_{\infty} (kPa)$ | $101.3 \pm 0.01$ |
| $T_S (K)$          | $350 \pm 2.0$    |

Table 1: Validation test conditions

expected due to issues SA has with shear flows, especially plane jets[13]. All of the values were normalized utilizing equation (13), where  $D$  is the width of the jet. Figure 2 shows a comparison of the results from the flow models at  $X = 4.75$ .

$$\phi = \frac{T - T_{\infty}}{T_S - T_{\infty}} \quad (13a)$$

$$X = \frac{x}{D} \quad (13b)$$

$$Y = \frac{y}{D} \quad (13c)$$

$$V = \frac{U}{U_{\infty}} \quad (13d)$$

$$V_S = \frac{U_S}{U_{\infty}} \quad (13e)$$

Grid independence is demonstrated by testing the temperature at a few locations with various grid sizes and geometries, as shown in table 2. Very little variation in simulated temperature over such a wide variety of cell counts shows that the result is essentially grid independent. The grid employed is an unstructured hexagonal mesh with refinement located near the jet and down stream of the jet.

The final validation is based on comparing the simulation against the experiment carried out in this study. This is done in figure 3. The simulation matches the experiment closely, with the exception of very close to the wall. This is to be expected as the flow models used, even with enhanced wall effects, have difficulty perfectly modeling the near wall conditions.



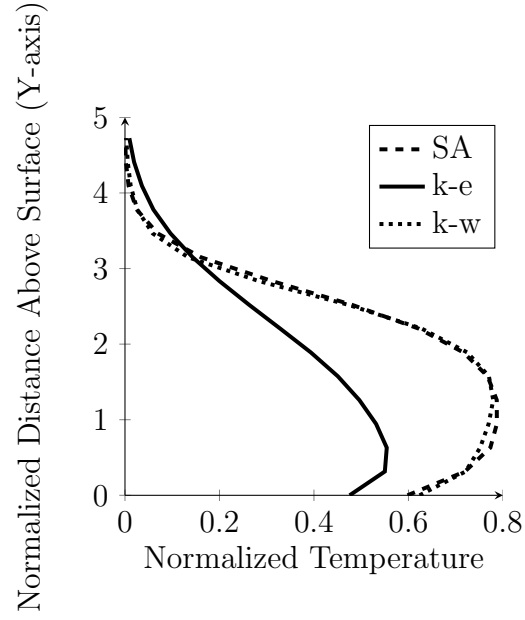


Figure 2: Validation of the simulation: local temperature using three flow models at  $X = 4.75$

| Location (x,y) (mm) | 0,1   | 10,5  | 15,5  | 30,10 |
|---------------------|-------|-------|-------|-------|
| Cell Count          |       |       |       |       |
| 57660               | 337.7 | 323.3 | 321.6 | 310.1 |
| 83888               | 337.7 | 323.3 | 321.6 | 310.1 |
| 166352              | 337.7 | 323.2 | 321.5 | 310.1 |
| 366168              | 337.7 | 323.2 | 321.5 | 310.1 |

Table 2: Grid Independence Study, local static temperature (K)

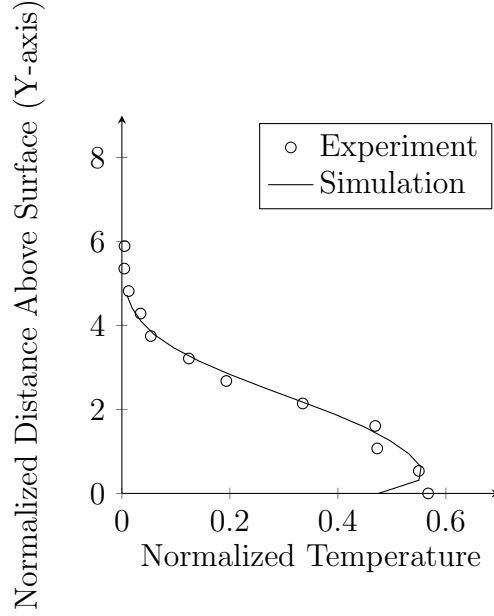


Figure 3: Validation of the simulation: local temperature - experiment versus simulation at  $X = 4.75$

#### 4. Inverse Solution Methodology

The basis for the solution strategy was described in the paper by VanderVeer and Jaluria [9], which was intended to solve the inverse plume in a crosswind. This methodology solved for the 2-D location and source strength of a plume in a crossflow. The jet in a crossflow solver must handle one additional parameter, the jet velocity. Therefore the methodology is required to be modified. For the sake of completeness, the original methodology will be briefly described here.

The method assumed that we can neglect the variations in density, variations in thermal buoyancy, and thermal radiation. If this is true, then the temperature of a given location ( $T(\mathbf{r})$ ) is linearly dependent upon the source temperature ( $T_S$ ) as shown in equation (14). The two parameters  $m$  and  $b$  are both location ( $\mathbf{r}$ ) dependent and may be calculated knowing a unique set

of simulations  $A$  and  $B$ .

$$T_S = m(\mathbf{r}) T(\mathbf{r}) + b(\mathbf{r}) \quad (14a)$$

$$\mathbf{r} = r(x, y) \quad (14b)$$

$$\mathbf{r}_i = \mathbf{r}_0 + \Delta_i \quad (14c)$$

$$m(\mathbf{r}) = \frac{T_{SA} - T_{SB}}{T_A(\mathbf{r}) - T_B(\mathbf{r})} \quad (14d)$$

$$b(\mathbf{r}) = T_{SA} - m(\mathbf{r}) T_A(\mathbf{r}) \quad (14e)$$

There are a few quick terms to make the following explanation easier. Sample point: local static temperature from within the domain at a particular location. Datum point: a selected sample point whose location will be defined as  $\Delta_0 = (0\text{ mm}, 0\text{ mm})$ . Search shape: relative location and pattern between a set of sample points and a datum point.

Start by selecting  $n$  sample points and  $d$  simulations. The method attempts to minimize the discrepancies between all of the sample points' calculated source temperature and the datum point's calculated source temperature to get a predicted source temperature. Then minimize the discrepancy between the predicted source temperature and a new set of sample point calculated source temperatures to get the corrected location and temperature. The following equations mathematically describe the previous text. Equation (15) is a minimized at  $\mathbf{r}_{SP}^*$  and the location  $\mathbf{r}_{SP}^*$  is used to calculate the predicted source temperature in equation (16). Then equation (17) is minimized at  $\mathbf{r}_{SP}$  and the location  $\mathbf{r}_{SP}$  is calculated with equation (18) used to calculate  $T_{SP}$ , where  $a$  is the number of sample points used in the prediction step and  $n$  is the total number of sample points.

$$F(\mathbf{r}) = \sum_{i=1}^a [m(\mathbf{r} + \Delta_i) T(\mathbf{r}_i) + b(\mathbf{r} + \Delta_i) - m(\mathbf{r}) T(\mathbf{r}_0) - b(\mathbf{r})]^2 \quad (15)$$

$$T_{SP}^* = \frac{1}{a} \left\{ \sum_{i=0}^a [m(\mathbf{r}_{SP}^* + \Delta_i) T(\mathbf{r}_i) + b(\mathbf{r}_{SP}^* + \Delta_i)] \right\} \quad (16)$$

$$F_{mod}(\mathbf{r}) = \sum_{i=a}^n [m(\mathbf{r} + \Delta_i) T(\mathbf{r}_i) + b(\mathbf{r} + \Delta_i) - T_{SP}^*]^2 \quad (17)$$

$$T_{SP} = \frac{1}{n-a} \left\{ \sum_{i=a}^{n-a} [m(\mathbf{r}_{SP} + \Delta_i) T(\mathbf{r}_i) + b(\mathbf{r}_{SP} + \Delta_i)] \right\} \quad (18)$$

The number and pattern of the sample points (i.e. the search shape) was optimized in a paper by VanderVeer and Jaluria [10]. In it three sample points were identified as being enough to solve the inverse plume problem. The three points were later shown to not handle error very well and the optimization process was continued to include nine sample points. They are shown in table 3.

| Sample Point | $\Delta$ (mm) |
|--------------|---------------|
| 0            | (0.0, 0.0)    |
| 1            | (1.7, 3.5)    |
| 2            | (2.8, 0.6)    |
| 3            | (0.5, 1.1)    |
| 4            | (2.1, 1.0)    |
| 5            | (2.3, 2.0)    |
| 6            | (1.2, 0.8)    |
| 7            | (3.1, 0.7)    |
| 8            | (0.8, 2.1)    |

Table 3: Search shape results of averaging over the domain with increasing the sample size to 9

Going back to the inverse jet in a crossflow problem, Knight et al. [14] used a quadratic response surface model to determine the jet temperature and jet velocity. Unfortunately, the response surface model used is two dimensional and would have difficulty applying directly to the current model. The response surface models effectiveness does demonstrate the quadratic nature of the jet velocity and linear nature of jet temperature with respect to local temperature.

The method starts similar to the original method, by selecting  $n$  sample points and  $d$  simulations spanning the thermal and velocity region of interest. Guess a jet velocity ( $U_S$ ) so the linear parameters are not dependent upon the jet velocity during the minimization. Calculate the parameters  $m$  and  $b$  from equation (14). Find the minimization of equation (15) in which we can find a predicted temperature from equation (16). We can develop an equation similar to equation (15) for the jet velocity. Starting with equation (19), we can minimize the discrepancy between the predicted jet velocities resulting in equation (20). As would be expected there are three parameters

needed  $(\Gamma_0, \Gamma_1, \Gamma_2)$ . The minimization of  $G$  allows  $U_{SP}$  to be calculated from equation (21). This new  $U_{SP}$  is then used to calculate a new set of linear parameters  $m$  and  $b$ . The process repeats itself until both the change in  $U_{SP}$  and the change in  $T_{SP}$  is smaller than some small residual. That is to say  $U_{SP} < \epsilon_U$  and  $T_{SP} < \epsilon_T$ , where  $\epsilon_U$  and  $\epsilon_T$  are small values such as  $0.01 \text{ m/s}$  and  $0.1 \text{ K}$  respectively. A flowchart of the methodology is shown in figure 4.

$$U_S = \Gamma_2(\mathbf{r}) T(\mathbf{r})^2 + \Gamma_1(\mathbf{r}) T(\mathbf{r}) + \Gamma_0(\mathbf{r}) \quad (19)$$

$$\begin{aligned} G(\mathbf{r}) = \sum_{i=a}^n & [\Gamma_2(\mathbf{r} + \Delta_i) T(\mathbf{r}_i)^2 \\ & + \Gamma_1(\mathbf{r} + \Delta_i) T(\mathbf{r}_i) \\ & + \Gamma_0(\mathbf{r} + \Delta_i) \\ & - \Gamma_2(\mathbf{r}) T(\mathbf{r}_0)^2 \\ & - \Gamma_1(\mathbf{r}) T(\mathbf{r}_0) \\ & - \Gamma_0(\mathbf{r})]^2 \end{aligned} \quad (20)$$

$$\begin{aligned} U_{SP} = \frac{1}{n-a} & \left\{ \sum_{i=a}^{n-a} [\Gamma_2(\mathbf{r}_{SP} + \Delta_i) T(\mathbf{r}_i)^2 \right. \\ & + \Gamma_1(\mathbf{r}_{SP} + \Delta_i) T(\mathbf{r}_i) \\ & \left. + \Gamma_0(\mathbf{r}_{SP} + \Delta_i)] \right\} \end{aligned} \quad (21)$$

## 5. Results and discussions

Due to the new methodology, an incremental approach to solving the problem was used. Work from the simplest steps and moving towards the more difficult cases, ending with the experimental results. The first five steps are of simulated data only, as to separate methodology error from experimental error. Twenty-four selected cases are used to demonstrate the capabilities and weaknesses of the described methodology. The cases are labeled A-X, the conditions of the selected cases are described in table 4 with the constant parameters in table 5. For example, case ‘T’ has a datum at  $20 \text{ mm}$  downstream,  $3 \text{ mm}$  above the surface, jet velocity of  $1 \text{ m/s}$ , and jet temperature of  $425 \text{ K}$ .

The first two steps utilize a single sample point, however, the rest use the nine sample points listed in table 3.

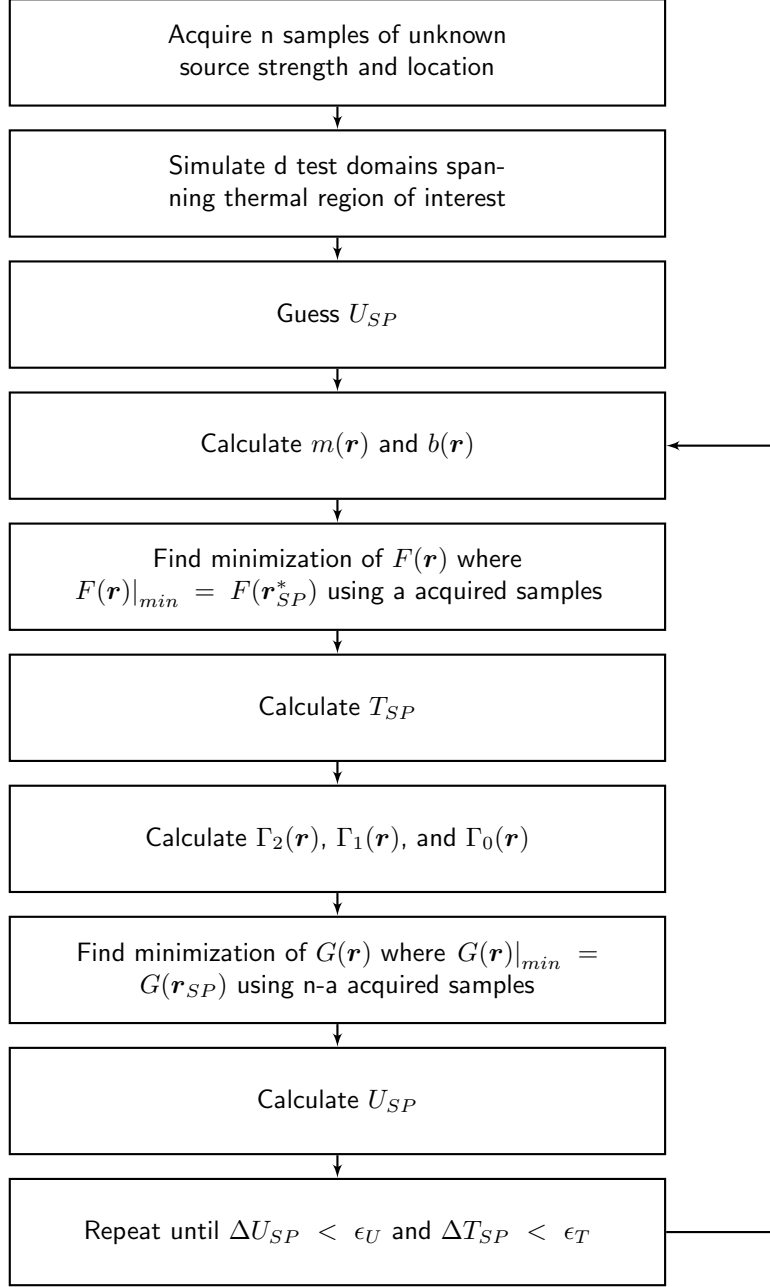


Figure 4: Flow chart of the predictor - corrector methodology for a jet in a crossflow

|                |     |     |     |     |     |     |
|----------------|-----|-----|-----|-----|-----|-----|
| $U_S$ (m/s)    | 1   | 1   | 2   | 2   | 4   | 4   |
| $T_S$ (K)      | 375 | 425 | 375 | 425 | 375 | 425 |
| Location (x,y) |     |     |     |     |     |     |
| 10 mm, 1 mm    | A   | B   | C   | D   | E   | F   |
| 10 mm, 3 mm    | G   | H   | I   | J   | K   | L   |
| 20 mm, 1 mm    | M   | N   | O   | P   | Q   | R   |
| 20 mm, 3 mm    | S   | T   | U   | V   | W   | X   |

Table 4: Several sampled case parameters

| Parameter        | Value |
|------------------|-------|
| $U_\infty$ (m/s) | 2     |
| $T_\infty$ (K)   | 293   |
| $P_\infty$ (kPa) | 101.3 |

Table 5: Simulation test conditions

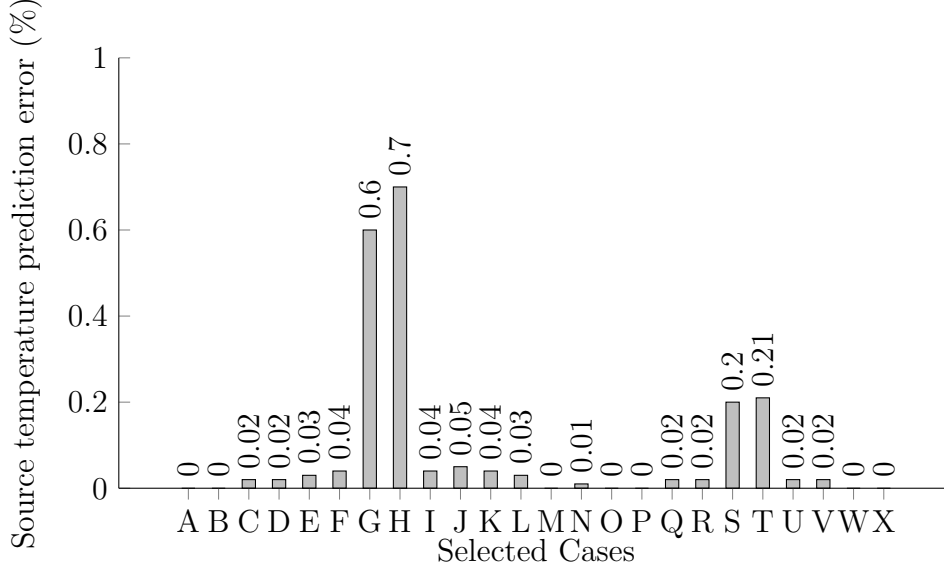


Figure 5: Error in the prediction of  $T_S$  from several sampled cases within the jet with  $r_S$  and  $U_S$  known (min:0%, max:0.7%, median:0.02%) [15]

### 5.1. Source Location and Velocity Known

In the first condition of unknown source temperature, the methodology breaks down to a linear equation, equation (14a). Due to the simple linear equation a single sample point is needed to solve the inverse problem. The results of the selected cases are shown in figure 5. The error is typically less than 0.1%. There are four cases with large error and they are located outside or near the edge of the jet. The temperature at these locations is near the ambient temperature and the Matlab polynomial curve fit has issues with the data points very similar to each other. The error is calculated using equation (22).

$$error_{temp}(\%) = \frac{|T_{SP} - T_S|}{T_S - T_\infty} \times 100 \quad (22a)$$

$$error_{x-location}(\%) = \frac{|x_{SP} - x_S|}{x_S} \times 100 \quad (22b)$$

$$error_{y-location}(\%) = \frac{|y_{SP} - y_S|}{y_S} \times 100 \quad (22c)$$



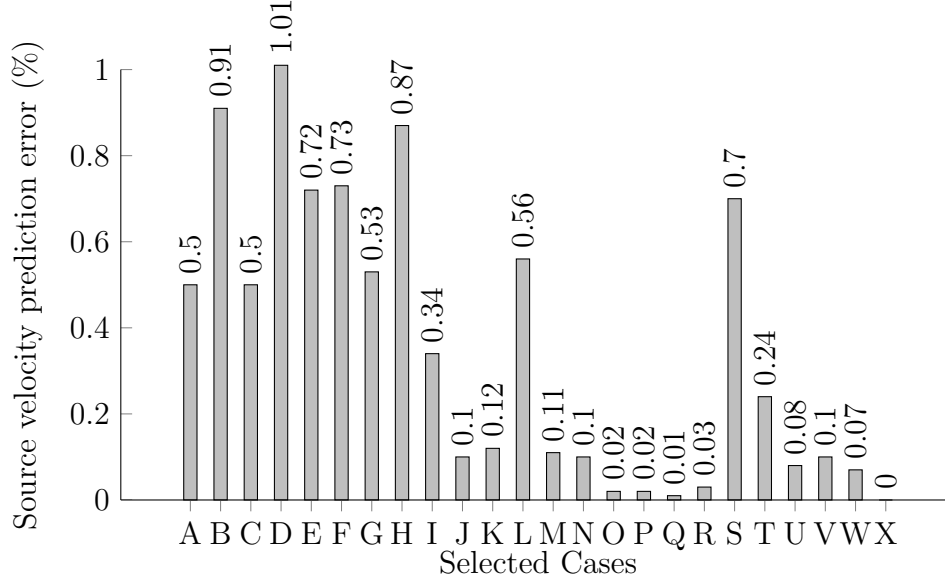


Figure 6: Error in the prediction of  $U_S$  from several sampled cases within the jet with  $r_S$  and  $T_S$  known (min:0%, max:1.01%, median:0.18%)[15]

### 5.2. Source Location and Temperature Known

The second case is of unknown jet velocity and is also simple and breaks down to equation (19). As the quadratic equation does not perfectly model the velocity profile, the error is still low, as shown in figure 6. The error is less than  $\approx 1\%$ , which is generally less than that of any experimental error.

### 5.3. Source Location Known

With only the jet location known, the method would ideally breakdown to the response surface model of Knight et al. [14]. However, the methodology has the jet temperature and jet velocity solved iteratively, and must still be solved in such a manner. The errors of the selected cases are shown in figures 7 and 8. Several of the cases had a decrease in error due to the increase from one sample point to nine.

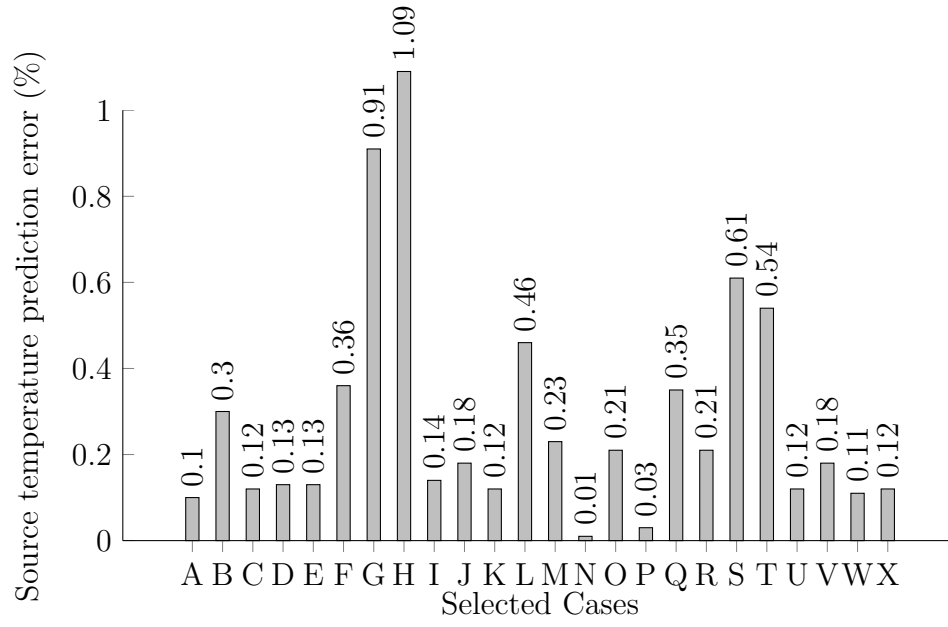


Figure 7: Error in the prediction of  $T_S$  from several sampled cases within the jet with  $r_S$  known (min:0.01%, max:1.30%, median:0.21%)[15]

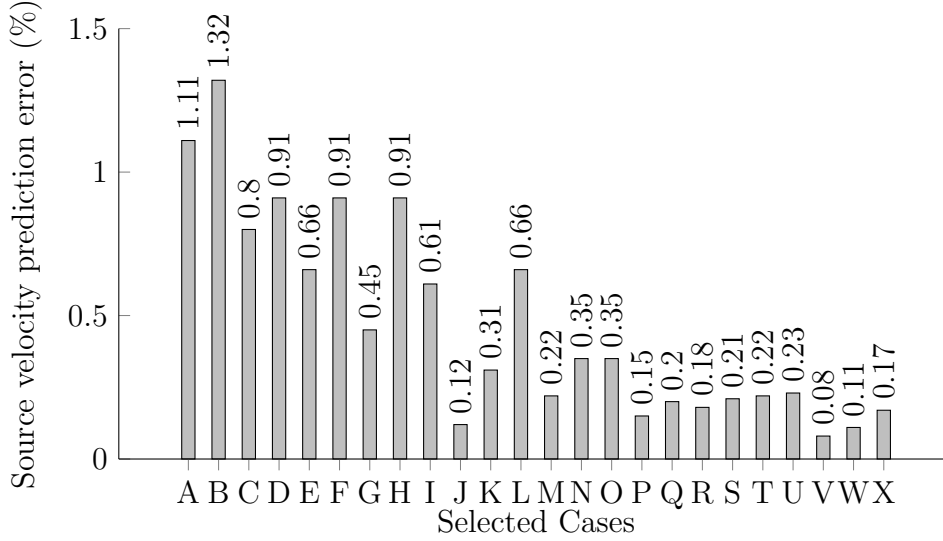


Figure 8: Error in the prediction of  $U_S$  from several sampled cases within the jet with  $r_S$  known (min:0.08%, max:1.32%, median:0.33%)[15]

#### 5.4. Source Elevation Known

Keeping with the incremental stepping, only the source elevation is known. As with the previous step, nine sample points are used. The error results of predicting the source temperature, velocity, and axial location are shown in figures 9 to 11. The max error increases significantly to 9%, but typical values are much less and reasonable for inverse problems.

#### 5.5. Source Location and Strength Unknown

The final incremental step is not possible with the current methodology and search shape. The underlying, self-similar characteristics of a jet in a crossflow creates an indiscernible infinite set of possible solutions. It is possible that a significant increased sample size would alleviate the problem, but the goal of this work was to utilize very limited domain information.

As an example, a set of alternate solutions is shown in table 6 for case ‘V’. This list was not meant to be comprehensive, but rather illustrative of the problem. The last column is the percent error between the various solutions.

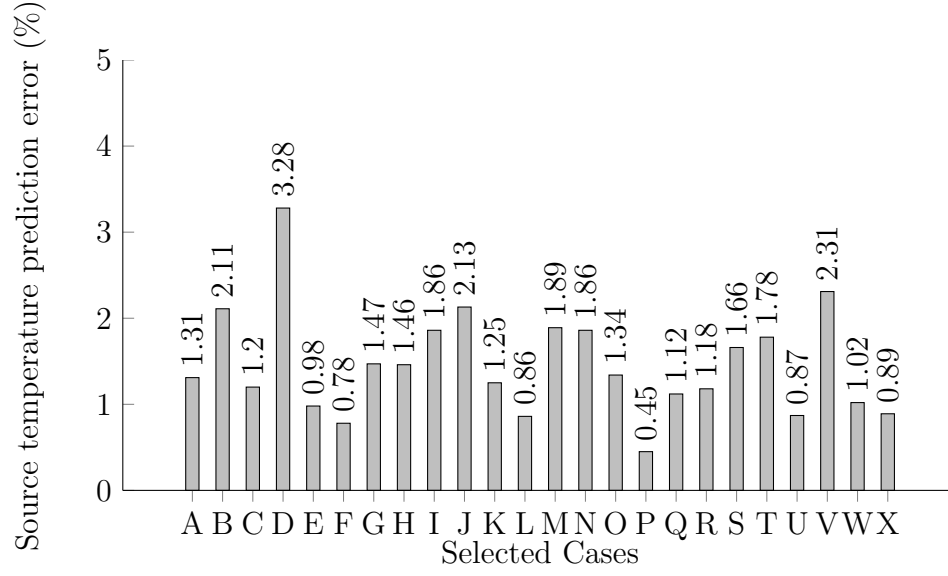


Figure 9: Error in the prediction of  $T_S$  from several sampled cases within the jet with source elevation known (min:0.45%, max:3.28%, median:1.32%)[15]

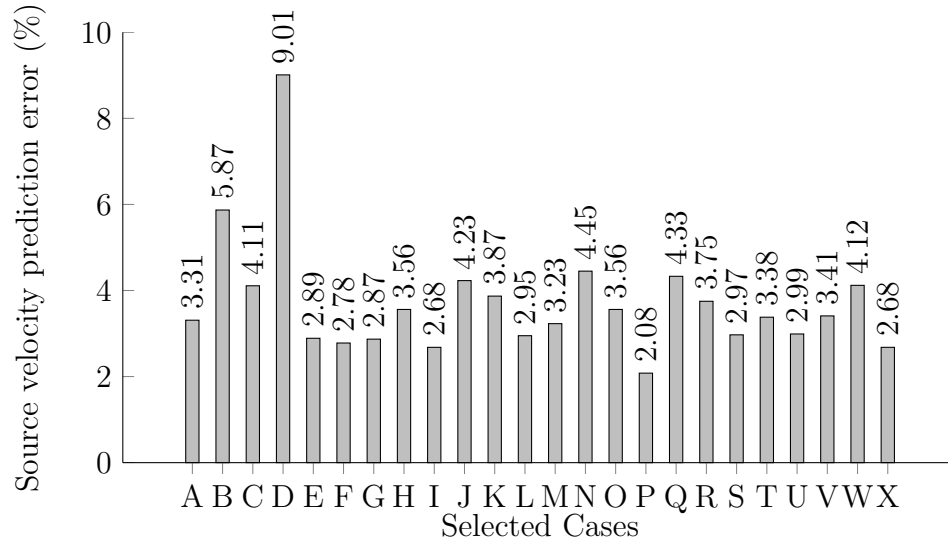


Figure 10: Error in the prediction of  $U_S$  from several sampled cases within the jet with source elevation known (min:2.08%, max:9.01%, median:3.40%)[15]

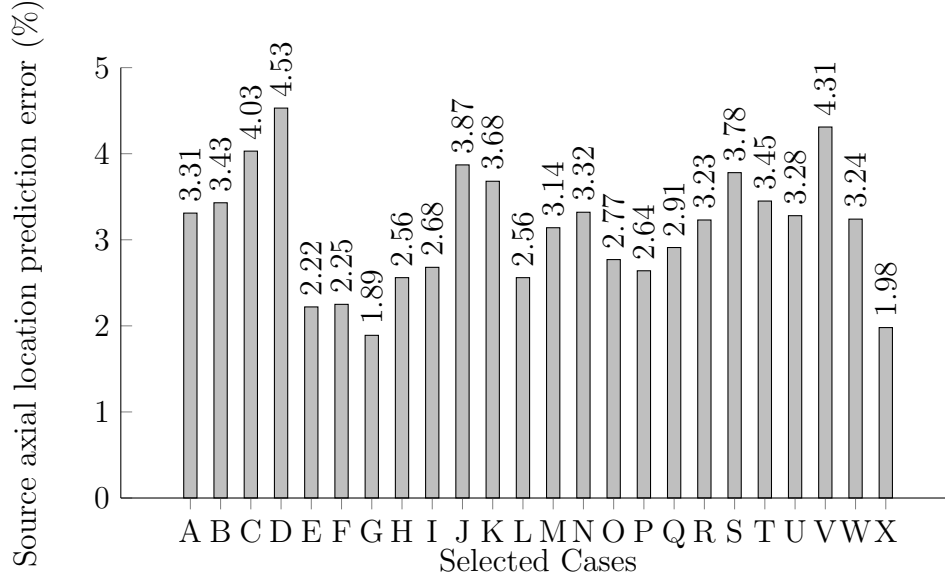


Figure 11: Error in the prediction of  $x_S$  from several sampled cases within the jet with source elevation known (min:1.89%, max:4.53%, median:3.24%)[15]

| Axial<br>Loc. ( $mm$ ) | Elev.<br>Loc. ( $mm$ ) | Jet<br>Temp. ( $K$ ) | Jet<br>Vel. ( $m/s$ ) | Search Shape<br>Error (%) |
|------------------------|------------------------|----------------------|-----------------------|---------------------------|
| 20.0                   | 3.0                    | 425                  | 2.0                   | 0.00                      |
| 10.0                   | 3.7                    | 375                  | 2.5                   | 0.60                      |
| 12.0                   | 5.1                    | 375                  | 3.0                   | 0.34                      |
| 14.8                   | 2.7                    | 400                  | 2.0                   | 0.30                      |
| 16.3                   | 4.5                    | 400                  | 2.5                   | 0.26                      |
| 19.5                   | 5.9                    | 400                  | 3.0                   | 0.29                      |
| 21.8                   | 5.0                    | 425                  | 2.5                   | 0.24                      |
| 25.6                   | 6.5                    | 425                  | 3.0                   | 0.34                      |
| 25.2                   | 3.2                    | 450                  | 2.0                   | 0.13                      |
| 26.8                   | 5.4                    | 450                  | 2.5                   | 0.25                      |

Table 6: Example alternative solutions[15]

| Parameter          | Value           |
|--------------------|-----------------|
| $T_{\infty} (K)$   | $297.6 \pm 8.0$ |
| $P_{\infty} (kPa)$ | $100.6 \pm 0.6$ |
| $U_{\infty} (m/s)$ | $2.0 \pm 0.02$  |

Table 7: Experimental test conditions

### 5.6. Experimental Results

Due to the last step we will only consider searching for the source temperature, velocity, and axial location for the experimental cases. The experimental conditions are listed in table 7. A select few results are shown in table 8. A table format was chosen to represent the data, as the error is significant and would make a chart difficult to read. The axial direction error has a minimum of 5.4%, a maximum of 18.7%, and a median of 10.5%. The jet velocity error has a minimum of 10.4%, a maximum of 21.8%, and a median of 14.75%. The jet temperature error has a minimum of 8.64%, a maximum of 13.7%, and a median of 9.99%.

A graph of actual vs predicted source temperature and velocity are shown in figures 12 and 13 respectively. It is easily visualized that this methodology predicts the source velocity significantly better than source temperature. The large percentages are due to the velocities being small. Often, the predicted error is close to the experimental error of the system.

## 6. Conclusions

A predictor-corrector method was modified and applied to the inverse jet in a crossflow problem. The inverse method was previously developed and the search shape was optimized. Four parameters were searched for source temperature, velocity, axial location, and elevation. The new methodology functioned accurately for 4 out of 5 steps, with all four parameters unknown being the exception. There was not enough information to accurately discern the solution from the infinite solution set. Other than the exception, the methodology performed well with approximately 9% even in the worst cases, and typically performing much better.

The step utilizing experimental results showed a greater error. This indicates that this methodology, when applied to the jet in a crossflow problem,

|                |   |       |       |       |       |
|----------------|---|-------|-------|-------|-------|
| $U_S$ (m/s)    |   | 1.0   | 1.0   | 4.0   | 4.0   |
| $T_S$ (K)      |   | 375   | 425   | 375   | 425   |
| Location (x,y) |   |       |       |       |       |
| 10 mm, 0 mm    | X | 10.9% | 11.7% | 11.1% | 13.8% |
|                | U | 21.8% | 18.6% | 10.4% | 11.9% |
|                | T | 8.70% | 9.87% | 10.7% | 12.9% |
| 20 mm, 0 mm    | X | 8.65% | 7.65% | 5.40% | 9.81% |
|                | U | 18.7% | 19.8% | 11.5% | 12.1% |
|                | T | 8.81% | 10.7% | 9.65% | 10.2% |
| 30 mm, 0 mm    | X | 10.6% | 10.4% | 18.7% | 9.78% |
|                | U | 15.8% | 17.9% | 12.3% | 13.7% |
|                | T | 9.61% | 8.64% | 13.7% | 10.1% |

Table 8: Error in predicting source axial location ( $x_S$ ), source strength( $U_S$  and  $T_S$ ) from a few sample cases within the jet, search shape with 9pts, utilizing experimental data[15]

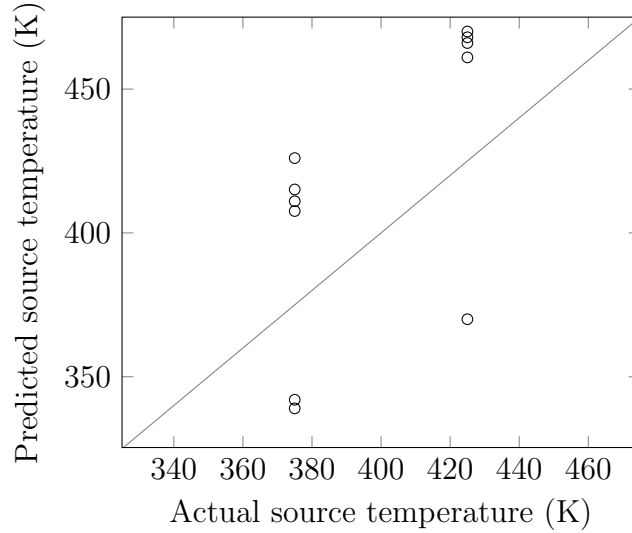


Figure 12: Actual vs predicted source temperature for several cases within the jet

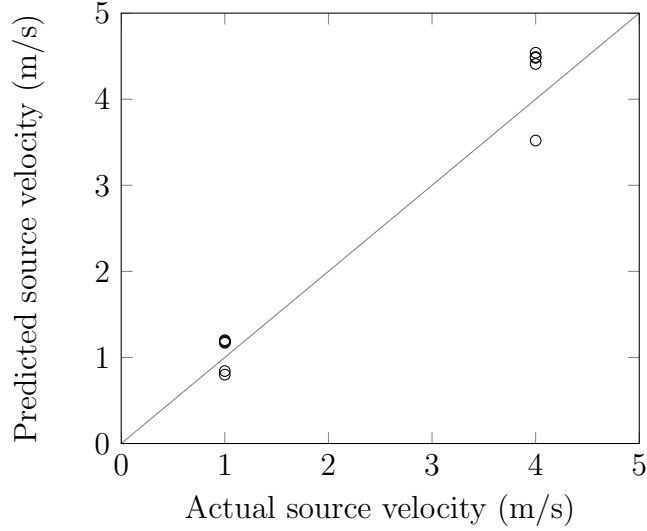


Figure 13: Actual vs predicted source velocity for several cases within the jet

is error sensitive. The error was as large as 21.8% for the jet velocity and as small as 5.4% for the axial location of the jet.

Previous optimization of the search shape indicates that the methodology is sensitive to the search shape[10]. More work on the inverse jet in a crossflow problem, by optimizing the search shape specifically for the jet problem, may reduce the overall error.

The methodology has demonstrated its capabilities and limitations, and much more work may be done regarding the limitations. There are several improvements still needing to be made, such as three dimensional capabilities, transient capabilities, and possibly the ability to differentiate multiple sources. These features, if implemented, would have significant applications to fires in urban environments and pollution in industrial areas.

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