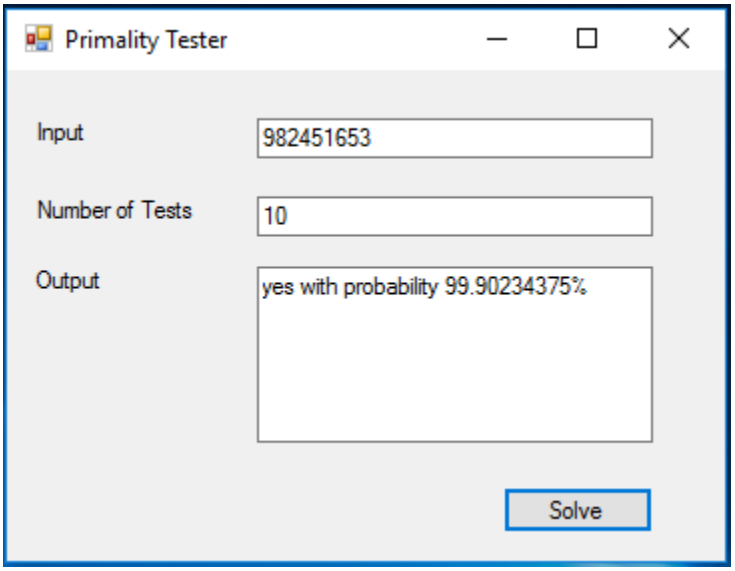
Joshua Van Steeter

Project 1: Primality Tester

1. Here is a screen shot of my working project



2. The code that I wrote:

```
private void solve_Click(object sender, EventArgs e){
        // retrieve the inputs from the GUI
        long N = Convert.ToInt64(input.Text);
        int k = Convert.ToInt32(ktests.Text);
        Random random = new Random();

        int testsPassed = 0;
        // run the number of tests specified in the input
        // O(c)
        for(int i = 0; i < k; i++)
        {
           // generate a random number between 2 and N
           long a = random.Next(2, Convert.ToInt32(N));
           // run the modular exponentiation function a^(N-1) % N
           long result = modular_exp(a, Convert.ToInt64(N - 1), N);
           // if the result of modular_exp is 1 then it passed the test, you may run another test,
            // it may be prime
           if (result == 1)
           {
              testsPassed++;
           }
           // if the result of modular_exp is not 1 then N is not prime, display "no" and quit testing
           else
           {
```

```
            output.Text = "no";
            return;
        }
    }
    // if all tests have passed, calculate the percent accuracy as 100 - (100/2^k) and display results
    double percent = 100 - (100 / (Math.Pow(2, testsPassed)));
    output.Text = "yes with probability " + Convert.ToString(percent) + "%";
}

private long modular_exp(long value, long exponent, long N)
{
    // the function for modular exponentiation
    // for every test of solve_Click() this function will fun log2(n) times which will halt after at most
    // n recursive calls, at each call it multiplies n-bit numbers
    // giving us a total run time of O(n^3)
    if (exponent == 0)
    {
        // base case, when an exponent is 0 the result will always be 1
        return 1;
    }
    // the recursive call
    long z = modular_exp(value, exponent / 2, N);
    if (exponent % 2 == 0)
    {
        // if the exponent is even return z^2 mod N
        long result = ((z * z) % N);
        return result;
    }
    else
    {
        // if the exponent is odd return x*z^2 mod N
        long result = ((value % N) * ((z * z) % N) % N);
        return result;
    }
}
```

3. The Fermat tests run in constant times, merely checking if the result of the modular exponentiation was equal to 1 or not, and doing this a constant number of times equal to the input number of tests. The modular exponentiation function runs in O(n3) time which I elaborate a little more on in comments in the code. Bringing our total run time to O(n3).

4. As each test is passed the likelihood for error decreases by 50%. Therefore the error to calculate percent probability of success out of 100% is 100 – (100/2^k)