P2110A: MATLAB Tutorial 2 activity

September 25, 2018

For today's task, we will show the motion of a pendulum by numerically solving it's equation of motion using Euler's method. If you recall, the equation of motion for a pendulum undergoing simple harmonic motion is:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta \tag{1}$$

Euler's method equation applied to the pendulum's motion can be written as:

$$\theta_{i+1} = \theta_i + \omega_i \Delta t \tag{2}$$

The algorithm for implementing it goes as:
1. Declare variables and arrays.
2. Initialize the arrays (set to zero to begin with)
3. Do the calculation
For each time step i, calculate ω and θ at time step i+1

- $\omega_{i+1} = \omega_i - (g/l)\theta_i \Delta t$
- $\theta_{i+1} = \theta_i + \omega_i \Delta t$
- $t_{i+1} = t_i + \Delta t$

4. Store the results. Repeat for the desired number of time steps.

Your Task

1. Implement Euler's method to solve the pendulum's equation of motion and plot the position as a function of time. Assume $g = 9.8[\text{m/s}^2]$, $l = 1[\text{m}]$, $\theta_0 = 0.2[\text{rad}]$, and the iteration step size is 0.01s for 1000 steps. Open the script called `Euler_pendulum.m`. You will see steps 1 and 2 of the algorithm are taken care of. What you will need to do is write the code for steps 3 and 4. After completing these steps, run the script by typing `Euler_pendulum` into the command window.

2. Look at the plot. Does this make sense to you? Why or why not?

3. Another approach for solving differential equations of this type is the Euler-Cromer method. It is identical to Euler's method, except the line $\theta_{i+1} = \theta_i + \omega_i \Delta t$ becomes $\theta_{i+1} = \theta_i + \omega_{i+1} \Delta t$. Now, open `EulerCromer_pendulum.m`, and again fill out steps 3 and 4 of Euler's method algorithm, with the adjustment for the θ_{i+1} line. (Note: This code should be very similar to Euler_pendulum.)

4. Run both scripts - you should see 2 plot figures appear. How are they different? Which do you think is a better physical representation of the pendulum's motion?

5. *Optional:* Look up the second order Runge-Kutta method Verlet method. How do they differ from the Euler/Euler-Cromer method?