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MEDIEVAL NUMEROLOGY
A Book of Essays

edited by
Robert L. Surles



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INTRODUCTION

The history of symbolic numbers is complicated and difficult to disentangle. This part of the history of ideas still awaits its Columbus.

Gunnar Qvarnström

Numeracy must surely predate literacy and any notion of the recorded history of man. Numbering, counting and enumerating are requisite means for intellectual orientation, and numerology, which the Swedish philologist Gunnar Qvarnström has called "the semantics of number," (*Poetry and Numbers*, p. 14) is as traditional and far-reaching as Greco-Roman classicism and as old and arcane as Germanic runes. It is currently felt that metaphor, or any other figurative device should serve to "concretize" abstract ideation into a concept more readily accessible to general comprehension. Another current generalization is the thought that mathematics may be understood as a language sufficient unto itself without need for interpretation into any other idiom. Yet it must here be said that neither of these understandings were fastly maintained in earlier times. The history of number symbolism is traced in the work of Vincent Hopper, whose views have come to define medieval number philosophy for the modern scholar. Beyond what Hopper calls "elementary" numbering, derived from certain fixed natural groups, i.e the hand for five, the figure of a man for twenty (with both hands and feet counting), etc., the Babylonian science of astrology saw divine orders of numbers in astronomical cycles and constellations. Yet another view of number grew out of the Pythagoreanism of Greece, fixing the relationship of numbers to one another and to the Cosmic Order. The dominant medieval attitude towards number symbolism in the West, however, was the Christian—which, in its turn adopted and adapted the majority of Plato's concepts. Greatly influential in accepting the coalescing body of number lore into the present era was the great scholar of biblical exegesis Philo Judaeus, whose writings established the principle of an allegorical interpretation of Hebrew scripture. This early model of commentary was quickly followed by the Fathers of the Christian Church. Isidore of Seville (*De Numeris*) tells us that all numbers found in Holy Scripture must carry a secret meaning, and Augustine elaborately details a few instances.

Thus, the influence of numerology in the medieval period on thought and literature, and, indeed, one must say on all art can hardly be undervalued nor (perhaps) properly appreciated in modern times. It is not difficult to suppose that the artist or thinker, with such examples before him, would strive to imitate in the microcosm of his work those creative procedures which he had come to believe God had used in the macrocosm of His creation. Nonetheless, whether or not the use of number would be discernible or properly appreciated by his "audience," while significant, might not have been of first importance to his craft. What must have been of transcendental value to the artist would be the belief that, since the thing he wished to create had been "ordered" in the proper manner, it would be good and in harmony with all creation.

Following the first comprehensive work on number usage (Hopper, 1938), a few general considerations on the same topic emerged (Curtius, in his "Excursus XV," 1953; Qvarnström, 1966; and Menninger, 1969). In addition to these, studies on the appreciation of medieval numerology tend to concentrate on a single author or work. Among the significant works in this regard are: Suheyla Bayrav's *Symbolisme médiéval: Béroul, Chrétien*, (1957); Michael S. Batts' "Numbers and Number Symbolism in Medieval German Poetry," *Modern Language Quarterly* (1963); Marien-Sofie Røstwig's *The Hidden Sense: Milton and the Neoplatonic Method of Numerical Composition*, (1963); Alistair Fowler's *Spenser and the Numbers of Time*, (1964); and Eleanor Webster Bulatkins's *Structural Arithmetic Metaphor in the Oxford Roland* (1972). Furthermore, necessary to the library of "medieval numerologists" are the following collections: *Number Symbolism*, ed. Christopher Butler (1970); *Silent Poetry: Essays in Numerological Analysis*, ed. Alistair Fowler (1970); and *Essays in Numerical Criticism of Medieval Literature*, ed. Caroline D. Eckhardt (1980). Many of these more recent studies concern themselves with what might be called literary allegoresis, wherein the numbers are used to restate metaphorically some basic idea inherent in the content of the work at hand. While grasping the "hidden meaning" is not necessary to following the story line, its appreciation (like that of any other poetic conceit) serves to strengthen the bond between author and reader/audience, and (as mentioned earlier) to strengthen the link between creator and Creator. Another use of number as poetic device is often termed "structural arithmetic metaphor." This entails no mention of number itself, but concerns an ordering of poetic form according to a preconceived

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arithmetic pattern. With such a device, an author might have "graphed" his work with rule, compass and line before writing any actual text. That the hero, the heroine, the adversary, the tryst scene, etc., occur at given verse numbers might well have connotative value and serve as metatext. Such number usage does not merely constitute an external frame, but becomes indeed a participant in the action of the work. Although no *ars poetica* has come down to us describing such a usage, the evidence of a great deal of research is sufficient to justify an assumption that the practice was viable throughout medieval narrative poetry.

In that the great majority of medieval numerical usage is based on the Christian interpretation of Pythagoreanism, our collection opens with "God as Monad: The Philosophical Basis of Medieval Numerology" by Lawrence P. Schrenk wherein he speaks of the particular attraction that number holds for philosophers and theologians. There follows a series of studies on artistic constructional geometry (also called "structural arithmetic metaphor" and the significance of line-count in textual creation, the first of which is Kay Slocum's "Speculum musicae: Jacques de Liège and the Art of Musical Number." Prof. Slocum relates medieval numerological concepts to the necessarily mathematic component of music theory. Four essays follow to exemplify narrative poetic text as "drawn" before written: The first of these, "The Form of the Phoenix: A Harmony of Number, Symbol, Proportion, and Unity" by Robert D. Stevick, is able to show sectional divisions of this Old English poem as crafted according to a topically numeric template, needing nothing more elaborate than rule, line and stylus. Joan Helm's "Erec and Enide: Cosmic Measures in Nature, and the Hebrew Heritage" is an amazingly complex (and demonstrably credible) explication of geometric design in early French narrative poetry of the twelfth century. Betty Vanderwielen's essay "The Significance of Numbers in the Structuring of Dante's *Commedia*" provides a summary background and coetaneous understanding of symbolic number usage in the most "numerically" crafted work of literature recognized by modern scholarship. As bridge between numeric structure in the geometric construct and number usage as single-number symbolism within a literary work, it seems appropriate to continue with that "consummate craftsman" Dante: with John Secor's "Three Dances of Three: The *Imago Trinitatis* in Dante's *Commedia*." In reading this study, one should think back to the opening essay by

Shrenck, since Secor's work follows equally well the triune nature of Christian mythology as it does the literary analogue of Dante's work. To continue with the rather ubiquitous "three" of medieval literary pervasion, the ternary system is further examined and exemplified in Anne Berthelot's essay on twelfth-century French narrative poetry—"Numbers as Symbols and Numeric Inflation in the Grail Family." Here we find specific numbers to be readily understood by the "audience" (again "3" being first among them) to become in time mere rhetorical figures—devoid of innate meaning. Often a large number is more than merely the integer following its immediate antecedent: a number like 153 might just be (to a numero-literary artificer) more than $152 + 1$. In "Gold and Grace in Hartmann's *Gregorius*," Aaron E. Wright is able to show us 12/13th-century German narration that draws on biblical number-count to create the understanding of Grace in folktale analogues. And there is number usage to demonstrate not only the expected order but things gone awry. A significantly-used number other than expected would surely "jolt" the reader out of connectivity with the expectations of Christian numerology. Conrad Haderlein's "Two, Three and Eleven: Disharmony, Disorder and Disarray (Emblematizing Social and Ideological Change in Thirteenth-Century Iconography and Poetic Documents)" will introduce the terms transgression and numerus deficiens as they represent the unexpected. Numerological composition is not a phenomenon confined to medieval texts; although the concepts were developed during that period, their usage continued into later times. Since Kent Hieatt's consideration of number symbolism in Spenser's *Epithalamion* in 1960, many studies have searched English poetics for traces of what, in Milton's words, might be called this "little-known tradition, not obvious, nor obtrusive, but retired." One such scholar Kate Frost treats a unique textual signature in the next study of this collection "John Donne, the Number 23, and the Tradition of Spiritual Autobiography." Finally, in our last essay we leave literature, poetic design and metaphor for another field of investigation. Jens Ulff-Møller's "Remnants of Medieval Symbolic Number Usage in Northern Europe" treats the pragmatic employment of number for real-world accounting and measuring purposes and speaks of a forgotten medieval number usage coincident with the traditional Western systems of the time.

Numerology has always been an attractive topic for medieval interests and will surely continue to be so. It is our hope that the present collection of investigations into medieval number usage will

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serve both as repository and example of the diversity of interests and "modes of attack" a number-conscious observer might employ for the appreciation of work crafted long ago.

Robert L. Surles

God as Monad: The Philosophical Basis of Medieval Theological Numerology

Lawrence P. Schrenk

The influence of numerology in the medieval period can hardly be overestimated. Numbers were central in the medieval attempt to render the world intelligible to the human mind, and mathematics could serve as a model for understanding not only the physical world but such diverse subjects as ethics and art.¹ Here we shall consider one quite narrow aspect of the use of numbers in the Middle Ages: the theological use of numerology and its philosophical basis. In particular, we shall examine three issues: the role of number in a philosophical theology, mathematics as a model for theology, and numbers as an analogy for creation. It will be impossible, in this very limited space, to analyze thoroughly, or even mention, every thinker who contributed to the history of medieval philosophical numerology; rather, we shall explore three key figures who contribute to our understanding of the development of these themes from their Greek origins to the twelfth century. First, we shall consider Nicomachus of Gerasa, the Greek Neopythagorean, as an important predecessor of the medieval view, then, Macrobius, an early western representative of the Neoplatonic numerical tradition, and, finally, Thierry of Chartres, a twelfth-century thinker.

Much attention must be paid to the early Greek background of philosophical numerology. Clearly, the Pythagoreans were interested in such a philosophical project, though one must be careful not to over-emphasize the scientific element in their thought, and should note that under Pythagorean influence Plato developed his mathematical inclinations.² In the so-called "Lecture on the Good", for instance, it is reported that Plato's audience left in disgust after they found the lecture to consist entirely in mathematics. The continued influence of mathematics in the Platonic tradition can be seen in Xenocrates' definition of

the soul as number.³ Such a mathematical understanding of the world endures throughout the later Platonic tradition.⁴

Why, however, does a mathematical outlook have such a great attraction for these thinkers? Why do philosophers attempt to either reduce the world to numbers or to at least understand the world in terms of them? I should like to suggest, as others have before, that, because of the level of certainty associated with it, mathematics is an enticing model for philosophy. Modern philosophers often depict their enterprise as 'scientific' because science is, for the modern world, the model for certitude; mathematics held a similar position for the ancients.⁵

Nicomachus of Gerasa

With these considerations in mind, let us turn to the first of our philosophers, Nicomachus of Gerasa. Little is known of Nicomachus other than his city of origin, his Pythagorean inclinations and his approximate dates (1st-2nd century A.D.).⁶ His influence, however, was widespread, especially through his *Introduction to Arithmetic*, which was known to the West in the translation by Apuleius. Although this Latin version is not extant, it was known to both Cassiodorus (*Inst. 140.16-20* [Mynors]) and Isidore (*Etym. 3.2.1* [Lindsay]), and it was adapted by Boethius for his own *De institutione arithmeticæ*.⁷ Thus, this treatise is an apt beginning for an understanding of medieval philosophical numerology.

The *Introduction to Arithmetic*, as the name suggests, is not a work of philosophical theology. It does, however, have certain theological implications. For instance, when deciding which of the studies (later known as the *quadrivium*) is superior, our author says:

[It] is arithmetic, not solely because we said that it existed before all the others in the mind of the creating God like some universal and exemplary plan, relying upon which as a design and archetypal example the creator of the universe sets in order his material creations and make them attain their proper ends; but also because it is naturally prior in birth, inasmuch as it abolishes other sciences with itself, but is not abolished together with them.

(*Intro. 1.4.1*)

The implication of the passage is that mathematics is necessarily central in comprehending the world. Two reasons are suggested: first, that mathematics was in fact the divine guide in creation, but also that other sciences are reducible to it. Specifically, Nicomachus is drawing upon

a later Platonic development that envisioned Plato's Forms as ideas in the mind of God;⁸ however, he has "mathematicized" these Forms and speaks of number as the ordering principle for the universe:

All that has by nature with systematic method been arranged in the universe seems both in part and as a whole to have been determined and ordered in accordance with number, by the forethought and the mind of Him that created all things; for the pattern was fixed, like a preliminary sketch, by the domination of number pre-existent in the mind of the world-creating God, number conceptual only and immaterial in every way, but at the same time the true and the eternal essence, so that with reference to it, as to an artistic plan, should be created all these things, time, motion, the heavens, the stars, all sorts of revolutions. (*Intro. 1.6.1*)

For Nicomachus any ordering must be fundamentally mathematical, and thus any rational understanding of creation must then be dependent on an understanding of the nature and origin of number.

Before beginning an analysis of number, we must note a curious fact about Greek mathematics; for the Greeks, *one* was not a number, rather the number series began with "two." The Greek word which we translate as "one" (*hen*) could equally well be translated as "unity" or "the unit". For the Greeks "one" was not, in the strict sense, a number but rather the 'unit' of numbers; it is that "identification" out of which number is generated. A particular number series, for instance the 'square' numbers, are derived from the unit; thus Nicomachus says, "for the first [in the square numbers], 1, is potentially the first square, the second 1 plus 3, is the first *in actuality*; the third, 1 plus 3 plus 5, is the second *in actuality*" (*Intro. 2.9.3*). Nicomachus draws upon the distinction between actuality and potentiality as a pattern for analyzing the unit, for, while it cannot itself be a number, it must be the principle of numbers: "Now unity is potentially a perfect number, but not actually, for it is [prime and incomposite] not by participation like the rest, but it is the primary number of all and alone incomposite." (*Intro. 1.16.8-9*) All numbers are implicitly, i.e. potentially, in the unit.

With such a philosophy of mathematics in hand, it is possible to see an analogy between God and the monad. There is a close parallel between the cosmogonical relationship of God and the world, and the arithmetical relationship between unity (or the monad) and numbers—each stands at the beginning of a creation series. While the extant

writings of Nicomachus do not draw this parallel explicitly, it is confirmed by a *testimonium* to his views: "Nicomachus says that God coincides with the monad, being seminally all natural things as the [monad] in number. . . ."⁹

The monad is used as (at least) an analogy for God—if it is not actually identified with God.¹⁰ Creation is then parallel to the expansion of the number series, and as the number series is already implicit in the monad, so creation is implicit in the mind of God.

Macrobius

When we move from the Greek Nicomachus to the Latin Macrobius,¹¹ we turn to a Neoplatonic tradition in which the supreme ontological principle is already designated as the One (to *hen*). Thus, linguistically the monad, as numerical unit and generator of number, is quite naturally assimilated to the supreme ontological principle. Macrobius inherits this tradition but, in his commentary on Cicero's *Somnium Scipionis*, actually associates the monad with all three of the Neoplatonic hypostases: One, Mind and Soul.¹²

This monad, the beginning and end of all things, yet itself not knowing a beginning or ending, refers to the supreme God, and separates our understanding of him (the One, without number) from the number of things and powers following. . . . It is also that Mind which, unaware of the changes of time, is always in one time, the present. . . . Then, too, by giving a little thought to the matter, you will find that the monad refers to the Soul. Indeed, the Soul is free from contamination with anything material, owing itself only to its Creator and to itself, and being endowed with a single nature. . . . (*Somn. Scip.* 1.8-9).

This triple association is important, for unity as represented by the monad is not a property of only one hypostasis; rather, it is identified with all "Divinity", and it emphasizes the simplicity of the Divine. Macrobius is clearly in the tradition of Nicomachus. Unfortunately, he has little else to say concerning the exact relationship between God and the monad.

Macrobius does, however, make some quite suggestive remarks concerning a literary theory which might justify the use of numerology in describing the Divine. Near the beginning of the commentary (chap. 2), before he approaches the *Somnium Scipionis* itself, Macrobius describes the function of different types of fiction.¹³ To begin with, stories can either merely "gratify the ear", such as the comedies of

Menander, or they can "encourage the reader to do good works." (*Somm. Scip.* 1.2.6-8) The class of such virtuous stories can be further subdivided into those which are purely fictitious, such as the fables of Aesop, and those which rest "on a solid foundation of truth." (*Somm. Scip.* 1.2.9) The latter, called fabulous narratives (*narratio fabulosa*), include, for instance, the stories of Hesiod and Orpheus. This class is yet again divided into the fabulous narratives which are base and unworthy of divinities and the fabulous narratives in which "a decent and dignified conception of holy truths, with respectable events and characters, is presented beneath a modest veil of allegory." Only these latter are used by philosophers. But we find that, although these 'worthy' fabulous narratives are quite appropriate for the more mundane philosophical matters "about the Soul, or about spirits having dominion in the lower and upper air, or about gods in general" (*Somm. Scip.* 1.2.13), they are not adequate for treating the highest ontological principles:

When the discussion aspires to treat of the Highest and Supreme of all gods, called by the Greeks the Good (*tagathon*) and the First Cause (*proton aition*), or to treat of Mind or intellect, which the Greeks call *nous*, born from and originating in the Supreme God and embracing the original concepts of things, which are called Ideas (*ideai*), when I repeat, philosophers speak about these, the Supreme God and the Mind, they shun the use of fabulous narratives. When they wish to assign attributes to these divinities that not only pass the bounds of speech but those of human comprehension as well, they resort to similes and analogies. (*Somn. Scip.* 1.2.14)

Here one might see a justification of the use of numerology as an instance of such analogies (*exempla*) or simile (*similitudines*) which is appropriate for expressing the attributes of the highest philosophical principles. The highest hypostasis can only be described as 'the One'. If I am correct in this analysis Macrobius is attempting to incorporate numerology within his 'literary theory'. Numerology is a very peculiar type of *fabula*, one that is adequate for divine allegory. If, as has been suggested (Mras 232-286), Macrobius is here dependent on Porphyry's lost commentary on the *Republic*, then this analysis of analogy may also have been taken over from the Greeks.

Although Macrobius does not explicitly discuss numerology in the context of his classification of fables, elsewhere he does claim that

numbers are adequate for such an analogical task: "in the progress of our thought from our own plane to that of the gods [numbers] present the first example of perfect abstraction" (*Somn. Scip.* 1.5.4). Macrobius envisions several levels of abstraction: first, we abstract from bodies to their surfaces, then from surfaces to lines, and finally from lines to numbers.¹⁴ Thus, "the first perfection of incorporeality is in numbers, and this is the common perfection and fullness of all numbers." (*Somn. Scip.* 1.5.15) This theory of abstraction would seem to underlie the adequacy of numerology for philosophical theology, and it seems reasonable to understand such numerology as the analogy or simile to which the human mind must resort in attempting to comprehend the Divine.

Thierry of Chartres

Finally, we must turn to Thierry of Chartres.¹⁵ Here we find a continuation in the Christian tradition of the identification of God and monad: "No mutability is in God, no plurality. Thus God is also unity (*unitas*)."¹⁶ Thierry also speaks of God as the *forma essendi*, i.e., the form of being: "God is the *forma essendi* of all things. For God participates in nothing." Everything, however, does participate in God, he is the 'form of forms'. Unity grounds the creation of numbers, and the creation of numbers is the creation of pluralized reality; it is the role of mathematics to grasp the procession from the monad, the unfolding of simplicity (*explicatio simplicitatis*).

The extension of this numerological analysis of God into a Christian context does, however, raise certain problems. Though it is, of course, easy to admit that God is unity, the Christian notion of a Trinity becomes immediately problematic. How can there be a seeming triad in unity? The response made by Thierry is striking in its refusal to abandon the mathematical analysis of reality even when such an analysis might seem quite out of place. The answer must be found within a peculiar philosophy of mathematics. Thierry's solution is to distinguish two different types of multiplication, one of which results in plurality and one which does not. Thus, there is no plurality in God. The existence of a divine Trinity is not a plurality but what he would call a repetition of unities.

In the Son, however, there is said to be unity. Equality is said to be where there is neither more nor less beyond that which happens to the existence of the thing. Primary equality is repeated in the multiplication of unity in itself. For unity is nothing if not unity.

For if there should be more or less, plurality is there repeated and thus from a multiplication of unity in itself something other than unity would proceed.

Thierry avails himself of a mathematical distinction between a repetition of equality and a multiplication of unequal entities which results in a plurality, in order to explicate a theological problem.

I am not interested in exploring the philosophical implications or adequacy of this solution here. Rather, I would like to emphasize the importance of this *sort* of solution for appreciating the importance of numbers in a medieval, Platonic understanding of the world. The problem which Thierry confronts might (to a modern) suggest the need to abandon the monad as a model of divinity, or at least to restrict the application of this analogy to the Godhead and reject its applicability to the Trinity. Thierry, however, refuses to abandon the identification of God and monad and insists instead on finding an arithmetical interpretation which allows for a Christian Trinity.

Conclusion

In this brief survey of Nicomachus, Macrobius and Thierry, we have seen the development in the use of the monad analogy for God from the world of Greek antiquity to the Christian Middle Ages. Notwithstanding the numerous permutations, such as Thierry's theory of multiplication, a strongly homogeneous tradition persists. This tradition is characterized by two features: first, an adherence to a mathematical understanding of reality and, second, the adherence to a set of core doctrines, such as the simplicity of God or the location of the Platonic Forms in God.

Notes

1. For a discussion of many of these other aspects of medieval numerology, see Peck.
2. The influence of Pythagoreanism on Plato must be reevaluated in light of Burkert. I do not mean to suggest the traditional interpretation of the philosophy of early Pythagoreanism which regards Pythagoras as an early scientist.
3. Cf. Merlan (11-33).
4. On the role of mathematics in later Greek Platonism, one can consult O'Meara.
5. For a recent criticism of the excessive influence of mathematics on contemporary philosophy, see Rota.

6. On these issues, see the introductory essays by Robbins and Karpinski in D'Ooge. Little attention has been directed at the writings of Nicomachus, but one might consult Hadot, O'Meara, and Tarán. Tarán's study contains information on the ancient commentaries on Nicomachus.
7. I use the text of Hoche and shall quote from the English translation by D'Ooge.
8. Several studies have tried to trace the origin of this location for Platonic Forms. Cf. Armstrong, Jones, and Rich. While it is clear that numbers are closely connected with Nicomachus' Platonic ontology, their exact role is not made explicit. Are numbers merely analogates for creation or are ontological principles in fact reducible to numbers? O'Meara (16-7) has argued that Nicomachus takes the more radical position and actually reduces ontological principles to numbers: "The intention of Nicomachus is, I think, clear: to reduce the Platonic Forms, the models of the universe, which he identifies with Aristotelian categories, to the formal properties of number; the universe is organized according to the various properties of number (p. 17)."
9. All translations of Macrobius are from Stahl.
10. While O'Meara (note 8, above) may be correct in taking Nicomachus to be reducing the Platonic Forms to numbers (and numerical properties), it is not clear that the Greek philosopher took the further step and reduced God to the monad.
11. For an overview of the philosophy of Macrobius, see Gersh.
12. There is a parallel in the Greek tradition where Proclus regards the head of each series as a monad and thus would also describe Mind and Soul as monads.
13. This literary theory and its medieval repercussions are discussed extensively in Dronke, (*Fabula*). He does not, however, make the connection with numerology.
14. A similar doctrine is to be found in *Didaskalikos* 10. The Greek use of abstraction has not been thoroughly studied, but one might consult Weinbert.
15. On Thierry in general, see Dronke, ("Thierry of Chartres"); on numerology and Thierry, see Jeauneau and Brunner, ("Creatio").

Speculum musicae: Jacques de Liège and the Art of Musical Number

Kay Brainerd Slocum

The *Speculum musicae*, written by Jacques de Liège in the early fourteenth century, is not only the largest surviving medieval treatise on music, but is viewed as being "without parallel in its scope and cogency" (Hammond) as a statement and summary of the theory and practice of music in the medieval world. It is well known that the *raison d'être* of the *Speculum* was the "refutation of modern errors and the vindication of traditional authority (Hammond)." Jacques had determined that his mission in life was to write a treatise exposing the errors of the composers of the *Ars nova* and to demonstrate the validity of the teachings of the *Ars antiqua*.

The composers of the early fourteenth century had presented the musical world with a revolutionary doctrine: they had introduced new techniques of rhythmic organization which admitted the possibility of duple meter. Their activities, which they designed as an *Ars nova*, were viewed as a challenge directed not only to the traditional musical establishment but to metaphysical doctrine as well. The encyclopedic *Speculum musicae* was a response to the *moderni* which explored traditional musical thought and provided a thorough explanation of the philosophical rationale for the compositional procedures of the *Ars antiqua*.

Jacques has been denigrated by historians of music as a reactionary figure—a crotchety old man who was tied to the past and who was unalterably opposed to progress in the musical art. An analysis of the *Speculum musicae* as set within the academic tradition has shown, however, that the theorist was not simply an isolated, anachronistic figure who decried the music of the *Ars nova*—he was a product of a university milieu which emphasized the role of music and its sister discipline, mathematics, within the context of the liberal arts. He was, therefore, deeply convinced on a philosophical and theological basis that music should be composed according to certain specific principles.

In order to understand Jacques' views, it is important to examine his philosophy of music within the context of medieval number theory. This study will, therefore, address the following issues: the role of number in Jacques' thought concerning music theory, the sources of his views, and the influence of number theory in the *Speculum* as it relates to concepts of consonance, proportion, *tempus*, and meter. Viewed from this perspective, the *Speculum musicae* will emerge as the creation of an individual who was thoroughly grounded in philosophy and the liberal arts, and who was responding to a challenge from outside that tradition.

Medieval Number Theory

The tradition of medieval number theory which provided the basis for the philosophy of music presented in the *Speculum musicae* developed from three sources: mathematically based musical theory, originating in the teachings of Pythagoras; doctrines concerning order, congruence, measure, number, and weight derived from the writings of Augustine; and the mystical interpretations of number advanced by the neo-Platonists and by pseudo-Dionysius the Areopagite. From these origins medieval thinkers developed a view that there was an order or harmony in the cosmos which had a universal character and appeared in all domains in the visible world as well as the world beyond. The organization of these truths, that is, their relationship to each other and to God, was made possible by the principles of the Pythagoreans, who, according to Aristotle, "...took the elements of numbers to be the elements of all things, and the whole heaven to be harmony and number (Hope 15)."

This concept was expanded by Nicomachus of Gerasa, writing in the first century A.D., to include a Creator who had fashioned everything in the universe according to a pre-determined numerical plan:

...the pattern was fixed like a preliminary sketch, by the domination of number pre-existent in the mind of the world-creating God, number conceptual only and immaterial in every way, but at the same time the true and eternal essence, so that with reference to it, as to an artistic plan, should be created all these things, time, motion, the heavens, the stars, all sorts of revolutions (Heath I, 67).

This cosmic plan was viewed as a graded progression from microcosm to macrocosm which emanated from a First Cause in a way

resembling the progression of the celestial spheres from the *Primum Mobile*. This pattern existed in the mind of God as a design for his material creations. Number was, therefore, the true and eternal essence of the divine plan.

This view of the universe was incorporated into Christian theology along with other forms of numerical speculation. Just as the early Christians undertook the reconciliation of Greek philosophy and oriental mysticism with the Christian religion, they coordinated number theories and used them to serve the true faith; within the Church all contradictions were resolved and reduced to the lucid harmony of ultimate truth (Hopper 9).

Drawing upon this tradition, medieval thinkers were concerned with establishing the numerical relationship of the supramundane, ecclesiastical, and temporal worlds. Philosophers and theologians, authorized by the writings of Augustine, believed that the purely mathematical aspects of number were of divine origin. Mathematics became, therefore, the key to the pattern or form of the world; in another sense it was the method by which the divine intellect became intelligible to human comprehension. Thus, numbers were viewed as divine symbols which should be employed in seeking union of the soul with God.

The principal beliefs concerning mystical number symbolism were derived from the writings of the fifth-century philosopher pseudo-Dionysius the Areopagite. His works posited a procession of emanations from the Superessential, which was unknowable and incomprehensible, to essence, to universals, and thence to individuals. It was impossible for man to know the Superessential until he had become incorruptible and immortal; hence the necessity for symbols. According to the Dionysian treatise, *On Divine Names*:

...we employ...appropriate symbols for things Divine: and then from there we press on upwards according to our powers to behold in simple unity the Truth perceived by spiritual contemplations, and leaving behind us all human notions of godlike things, we still the activities of our minds, and reach...into the Super-Essential Ray, wherein all kinds of knowledge have their pre-existent limits (p. 58).

The symbols "appropriate for things Divine" were believed to be numerical. Further, the Superessential One contained the One existing

as well as every number, but He might be viewed as both unit and triad. As pseudo-Dionysius argued:

...in almost all consideration of Divine things we see the Supreme Godhead celebrated with holy praises as One and as Unity....And It is called the Trinity because Its supernatural fecundity is revealed in a Threefold personality, wherewith all Fatherhood in heaven and on earth exists and draws Its name (p. 56).

The First Cause was both Beauty and Goodness, and all creation was essentially good, since all proceeded from the Monad, which was Unity, and might thus be symbolized as diversity, or the Duad. It was represented by the state of this life, from which the mystics looked forward to a time when the human soul would be absorbed into the divine Triad. Until then, however, they believed that man must employ symbols which would aid in his progress toward knowledge of the Godhead. The symbols which they accepted as the most exact representation of Divinity were numbers; hence, they were committed to finding evidences of significant numerical representation throughout the universe. Hugh of St. Victor, for example, found multiple instances of the Trinity. He pointed out that the Testaments were each divided into three parts: law, prophets, and hagiographers in the Old Testament, and evangelists, apostles, and fathers in the New.² Three hierarchies existed which were composed of God, angels, and man.³ There were three theological virtues⁴ and a triple triplicity of angels imaging the Trinity.⁵ Hugh also adhered to the Dionysian distinction between unity and diversity, finding evidence in the dichotomy between heaven and earth, invisible and visible, angels and man, prelates and subjects, contemplative and active, spirit and flesh, Adam and Eve, perfect and imperfect.⁶ In pronouncements such as these number was viewed as abstraction, symbol, or idea; it became part of a philosophical system, rather than simply a summation of concrete units.

These symbolic numbers formed an essential feature of the eternal pattern, and were seen to be an important part of the church triumphant; hence, it was necessary that they be manifested in the offices and monuments of the church on earth. Evidence of this may be found in the "frozen eloquence" of cathedral architecture, and it was the conviction of medieval thinkers such as Bacon, Aquinas, and Bonaventure, as well as the music theorist Jacques de Liège, that the music composed for use in the divine service must also reflect the eternal

order. To this end they all agreed with Bacon that "...the gate and key is Mathematics, which the saints discovered at the beginning of the world...and which has always been used by all the saints and sages more than all the sciences (Bacon I, ch. 1, p. ll6)."

Number Theory in the *Speculum musicæ*

This tradition of medieval number theory provided several concepts which became basic principles for Jacques de Liège. From this tradition he assimilated the view that there was an order or harmony in the universe which was manifested in the sensible world; it was his contention that music should reflect this harmony, becoming an audible representation of the divine plan. In the *Speculum Musicae* Jacques adopted the general concepts of medieval speculation concerning numbers, but he took great care to formulate a solid mathematical foundation for his views. His essential purpose was to show how music and number were related, and how music provided, in sound, an apprehensible reflection of the transcendent numerical order. Hence, he began his work with a detailed discussion of the various integers and of arithmetical, geometric, and harmonic progressions and proportions. Jacques emphasized the value of mathematical erudition, for he believed that there could be "no truth, no thorough knowledge, no proper philosophy" without it.⁷ Following Boethius, he adhered to the division of mathematics into two categories: arithmetic, which concerned primarily number in its abstract and absolute sense, and music, which dealt with number as "applied" to sound.⁸

Jacques devoted a great deal of effort to the demonstration of mathematical calculations, showing, for example, the divisions of numbers, the construction of multiplex and sub-multiplex numbers, and the generation of superparticular and subsuperparticular numbers. These matters pertain essentially to his discussion of mathematical theory. There are, however, several basic numerical concepts which apply directly to his musical philosophy.

Unity was the first principle of all things; it was that quality which defined the essence of an object.⁹ Unity, furthermore, transcended living things; it extended beyond existence to divinity,¹⁰ for God was the truest, most simple, most actual unity, forever unchangeable and eternal.¹¹ Unity was thus the root of equality and identity, unalterable and indivisible.

The number two represented the opposite principle, receding from unity in alterity, mutability, diversity, and division. Breaking away

from unity, it departed from the utter simplicity which was God. The binary principle was thought to be disgraceful, for it was the first to recede from unity and was thus the principle of division.¹² Since it receded from the One, it signified sin, which deviated from the First Good. Furthermore, two could be divided by two; it was corruptible and transitory.

In the number three, however, was found perfection; it was indissoluble and incorruptible; it could not be divided by two, since unity existed in its midst. Calling upon the authority of Aristotle, Jacques recounted that three contained unto itself three dimensions: longitude, latitude, and depth, not simply lines and surfaces. Furthermore, God, who was most perfect within Himself, and from whom all perfection proceeded, proved that perfection existed in the number three since He reached perfection in the Trinity. He, who created everything according to measure, weight, and number, rejoiced in odd number.¹³

Bringing additional material to bear, Jacques noted that the Pythagoreans concurred in the belief that the principle of unity was echoed in the number three. It represented all reality, because it had a beginning, a middle, and an end. Unity and diversity combined within the triad to form harmony; here the first and second principles were linked into a single complete order (Hopper 42). This concept had provided Christian theologians with an obvious analogy: the Pythagorean triad was reflected in the Christian Trinity.

The number four also held great meaning for Jacques, who demonstrated how it was reflected in the four elements of the earth, the four humors of mankind, and the four cardinal virtues. It was, furthermore, the number which completed the decad ($1+2+3+4=10$).¹⁴

These ideas, which Jacques drew from medieval number theory, became basic principles in the musical philosophy of the *Speculum*, for he viewed numbers as symbols which could represent the macrocosm in the microcosm: numbers reflected the divine plan of God in the sensible world. The numbers which held the greatest significance were the members of the tetrad, for each embodied a specific principle. The number one represented the first principle of all things: it was evident in all objects, but extended beyond existence to God, who was the truest unity. The number two represented opposition, for it receded from unity, or God; it was, therefore, imperfect. The number three, however, embodied perfection, for it echoed the Trinity. The number four was reflected in the sensible world in a variety of ways: it was,

furthermore, the integer which completed the decad. These concepts were of primary importance in musical number theory as developed in the *Speculum musicæ*, for it was the relationship between these integers, according to Jacques, which determined the values of musical proportion and temporal mensuration.¹⁵

Music and Number

Jacques believed that numbered sound was the principal subject of music, especially as number pertained to harmonic modulation, or proportion, as described in formal theory.¹⁶ He explained that concepts of numerical proportion could be applied to the relationship of two sounds which occurred together, for musical intervals could be thought of as ratios between numbers; proper numerical ratios, furthermore, formed the basis for musical consonance.¹⁷

Since, according to Jacques, all music should be founded upon reason and speculation, the real *musicus* was that man who possessed the ability to judge intervals, modes, rhythms, and melodies according to a true understanding of the numerical properties of music.¹⁸ It was important, therefore, for him to refer consistently to number, for there was an innate connection between proportion and number: it was not possible to probe the proportions of musical consonance, nor to judge them as better, more certain, or truer, without reducing them to their numerical equivalents. Hence, the study of number was basic to the accurate perception of music.¹⁹ It was the responsibility of the *musicus* to discern the way in which consonance was founded in proportion; he should not, however, look at dimensions only, as did the *mathematicus*, but should be concerned with quality, order, connection, and proportion.²⁰

Essential to this understanding was the knowledge of Pythagorean number theory, a doctrine which held that all matters concerning musical intervals could be explained by arithmetical methods. Since the Pythagoreans dealt only with that aspect of musical sound which could be described numerically, they gave their attention to musical intervals, because these were readily expressed as ratios of integers (Crocker 192).

The Pythagoreans were committed to the study of particular kinds of musical intervals, and, in addition, the basis of integer-arithmetic injected a certain hierarchy of value into their perception of these intervals. The numbers at the beginning of the integer series became ideal forms, or prototypes, of the larger ones. Because they had fewer

elements, these numbers were simpler, and hence better. The basic proportion established between them became the musical ratios which must be manipulated by reason in order to discover the proportions of all other musical intervals.

The Pythagoreans found that the ratios 2:3 and 3:4, when combined, formed an octave, which in turn formed a double octave with the first ratio. This is the only case in the integer series where two consecutive ratios, when multiplied, combine to produce the ratio immediately preceding [(2:3) x (3:4) = (6:12) or (1:2)]. For the Pythagoreans, this was an example of the unique quality of the small numbers. This fact, together with the observation that the sum of the tetrad was ten, the perfect number, had great impact; the tetrad became a cornerstone of Pythagorean musical theory, fixing the number of consonances.

In addition to the speculative, mathematical method of defining musical theory, Pythagoras also utilized empirical procedures. According to tradition, he determined the consonances by striking sounding hammers of precise weights, and the resulting intervals confirmed his mathematical calculations. Thus, Pythagorean numbers were not merely ideal; they were exemplified in the sensible world through an experimental process which showed that the ratios, 1:2, 2:3, and 3:4, actually sounded more harmonious than other intervals. Thus, the arithmetical method was confirmed by empirical evidence.²¹

Jacques recounted the Pythagorean legend not simply to embellish his own work, as was often the case in medieval treatises,²² but to underscore an important philosophical point, that is, the founding of musical theory on scientific principles.²³ Jacques, however, was careful to mention that Pythagorean theory was only a secondary verity which emanated from the *prima veritas*, the deity Himself, and was a reflection of the divine order.²⁴

Mathematical theory and empirical method were both reflected in the Pythagorean classification of the intervals according to the degree of simplicity of their numerical ratios, for the simpler intervals not only fulfilled the requirements of number theory, they also provided the most effective musical harmony. There were three kinds of intervals: (1) multiple intervals, the ratios of which can be expressed by the formula $n:1$, n being positive and whole; (2) superparticular intervals with the formula $(n+x):n$, in which $x = 1$; and (3) superpartient intervals with the formula $(n+x):n$, in which x is larger than 1.

Multiple intervals include the *diapason* (octave), which has a ratio of 2:1, the *proprio dupla*; the *diapason cum diapente* (twelfth), which has a ratio *proprio tripla*; and the *bis diapason*, (double octave), which has a ratio of 4:1, the *proprio quadrupla*.

Superparticular intervals comprise the *diapente* (fifth) which has a ratio of 3:2, the *proprio sesqualtera*; the *diatesseron* (fourth), which has a ratio of 4:3, *proprio sesquitertia*; and the *tonus* (major second), which has a ratio of 9:8.

The ratios of the superpartient intervals, which include the *ditonus* (major third), the *semiditonus* (minor thirds), the *tritonus* (tritone), the minor second (*semitonium*), the major sixth, (*tonus cum diapente*), the major seventh (*ditonus cum diapente*), and the minor seventh (*semiditonus cum diapente* or *duplex diatessaron*), become increasingly complex. For this reason they were viewed as less harmonious.

Jacques dealt in detail with all of these intervals, classifying and describing them as consonances or dissonances according to their ratios. Following the Pythagoreans, he defined consonant intervals according to the proportions which existed between the numbers of the tetrad, for he believed that these relationships possessed more integrity, unity, and concord. Hence, all multiple intervals were consonances, and so were all superparticular intervals, with the exception of the major second.²⁵ The other intervals were considered to be dissonances. Thus, the consonances comprised the octave or *diapason* (2:1), the twelfth, or *diapason cum diapente* (3:1), the double octave, or *bis diapason* (4:1), the fifth or *diapente* (3:2), and the fourth, or *diatesseron* (4:3).²⁶ Jacques defined these five intervals as consonances according to the following criteria: the less the larger number departed from the smaller number, or the more "equality" there was between the two numbers, the more harmonious the consonance produced by the proportions. Jacques, however, thought that the Ancients had ignored the best and most perfect of all consonances, the unison, which fulfilled these requirements most accurately and completely.²⁷

In Jacques' classification of intervals the unison was the first major consonance, which he defined as the foundation of all the others. The unison provided the most perfect concord because its tones were not unequal but equal sounds, one tone being neither higher nor lower than the other; thus, according to Jacques, it exuded no dissonance whatever. Furthermore, the unison was the most perfect consonance because it was closest to fundamental numerical unity (Smith II, 31).

This emphasis upon the unison as an ideal was undoubtedly a reflection of a general religious value, representing for Jacques the identity of musical expression with God, the One, the Creator.

According to Jacques, the value of the numerical proportions, and therefore of their respective consonances, was determined by the degree of their equality. Thus, he believed that the *diapason* produced the most perfectly concordant tones because it most nearly approached the unison (1:1) in closeness of proportion (2:1).²⁸ Multiple proportions were especially tied to the concepts of integrity, unity, and concord, since they most readily returned to basic unity.²⁹ Thus, the proportion 2:1 was to be preferred, since it could be easily reduced to 1:1.

The *diapason* was a most important consonance not only because of the perfection of its proportional relationship but also because it contained within itself the other consonances, that is, the fifth (*diapente*) and the fourth (*diatesseron*). According to Jacques, it produced them in the same way that basic unity provided the source of all numbers. Unity came before all other numbers; in a similar manner the *diapason* preceded the other consonances in value and provided their foundation.³⁰

Proportions of the multiple type (including the *bis diapason*) were closer to equality than proportions of the superparticular type, for multiples were a natural series of numbers related to the first number, that is, one or unity.³¹ Superparticular proportions, on the other hand, were not related to the first number in a natural numerical series, but rather began in plurality. These proportions were ranked according to their proximity to equality.³²

The *diapente* was defined as a consonance of unequal tones in sesqualtera proportion: 3:2. It was a superparticular proportion which contained the number to which it was compared, and also a half of it ($2+1=3$). Just as the *diapason* preceded the other consonances defined by multiple proportion in the order of hierarchical perfection and ontological goodness, the *diapente* was the most perfect among those founded on superparticular proportionality, specifically the *diatessaron* and the *tonus*.³³

The *diatessaron* was of sesquitertien proportionality (4:3). Jacques remarked that the fourth was a consonance of unequal voices which sounded together harmoniously in a way that was pleasing to the ear.³⁴ In fact, he pointed out that certain musicians, the monks of Saint Gall, for example, seemed to prefer this consonance, and used it regularly in

their discant, although "it was more imperfect and uncivilized than the *diapente*, in the judgment of the ear as well as the intellect." This consonance, in his opinion, sounded better when used above a fifth rather than when used simply by itself, because the outer voices would then produce a *diapason*: hence it was an excellent combination for use in completing a mensural cantus.³⁵

The other simple type of proportion, superpartient, departed from the simplicity and completeness of the first two species, and did not contain musical harmony. Jacques wrote that all the discords were based upon superpartient proportionality; he believed they were discordant because their numerical proportions yielded values quite remote from the basic unity and simplicity characteristic of the concordant consonances; thus, dissonance was a result of the imperfection of the relative proportions.³⁶

Jacques concluded, therefore, that only proportions of the multiple and superparticular types could produce musical consonances. Those produced by multiple proportions were the most harmonious, for they were the most easily perceived by virtue of their simplicity, and their proportions were nearest to equality. Consonances represented by the superparticular proportions followed these in value, and again the proportions closest to simplicity and completeness sounded the most harmonious. Jacques remarked that it was appropriate that the consonances of superparticular proportion were three in number, for three provided the primary principle: everything perfected in three imitated the Trinity of Divine persons.³⁷

Thus, the hierarchical order of consonances after the unison proceeded as follows: the octave (2:1), the twelfth (3:1), the double octave (4:1), the fifth (3:2), and the fourth (4:3). As we have seen, this gradation was determined according to the degree of proximity of each ratio to unity; in musical terms, by the closeness of each interval to the unison. In addition, this hierarchy had a philosophical dimension, as Jacques was careful to demonstrate. Each form was defined as more or less "perfect" according to the ease with which it might be reduced to its primary structure; the more simple represented the more perfect.³⁸ Perfection proceeded from the integrity and totality of the whole based upon both material and form, but principally upon form.³⁹ The degree of integrity of form, furthermore, determined the goodness of an object; this goodness could be discerned within a musical composition by applying the canons of numerical value.⁴⁰ This

philosophical matrix provided the justification, indeed the necessity for principles of musical composition—principles which the composers of the *Ars Nova* refused to observe.

The *moderni* deviated from the metaphysical structure which provided the philosophical foundation for traditional methods; their compositional procedures departed radically from the practices of the past, as represented most appropriately in the music of Franco of Cologne.⁴¹ According to Jacques, their music lacked fundamentals;⁴² he was distressed because they failed to observe established principles of composition in two important areas: harmony, or consonance, and measured time, or rhythm.

The Moderni and Consonance

Jacques objected to the ways the *moderni* utilized harmony in their composition of discant, for he believed that highly specific rules should obtain.⁴³ According to Jacques, the added voice should match the tenor note for note, and concord should be perfectly observed, with the *diapente* (fifth) receiving special prominence.⁴⁴ The resulting discant would produce one sweet sound from several distinct ones—not merely simple unity—but harmonious and concordant mixtures.⁴⁵ Three or four consonances should be used to create this euphony: the unison, fourth, fifth, and octave.⁴⁶ These were the intervals which represented the ratios of perfect form.⁴⁷

In his opinion, music for church ceremonies should be constructed in this manner, for it must be perfect, having neither "wrinkle" nor "blemish"; discords produced disgrace.⁴⁸ The methods used by the *moderni* in composing discant offended the intellect and the senses; whereas music should produce delight, it induced sadness instead. Jacques saw this as abuse, rudeness, even beastiality! "It seemed as if an ass was dressed as a man, a nanny goat as a lion, a bird as a fish, a snake as a salmon," for consonances and distinct dissonances were not separated from one another.⁴⁹

He also had criticism for those who did not know how to sing steadily and well: they dared to sing discant, and they substituted *cantus mensurabilis* for *cantus planus*.⁵⁰ The *moderni*, furthermore, not only failed to observe the proper methods, they sang too wantonly, and they inserted many superfluous voices. Some used too much "hocketing"; they fragmented consonances, they danced in inappropriate places, they barked like dogs; and like insane people, fed by confused and distressing digressions, they used harmony of a remote nature.⁵¹

This strange harmony occurred when singers discanted wantonly, using their windpipes in a "certain way." A more natural and moderate harmony should be used in the church of God, for it exhibited the most clarity, producing concord with the tenor in discant.⁵² Jacques chastized the new composers for dismissing the "ancients," and for using new methods and new musical forms. Their music was too difficult to sing, especially with regard to rhythm and meter.⁵³ Once again they had abandoned traditional practice, ignoring the canons of *tempus perfectum* and *tempus imperfectum*.

Tempus and Meter

The innovations of the composers of the *Ars nova* in the realm of rhythm, or "measured time", represented their strongest challenge to established musical tradition. By introducing the revolutionary concept of duple meter, they had abandoned the perfection of ternary theory, which was the rhythmic reflection of the Trinity.

Tempus perfectum, according to the "Ancients," was a system in which the *recta brevis* could be divided into three equal, or two unequal parts,⁵⁴ thereby reflecting the ternary number system. The *moderni*, by contrast, introduced a concept of notation which allowed the *brevis* to be divided into two equal parts as well as into three.⁵⁵ The traditional system allowed this division only within the category of *tempus imperfectum*,⁵⁶ whereas composers of the *Ars nova* mixed the two categories, making imperfect and perfect interchangeable. Citing Averroes, "the Commentator," Jacques asserted that these divisions must be distinguished according to the rules of logic, and that the distinction must be formal as well as material.⁵⁷

Amplifying his argument with a quotation from Aristotle, Jacques stated that whatever was called perfect must be based upon the ternary system, for that which was binary was imperfect.⁵⁸ As we have seen, the binary number was called infamous and imperfect, not only because it receded from unity, but also because it contained two parts.⁵⁹ If *tempus perfectum* were divided into two equal parts, it would become imperfect. Even if the binary series were continued to infinity. Jacques pointed out, it would still be imperfect, for the formal distinction of imperfection is the division into two.⁶⁰

Following Aristotle, Jacques declared that the perfection of all material things depended upon the ternary principle, for they possessed three dimensions—longitude, latitude, and depth. God, furthermore, who was most perfect in Himself, and from whom all perfection

proceeded, proved that perfection consisted of the ternary principle, since He reached perfection in the Trinity, which embodied also the greatest simplicity and unity. In all other things—angels, men, and other creatures—there was a vestige of the Trinity, since they were created in the image of God.⁶¹ All of this was brought to bear on mensural music, for Jacques believed that music should be fashioned from perfection and should echo the Trinity, which the imperfect notes and melodies of the new composers failed to do.⁶²

He utilized the tenets of his doctrine to refute several arguments which the *moderni* had brought forth to justify their methods of competition.⁶³ One theorist claimed that music could be crafted from imperfections before God assumed human form; it was, therefore, a valid procedure. Jacques insisted, however, that God had always existed as Three and One and will be so forever. Thus, only music based upon the ternary principle could be reduced to His perfection; melody which utilized imperfection was absolutely unacceptable.⁶⁴

Another argument advanced the theory that since God is three in person but only one in substance music should not necessarily reflect three more than one. For Jacques, however, natural melody referred to both the divine Trinity and Its unity, for the number three was the principle of perfection and distinction within the Trinity, whereas unity provided the foundation for the concord required in ecclesiastical music; this concord reflected the simple unity of the divine essence.⁶⁵

A third argument claimed that God was three and one regardless of whether music was composed using perfections or imperfections. Jacques replied that God was intrinsically perfect and that aspects of the material universe were ordered according to His perfection;⁶⁶ music, in his opinion, must exhibit this order. Jacques' responses were derived from the basic philosophical premise of the *Speculum musicae*: all being participated in the Divine Exemplar; all good in the prime Good which was good in essence; all truth in the prime verity which was God. All multiplicity proceeded from unity just as all being derived from the prime Being.⁶⁷ The *Speculum* was an attempt at demonstrating the ways in which music, properly understood, should reflect the Divine Exemplar.

Utilizing numerical theory and Aristotelian logic, Jacques formulated a defense against the compositional procedures of the younger generation. He drew upon sources from the ancient world as well as traditions characteristic of his era. His work was a "mirroring"

of the musical philosophy of the thirteenth century, and of the university intellectual tradition of which he was a product.

Notes

1. The bibliography concerning Jacques de Liège is limited, despite his importance. This is due, in part, to the lack of a complete published edition until comparatively recently, when Bragard's seven-volume edition became available. Prior to this time only the sixth and seventh books of the *Speculum* were readily accessible, in Coussemaker. There is also an English translation of part of the seventh book in Strunk's work.
2. Hugo of St. Victor. *Exegetica*. PL 175. cap. VI. "Utrumque Testamentum tribus ordinibus distinguitur: Vetus Testamentum continet legem, prophetas, agiographos. Novum autem Evangelium, apostolos, patres."
3. Hugo of St. Victor, *Commentariorum in Hierarchiam Coelestem S. Dionysii Areopaoite*. PL 175. cap. 111. col. 929-930. "Tres sunt hierarchie, in quarum descriptione theologus et narrator hierarchium et potestatum sacrarum quae in coelo et in terris sunt. Dionysius, opus consummatum explicit. Prima principialis omnium est Trinitatis....Secunda hierarchia in angelica natura formata est....Tertia, et ultima hierarchia in humana natura ordinata est..."
4. Hugo of St. Victor, *Summa Sententiarum*. FL 176, cap. I, II, III. col. 42-43. "De Fide...De ape et charitate."
5. Hugo of St. Victor, *Summa Sententiarum, Tractatus Secundus*. PL 176, cap. V. col. 85. "Et inveniuntur in istis ordinibus tria terna esse; et in unoquoque tres ordines, ut Trinitatis similitudo in eis prae aliis creaturis impressa videatur. Sunt enim tres ordines superiores, tres inferiores, tres medii."

6. Hugo of St. Victor, *Allegoriae in Vetus Testamentum*. PL 175, col. 635, 639. "Coelum designat summa, terra ima; coelum invisibilia, terra visibilia; coelum angelos, terra homines; coelum spiritualia, terra corporalia.... Sic illa coelestium spirituum angelica natura, creaturas universas, et coelestis patriae mansione, et conditionis suae dignitate superat, et eis superemicit. Terra significat hominem...coelum significat praelatos, perfectos, contemplativos. Terra autem significat subditos, imperfectos, activos. Coelum igitur angeli terra homines: coelum praelati, terra subjecti; coelum perfecti, terra imperfecti; coelum contemplativi, terra activi...Adam significat spiritum. Eva, carnem."
7. *Speculum* 1: 30 -...Boethius scientias commendet mathematicas, ut sine illis nullum dicat inventorem esse veritatis, nullum dicat esse recte sapientem, nullum dicat esse recte philosophantem."
8. Ibid., p. 31. "Eodem modo exponatur musicum esse de numero vel multitudine ad aliquid, vel quia est de multitudine concreta vel contracta ad alias res speciales, ut ad sonos, quantum ad musicum sonoram..."
9. Ibid., p. 91. "Unitas igitur est qua unaquaeque res est id quod est."
10. Ibid., p. 92. "Etiam se extendit ad ens omnino simplex, illimitatum et infinitum, extra omne genus existens, ut est unitas divini esse."
11. Ibid. "Deus enim verissime unus est, sicut suum esse est verissimum esse simplicissimum, actualissimum, omnino immutabile, omnino aeternum."
12. Ibid. p. 142. "Quia dictum est unitatem radicem primam aequalitatis vel idemperitatis, immutabilitatis, inalteritatis esse et indivisionis, et quod oppositarum conditionum binarius princeps est, haec aequaliter declaremus: Repugnat unitati mutabilitas, alteritas, diversitas et divisio, quia formale sibi est indivisionem, ut supra dictum est, importare. Alteritatis et divisionis est...Merito igitur binarius alteritatis principium est qui immediatus ab unitate recedit; primus omnium unitati est dissimilis, primus omnium ab unitate disiungitur. Sicut igitur recedens a simplicissimo (eiusmodi Deua est) ad aliquam accedit compositionem vel componibilitatem, ut sit, in se, compositum compositione aliqua de prius tactis, vel alteri componibile, sic recedens ab unitate in aliquam cadit multitudinem, divisibilitatem, mutationem et alteritatem; nihil enim

- sibi ipsi alterum sed idem est; sed prima alteritas in duobus ab invicem distinctis est."
13. Ibid., VII: 30. "In tantum autem ternareitas perfectionem designat ut sola corporeitas perfecta dicatus quantitas, secundum philosophum, primo, Coeli et Mundi, quia claudit in se tres dimensiones: longitudinem, latitudinem et profunditatem; non sic linea, non superficies. Item Deus, qui est perfectissimus in se et a quo omnis procedit perfectio, sic approbat in ternario perfectionem consistere ut in ipsa perfectissima sic Trinitas cum simplicissima tamen unitate. Et in aliis etiam omnibus aliqualiter trinitas contineatur quia creatura omnis vel est producta ad Dei imaginem, ut sunt Angeli et homines, vel ad Dei vestigium ut ceterae creaturae. Omnia enim in numero, pondere fecit et mensura. Numero Deus impari gaudet. In Angelis tres sunt hierarchiae quae quaelibet per ternarium distincta est."
 14. Ibid., V1:53. "Iam igitur de quaternarii perfectione dicamus quae occurunt. Artius tunc extrema iunguntur cum medietad geminatur: sic autem est de quaternario. Qua in re, artifex natura, iuddu Creatoris, non immerito hunc sibi numerum ascivit quo insolubili colligationis vinculo quattuor elementa sibi repugnantia sub quadam proportione devinxisset. Merito, inquam, hic numeris musicae harmoniae attributus est cuius dispositione fere tota harmonia videtur conatitui. Hic est numerus qui inferiora superioribus conciliat corporisque statum in quattuor humoribus iugabili proportionum competentia foederat, bestiales hominum motus comprimit, mores in quattuor virtutibus cardinalibus componit, iras daemonum mitigat vel potius fugat, etiam iosma terram sonorum suorum <concentibus> coelo quodammodo associat. Hic est numerus quem, si praecedentibus se terminis naturalibus, idest uni, duobus, tribus adiunxeris.....denarii summam conficies, ut ex hoc etiam eius perfectione in decahordo assuescamus psallere psalterio quatenus Deo nostro jocunda decoraque sit ludatio."
 15. Ibid., I: 31. "Eodem modo exponatur musicam esse de numero vel multitudine ad aliquid, vel quia est de multitudine concreta vel contracta ad alias res speciales, ut ad sonos, quantum ad musicam sonoram, vel ad alias res, quantum ad alias musicae species, vel quia primo et principaliter considerat de numero ad aliquid, ad quem habet recursum ad iudicandum de proportionibus

- rerum ab ipsa speculandarum vel mensuris, et cum de numero etiam ad aliquid tractet arithmeticas, subalternabitur musica arithmeticae."
16. Ibid., p. 66. "subiectum autem huius musicae sonus est numeratus vel sonorum numerue invicem collatorum ut concurrunt scilicet ad aliquam hermonicam modulationem faciendum, quae vel est ratio formalis in obiecto suo, vel ipsius propria passio."
 17. Ibid., p. 191. "...consonantiae autem harmonicae in proportionibus sunt numerorum ad aliquid."
 18. Ibid. p. 67. "Hanc igitur consonantiam ex numeratis sonis invicem collatis proportionatisque nascentem sonora musica principaliter considerans tractare quaedam habet de sonis, quaedam de numeria, quaedam de proportionibus. Subest enim scientiae naturali, quoad sonum, et arithmeticae, quoad numerum. Sunt enim soni consonantiae materia: quos invicem habet comparare et quam reddant harmoniam. quam faciant consonantiam. discernereque <distincto distinctum> et ipsorum musicus scire debet proportionem."
 19. Ibid., p. 102. "Adhuc considerat musica de numero ad aliquid, qui ad sui consistentiam alium requirit numerum ad quem refertur, quia inter numeros sic comparatos quaedam innascitur habitudo, quaedam proportio aequalitatis vel inaequalitatis, et non solum inter res speciales ad invicem comparatas. Nam musicarum consonantiarum proportiones, secundum Boethium, non melius, non certius et verius probari possunt quam prout reducuntur ad proportiones numerorum ad aliquid."
 20. Ibid., p. 35. "Musicus autem non ad dimensiones tantum inspicit, sed ad qualitates et ad ea secundum quae, ut inter se comparantur, innascitur ordo, connexion, proportio. Nec inspicit musicus dimensiones illorum absolute, nec quantitatem continuam, sed dimensiones invicem numeratas, ad aliquam connexionem, ordinem et proportionem applicatas."
 21. Ibid., IIa: 15. "Unde Pythagoras fundamentales consonantiarum proportiones in ponderibus repperit malleorum sonantium. Nam, antequam malleos sonantes ponderasset, audiebat sonorum mixtiones et consonantias, sed in qua fundarentur proportione (et quod hic determinatae <sunt>, ista, scilicet diapason, in dupla, illa in alia, ut diapente in sesquialtera, sic de aliis) nescivit donec illos ponderasset malleos et ipsos ad invicem, ut expediebat, combinasset."

22. Ibid.. V: 8 ".....videatur qualiter harum Pythagoras proportiones adinvenerit, unde multum musicae crevit scientia quae certis et veris sicut et ceterae fundari debet principiis."
23. Ibid., p. 9. "Tandem igitur nutu Dei, qui est prima veritas a qua omnis secundaria procedit veritas, Pythagoras, cum iter ageret, fabros malleis quinque super incudem percutientes audivit."
24. Ibid., IIa:20. "Hoc autem erit unum quod postea diligenter conabimur inquirere: in qua scilicet determinata proportione in numeris quaelibet consistat consonantia quoad consonantiae voces extremas et, si possumus, qui <sint> primi numeri proportionum illarum."
- Ibid., p. 38. "Musicales consonantiae fundari debent in proportionibus quae magis accedunt ad integratatem, unitatem et concordiam. Hae autem sunt proportiones multiplices et superparticulares."
25. Ibid., p. 17. "Consonantia autem specialiter et appropriate dicta est illa quae dicitur de distinctis sonis concorditer et conformiter auditui se facientibus et quorum mixtio in proportione simplici, multiplice, vel superparticulari fundatur, et, hoc modo solum, videntur antique musici consonantiam accepisse qui non posuerunt nisi quinque consonantias, scilicet diapason, diapente cum diapason, bis diapason, diapente, et diatesseron."
26. Ibid., p. 29. "Unisonus potest sumi pro sono uno continuo et indistincto, et, hoc modo, non est consonantia, ut patet ex dictis. Alio modo sumi potest pro pluribus sonis numeratis et distinctis, aequalibus tamen, quorum unus non eai alius, et, hoc modo, ipsum sumendo, tenetur a multis unisonus esse consonantiam veram et omnium primam. Competunt enim aibi ea quae sunt de intrinseca ratione consonantiae simpliciter, generaliter et absolute sumptae. Haec enim ad consonantiae natural sufficere videntur, ut sit quaedam harmonica modulatio, id est permixtio sonorum distinctorum in medio et apud auditum ad certam reducibilis proportionem. Unisonus autem diatinctos requirit sonos, licet sint aequales. Aequalitas autem stat cum distinctione nec excludit eam, et non est dubium quin distincti soni aequales simul producti (puta se duo vel plures in eadem cantent gravitate) invicem misceantur....Item talis mixtio in certa fundatur proportione, scilicet in aequalitate quae est vera et prima omnium proportionum aliarum. Omnis enim inaequalitas, ut ostensum est, ab aequalitate nascitur et in ipsam

- resolvitur et tanto perfectior et maior dicitur proportio, quanto in immediatus nascitur ab aequalitate."
27. Ibid., p. 31. "Nec solum videtur unisonus consonantia, sed prima omnium simpliciter et summa, ut, sicut aequalitas omnem inaequalitatem antecedit, sic consonantia, super aequalitate fundata cuiusmodi est solus unisonus, praecedat omnem consonantiam super inaequalitate fundatam. Etsi propter quod unumquodque tale et illud magis diapason dicatur prima inter omnes consonantias vocum inaequalium, quia dupla proportio immediatus sequitur et resolvitur in ipsam aequalitatem....ipse unisonus super vera aequalitate fundatus erit simpliciter prima omnium consonantium, quia et voces suae summe uniuntur. Et sicut aequalitas fundamentum est omnium proportionem, sic et unisonus omnium consoniarum. Unde, supra tenorem, in qua sunt voces solum unisonum inter se habentes omnis cantus et discantus, et consonantia fundari potest."
28. Ibid., p. 45. "Concordi satis sententia tenuerunt Antiqui diapason inter consonantias sonorum inaequalium, de quibus loquuntur, esse primam et optimam. Et quod sit prima via perfectionis, probatur per hoc, quia fundatur in prima proportione numerorum inaequalium cuiusmodi est dupla proportio, quia etiam in mediatione sue simplicitatis et perfectionis sequitur unisonum qui in vera fundatur aequalitate. Dupla enim proportio immediatus aequalitatem respicit et unitatem, et, per easdem rationes, probari potest diapason esse optimam et perfectissimam consonantiam."
29. Ibid., p. 48. "Diapason itaque, secundum hoc, est simplex consonantia, imo simplicissima, inter consonantias sonorum inaequalium, quia dupla proportio, cum sit prima omnium proportionum terminorum inaequalium, et immediatus in aequalitatem reducitur, sicut ab ipsa nascitur."
30. Ibid., p. 46. "Dicit enim quod, sicut unitas est principium crementi et diminutionis in numeris, sic diapason in consonantias, quia non habet aliquam proportionem sibi oppositam. Ut aliae... sicut igitur in prima figura, unitas omnem antecedit numerum, et, per se sumpta, cui apponatur, non habet, sic, in secunda figura, diapason ceteras praecedat consonantias. Quia eius fundamentalis, ut est dictum, proportio immediatus et prius simplicem sequitur unitatem et per prius reducitur ad aequalitatem, nec habet aliquam sibi oppositam proportionem."

- Ibid., p. 49. "Supposito enim quod diapason in se contineat diapente et diatessaron, quae, secundum sub et supra, praecise reddant diapason...."
31. Ibid., p. 26. "Si in multiplicibus, vel in prima proportione numeri multiplicitatis, scilicet in dupla, sic est diapason, vel in secunda, acilicet in tripla, sic diapente cum diapason, vel in tertia scilicet in quadrupla, sic est bis diapason."
32. Ibid., p. 74. "Praecedat haec consonantia ordine perfectionis et bonitatis ceteras consonantias in superparticularibus fundatas proportionibus, scilicet diatessaron et tonum (sicut diapason fundatas in proportionibus multiplicibus de quibus iam dictum est), quia fundatur in prima proportione superparticularitatis quae perfectior est ceteris, sicut diapason in prima multiplicitatis ad quam immediatus sesqualtera reducitur proportio."
33. Ibid., p. 73. "Dispente est consonantia vocum inaequalium concorditer sonantium in sesqualtera consistens proportione, per hoc, quod dicitur "concorditer sonantium," convenit dispente cum consonantibus quarum voces, simul prolatæ, concordant et auditui placent, est enim dispente, secundum Ptolomaeum, de numero consoniarum quarum voces sunt consonae, a consonande, idest a concordando, dictæ, et non a simul sonando solum, sed, per hoc, quod additur, "in sesqualtera consistens proportione," ab omnibus distinguitur consonantibus. East autem sesqualtera proportio nascens habitudo inter terminos inaequales, quorum maior minorem habet totum et eius medianam partem praecise, ut sunt tria et duo, qui sunt minimi et primi termini huius proportionis, vel VI et IIII, qui sunt primi secundarii termini dictæ proportionis."
34. Ibid., p. 81. "Diatessaron est sonsonantia vocum inaequalium concorditer sonantium in sesquitertia consistens proportione. Per hoc quod dicitur "concorditer sonantium," convenit haec consonantia cum his quarum voces dulciter se faciunt apud auditum, quae appropriato nomine non modo consonantiae sed concordantiae dici possunt."
35. Ibid., p. 82. "Hac consonantio quidam libenter utuntur, ut Galenses. Sui enim discantus saepe diatessaron resonant, licet simpliciter loquendo rudior et imperfectior sit quam diapente: quod probat auditus, quod iudicat intellectus, quia proportio aesqualtera perfectior est quam sesquitertia, et per prius ad aequalitatem

- reducitur. Etiam pars media alicuius totius maior est parte tertia, ut ipsius senarii trinareitas quam dualitas; sed distincta hominum genera in distinctis consonantiis et cantibus delectantur."
36. Ibid., p. 26. "Si vero fundentur in proportionibus superpartientibus, sic, exceptis diapente, diatessaron et tono, sunt omnes consonantiae quae via imperfectionis praecedunt diapason, ut comma, diesis, apotome, semiditonius, ditonus, tritonus, semitonium cum diapente, tonus cum diapente. aemiditonua cum diapente. ditonus cum diapente. Illae omnes, ut patebit infra, in proportionibus fundantur superpartientibus distinctis."
 37. Ibid., IV; 84. "Et, secundum hoc, multiplicitum proportionum consonantiae in senario sunt numero qui perfectus est. De tribus enim, secundum philosophum, primo dicitur omne; omne autem et totum et perfectum idem, et omnis perfectio in tribus consistat ad imitationem perfectissimae Trinitatis in Divinis personis. Unde, in creatura qualibet trinitis repraesentatur divina per partes imaginis expressius, vel per partes vestigii imperfectius et obscurius."
 38. Ibid., p. 57. "Ad ordinem autem perfectionis qui sumitur a forma reduci videtur ordo simplicitatis formalis et maioris et maioris actualitatis. Quod enim ex parte formae simplicius eat hoc actualia ens et perfectius."
 39. Ibid. "Item quod aliquid trahit perfectionem ab integritate vel totalitate a toto composito ex materia et forma videtur procedere principaliuia tamen a forma."
 40. Ibid. "Item quod a bonitate trahat res perfectionem suam maiorem vel minorem si bonum convertatur cum ente, reducitur talis perfectio ad illam quae a forma est tanquam ad principium quolibet ad totum compositum tanquam ad principium quod."
 41. Ibid., VII: 5. "Sane, cum dudum venerabiles viri de musica plana tractaverint, ut Tubalcain ante diluvium, et inde quam plures de quibus supra meminimus, de mensurabili etiam multi inter quo eminet Franco Teutonicus et alias quidam qui Aristoteles nuncupatur, nunc nostris temporibus novi recentesque venerunt de mensurabili tractantes musica, parentes suos doctores antiquos parum reverentes, quin potius! illorum bonam doctrinas in aliquibus mutantes, corruptentes, reprobantes, annulantes factis, quidquid verbis protestentur, cum curiale bonique moris sit illos in bene dictis imitari, in dubiis ipsos excusare, ipsos exponere."

42. Ibid. "Nom plerumque novae doctrinae, etsi prima fronte exterius niteant, bene tamen examinatae interius solidis carent fundamentis, abiiciuntur nec diu durant."
43. Ibid., p. 9. "Possunt autem voces discantus ad voces comparari tenoris cum quibus debent concordare, et tunc tales cantus discantus dicitur.... Tales voces rationem cantus habent. Et eodem modo dicendum est de vocibus tenoris ut ad voces comparantur discantus et simul cum illis dicuntur, vel ut per se et divisim decantantur."
44. Ibid. "Et cum supra tenorem unum fit unus discantus, perfectiores et pauciores observari debent concordiae et praecipue diapente."
45. Ibid., p. 11. "Discantat igitur qui simul cum uno vel pluribus dulciter cantat ut ex distinctis sonis quasi sonus unus fiat, non unitate simplicitatis, sed dulcis concordisque mixtionis."
46. Ibid. "...tres vel quattuor sunt consonantiae quibus in diaphonia sive discantu principalius, saepius et confidentius utimur, idest unisono, diatessaron, diapente et diapason."
47. Ibid., p. 13. "Sed hoc in generali dico quod, sicut forma dat esse, et bona et perfecta forma supra debitam materiam bonum et perfectum esse, sic bona ordinatio vocum concordam inter se habentium bonum facit discantum efficientem ut est peritus et discretus discantator vel discantuum compositor qui novit consonantiarum naturas et qualiter in discantibus debeat ordinari, quae aptiores sint in principio, quae in medio, quae in fine, qualiter alternatim sub et supra variari debeat."
48. Ibid., p. 22 "...in ecclesia sancta Dei militante. in qua nec ruga debet esse. nec macula, non verentur discantare.... Unde provenit ut saepe in ea <maculam> inducant discordiae."
49. Ibid., p. 2. "Heu pro dolor! hie temporibus aliqui suum defectum inepto proverbio colorare moluntur: "Iste est, inquit, novus discantandi modus," novis acilicet uti consonantiis. Offendunt hi intellectum eorum qui tales defectus agnoscent. Offendunt sensum nam inducere cum deberent delectationem, adducunt tristitiam. O incongruum proverbium! O mala coloratio, irrationabilis excusatio! O magnus abusus, magna ruditas, magna bestialitas, ut asinus sumatur pro homine, capra pro leone, ovis pro pisce, serpens pro salmoni! Sic enim concordiae a discordiis distinctae sunt nullatenus una sit alia."

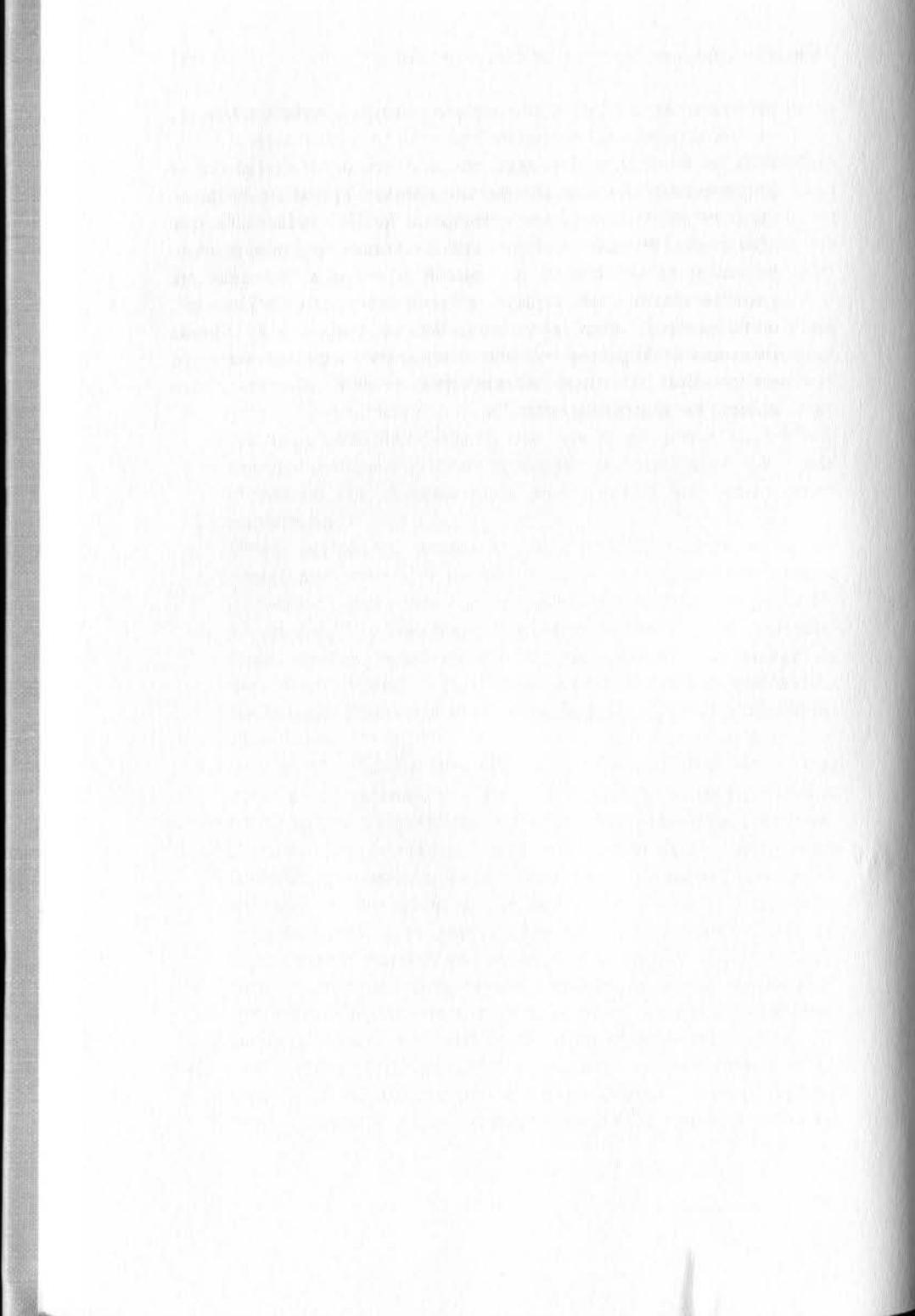
50. Ibid. "Etiam audent discantare qui nesciunt bene firmiterque cantare, cum tamen discantus sit quidam cantus praesupponatque cantus mensurabilia cantum planum."
51. Ibid. "Sunt autem aliqui qui, etsi aliqualiter discantare noverint per usum, modum tamen non observant bonus. Nimis lascive discantant, voces superflue multiplicant. Horum aliqui nimis hoketant, nimis voces suas in consonantiis frangunt, scandunt et dividunt, et in locis inopportunis saltant, hurcant, iupant et, ad modum canis, hawant, latrant et, quasi amentea, incompositis et anfractis pascuntur vexationibus, harmonia utuntur a natura remota."
52. Ibid. "... quaedam a natura remota, quando quis nimis lascive cantat vel discantat, organizans subtilitate arteriarum, quadam superfluitate voces multiplicat; alia naturalis quae harmoniali mediocritate ducitur, et haec in ecclesia Dei habetur authentica, cum sit simplex et modesta... idest tenor cum discantu concordet."
53. Ibid. "Sed utuntur novo cantandi modo, antiquum dimittunt; imperfectis nimium utuntur, in semibrevis, quas minimas vocant, delectantur, a se repellunt cantus anticos organicos, conductos, motellos [sic], hoketos duplices, contraduplices et triplicea, nisi quod aliquo illorum inaerunt in motetis, vel motellis [sic] suis. Subtiles et difficiles ad cantandum, ad mensurandum discantus componunt. Et hi, sicut ab Antiquis: Francone, Aristotele et ceteris. in <cantandi> se separant modo, sic et in modo notandi cantus suos mensuratos, ut tangetur infra."
54. Ibid., p. 27. "Tempus, inquiunt, perfectum secundum Antiquorum considerationem est quaedam aequa proportio in qua recta brevis in duas partea inaequales aut in tres aequales dividitur, quae ulterius dividi vel discerni nunquam possunt."
55. Ibid. "Insufficiens est, ut aiunt, quia tempus perfectum, sicut est divisibile in partes duas inaequales vel in tres aequales, ita aequa bene divisibile est in duas partes aequales, eodem valore temporis nihilominus remanente. De hoc autem intentionem non facit dicta temporis descriptio."
56. Ibid., p. 30. "Ad confirmationem dictorum maioremque excusationem Antiquorum, amplificemus hic causam vel causas quare tempus illud perfectum dicatur quod est divisibile in tres partes aequales et quare sibi repugnat parti in duas partes aequales et, ex alia parte, quare tempus rationabiliter dicitur imperfectum cui,

- secundum suam propriam rationem, ut a perfecto distinguitur, competit duas in se continere partes aequales, non tres."
57. Ibid., p. 29. "Non valet ista responsio quia, qui sic dicunt, materialiter et non formaliter tempore perfectum et imperfectum inter se distinguunt, cum tamen materia principium distinctionia non sit, quia in fundamento naturae, idest in materia, nihil est diatinctum, secundum Commentatorem."
58. Ibid. "Videtur quod haec illis competant iuxta suam denominationem "perfecto", ut salvetur sua denominatio, quia "perfectum" dicitur, consimiliter et "imperfecto." Nam secundum philosophum primo *Coeli et Mundi*, secundum Pythagoricos ut ibidem dicit, hanc praedicationem, scilicet *omne orimum de tribus*, dicimus. Hoc est dictum quod primo dicitur *omne de tribus* modo secundum eundem philosophum in sua *Metaphysica: omne totum et perfectum* idem.
- Secundum hoc igitur tempus illud dicitur perfectum quod a ternario denominationem suam sumit, et illud imperfectum quod a binario."
59. Ibid. "Binarius enim, ut supra vieum est, infamis st imperfectus dicitur, non solum quod primo recedit ab unitate, sed etiam quia sic in se continet duas partes, quod omnino sibi repugnat continere tres."
60. Ibid. "Da igitur quod tempus perfectum in duas partes aequales sit divisibile, ipsum erit imperfectum. Oportet igitur ut ad aliquod perfectum reducatur. De quo quaeram an in partes duas aequales divisibile sit vel non. Si sic, ipsum erit imperfectum et ad aliud reductibile quod sit perfectum sicut prius et sic vel erit processus in infinitum, vel potius fuit etandum in primo quod scilicet repugnat tempori perfecto, ut ab imperfecto formaliter diatinguatur, in duas esse divisibile partes aequales, quae est ratio temporis imperfecti."
61. Ibid. "Item Deus, qui est perfectissimus in se et a quo omnis procedit perfectio, sic approbat in ternario perfectionem consistere ut in ipsa perfectissima sit Trinitas cum simplicissima tamen unitate. Et in aliis etiam omnibus aliqualiter trinitas contineatur quia creatura omnis vel est producta ad Dei imaginem, ut sunt Angeli et homines, vel ad Dei vestigium ut ceterae creaturae."
62. Ibid., p. 60. "In tantum autem tactus doctor imperfectas notulas et cantus ex imperfectis compositos nititur approbare ut Antiquos

- increpare videatur qui cantum naturalem ex perfectis confectum ad summam referunt Trinitatem."
63. Ibid. "Dicit enim quod, antequam Deus carnem assumeret humanam, salva Dei essentia, ex imperfectis cantus esse poterat. Item sum similiter sic Deus trinus est in personis, sicut unus in substantia, non plus debet cantus naturalia referri in Trinitatem divinam quam in unitatem. Item sive cantetur ex perfectis sive ex imperfectis, neque plus neque minus Deus est trinus et unus.
64. Ibid. "Sed quid facit ad hoc propositum loqui de ipsa Verbi incarnatione? Ante enim <illam> ineffabilem naturae humanae a persona Verbi assumptionem et post, vere fuit Deus trinus et unus, et hoc ab aeterno est et erit in aeternum sine fine. Et cum cantua ex perfectis ratione ternarii in summam primam et perfectissimam reducatur Trinitatem, non sic cantus ex imperfectis cum in summa illa Trinitate nulla prorsus cadat vel cadere possit imperfectio.
- Omnis perfectio a summa et prima procedit perfectione et per consequens omnis ternarius, trinitas vel ternareitas ratione perfectionis quam importat iure in primam summam et perfectissimam reducitur Trinitatem. Quando ergo dicitur quod, antequam Deus carnem assumeret de Virgine, poterat esse cantus de imperfectis, dicendum quod verum est sed de facto, non secundum fundamenta illius artis quae in perfectione ratione ternarii radicatur."
65. Ibid, p. 61. "Sed dicitur ulterius: '<Cum similiter sic> Deus trinus est in personis, sicut unus est in substantia, non plus debet cantus naturalis ex perfectis confectus referri in divinam Trinitatem quam in ipsis unitatem.' Dicendum quod cantus naturalis ex perfectis compositus refertur et in divinam Trinitatem et in ipsis unitatem! In Trinitatem ratione perfectionis et diatinctionis quam importat ternaria, in ipsam vero unitatem ratione concordiae quae in huius modi cantibus requiritur, nam concordia dicitur distinctarum vocum vel distinctorum cantuum in unum <redacta> concordia. In illa autem summa Trinitate, summa est concordia cum qua simplicissima stat unitas ipsius divinae essentiae."
66. Ibid. "Dicitur ulterius: "Sive cantetur ex perfectie sive ex imperfectis, nihil nimis est Deus trinus et unus". Verum est quia Deus sic in se veraciter et realiter intrinsece perfectua est ut ad

nihil extra se, idest ad creaturam quacumque, realiter referatur, realiter dependat vel ordinetur."

67. Ibid., p. 60. "Dicendum quod non sunt increpandi illi qui cantus ex perfectis compositos in summam referunt Trinitatem. Unumquodque enim convenienter principium in illud reducitur a quo effective vel etiam exemplariter oritur, ut omne bonum in primum bonum quod est bonum per essentiam, et alia per quandam participationem: omne verum in primam veritatem quae Deus est, omne ens in primum ens et omne esse in primum esse. Omnis enim multitudo procedit ab uno et omne esse a primo esse quod est esse Dei. Ab illo aiquidem derivatur esse in aliis, his quidem clarioris, his autem obscurius."



The Form of *The Phoenix*: A Model of Its Number, Proportion, and Unity

Robert D. Stevick

The text of the Old English poem *The Phoenix* is preserved uniquely in the Exeter Book where its sectional divisions—or fitts—are unnumbered (as they are also in several of the long texts of Old English) but they are prominent and unquestioned. Even though they are mentioned in the introductions to the principal collective and separate editions of these poems,¹ they are not incorporated into the printed texts. Like others, the sectional divisions in the manuscript that preserves the text have remained virtually ignored apparently for never having been understood. What I shall show in this paper is that the sectional divisions of *The Phoenix* (like those of *Andreas*, *Christ II*, *Christ III*), and others can be predicted in a rectangular model laid down to embody a simple arithmetical ratio—that is, from a modular plan; the prediction is carried out by the same elementary methods of constructive geometry that are also used in architecture, manuscript illumination, and any of the other early medieval trades that "use measure." While most long poems in Old English are incomplete, *The Phoenix* has no lacunae that would change the count of lines from what it presumably was when the text left the poet's hands; any discontinuities in syntax, meter, or sense can be made good by emending single forms or, as in verse line 155, by supplying one word (*wyn*, which may have been represented in the exemplar by a runic symbol). The striking thing about the sectional divisions of *The Phoenix* is the prominent use of modular grouping of lines of text. The sectional divisions follow lines 84, 181, 264, 349, 423, 517, 588, and 677. It takes only simple arithmetic to discover that the lengths of the sections, measured in metrical lines, conform extensively to a modulus 84: that is the length of the initial section, the average of the first seven, and the average of smaller groups as well: see table 1. Such extensive recurrence of such a large number is not likely to result from chance. It is statistically not predictable that six modular groupings should occur in a text having only eight sections, with thirty-six possible

groupings (including groups consisting of one section only). If it is also considered that the modular groupings are confined to the first seven sections of the text, it will appear even less probable that the same large factor 84 should occur so many times without design.

Section(s) I	contain(s)	1 x 84	lines	(= 84),
III-IV	"	2 x 84	"	(= 168),
V-VI	"	2 x 84	"	(= 168),
III-VI	"	4 x 84	"	(= 336),
(II..VII)	"	2 x 84	"	(= 168)),
II-VII	"	6 x 84	"	(= 504),
I-VII	"	7 x 84	"	(= 588).

Table 1. Occurrence of modulus 84 in *The Phoenix*.

Section Number of lines in section groups

[I]	84
	181
[II]	97 264
	180 349
[III]	83 265 423
	168 339 517
[IV]	85 242 433 588
	159 336 504 677
[V]	74 253 407 593
	168 324 496
[VI]	94 239 413
	165 328
[VII]	71 254
	160
[VIII]	89

Table 2. Lengths of sections of *The Phoenix*.

Table 2 shows the sums of lines for all sectional groups of the poem. Quite apart from statistical considerations, there are arithmetical properties of the modular number itself which, in relation to the larger aspects of the poem's divisions, seem to put the modulus and the whole formal scheme on the side of artistic contrivance, altogether apart from chance or "natural" shaping. The number 84 has been known at least since the time of Nichomachus as a solid number—that is, a number

that is the sum of a series of triangular numbers. Triangular numbers are the successive sums of the series of natural numbers. Thus:

Natural	1	2	3	4	5	6	7	8	9	...
Triangular	1	3	6	10	15	21	28	36	...	
Solid	1	4	10	20	35	56	84	...		

The modular number, which is limited to governing the first seven sections of the text, is the seventh of the solid numbers. All this understanding of numbers was transmitted to the Middle Ages by Boethius.² Yet further, the modular 84 also dominates the harmonic sequence 12, 21, 84—one of the few in this range of whole numbers.³ By the nature of this series (also explained by Boethius), the first two are factors of the third; in this instance the relations of the first two numbers to the third are the multipliers 7 and 4, respectively. That is to say, the modular number is not only another assertion of seven, according to medieval number theory, it also heads a harmonic sequence whose factors are 7, 4, 1. These characteristics of natural 7, the seventh solid number, and the factors of the harmonic sequence offer a good lead on how to construct a model of the poem's sectional divisions. Still another lead to discovery of the formal plan may lie in the disposition of the modular divisions of the text of the manuscript. As shown above, the module occurs in groups of consecutive sections that number 1, 2 twice, 4, 6, 7, and no other. Now, there is only one rectangular configuration that satisfies all the surface numerical properties so far described and at the same time satisfies just the set of numbers of modular section-groups. It is a rectangle in the ratio 7 : 4 (the same numbers, along with 1, that are factors in the harmonic sequence ending with 84). It further requires specifically that a single modulus at one end of the seven be marked, leaving six, together with the unique occurrence of four divided as two and two. In terms of the concept of the formal scheme, or in terms of the derivation of it, this disposition of the modular groups is set as soon as, from the ratio 7 : 4 laid down in a rectangular frame, or premise, the initial division is made, setting the initial modular marker at one module from the long end of the rectangle. How the 7 : 4 ratio is related then to the eighth section of the poem will appear presently; the relation is utterly simple to model and very accurate to compute graphically. While all the aspects of modular eighty-four just noted are strictly arithmetical, hence independent of symbolic significance that may well attach either to the modulus itself or to its factors, the symbolism of numbers may offer a

further lead toward understanding the form of the poem. *The Phoenix* is unlike the other Anglo-Saxon vernacular religious narratives in being formally a symbolic text. It is not a narrative of a biblical person or a saint which carries (or may carry) symbolic import. Rather, it is a narrative of a legendary bird followed by an explication of the symbolic significance of the narrative. Symbol and significance are told in sequence, and at nearly equal length. It should not be surprising, then, to find symbolism apparently attached to the form of the poem as well.⁴ There is no question that the topic of the concluding section of the text is the eternal joys of the blessed. From the onset of the explicit interpretation of the phoenix fable, a little more than half way through the poem, there is a progress from the expulsion from Eden, through the sacrifice of Christ, to the certainty of resurrection and Judgment. Finally, the chosen spirits (like the resplendent birds around the phoenix) follow Christ, exultant forever: a diadem adorns each of the righteous, where enduring joy, eternal and renewed, shall never diminish; the company of spirits praise the Saviour, sing praise of the Creator, adore the Prince with voices united (lines 591-621). There is also no question that this topic is delayed until this final segment of the poem. The fable is recounted in a generally straightforward way in the first 380 lines of the text, paraphrasing the Lactantian *De ave phoenice*. The conventional and plain interpretation is then developed, beginning at line 387. The explication is direct and methodical. The nature of this bird signifies the servants of Christ, how they gain bliss in the heavenly home; the departure of the phoenix from paradise is like our first parents giving up Eden in great need to become rejuvenated; the betokening of the fiery death and rebirth of the bird is the gathering of body and spirit together before Christ when fire has devoured this world. Lest anyone think I write this lay in falsehood, hear the saying of Job who, "as the phoenix, after resurrection will be allowed life renewed, joys with the Lord" (lines 558-560). And yet further: God's prophet spoke thus "concerning his resurrection in eternal life so that we can the more surely understand the glorious meaning that the resplendent bird signifies through its burning. Remnants of bone, ashes and cinders, it gathers all after the burning, the bird brings them then in its feet to the dwelling of the Lord, toward the sun. There they remain many years [which is to say "it" remains—the phoenix signifying all the blessed]. . . . So now, after death, through the might of the Lord together souls with body will journey beautifully adorned

most like the bird in blessedness with the finest fragrance to where the righteous sun shines beautiful over the host in the city of glory" (lines 572-588). It is exactly here, at the opening of the eighth section, that the unceasing joys of the blessed after Judgment become the final subject of the poem. There has been no haste in getting to this topic, no economy; neither has there been delay for a hymn of praise by the righteous or a lamentation by the wicked; there has been instead a methodical explication followed by an overt restatement of the commoner symbolic values of aspects of the phoenix story, clustering around the doctrine of the Resurrection. In brief, the eighth section of the text, which is the last, has to do chiefly with the joys that will endure forever after the Judgment is over. After the number seven, there will arise eight itself relying on its royal power. For He is the true eight, who has borne the sin of the world. It is the first; it is the eighth; it is the last. It will be the last in this way, that it is everlasting.⁵ The eighth day is sequel to the Resurrection and the Judgment that have been made possible by the advent of the Redeemer and the resurrection of the Saviour, coming to pass when all other days have come to an end. The first seven sections of the text concern the temporal world. The eighth section has to do with eternity. Eight is a commonplace symbol for eternity, seven for the ages of the world. If the formal plan of the poem is to express this number symbolism, a way must be found both to divide the text so that the eighth section is separate from the seven that precede it, and to integrate the eighth section with the seven that it follows: it must be at once the first, the eighth, and the last.

Following all the leads that have emerged from considering the modular arrangement of text, the modular number, and the symbolic aspects of the narrative, a model for the sectional division of *The Phoenix* can be constructed as illustrated in Fig. 1. On a 7 x 4 rectangle, laid out using a modulus measuring 84, the first module (84) (Fig. 1a), a geometrical derivation, offsets the modular scheme for the second sectional division (Fig. 1b), but modular measures are marked off from that division for the fourth and sixth sectional divisions (Fig. 1c, d). The two remaining divisions derive geometrically from divisions already made (Fig. 1e, f).⁶ And then the overall length is set as equal to the diagonal measure of the source rectangle (Fig. 1g). As simply as that all the quantitative divisions of the text can be explicated. Understanding of this form, though, does not necessarily follow from

a formula for its replication. Without an understanding of the inner logic of the schema for this poem, there can be no understanding of the form itself.⁷ Let us review the derivational steps now with an eye to the formal relations they embody. There is a boldness in the opening maneuvers that is easy to miss amongst the arrays of numbers and the webs of arcs and lines in Fig. 1. It is the directness with which "seven" is asserted, an opening declaration that seven will be the governing number of this poem. Seven modules embrace the first seven sections (and not eight). And as if to make the modularity unmistakable, the first modular division is selected as the first sectional division. (The *Andreas* poet selected the final modular division as the last sectional division.) That leaves six modules to be divided into six sections. How was it done? The first of these divisions is made from the simple ratio 2 : 1 embodied in the sides of a right triangle. The next two are made merely by copying the measure of the width and the half-width of the rectangle, in turn. The last two divisions are made by simple development from the ratios 2 : 1 and 7 : 4. That is, among the lengths of the first seven sections of the poem, four are set directly from the modular grid of a 7 x 4 rectangle, the other three divisions being made by utilizing the properties of ratios already entailed. The eighth section of the poem follows the seven that are governed by the module; it is the first not to be governed by the module, it is the eighth, it is the last. It is separated from the seven by not being a part of the modular plan, yet it is integrated with the seven by being set directly from the whole of the 7 x 4 modular rectangular frame. One more aspect of the plan may reflect mathematical symbolism, too. I have shown elsewhere⁸ that *Andreas* and *Christ II* have sectional divisions fully derivable within a model that begins with a rectangular frame with a proportioning of 3 : 2. In modular terms, the frame presents six squares. Six was known as a perfect number for being the sum of its factors. The next perfect number is 28, the same as the count of squares in a rectangle proportioned as 7 : 4. Here are three poems, then, whose sectional divisions can be modeled completely and accurately from modular grids which contain a perfect number of modular units. My earlier essay on some formal aspects of *The Phoenix*⁹ stopped short of providing a model of how all the mathematical consonances among the manuscript divisions of the poem may have been achieved. Even more, the conclusion steered away from the inference that this Anglo-Saxon poem of the tenth century would likely

have had an integrated schematic basis that can be readily understood and replicated. "My not having found it is not my only reason for supposing at this stage that a grand template for layout of the poem was never made." The other reasons are literary ones argued by others: that the poet failed to provide a comprehensive allegory, giving us only a circumambulation in place of a fourfold allegory, or that the poem renders the relationship between beauty and salvation which unites differing allegorical perspectives.¹⁰ And so, "If we do not find in this poem a rigid, foursquare development of theme and symbol, we may also not find the use of number and proportion to have had an algorithm in a single geometrical figure, and for the same reason." However, it is now apparent that the form of *The Phoenix* almost certainly did begin in something similar to a template—or a model something like the one proposed here. The perfect congruence of the geometrical model with the contemporary divisions of the manuscript text of the poem is very good circumstantial evidence that the form of *The Phoenix* did originate in a mathematical model. The further congruence of numerical features of the text with prominent patterns in Boethian (and classical) number theory redoubles that evidence. And the match-up of number symbolism with the disposition of the topical materials, within the eight sections of the text, provides confirming evidence of yet another kind. In view of this variety of evidence, it is hard to account otherwise for the sectional divisions of this text, except impressionistically. By correlating the measures of sectional divisions with each other in groups or clusters, a formal scheme becomes immediately apparent: there is an unmistakable system of relationships among all parts of a poem and of the parts to the whole extension. The plan of *The Phoenix* is as ingenious a manipulation of quantities as the poems of Hrabanus Maurus are ingenious manipulations of the literal elements of written language. The fact that formal plans of *The Phoenix* and other texts were manifest in the sectional divisions themselves was difficult to discover exactly because the plans of these texts were the opposite of such familiar forms as those of the sonnet or rhyme royal. First, they are large forms, coextensive (as they must be) with whole texts that run into hundreds of lines. Next, they did not manifest themselves in patterned recurrences of phonological features—couplets, stanzas, linked stanzas, bob and wheel. Patterns of that kind are comprehensible (and expressible) in simple arithmetical terms: seven-line stanzas with rhyme linking the first four lines

alternately, the fifth to one of the alternatives, and a new rhyme in the final pair; or stanzas linked in fives, or sevens, or twelves, or twenty-fives, and so on. By contrast, the forms of these long poems in Old English did not use recurrence of this kind, and for that matter did not use recurrence of such elements as opening formulas (lexical) or syntactic patterns or concluding syllable patterns (*cursus*).¹¹ Rather than being manifest in recurrences of linguistic patterns, the forms are manifest in the divisions—in the demarcations—of the complete text. That brings up the last opposition between these forms and later ones: measures of section-lengths combine in relations that are predominantly geometrical rather than arithmetical in nature. They are conceived as quantity that is continuous—to be divided—rather than quantity that is discrete—to be aggregated.¹²

This last consideration brings up what (to me) is the least expected yet thoroughly demonstrable understanding of the divisions of the poetic texts. The relations among the manuscript's divisions of this text—and others—imply unmistakably the notion that a poetic text was conceived as a continuous quantity—that is, as a magnitude (just as the forms of most page designs imply it in a much more obvious way). Magnitude is continuous quantity which (as noted above) is dividable, rather than discrete and only capable of increase. And if it is fixed (or "at rest"), as it can only be in books, magnitude is the subject of geometry (magnitude that is moveable is the subject of astronomy). Small wonder, then, that the ratios among segments of the manuscript texts of poems incorporate irrational numbers such as ϕ and ϕ^2 and ϕ^7 (in modern notation) as well as "terms" or numbers such as 1 and 2 and 4 and 7. This is a concept of unity of verse composition hardly separable from a concept of unity in page decoration, and a concept of literary form as rigorous as it is remote from literature in Modern English. Understanding of how this concept of literary unity and form came into being may lie in tracing—or reconstructing—the early response in northwestern Europe to the codex, replacing the scroll, for texts of religious writings canonical and otherwise. As simple as was the change from scroll to codex in some respects, it presented problems and possibilities for esthetic concerns or for conveying symbolic meanings. To accommodate writing that proceeded from left to right, the scroll maintained a constant height for its surface, and the text-space writing was accordingly held to a fixed height. The scroll, though, did not present an area with a fixed width for the surface. The width of writing was regular, typically rather narrow columns, while

the opening between the rolls was unrelated to the height by any fixed factor. With only one dimension that was determinate for the area in view, there could be no fixed ratio with another dimension: the scroll did not present to view either surface or text-space in a constant proportion. On the other hand, the codex format required constant ratio for the rectangular areas of both surface and text-space. Whatever the dimensions chosen for the book, they embodied a proportion of two dimensions set at right angles one to another. Now, without a standard, or usual, or conventional, or canonical shape for books, the makers could allow the proportions of their codices to be random—determined solely by the size and shape of the skins from which the sheets were cut, or set to a size chosen merely by intuition, or shaped some other non-deliberate way. Or, those persons who were preparing the sheets of parchment could set the proportions of their sheets according to simple concepts of mathematics, those of geometry being the more appropriate. In Britain and Ireland, at least, many of these earliest books—the finest of them anyway—were given shapes for the sheets and folios that are of this latter kind. A very interesting parallel is to be found in the use of figures and formulas in a tradition of boatbuilding in western Norway that extends back to Viking times. The concept of shape of a boat is coded into proportional figures, according to Jon Godal. "When a boatbuilder of the old school relates information about the construction of a boat to a colleague or member of his family, he usually communicates a long list of figures. He works on the basis of an oral tradition independent of plans and drawings. He recalls a boat as a combination of figures and interpretations of these figures." And because there are different shapes for boats of different uses, when "a boatbuilder wishes to alter the shape of a boat, he may add to or subtract from the standard proportion figure," or "he may also change a boat's shape by moving the point of measurement."¹³ It is entirely possible that rectangular (and other) designs in early Irish-Northumbrian culture were understood, transmitted, and created in analogous ways. In this light consider then the metrical form of early Germanic poetry. The largest metrical unit was the line (as we now call it). So far as evidence has been found, there was no arithmetical scheme for combining lines into patterns—into formal units. The lines of verse occurred in sequence, one line succeeding another much as the strips of writing in a scroll merely succeed one another as discrete quantities. They obeyed the rules of

syntax of their language, and the meanings they expressed were arranged according to an expository scheme (typically that of narration). Yet the lines did not comply with any purely formal rules other than the rules that defined their internal structure. Without a fixed and constant scheme for grouping lines—such as the method that later produced stanzas, or that also linked stanzas, as in *Pearl*, or created sonnet sequences—there was another model for imposing a non-lineal form on the succession of metrical lines. That model was closely related to the one embodied in the forms used for making books, in which the object was whole, to be segmented into appropriate form, not an object built by increment of compositional units. At least we know this: that the shapes of vernacular poetic texts in Anglo-Saxon England (and the shapes of pages and framed decorations of books and in Ireland and England of the seventh to tenth centuries) are fully explainable as creations in accordance with mathematical principles found commonly in classical arithmetic and practical geometry.

Prior history apart, when a model predicts all the formal divisions of a poem, and does so with precision and with homology, there is good reason to believe it is the right one. We have had no such model in the past. When in addition the models of a series of poems (besides the models of ornamental pages) correspond in kind, in conception, in methods of construction, in cultural matrix as no other models have done, then (I believe) we have the best means yet available for understanding the forms of poems such as *The Phoenix*.

Notes

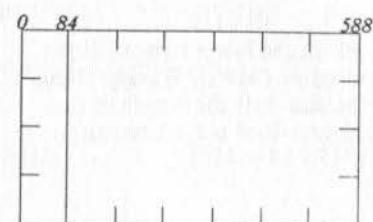
1. The principal collective edition is George Philip Krapp and Elliott Van Kirk Dobbie, *The Exeter Book*, Anglo-Saxon Poetic Records 3 (New York, 1935). N. F. Blake has edited *The Phoenix* (Manchester, 1964); also useful is Albert S. Cook's edition in *The Old English Elene, Phoenix, and Physiologus* (New Haven, 1919). *Christ III* is edited by Albert S. Cook in *The Christ of Cynewulf, A Poem in Three Parts: Advent, The Ascension, and The Last Judgment* (Boston, 1900).
2. The relation of Boethius's *De Institutione Arithmetica* to the *Introduction to Arithmetic* of Nicomachus is explained in the introduction, pp. 132-137, by Frank Egleston Robbins and Louis Charles Karpinski to Nicomachus of Gerasa, *Introduction to Arithmetic*, translated into English by Martin Luthor D'Ooge, University of Michigan Studies, Humanistic Series, vol. XVI (New

York, 1926); exposition of triangular and solid numbers in the text is on p. 241 ff.

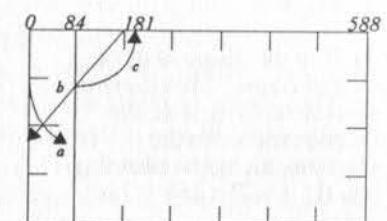
3. Numbers are in a harmonic sequence when the middle number in their reciprocals is the arithmetical mean of the other two. Thus the reciprocals $1/2$, $1/21$, $1/84$, when given a common denominator, are $7/84$, $4/84$, $1/84$; the arithmetical mean of 7 and 1 is 4. The other harmonic sequences in this range are 4, 7, 28 and 8, 14, 56, but neither of them includes the modular number (84).
4. This paragraph and the next two are drawn virtually unchanged from my essay "Mathematical Proportions and Symbolism in *The Phoenix*, *Viator*, 11 (1980), 95-121.
5. This is Byrhtferth's version of it, quoted from *Byrhtferth's Manual*, (Bodleian, Ashmole 328), ed. S. J. Crawford, Early English Text Society, o. s. 177 (Oxford, 1929), 214-215.
6. The computation, in modern notation, is as follows:
 - (a) The end of the first module is 84 (1×84) from the beginning, the end of the seventh module 588 (7×84) from the beginning.
 - (b) The arc (a) transfers the measure of two modules to become the hypotenuse of a triangle with one side one module in length; the other side is therefore $\sqrt{3}$ (in modern notation). The ratio $2:\sqrt{3}$ is then applied (b) to the modular measure 84 to yield 97. The measure 97 is then marked (c) along the planning line. More precise computation: 96.99.
 - (c) The measure of two modules, 168, is added to $84 + 97 = 349$.
 - (d) The measure of four modules, 336, is added to $84 + 97 = 517$.
 - (e) Simply, this sets $\sqrt{5} : 1$ and applies it to divide two modules (168) into 94 and 74. More precise computation: 93.91 and 74.09.
 - (f) The ratio of the overall frame (4:7) is invoked here, at the second module (168); the measure is $(1 + 4/7) \times 168 = 264$.
 - (g) Diagonal measure of a 7×4 rectangle is $\sqrt{7^2 + 4^2}$, or $\sqrt{65}$, which applied to the modular number yields 677; the measure is transferred to the planning line. More precise computation: 677.23.
7. "Inner logic" is a term I have refrained from using elsewhere. It does not refer to something appreciated by inspection or intuition.

- On the contrary, it refers here to an internal logic that can be fully explicated and will not be fully understood until it has been.
8. The Manuscript Divisions of *Andreas, Philologia Anglica*, Essays Presented to Professor Yoshio Terasawa on the Occasion of His Sixtieth Birthday, ed. Kinshiro Oshitari, et al. (Tokyo, 1988), 225-40, and Two Notes on *Christ II, Leeds Studies in English*, New Series XX (1989), 293-309.
 9. See note 4, above.
 10. See Blake (note 1, above), 34; Stanley B. Greenfield, *The Interpretation of Old English Poems* (London, 1972), 144; J. E. Cross, *The Conception of the Old English Phoenix*, in *Old English Poetry: Fifteen Essays*, ed. Robert P. Creed (Providence, 1967), 145; Daniel G. Calder, *The Vision of Paradise: A Symbolic Reading of the Old English Phoenix, Anglo-Saxon England*, 1 (1972), 168.
 11. There may be a qualification here in the use of *Eala* to begin the segments of *Christ I* that correspond to the beginnings of the antiphons on which each segment is based. Or maybe not. The sectional divisions do occur preceding a segment beginning *Eala*, but some of the *Eala*-segments do not begin at sectional divisions in the manuscript. The lengths of these segments are shorter than are the lengths of sections of poems generally in Old English. I do not regard the occurrence of $\text{---} \text{æs ofereode}, \text{---} \text{isses swa maeg}$ in *Deor* as an exception, because the poem is entirely unlike the ones with sectional divisions of the kind we are considering, whether in length, topic, or format.
 12. This distinction was transmitted by Boethius (and who knows by what other sources?). It is made in careful detail in Book 2, Section 3, of *Fundamentals of Music*, Translated, with Introduction and Notes by Calvin M. Bower, Music Theory in Translation Series edited by Claude V. Palisca (New Haven, 1989). It begins: "According to Pythagoras all quantity is either continuous or discrete. That which is continuous is called 'magnitude,' whereas that which is discrete is called 'multitude.' The properties of these are different and even opposite" (p. 53).
 13. *Measurements, Figures and Formulas for Interpretation of Western Norwegian Boats and Viking Ships*, paper read at the 25th International Congress on Medieval Studies, Western Michigan University, May, 1990.

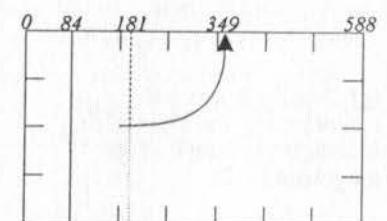
Figure 1. Given: A rectangle proportioned 7:4, with modulus 84.



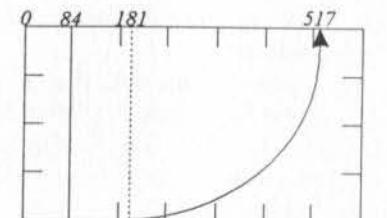
(a) Designate the end of the first module as the first division (84) and the end of the last module as another division (588).



(b) Draw an arc (a) with two-module radius as shown to intersect the next modular line; then run a line (b) as shown to set graphically the ratio 2: $\sqrt{3}$ at right angles; apply that ratio to the modular measure 84



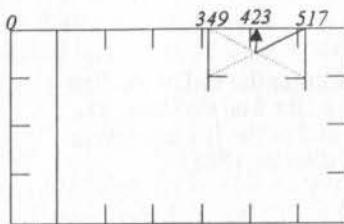
(c) to yield 97 beyond the first 84 (that is $84 + 97 = 181$). (c) Using the center line of the modular grid for reference, measure two modules down the sides from the second demarcation ($181 + 168 = 349$).



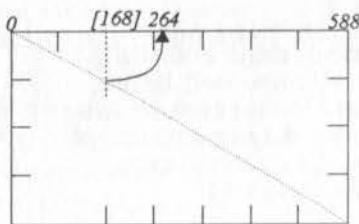
(d) Using the width of the frame for reference, measure four modules down the sides from the second demarcation ($181 + 336 = 517$).

Figure 1. Continued ...

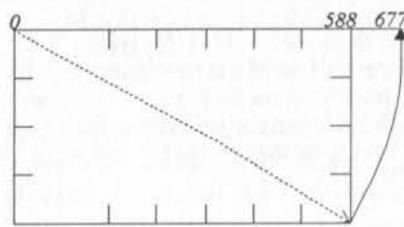
(e) In the lower two-module measure (349-517) copy along the side half the length of the diagonals of a 2 x 1 rectangle ($517 - 94 = 423$).



(f) Run the diagonal of the overall frame. Then copy the measure shown, from the second module to the diagonal, along the planning line ($[1 + 4/7] \times 168 = 264$).



(g) Finally, transfer to extensions of the sides of the rectangle the length of its diagonals (677).



Eric and Enide: Cosmic Measures in Nature and the Hebrew Heritage

Joan Helm

The manuscript known as Paris, BN fr. 794 contains four complete stories of the adventures of King Arthur and the Knights of the Round Table, and the unfinished original story of the Holy Grail. The first in the collection, *Erec et Enide*, composed in the twelfth century by Chrétien de Troyes, is believed to be the first written Arthurian romance. No original Chrétien manuscript has survived, although there are seven manuscripts which contain transcriptions of a full text of *Erec et Enide*, all differing slightly, while keeping to the same basic story. From a signature-type inscription at the end of one romance, the manuscript BN 794 is assumed to be the work of a scribe named Guiot. This early thirteenth-century version is sometimes referred to as the "Guiot" manuscript.

As Enide is introduced, we learn that she was Nature's most beautiful and perfect work. In this introductory description of Enide, the personified Nature is named three times. In Enide, Nature excelled herself, never again to achieve such a pinnacle of perfection. Nature bears witness that so fair a creature as Enide was never seen in all the world:

Capital.

olt estoit la pucele gente
car tote i ot mise s'antante
nature qui fete l'avoit

ele meïsmes s'an estoit
plus de .Vc. foiz mervelliee
comant une sole foiee
tant bele chose fere pot

car puis tant pener ne se pot
qu'ele poïst son essanplaire
an nule guise contrefaire

[The maiden was very beautiful
for Nature, who had made her,
had concentrated all her skill
on the task.]

Nature herself had marvelled
more than five hundred times
as to how on this one occasion
she had been able to make such a
beautiful thing

for since then, try as she might
she had never been able to duplicate
in any way her original model

De ceste tesmoigne nature
c'onques si bele criature
ne fu veue an tot le monde.

(*Erec* 411-23)

In the opening segments of the romance Enide does not appear. Rather, King Arthur has gathered together the Knights of the Round Table and announced his intention of celebrating and perpetuating a mysterious ancient ceremony known as the Kiss of the White Stag. According to this ancient custom, the King must needs bestow the Kiss of the White Stag upon the fairest maiden in the land. Prior understanding of the significance of this ceremony seems to be assumed, as we are told tantalisingly little. The Knights of the Round Table, however, must surely have been fully enlightened. In a speech to the King, an agitated Sir Gawain reveals that the significance of this ancient Pendragon rite had long been known to the members of the court of King Arthur.

Nos savomes bien tuit piece a [We have all known for a long time
quel costume li blans cers a the tradition associated with the
(43-44) white stag]

Erec, the hero of the romance, does not join the King in the hunt for the White Stag, which is presumably prerequisite to the Kiss. Lingering in the forest with Queen Guinevere, Erec witnesses an insult to the Queen and one of her damsels, which he is honour bound to avenge. With the restoration of the Queen's honour in mind, Erec arrives at the household of a poor vavasour, who offers him shelter. Erec is pleased to accept. The vavasour calls to his wife and his daughter, who is very beautiful. Thus the introduction of Enide into the romance was delayed until the scribe had reached the line number 396, significant to her part of the geometric model, to be developed below.

Each scribe, while retaining the basic story, was apparently at liberty to set out his work in the manner best suited to his purpose. The scribe Guiot structured the design of his manuscript to facilitate access to numbers in harmony with the philosophic symbolism of a specific geometric model. Two separate but harmonious portions of the model represent the union of Erec and Enide. In order to do this he arranged that each of his pages would contain three columns, each with a total of forty-four lines. Nothing could have been more simple. To know his line count at any given time, he had only to count his columns and multiply by forty-four, or count his pages and multiply by 132. It is the

Of this Nature bears witness
that a creature of such beauty
was never seen in the whole world.]

purpose of the present study to provide graphic representation of this organizational device.

The version of *Erec et Enide* in manuscript B. N. 794 consists of 6880 lines of poetry. The well known edition by Mario Roques contains 6858 lines, because two lines which were repeated by the scribe have been deleted. The 6880 occupied lines are contained in 156 columns each of 44 lines, with a remainder of 16 lines in the final column. Each of the vellum pages had been ruled with a sharp instrument, so that every column contains exactly 44 lines. Each page contains three columns, giving a total of 132 lines to each page. There are, therefore, 52 pages, or twenty-six folios with the full complement of 132 lines. The final 16 lines which make up the balance are in the first column of the fifty-third page, (fol. 27 r°).

$$52 \times 132 = 6864; \quad 6864 + 16 = 6880$$

Each Arthurian romance in BN 794 begins at the top of a new column, so that the scribe had only to count his pages and multiply by 132, or count his columns and multiply by 44. Only the balance of lines in the final column would need to be counted individually.

For his own convenience, the scribe could have kept a table of fifty-three numbers, signifying the line count at the end of each page. If his scheme had been to allow ten pages to a row, the table would have been as follows:

132, 264, 396, 528, 660, 792, 924, 1056, 1188, 1320,
1452, 1584, 1716, 1848, 1980, 2112, 2244, 2376, 2508, 2640,
2772, 2904, 3036, 3168, 3300, 3432, 3564, 3696, 3828, 3960,
4092, 4224, 4356, 4488, 4620, 4752, 4884, 5016, 5148, 5280,
5412, 5544, 5676, 5808, 5940, 6072, 6204, 6336, 6468, 6600,
6732, 6864, plus 16 lines.

The introduction of Enide, as stated, is delayed until the scribe has reached line 396. By some arrangement of the basic numbers such as that described above, and by his own familiarity with them, he would know that line 396 would be the last line on the third page. The time to introduce Enide has come, and she duly makes her first appearance at the top of the first column on the fourth page. Following the introduction of Enide, the poet describes the great amazement of Nature, when she views with wonder the result of her own work. Nature (whose work is the world of creation) testifies that Enide is the most dazzling creature she ever produced.

The numerical sequence 396 re-occurs at line 3960, which the scribe would know to be the last line on page thirty. At the bottom of the last column on the thirtieth page and at the top of the next column, Enide is behaving in a curiously cryptic manner, in perfect harmony with the idea of a concealed meaning. She is deliberately concealing herself. Because she does not wish to be recognised, she artfully draws her veil across her face, just as she would if she were doing it only because of the dust, or the glare of the sun.

Et la dame par grant veidie
por ce qu'ele ne volit mie
qu'il la coneüst ne veïst
ausi con s'ele le feist
por le chaut ou por la poldriere
mist sa guinple devant sa chiere.

[And the lady with great wisdom
because she did not desire at all
to be seen or recognised
just as though she were doing it
because of the heat or dust
put her veil across her face.]

Enide's action was not inconsequential. It was carried out as a result of her "grant veidie," or wisdom. The image of the veil to conceal the secret meaning of sacred mysteries is almost routine in medieval literature. An example is given by the Roman philosopher Macrobius in his *Commentary on the Dream of Scipio*. The words of Macrobius must be taken seriously in any study of *Erec et Enide*, because the Arthurian story teller acknowledges in the romance that he used the work of Macrobius as a source. Macrobius describes the strategies used by philosophers of Nature:

... because they realise a frank, open exposition of herself is distasteful to Nature ... her sacred mysteries are veiled in mysterious representations, so that she may not have to show herself even to initiates. Only men of superior intelligence gain a revelation of her truths.¹

We may assume that Enide, with her "grant veidie," was among those who possessed superior intelligence, and had gained a revelation of Nature's truths. Enide is introduced into the romance at line 396; she behaves cryptically at line 3960. There is yet another place at which we should logically hope to find a clue to her identity, and that is at the revelation of her name.

Enide's name is used for the first time in the romance in the line immediately preceding 1980, the last line on the fifteenth page, exactly one half of 3960. The occasion is the wedding of Erec and Enide. Enide's name is not revealed until the time of marriage. Some of the ancient and medieval mystery concerning the power of names seems to

be implied. Whether even Erec knew her name remains unclear, but certainly it was not public knowledge. It was necessary for Erec to call Enide by her proper name on the occasion of their marriage. Unless he did so, they would not be properly married:

Quant Erec sa fame reçut [When Erec received his wife
par son droit non nomer l'estut she had to be named by her right
name

qu'altremant n'est fame esposee for otherwise a woman is not
married

se par son droit non n'est namee if she is not called by her
proper name

Ancor no savoit l'an son nom
mes ore primes le set l'on
As yet her name was not known
but now for the first time they
knew it

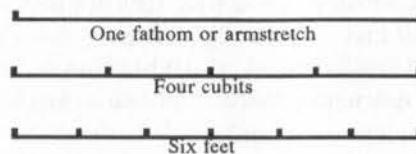
Enyde ot non au baptestire
L'arcevesques de Quantorbire
qui a la cort venuz estoit
les beneisi com il doit
Enide was her baptismal name
The archbishop of Canterbury
who had come to the court
blessed them as he ought]

The mention of the highest ecclesiastical dignitary in the land may have been no more than a judicious political bouquet, but it is also possible that the mystery surrounding names was still considered sacred and therefore under the jurisdiction of the church in spite of a widespread pagan background. Again, we learn Enide's name in the line immediately preceding line 1980, which is exactly one half of 3960. Thus Enide appears to be linked with the numbers 396, 3960 and 1980. If these are assumed to be part of a strategic plan, devised to assist us in our quest for her identity, two interpretations arise.

The first applies to a system of measures known as the German or Saxon units, unquestionably in use when the first written Arthurian romances appeared. Zupko, in *British Weights and Measures: A History from Antiquity to the Seventeenth Century*, wrote:

Both the Roman and German feet were used in Britain during the early Middle Ages but by the end of the period . . . prior to the Norman conquest, the German foot emerged as the primary unit.²

The German foot, which Zupko's research proved to be the primary unit, was part of a system in which the foot was always two thirds of the cubit.



The German foot was 13.2 inches in length. The corresponding cubit was 19.8, the sequence of the line number immediately preceding the revelation of Enide's name. Ten of these cubits made the English rod, pole, or perch of 198 inches, which continued to be used as a measure of land in our day, until the introduction of the modern metric system. The terms rod, pole and perch are synonymous. All have a length of 198 inches. Ten perches of this system would be 1980 inches, which is both one quarter of a furlong and the number sequence of the line immediately preceding Enide's name revelation. Readers of *Erec et Enide* will recall that Enide was the maiden worthy to take the sparrowhawk from a silver perch. It is not unlikely that the lines covering key scenes, such as name confessions, were inserted in their correct positions when the structure was planned, before the filling in of the rest of the story took place. As shown, ancient number systems were based on body ratios, in which the foot was consistently two thirds of the local cubit. The cubit was believed to be the distance from the elbow to the tip of the middle finger. Two German or Saxon cubits would be 39.6," the sequence of Enide's introduction at line 396 and her cryptic behaviour at line 3960. The two cubit unit of 39.6 inches was the "gird," or yard which in turn was one half of the "fathom" or full armstretch of 79.2 inches. The fathom or armstretch therefore consisted of six feet, four cubits or two yards. The one exception seems to have been the Hebrew reed, which was not an armstretch of four cubits but a measure of six cubits. One hundred of the German or Saxon fathoms made the furlong of 7920 inches. A square with sides the length of this furlong had an area of exactly ten of the acres that we have come to think of as English.

Square one furlong 7920"

10 acres

Side one furlong 7920"

(One hundred fathoms or armstretches of 79.20")

At this point there is nothing to explain why Nature should have marvelled at the creation of Enide, or why Enide should have been considered the work of Nature at all. Nor is it yet clear why, efficient though this system may be, the scribe chose it to structure his manuscript line numbers. While seeking to understand the medieval attitude to number and measure, it is necessary to note Zupko's warning against the practice of making "forced linkages between ancient and contemporary systems of weights and measures on the one hand and the British system on the other."³ But as Berriman has shown in his *Historical Metrology*, the 19.8 inch cubit was certainly known in antiquity and is in fact well documented in ancient Mesopotamia.⁴ The question of whether there was unbroken continuity in the use of these measures would scarcely have entered the mind of the Western European crusader, pilgrim, or intellectual courtier coming into contact with the customs and culture of the East. Educated to the Platonic idea that a numerical World-Soul had been woven into the universe at the time of creation, he would have been predisposed to believe, even to embrace the idea that such was the case. A reference at line 1320, last line on the tenth page, takes the reader back to the first created man in the primeval garden. Erec, asking for the hand of Enide, offers to bestow upon her father a possession dating back to the time of the first created man:

sires seroiz de Roadan
qui fu fez des le tans

1320

[(You) will be lord of Roadan
which was made from the time
of Adam]

The implication is that this special number scheme, which included the 13.2 inch foot, and 7920 inch furlong, was inherited from the earliest age of man.

In order to understand the poet's method and objective, it is necessary to enter into the mind of the medieval artisan as he approached his work. Plato's *Timaeus*, which was one of the most influential documents in Western European intellectual circles in the twelfth century, provided powerful support for the concept of a numerical plan and pattern in Nature. At the Cathedral school of Chartres, a Neo-Platonic revival had flourished under such scholars as Chancellor Thierry, who was obsessed with the idea of interweaving earlier philosophies and beliefs into the service of conventional theology. In the *Timaeus*, Plato's maker of the universe sets the example to be followed by the maker of anything. Timaeus explained to his friends that when the universe or cosmos was created, the maker kept his eye on the eternally unchanging and used it as his pattern:

Clearly, of course, he had his eye on the eternal; for the world is the fairest of all things that have come into being and he is the best of causes. That being so, it must have been constructed on the pattern of what is . . . eternally unchanging.⁵

Here we have reference not only to a specific pattern in creation but a declaration that the pattern used was based on a vision of the eternally existent. Work carried out in emulation of the creator was good:

Whenever, therefore, the maker of anything keeps his eye on the eternally unchanging and uses it as his pattern for the form and function of his product, the result must be good.⁶

Belief that pattern was the one, or even part of the one used by the creator to structure the universe, would be sufficient to cause an artisan to emulate and preserve it. The words of Otto von Simpson in *The Gothic Cathedral*, provide a necessary reminder of the gulf that exists between the attitude of the modern scholar and the mind of the medieval maker:

The modern artist is free to create; we demand of him only that he be true to himself. The medieval artist was committed to a truth that transcended

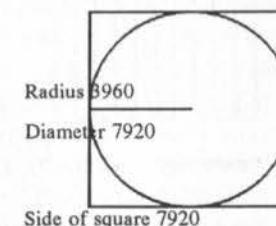
human existence. Those who looked at his work judged it as an image of that truth.⁷

Whether he saw it in terms of Platonic Eternally Existing Being, or according to ecclesiastical doctrine, or both, the medieval artist was committed to just such a truth. If the pattern of this truth could be found in Creation, this would explain to some extent why Nature marvelled when, in Enide, she saw that she had produced a result that she was never again able to duplicate.

The first application of the "Enide" numbers 3960, 3960 and 1980, shows them to be in harmony with German or Saxon measures. The alternative but harmonious interpretation of the "Enide" numbers is applicable to the work of Nature in the earthly creation. It is a fact that can be confirmed in any reliable Atlas or Encyclopaedia, that the radius of the Earth, the work of Nature, is 3960 of the miles known to us as British Imperial.⁸

In the opening lines of the romance, the reader is warned to make use of his intelligence, lest he underestimate or overlook what may conceal greater richness than he suspects. A knight or scholar knowing the terrestrial radius to be 3960 English miles would perceive a reason for Enide to artfully conceal her face with a veil at this point. Because the Hebrew stade is one tenth of a mile (at sixty reeds of 105.6 inches),⁹ it follows that the terrestrial radius of 3960 miles is also 39600 Hebrew stades. However, for the present it is Enide, the wonderful work of Nature who commands our attention.

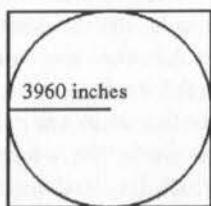
At line 3960, the number of miles in the terrestrial radius, Enide with great wisdom "par grant veidie" is found concealing herself. The circle with radius 3960 will have a diameter of 7920, the same as the side of the square enclosing it.



The square on the diameter of the earth has sides of 7920 miles; this number sequence has already been seen as the 7920 inches of the English furlong, and the 79.20 inches of the German or Saxon

armstretch or fathom. Ten of these armstretches made a chain, and ten chains made the furlong. A further decimal reduction produced the old "link" measure of 7.92 inches. The link, which was one thousandth of a furlong, was also two handbreadths of 3.96 inches.

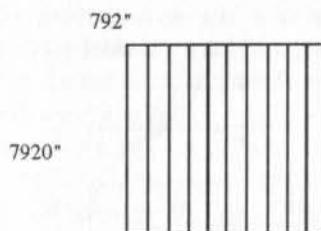
It has been noted that a square on a furlong of 7920 inches has an area of exactly ten acres. This means that a square on the earth diameter would have an area of ten English acres, if the miles were reduced to inches.¹⁰



Area of square when miles are reduced to inches, 10 English acres.

A. E. Berriman in his *Historical Metrology* referred to the English acre as "the most intriguing of ancient measures."¹¹ It was in terms of the acre that farm land was recorded in the famous survey commissioned by William the Conqueror.

A square of ten acres could be conveniently divided into ten single acre strips. This would have greatly facilitated the practice of tithing.



Each strip would then measure one furlong by a tenth of a furlong, which is the definition of an English acre given by Berriman.¹² This is identical in proportion to the acre described in units of 198 inch rods by Zupko: In plowing, Saxon peasants worked a section of land that consisted of forty rods in length, (the furlong) and four rods in width.¹³

The perimeter of this acre would be 17424 inches: $7920 + 792 + 7920 + 792 = 17424$ inches.

Before King Arthur bestowed the ceremonial Kiss of the White Stag upon Enide, he invited his assembled courtiers to consider his conviction that her beauty was so perfect, that she must have come from that place where heaven and earth unite. In the Guiot manuscript, King Arthur makes the specific reference to a meeting place of heaven and earth at line 1742:

puis dist as chevaliers seignor	[Then he said to the knights,
que dites vos. que vos	Lords
an samble	what do you say? how does it
ceste est de cors, de vis	appear to you
ansamble	this one is of body and of face
et de quanqu'estuet a pucele	together
et la plus gente et la plus bele	and of all that appertains to a
ne qui soit des la, ce me sanble,	damsel
ou li ciaxies et la terre ensamble	and the most gracious and the
1742	most beautiful
	seems to me
	where the heaven and the earth
	unite?]

Within the available line numbers, the perimeter of the acre matches the line number specifically mentioning Enide's special qualification and standard of merit.¹⁴

$$7920 + 792 + 7920 + 792 = 17424$$

"ou li ciaxies et la terre ensamble," line 1742

Although the earthly aspect of this model is emerging, it contains nothing as yet to indicate a symbolic union with the heavens. The original square was based on the diameter of the earth, before the miles were reduced to inches. The furlong is essentially an earth measure. The name furlong comes from the Old English furlang, from "furh" furrow, and "lang" long.¹⁵ The furrow refers to the cultivation of the earth. By this model, the measures involved would qualify to be thought of as earth measures.

The measures associated with Enide's contribution are geometric in the sense described by Martianus Capella's bridesmaid Geometry, in *The Marriage of Philology and Mercury*. To the assembled guests at the

wedding, the bridesmaid Geometry revealed the nature and purpose of her identity:

I am called Geometry, because I have often traversed
and measured out the earth.¹⁶

The maiden counted worthy to receive the Kiss of the White Stag was linked by the scribe to measures that qualify to be thought of as geometric in the true sense, that is, earth measures. Why then was King Arthur so confident that she came from the meeting place of earth and heaven?

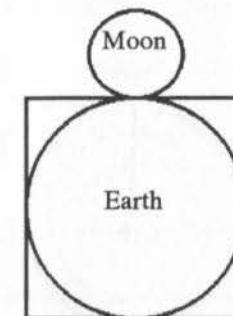
Before the heavenly part of the union can be seen, it is necessary to have a clear picture of the twelfth-century view of the universe. It was believed that the dividing line of the earthly, temporal region, and the heavenly imperishable realm, lay on the pathway of the moon. Hugo of St. Victor in his *Didascalicon* wrote

. . . . astronomers (mathematici) have divided the world into two parts: into that namely, which stretches above the sphere of the moon, and that which lies below it.¹⁷

Magister Hugo went on to say that everything above the pathway of the moon was called Elysium because of the perpetual tranquility of its light and calm. The region he called sublunary extended from the moon down to the earth. This was known as "infirnum" from the confusion and instability therein. Macrobius also recorded a tradition which taught that,

. . . the realm of the perishable begins with the moon
and goes downward.¹⁸

Macrobius wrote that, according to this tradition, souls falling from heaven to earth die when they reach the line of demarcation that is the moon's path. Souls rising from earth to heaven begin to live when they ascend to the region above the lunar pathway. By this reasoning, if the heavenly temple called the moon were lowered to meet the earth, the infernal region in between would disappear. If the earthly temple and the heavenly temple were brought together, the region of perpetual tranquility and light described by Hugh of St. Victor, would extend right down to earth. Keeping the measurements of the earth model already shown, it is possible to effect a symbolic union of heaven and earth in this way, by bringing the heavenly temple and the earthly temple together. What Hugh of St. Victor called the infernal region separating heaven and earth would be eliminated.



The union of heaven and earth was symbolically represented by one of antiquity's most consistent traditions. The concept was simple. While the square symbolised the elements and attributes of earth, the circle was used to represent everything with eternal or heavenly connotations. Although the perimeter of a square may be accurately calculated to be four times the length of its side, the circumference of a circle having the same side as its diameter is incommensurable, because the ultimate limit of Pi remains unknown. Alain de Lille, in his *Sermo de Sphera Intelligibili*, went so far as to identify divinity with the circle. He taught that

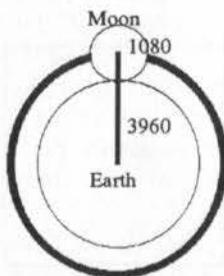
God is a circle whose centre is everywhere and
whose circumference is nowhere.¹⁹

The problem of bringing square and circle into unity was never, in antiquity, an idle mathematical diversion. By virtue of this simple symbolism, the union of square and circle was seen to represent the transcendence of life over death, eternity over time, heaven over earth. If the radius of the earth 3960, and the radius of the moon 1080 are combined, a new circle is formed. The radius of the new circle will extend from the centre of the heavenly circle to the centre of the earth. By the combined radii, it would be symbolically correct to say that heaven and earth have been spanned. The new circle has a radius of 5040, the combined radii of the earth 3960, and the moon 1080.

Although we can never know the exact circumference of the circle because of the irrational nature of Pi, it is possible to reach a rational approximation using the ratio 22: 7, the method approved by Macrobius. The combined radii is doubled to give the diameter, which is then multiplied by twenty-two and divided by seven.

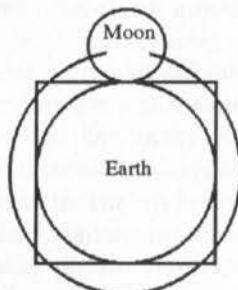
$$1080 + 3960 = 5040 \quad 5040 \times 2 = 10080$$

$$10080 \times 22 = 221760 \quad 221760 \div 7 = 31680$$



Combined radii 3960 plus 1080 equals 5040.
Circumference of circle 31680 miles.

The circumference of the circle using this method is 31680 English miles or 316800 Hebrew stades, exactly the length of the perimeter of the square surrounding the earth circle. In terms of the numerical sequence 31680, it can be said that the perimeter of the square and the circumference of the circle have been brought into a form of unity.²⁰



Perimeter of square = 31680
Circumference of outer circle = 31680

This union of heaven and earth might be thought of as the "Enide" contribution to the Arthurian romance. It has been reached by following the clues linking Enide to the beautiful but temporal work of Nature.²¹ The specific reference to Enide as the maiden from the meeting place of heaven and earth was found at line 1742. Within the available line numbers, this is the sequence of the 17424 inches in the perimeter of the acre as displayed in the model reflecting the work of Nature.

Geometry, who wore her shoes out traversing and measuring the earth, has no reason to find fault.

It was not the author's purpose that his "conjointure" should be interpreted only on a secular level. In the introduction to his *Erec et Enide* he stated plainly that his objective was to recount a tale that would be remembered so long as Christianity endures:

Des or comancerai l'estoire [Now I will begin the story
qui toz jorz mes iert an mimoire which will always remain in
memory
tant con durra crestianez. so long as Christendom endures.]

23-25

In von Simpson's words, the medieval artist was committed to a truth that transcended earthly existence, and those who observed it judged it as an image of that truth. In terms of Western European conventional theology, the truth that transcended earthly existence would have been the One whose story would be told so long as Christendom endures.

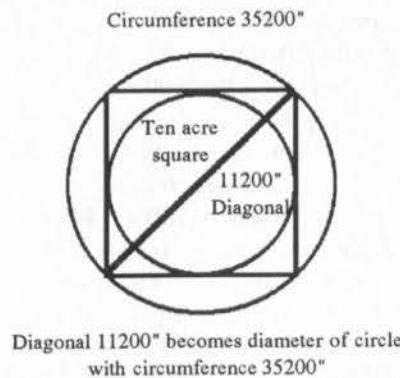
The puzzling fact that the story contains little or no ostensible reference to Christianity has been the topic of a considerable amount of scholarly discourse. Significant to the present discussion, when the numerical equivalents of the Greek letters used to spell the name of the Christian Redeemer are added together, the result is 3168. This is the sequence of the number of miles in the square and circle producing the union of heaven and earth shown in the model:

Lord	Kappa	20
	Upsilon	400
	Rho	100
	Iota	10
	Omicron	70
	Sigma	200 = 800
Jesus	Iota	10
	Eta	8
	Sigma	200
	Omicron	70
	Upsilon	400
	Sigma	200 = 888
Christ	Chi	600
	Rho	100
	Iota	10
	Sigma	200

Tau	300
Omicron	70
Sigma	200 = 3168

In twelfth-century Western Europe, such theological interpretation would have been seen as the fulfillment of earlier traditions. This model may be thought of as the key. The Arthurian romances and the Grail story in the Guiot manuscript are variations on the single theme of the quest for unity between heaven and earth. For example, in *Le Chevalier au Lion*, the marriage of the hero Yvain to a beautiful Christian lady²² is mediated by a damsels significantly named Lunette. The sequence of the lunar diameter 2160 is thus marked by a capital. The occasion is the celebration of the marriage, or beautiful union, made possible by Lunette. Soon after the marriage of Yvain and the beautiful Christian lady discord appears, but with the help of the ever-faithful Lunette, unity is restored in the lines corresponding to the circumference of the moon.

Further, tradition has it that King Arthur would not begin his meal until some new marvel had been shown, and, in similar fashion, let us continue the "numeric feast": If the diagonal of the ten acre square is used as the diameter of a new circle, yet a new numerical marvel emerges. The circumference of the new circle will be 35200 inches.²³



If the circle of circumference 35200 inches is expressed as a square, the area of the square in terms of the "English" acre is, curiously, 12.345679. The first and most obvious characteristic in the sequence is the absence of the number eight. However, if the number is

multiplied by the missing eight, an unbroken sequence emerges, leading all the numbers in perfect harmony back in the direction of the One.

$$12345679 \times 8 = 98765432$$

The most disinterested observer can hardly fail to acknowledge the special harmonizing power of this number, in relation to nine and its multiples.

12345679 x 9	=	111 111 111
12345679 x 18	=	222 222 222
12345679 x 27	=	333 333 333
12345679 x 3	=	444 444 444
12345679 x 45	=	555 555 555
12345679 x 54	=	666 666 666
12345679 x 63	=	777 777 777
12345679 x 72	=	888 888 888
12345679 x 81	=	999 999 999

In terms of number symbolism, the following application unites the alpha and the omega in this perfectly patterned sequence in which the beginning and the end are One:

$$12345679 \times 999,999,999 = 12345678987654321$$

While Enide was strategically introduced into the romance at line 396, Erec makes his first appearance in a sentence beginning at line 81. King Arthur and his knights have gone hunting for the White Stag, which must be apprehended before the Kiss can be bestowed. Guinevere lingers behind, accompanied by a damsels of fair and gentle birth. They are joined by Erec.

Après les suist a esperon

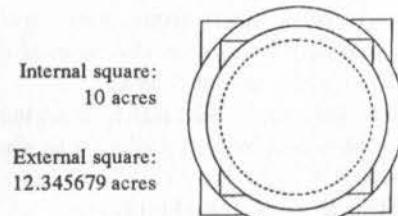
uns chevaliers Erec a non
de la Table Ronde estoit...

line 81[They were followed on
horseback
by a knight named Erec
who belonged to the Round
Table]

At line eighty-one the introductory description of Erec begins. The significance of this number in relation to Erec lies in its reciprocal. That is, one divided by eighty-one, which produces the numerical sequence shown above as 12.345679 acres.

One divided by 81 equals 0.012345679

If this is a strategic device both generally concealing and revealing (to the searching few) a clue to Erec's identity, it would provide logical explanation as to why the pair are said to be so perfectly suited. The "Erec" square and circle of 35200 inches is a perfect frame for the "Enide" square and circle of 31680.



The perfect union of Erec and Enide:
External square and circle 35200 inches
Internal square and circle 31680 inches

Line numbers relating to Erec apply well to Hebrew standards. The suggestion of a Hebrew influence in the first Arthurian romances is not new.²⁴ In the twelfth century there were few localities so richly endowed with access to secrets of the Jewish faith as the region of Troyes. The great Rabbi Solomon (Sheloma) Ben Isaac, known as Rashi, had set up in that region what has been described by the contributor to *Encyclopaedia Britannica* as: "the most important centre of Rabbinic learning in central Europe, a fountain head of Rabbinic scholarship."²⁵

Erec's first battle was with Ydiers. The conflict was long and fiercely contested, until Erec finally emerged the victor. In defeat Ydiers requested to know the identity of the knight who had vanquished him. Erec replied that he would not conceal his name. His name is Erec:

... Jel te dirai [I will tell you]
ja mon non ne te celera [1056] I will never conceal my name
from you

Erec ai non.

Erec is my name.]

The name confession is centred on line 1056. This corresponds numerically to the 105.6 inches in the Hebrew reed. As shown in relation to Enide, the scribe had planned his structure so that key line numbers of the model would be easy to locate. We recall that line numbers at page endings proceed thus:

132, 264, 396, 528, 660, 792, 924, 1056 . . .

To accent this line number, Erec's name confession scene is found at the last lines on page eight and at the top of the first column of page nine. As noted, key lines such as this may well have been written in during the initial laying out of the overall structure. Erec is thus

making his first post-battle name confession at line 1056, corresponding to the 105.6 inches of the Hebrew reed, which consists of six cubits of 17.6 inches.

Hebrew reed
17.6" 17.6" 17.6" 17.6" 17.6" 17.6"

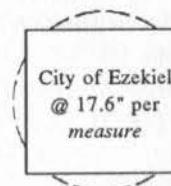
Length of reed 105.6 inches.

Erec's first post-battle name confession scene centered on line 1056
(From *Weights and Measures*, Encyclopaedia Britannica, 1973 ed.)

The question that now arises is whether Hebrew applications of the model and the number sequence 12345679 were available, or likely to have been known by a twelfth-century scholar in the region of Troyes. Available space allows only a brief consideration.

In the closing verses of the Old Testament Book of Ezekiel, the visionary city Jehovah-Shammah is described as having a perimeter of eighteen thousand measures: "It was round about eighteen thousand measures: and the name of the city from that day shall be, The Lord is there." (Ezekiel ch. 48 v. 35)

The 105.6 inch Hebrew reed, as shown, consisted of six 17.6 inch cubits. If the 17.6 inch cubit is applied as the unspecified "measure," the perimeter of the city would be 316800 inches. With a hundredfold increase, this application to Ezekiel's city is identical to the "Enide" square around the earth circle, when the miles are reduced to inches. The side of the city at this rating would be 79200 inches or ten furlongs, and the area one thousand acres.



Perimeter 18000 x 17.6" = 316800"
Jehovah Shammah, "The Lord is there."

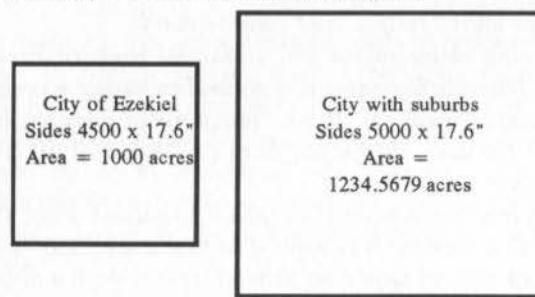
We are told that surrounding the hypothetical thousand-acre city, there were suburbs extending two hundred and fifty of the unspecified measures, in each direction:

And the suburbs of the city shall be toward the north
two hundred and fifty, and toward the south two

hundred and fifty, and toward the east two hundred and fifty, and toward the west two hundred and fifty.

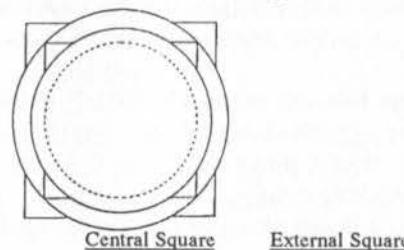
Ezekiel ch. 48 v. 17

In the application already shown, the city had sides of four thousand five hundred measures. The extension of two hundred and fifty in each direction would give sides of five thousand measures. If the measure of 17.6 inches is retained, the external square enclosing the city plus its suburbs now has sides of 88000 inches and a circumference of 352000. The area of the new square, including the suburbs, reproduces the above special sequence in terms of what we think of as English acres. The calculated area is 1234.5679 acres.



When the hero Erec and his sweetheart Enide are betrothed, we are told that never were two so beautiful "ymages" ever brought together by law or marriage. This was not seen as a man-made or mortal harmony. It excelled all natural human harmonies and unions. Erec and Enide were so perfectly suited that neither law nor marriage could ever create such a beautiful union:

onques deus si belles ymages [never were two so beautiful figures
[1495] n'asanbla lois ne mariages. ever brought together by law or
marriage.]



As Erec and Enide	10 acres	12.345678 acres
As Jehovah Shammah	1000 acres	1234.5679 acres

The union excelled anything that mortal man could devise. We learn this at line 1495—the sum of the numerical values of all the letters in the Hebrew alphabet. The harmony of Erec and Enide was a perfect union.

The above interpretation, if correct, demonstrates the great gulf that separates the medieval frame of mind from the twentieth-century approach to literary criticism. As von Simpson noted, the medieval artist was committed to a truth that transcended earthly existence, and he knew that his work would be judged as an image of that truth. Although no modern scholar would attempt to interpret a twentieth-century work by medieval standards of artistic or literary merit, attempts are still made to assess medieval work by modern methods. The above approach would be harmonious with the philosophy approved by Macrobius, when he described the *narratio fabulosa* by which

... a decent and dignified conception of holy truths, with respectable events and characters, is presented beneath a modest veil of allegory. This is the only type of fiction approved by the philosopher who is prudent in handling such matters.²⁶

Notes

1. Macrobius, *Commentary on the Dream of Scipio*, trans. with intr. William Harris Stahl (New York: Columbia UP, 1952) 86-87.
2. Ronald Edward Zupko, *British Weights and Measures: A History from Antiquity to the Seventeenth Century* (Wisconsin: University of Wisconsin Press, 1977) ll.
3. Zupko xiv Intr.
4. Berriman, A. E. *Historical Metrology: a new analysis of the archeological and the historical evidence relating to weights and measures* (London: J. M. Dent and Sons, Ltd., 1953), 29. Berriman gives a useful table comprising the most important linear measures of antiquity.
5. Plato: *Timaeus and Critias*, trans. with intr. Desmond Lee (1965; Harmondsworth, Middlesex, England: Penguin, 1979) 41.
6. *Timaeus*, Lee trans. 40-41.

7. Otto von Simson, *The Gothic Cathedral: Origins of Gothic Architecture and the Medieval Concept of Order*, Bollingen Foundation Series 48 (1956; Princeton, New Jersey: Princeton UP, 1974) xvii.
8. *The Readers Digest Great World Atlas* gives exactly this figure in the 1962 edition, (107). The National Aeronautics and Space Administration (N.A.S.A.) figures give polar and equatorial diameters which differ because the earth is not a perfect sphere. The mean diameter of these figures obtained by satellite is 7917.4 miles, giving a radius of 3958.7, a discrepancy of only 1.3 miles from the round figure of 3960. "Earth," *Encyclopaedia Britannica* 1973 ed.
9. "Weights and Measures," *Encyclopaedia Britannica* 1973 ed.
10. A square on the diameter of the earth with the miles reduced to inches is equal to a square of ten acres. This was my own independent observation, reached by following the clues laid down by the scribe Guiot in the manuscript.
11. Berriman explored the possibility of an earth-measured origin for the English acre, but apparently did not know the proposed model. He accepted a slightly smaller figure for the earth diameter, and used a different approach. Berriman wrote:
The English acre is the most intriguing of ancient measures because it is virtually equal to a hypothetical geodetic acre defined as one-myriad-millionth of the square on the terrestrial radius: if both acres are expressed as squares, the difference between the lengths of their sides is less than one part in 1200 (page 3).
12. Berriman *Historical Metrology*, 171.
13. Zupko, 11.
14. The connection of the perimeter of the single-acre strip with line 1742, "ou li ciex et la terre asanble," was in actual fact made by following the logic of the philosophy, and the clues laid down by the Guiot scribe within the text.
15. *The Oxford English Dictionary*, 1933 ed. Furlong: [O. E. furlang str. neut., f. furh, Furrow + lang , Long a.]
1. Originally, the length of the furrow in the common field, which was theoretically regarded as a square containing ten acres . . . furlong has always been used as a name for the eighth part of an English mile, whether this coincided

- with the agricultural measure or not. The present statute furlong is 220 yards, and is equal both to the eighth part of a statute mile and to the side of a square of ten statute acres.
16. Capella *Marriage of Philology and Mercury*, 2: 220.
 17. Hugh of St. Victor, *The Didascalicon: A Medieval Guide to the Arts*, trans. with intr. Jerome Taylor (New York: Columbia UP, 1961) 54.
 18. Macrobius 131.
 19. Alain de Lille, *Sermo de spharea intelligibili*, qtd. in intr. Alan of Lille: *The Plaint of Nature*, trans. and commentary James J. Sheridan (Toronto, Canada: Pontifical Institute of Mediaeval Studies, 1980) 25.
 20. The square and circle of 31680 miles from the earth and moon comes from John Michel's *The City of Revelation: On the Proportion and Symbolic Numbers of the Cosmic Temple*. Great Britain: Garnstone Press, 1972. Also London: Abacus, 1973 reprinted 1978. The model is given on page 73. This does not claim to solve the problem known as the squaring of the circle, which involves area. This model involves only the linear measures of radius, diameter, circumference and perimeter. (The area of a circle is always larger than that of a square with the same perimeter. Although there is insufficient space here to show the scribe's method of addressing the problem of area, future publications will show that he was well aware of the difference.)
 21. The clues relating to the work of Nature in the manuscript were independently observed to be applicable to the design given on page 73 in *The City of Revelation* by John Michel.
 22. De si tres bele crestiene [Of such a beautiful Christian lady
ne fu onques plez ne word or mention was never made]
parole 1148-49
 23. The mystery of the square root of two, like Pi, involved the irrational and demands a study to itself. $7920 \times 1.414 = 1198.88$
The number 11200 which yields the result given here is the closest in round figures.
 24. A study by Urban T. Holmes and Sister M. Amelia Klenke in 1959 supported the concept of Jewish influence in the Grail story. Holmes followed this in 1970 with a more general study on the works of Chrétien de Troyes, in which he wrote of

. . . the presence in eastern France and in the Rhine-land of Jewish schools and wealth. The present writer believes that this too had considerable influence on some writers, but the idea has been so vigorously opposed by many of our colleagues that we leave it as an open question.

Chrétien de Troyes, p. 28

25. "Rashi (Rabbi Solomon [Sheloma] Ben Isaac)," *Encyclopaedia Britannica* 1973 ed.
26. Macrobius *Dream of Scipio* 85.

The Significance of Numbers in the Structuring of Dante's *Commedia*

Betty Vanderwielen

Dante scholars have long recognized number symbolism as an important element in the composition of the *Commedia*. Not only are numerical allusions copiously interspersed throughout the *Inferno*, *Purgatorio*, and *Paradiso*, but the very divisions of the poem bespeaks numerological significance: three books, each divided into 33 cantos with the consequent incomplete number 99 transmuted into the 100 of completeness by the insertion of one prologue canto. (The number 3, of course, represents the Trinity; 33 the number of years of Christ's life on earth; and one signifies God as Unity).¹ Even the terza rima verse form emphasizes the number 3. In fact, as the famous Dante scholar, Charles Singleton, points out (82), the rhyme scheme invented by the poet reflects on a miniature scale the structure of the whole *Commedia*, for terza rima consists of 3-unit stanzas or terzines to which one extra line is appended at the end of every canto—a tiny, inverted image of the extra prologue canto added to the body of the three books.

Moreover, Singleton discovered another more complex numerical pattern in the center of the poem. By counting out the number of lines in each canto, he found that the exact center canto—*Purgatorio* 17—was neatly framed by the lengths of the three cantos preceding and the three following it: (14) 151, (15) 145, (16) 145, (17) 139, (18) 145, (19) 145, (20) 151. Now, for the people of the Middle Ages, the numbers from one to twelve had their own particular symbolic meanings; numbers beyond twelve were reduced to their smaller units or looked at in terms of their component parts. By adding the individual numerals of such a number, for instance, one could reduce it to its symbolic essence.² Singleton, therefore, added together the numerals which constituted the beginning and the ending lengths for the 7-canto pattern which he had discovered. Thus 151 becomes 1 + 5 +

1, which yields the number seven, a number which also represents the sum derived from adding the components of verse 70 ($7 + 0$) of *Purgatorio* 17, the verse which lies at the exact center of the whole poem. Then, by examining the content of the text contained in these cantos, Singleton concluded that what was thus numerically framed and set off by this repetition of sevens constituted "nothing less than the central pivot of the whole poem in terms of the action, in terms, that is, of what happens to the wayfarer Dante as he passes *through the center*"—what the Dante scholar refers to as the "conversion at the center" (85). As Singleton himself assures us, his discovery, far from being the final word on numerology in the *Commedia*, opens up entirely new possibilities for investigation. I would like to explore one of those possibilities in this paper. In adding together each of the canto lengths another pattern appears—or, more accurately, three other patterns united by the common factor that each has its midpoint in the middle canto (17) of its particular book (see Table 1). In the *Purgatorio*, of course, this pattern includes Singleton's own; but it also goes beyond it, encompassing not seven, but thirteen, cantos. While Singleton's center pattern is linked to the central content of the poem, I would suggest that the significance of this other triple pattern lies in the very essence of the numbers themselves, for the numbers which make up these patterns convey their own symbolic meanings, and their sums in each book represent the essential content of that book.

However, before we can accept the plausibility that any author actually would go to the trouble to introduce such complexity into the structuring of a lengthy poetic work, we should recapitulate briefly the supreme importance which numbers held for Dante and his contemporaries.

It is somewhat difficult for modern readers to appreciate how completely the "science" of numerology permeated the lives of the people of the Middle Ages. As Vincent Hopper explains in his book, *Medieval Number Symbolism*, "there was literally no reservoir of knowledge or inspiration on which this period could draw which was not impregnated with number philosophy" (89). Hopper traces these influences from an early sense of mysticism concerning numbers, through the association of numbers with astrology and their rise to religious significance in Babylonia, their inclusion in the Old Testament scriptures, and finally the re-emergence of Pythagoreanism which held numbers themselves to be inherently sacred expressions of fundamental

truths. All of these factors were prevalent when the early Christians writers began addressing the science of numerology. Whereupon, "instead of denying or neglecting what had gone before, the Church accepted number theory in all its forms, thus preserving and revitalizing them all" (93).

St. Augustine was perhaps the most famous of those writers who embraced, and subsequently Christianized, numerology:

When I think about the unchanging truth of numbers, and when I consider the province of numbers—their room or sanctuary, as it were, or whatever suitable name can be found by which we may designate the home or seat of numbers—I am far removed from my body... The same thing happens to me when I think as carefully and intently as I can about wisdom. Besides, I am very much amazed because these two things lie in the most secret and yet most certain truth—even by the testimony of the Scriptures, where number and wisdom are placed together.

(On Free Choice of the Will 2.11.122-123)

Augustine's writings, as Hopper elucidates (chap. 5), are filled not only with the philosophy of numbers but also with elaborate expositions on the symbolism of various numbers in scriptural passages. During this period, essentially pagan meanings were translated into Christian terms: one is no longer "First Cause" but specifically "God"; three is no longer any triad but specifically "the Trinity" and, by relation to the Triune God, becomes the number of perfection. Nor was Augustine the only proponent of this "science":

Many of them [the early Church Fathers], like Augustine, exhibit a manifest pride in their numerical learning, as well as a never-failing delight in discovering new instances of the numerical harmony of all things. It is not unusual in their writings to come upon extended commentaries on number science, commentaries which are not called for by the contexts. It becomes a commonplace also for them to launch into an encomium on a given number, digressing from the theme to point out as many as possible of the meanings or usages of the number in question. (Hopper 86-87)

Correlatively, it also becomes common practice for medieval writers to arrange the divisions of their books and texts in numerically symbolic ways, a practice Dante later follows.

These, then, were the reservoirs from which the numerological concepts of the Medieval Ages were drawn—Scripture, astrology, neo-Pythagoreanism, and the strong approbation of the Church Fathers. But perhaps nothing speaks so eloquently of the pre-eminence which number had attained in this period as does the illustration from a thirteenth-century bible which figures God as an architect.³ Pictured in the process of creating the world, He uses a drafting compass to impose on the planets and the entire universe an exact form and proportion which accords with numerical principles. As Butler explains in his book *Number Symbolism*: "the arithmologists believed that God himself thought of his creation, indeed thought it up, arithmetically" (10). In a sense, then, number is God's own language.⁴

Dante was, of course, a product of his age, and as such, the pervasiveness of number symbolism was a birthright which he wholeheartedly embraced. Even before he wrote the *Commedia*, his conviction of the metaphysical nature of numbers is overwhelmingly apparent in the *Vita nuova* with its association of Beatrice with the number nine.⁵ At one point Dante connects Beatrice's death with the number nine and offers the supposition on astrological grounds that "this number was friendly to her." Then he goes beyond astrology and enters theological and metaphysical realms:

But more subtilly thinking, according to the ineffable truth,
this number was herself, I speak by similitude, & that I mean
thus.

The number three is the root of nine since, without any
number, multiplied by itself it makes nine as we see plainly
that three times three make nine. Then if three by itself is
the factor of nine, and the (factor or) maker of miracles is
three, that is Father, Son, & Holy Spirit which are three &
one. This lady was accompanied by this number of nine, to
give to understand, that she was a nine, that is a miracle,
whose root that is of the miracle, is only that wonderful
Trinity. (29.14-24)

When we understand, therefore, the incredible significance which numbers held for Dante, it is no longer so astounding that he would place purely numerical patterns at the center of each of the books of the *Commedia*. As Russell Peck says in his essay "Number as Cosmic Language": "Indeed, admiration of the number in things is a form of praise. In discovering form, one exercises the highest of his God-

created rights—the grace of participation in the divine ratio" (17-18). If such participation resulted from "discovering form," how much greater participation must Dante have felt in *creating* form using God's own language of number.

It is impossible, of course, to state definitively how Dante wished his numerical pattern to be interpreted; nevertheless, based on the most common assignations of specific numbers throughout the medieval period, I would like to suggest a possible reading.

As Table 1 indicates, the pattern in the *Inferno* consists of five cantos, the sum of whose lengths compute out to 7, 10, 10, 10, 7 and whose collective sum is 44. Christian medieval number symbolism held that four represented the human being. Theophilus of Antioch justifies this connection through a somewhat convoluted explanation of the ordering of creation:

Similarly the three days prior to the luminaries are types of
the triad of God and his Logos [the creative Word] and his
Sophia [the 'spirit borne over the water']. In the fourth place
is man, who is in need of light—so that there might be God,
Logos, Sophia, Man. For this reason the luminaries came
into existence on the fourth day. (2.15)

More specifically, four was considered the number of the bodily part of men and women since medieval science taught that people were composed of the four elements: "There are four Elements, and original grounds of all corporeal things, Fire, Earth, Water, Aire, of which all elemental inferiour bodies are compounded" (Agrippa 1.6). In addition, four represented carnality by virtue of the existence of four humours in the body (phlegmatic, sanguine, choleric, and melancholic) and four bodily conditions (hot, cold, moist, and dry). The number 5, during the medieval period, also suggested worldliness and animality, principally because of its association with the five senses. Nor did it escape the notice of medieval theologians that the imperfect law of the Old Dispensation was contained in the Pentateuch—the first five books of the Old Testament. Just as the Old Testament law was replaced by the New Dispensation, so, Augustine explains in his homily on the Gospel of St. John, the five senses of the flesh dominate until "understanding, imbued by reason, begins to rule the soul" (21-22).

Therefore, Dante's use of five cantos which equal the number 44 (double numbers were not added together but treated as doubles, i.e. twice as much "4-ness") signifies a preponderance of body-ness, of

sensuality. What, in one phrase, could more aptly describe the *Inferno*? It is, indeed, a place where humans exists in their bodily, sensual, animalistic part.

The particular canto-length sums which Dante chose to make up this pattern in the *Inferno* are also pregnant with meaning. Both 7 and 10 had various different connotations in the Middle Ages (cf. Peck 61-62), but our understanding of them here must be influenced by the symbolism of four and of five. Thus, the most obvious meaning for seven is the human being who is composed of both body and soul.⁶ As Honorius confirms:

Tria et IIII fiant VII. Anima habet tria, scilicet memoriam, intelligentiam, voluntatem. Corpus autem constat ex III elementis. Quali ergo amore anima corpori jungitur, tali dilectione justum est ut proximis conjungamur.

("Speculum Ecclesiae: Dominica in Quinquagesima" 873C)
[Three and four make seven. The spirit contains three things, namely memory, intelligence, and will. The body, on the other hand, is composed from four elements. Therefore, as the spirit is joined to the body by love, so it is fitting that we are joined together very closely by choice.]

The number 10, associated as it is with the number five (Pentateuch), signifies the law (10 Commandments) of the Old Testament. And, indeed, it is the Old Testament mode of justice (an eye for an eye, a tooth for a tooth) which is the governing principle in the *Inferno*, for souls in Dante's hell—all those who rejected the forgiveness of the New Testament—receive punishment in a manner and measure commensurate to their sins.

The juxtaposition of the two numbers seven and ten also invokes special connotations. For one thing, medieval people traditionally visualized the 10 Commandments as being divided into two tablets, one containing the first three precepts which relate to the love of God, the other the remaining seven injunctions concerned with behavior to other humans.⁷ Then, too, according to Augustine, the 7 + 3 combination of 10 also signified the union of the person (7) with his Triune God (3) ("Against Manichaeus," chap. 10). As Hopper summarizes the issue: "Ten had long been recognized as the image of unity, but it was Augustinian Pythagoreanism that produced it by adding the Trinity of the Creator to the heptad of the created" (Hopper 85).

Because of these two familiar connotations, by placing the seven and the 10 together Dante entices the medieval mind to supply the missing three—and that is precisely what is missing in the *Inferno*: the presence of God. Of course, theology teaches that actually nothing can exist in the complete absence of God. The sense in which God reveals his presence in Hell, then, is through His persona of Divine Justice. Thus, at the center of the *Inferno*, the number 10—the number of Old Testament justice—echoes exactly three times.

The *Purgatorio* is made up of 10 cantos, followed by 13 cantos which form a pattern, followed by another 10 cantos. The 10 is, once again, not only the number of completion but also of Divine Justice, for Purgatory, like Hell, is also a place where men and women must pay for their sins. Therefore the number pattern in the *Purgatorio* bears a close relationship to that of the *Inferno*, especially if the former is broken down into three sub-patterns (Table 2). Now it becomes obvious that the two framing sets of 7,10,10, 7 mirror the *Inferno* pattern—except that the center 10 is missing. The third, middle sub-pattern can be seen as an elaboration of the other two with a new number at the center: 7,10,10,13,10 10,7.

If we look at this 13 in its separate parts as a 1 and a 3, it becomes the perfect representation of one God in three Divine Persons, which when multiplied together ($1 \times 3 = 3$) is still the Triune God. Thirteen is also the "gathering together" of 10 and 3, and as such represents the union of the Old Testament and the Trinity, which in turn represents the New Testament.

This 13 at the center, then, is what distinguishes Purgatory from Hell. For here in Purgatory, punishment for sin is tempered by the forgiveness of the New Testament and becomes a means of purification. Here the bodily, animalistic part of human nature is expurgated so that a perfected being can arise to take his or her place with the angels. Indeed, in the *Convivio*, Dante explains how each of the nine orders of angels contemplates a different aspect of the Three Divine Persons and he interjects: "I say that of all these Orders a certain number were lost as soon as they were created, to the amount perhaps of one tenth, and in order to replace these mankind was afterwards created" (2.6.90ff).

In the *Purgatorio* pattern, the number 7 appears four times because the bodily part of the human being is not yet perfected. The number 10—which appeared three times in the *Inferno* pattern—here appears

nine, or 3×3 times (including its presence in the 13 as $10 + 3$). Ninety (the sum of the nine tens) is "one tenth" short of 100 (the 10 of completion, self-multiplied), for it is here in Purgatory that humans are being refined into the fitting replacement for those fallen angels. Of the three subdivisions of the *Purgatorio* pattern, the framing sets each add up to 34—the $3 + 4$ (soul-body) combination which equals the 7 of the human being. The main subdivision adds up to 67, which added together equals the 13 of the Trinity and the New Testament.

The total number of the *Purgatorio* pattern, the number which expresses the essence of this book of the *Commedia*, is 121 (Table 1). It is possible to add this number ($1 + 2 + 1$) and get four, which corresponds to the 44 of the *Inferno* pattern. There the preponderance of bodiliness was intensified by the doubling of the four and the symbolism of the five (the number of cantos making up the pattern). While *Purgatorio* is still concerned with the bodily nature of the human being, that nature is no longer limited to reflecting upon itself. Influenced as it is by the 13 (the number of cantos in the pattern, as well as the center number), the bodily four of the human nature is in the process of being perfected.

Another, and perhaps more interesting, way of looking at the *Purgatorio* sum of 121 is to analyze its separate parts: 1, 2, 1. Traditionally, one is Unity, two is Duality. Certainly the *Commedia* indicates that Dante was concerned with many dualities—Church/State, matter/form, heaven/earth, angels/men, the light of reason/light of Faith, the active life/the contemplative life. Vincent Hopper points out that all these duads, which in the *Purgatorio* labor toward unification, finally achieve that unity in the *Paradiso* (165-192). But this essentially imperfect number two also stands for the Second Person of the Trinity who combines the perfect nature of a God with the corrupt nature of the human.

Hopper suggests that one particular member of the Trinity is "discretely" emphasized in each of the three realms of the *Commedia*: the Power of the Father is most evident in the *Inferno* and the Love of the Holy Spirit infuses the *Paradiso*, while the Son, whose Human and Divine Natures are perfectly united, becomes the ultimate role model in Purgatory, where humans strive to perfect their bodily natures and bring it into complete harmony with their spiritual natures. The number 1, 2, 1 then, can be viewed as a graphic representation of the dual nature of Christ (2) surrounded by the Unity (1) of each of the

other Divine Persons. Similarly, it visually depicts the dual body-soul nature of the human being enveloped, as it were, by the Unity of God. This resolution of dualities by immersion in Unity, then, may be seen as the essential numerical meaning of the *Purgatorio*.

While the numerical pattern of the *Purgatorio* repeated, expanded, and elaborated on that of the *Inferno*, the pattern of the *Paradiso* does not immediately project a similar relationship. Indeed, visually it appears as dissimilar to the other two patterns as Heaven is to Hell or to Purgatory. A closer perusal, however, reveals the *Paradiso* to be the completion of all that was struggling for fulfillment in the *Purgatorio* (Table 3). In the *Purgatorio* the number 7 (the human being composed of body and soul, $4 + 3$) appeared four times—the body predominated. In *Paradiso*, the number 7 appears three times—humans still retain their composite body and soul, but now, at last, the spiritual predominates.

Now also, the nine tens of *Purgatorio* reach their final completion in the 10 tens of the *Paradiso*. In this state of multiplying, or reflecting upon itself, 10 in its symbolic representation of Law now will be superseded by its other connotations: "Unity, perfection, all-inclusiveness" (Peck, 62). The one 3 ($10 + 3$) from the center of the *Purgatorio* is mirrored and multiplied in the two sets of 3's at the beginning and end of the *Paradiso*. Each set is made up of three 3s—the number of the Trinity reflexive upon Itself. Significantly, it is the perfected human (7 with the three predominant) who is thus surrounded and enveloped by completion (100) and by the Triune God.

As for the essential number of the *Paradiso*, the sum of the pattern numbers is 139 (Table 1). Added to itself ($1 + 3 + 9$), it becomes 13, the number of the New Testament, or the number of the Trinity combined with the unity and perfection of the 10. Taken in its separate parts, 1, 3, 9, it becomes God, Trinity, and Trinity reflected upon itself (or the nine could also symbolize the nine orders of angels, whose ranks the perfected human joins, in their office of contemplating the attributes of the Triune Godhead).

Thus, Dante has placed at the center of each of the books of his *Commedia* an intertwined numerical structure. Ultimately he has distilled out a sum which compresses the entire meaning of each book into one numerical phrase, as if he would synthesize his masterpiece into its simplest form and then translate it into God's own language of number. In the *Inferno* that sum is 44, the body reflected back upon

itself. In the *Purgatorio* that sum is 121, duality achieving perfection by inclusion in Unity. In the *Paradiso* that sum is 139, the Triune God in all its attributes.

Agrippa had the occult in mind when he wrote: "But he that knows how to joyn together the vocall numbers, and the naturall with divine, and order them into the same harmony, shall be able to work and know wonderful things by numbers . . ." (2.173). Certainly Dante knew how to order number into harmony with the divine to work his own magic in the *Commedia*.

TABLE 1
Canto Lengths and Sums

Inferno <u>canto/lines/sum</u>	Purgatorio <u>canto/lines/sum</u>	Paradiso <u>canto/lines/sum</u>
1 136 = 10	1 136 = 10	1 142 = 7
2 142 = 7	2 133 = 7	2 148 = 13
3 136 = 10	3 145 = 10	3 130 = 4
4 151 = 7	4 139 = 13	4 142 = 7
5 142 = 7	5 136 = 10	5 139 = 13
6 115 = 7	6 151 = 7	6 142 = 7
7 130 = 4	7 136 = 10	7 148 = 13
8 130 = 4	8 139 = 13	8 148 = 13
9 133 = 7	9 145 = 10	9 142 = 7
10 136 = 10	10 139 = 13	10 148 = 13
11 115 = 7	11 142 = 7	11 139 = 13
12 139 = 13	12 136 = 10	12 145 = 10
13 151 = 7	13 154 = 10	13 142 = 7
14 142 = 7	14 151 = 7	14 139 = 13
15 124 = 7	15 145 = 10	15 148 = 13
16 136 = 10	16 145 = 10	16 154 = 10
17 136 = 10 }=44	17 139 = 13 }=121	17 142 = 7 }=139
18 136 = 10	18 145 = 10	18 136 = 10
19 133 = 7	19 145 = 10	19 148 = 13
20 130 = 4	20 151 = 7	20 148 = 13
21 139 = 13	21 136 = 10	21 142 = 7
22 151 = 7	22 154 = 10	22 154 = 10
23 148 = 13	23 133 = 7	23 139 = 13
24 151 = 7	24 154 = 10	24 154 = 10
25 151 = 7	25 139 = 13	25 139 = 13
26 142 = 7	26 148 = 13	26 142 = 7
27 136 = 10	27 142 = 7	27 148 = 13
28 142 = 7	28 142 = 7	28 139 = 13
29 139 = 13	29 154 = 10	29 145 = 10
30 148 = 13	30 145 = 10	30 148 = 13
31 145 = 10	31 145 = 10	31 142 = 7
32 139 = 13	32 160 = 7	32 151 = 7
33 157 = 13	33 145 = 10	33 145 = 10
34 139 = 13		

TABLE 2
Relationships of Numerical Patterns of Inferno and Purgatorio

<u>canto</u>	<u>Inferno</u>	<u>Purgatorio</u>
	<u>sum</u>	<u>sum</u>
1		
.		
.		
.		
.		
.		
.		
.		
11	7	
12	10 } = 34 = 7	
13	10	
14	7 (7)	
15	7	10
16	10	10
17	10	13 } = 67 = 13
18	10	10
19	7	10
20	7 (7)	
21	10	
22	10 } = 34 = 7	
23	7	
.		
.		
.		
.		
.		
.		

33

TABLE 3
Relationship of Numerical Patterns of Purgatorio and Paradiso

<u>canto</u>	<u>Purgatorio</u>	<u>Paradiso</u>
	<u>sum</u>	<u>sum</u>
1	.	
.	.	
.	.	
.	.	
.	.	
.	.	
11	7	10 + 3
12	10	10
13	10	7
14	7	10 + 3
15	10	10 + 3
16	10	10
17	10 + 3	7
18	10	10
19	10	10 + 3
20	7	10 + 3
21	10	7
22	10	10
23	7	10 + 3
.	.	
.	.	
.	.	
.	.	
.	.	
.	.	

33

Notes

1. Almost any book dealing even superficially with number symbolism will verify these as the most common Christian associations assigned to the numbers 1, 3, 33, and 100. For a more detailed discussion of the framing structure of the *Commedia* see Hopper: 147-149.
2. Hopper explains: "All large numbers, as has been seen, are reduced to their roots for explanation. Strictly speaking, adding numbers sums up their significances into a single unit. Multiplying diffuses a property into a given number of directions or objects. Squaring gives extension. Cubing either produces solidity or, more often, gives height or godliness. The addition of one to any composite number unifies the whole, if a recognized limit is thereby reached. By the same token, the failure of a number to reach a recognized limit implies a defect or deficiency. Thirty-eight, for example, fails to reach the limit of 40, whereas 39 succeeds by adding unity: (82).
3. Reprinted in Eckhardt's book, the illustration originates from a mid-thirteenth-century French bible (Cod. 2554, fol. iv) housed in the Osterreichische Nationalbibliothek.
4. In a later century, Agrippa will express this relationship even more emphatically in his *First Book of Occult Philosophy* when he asserts that numbers have
 . . . the greatest, and most simple commixtion with the Ideas in the mind of God, from which they receive their proper, and most efficacious vertues . . . Again, all things that are, and are made, subsist by, and receive their vertue from numbers . . . By it is there a way made for the searching out and understanding of all things knowable (171-172).
5. Hopper says of this number: "Of all the number symbols of the Middle Ages, few were so specifically meaningful as the number 9, which is always first and foremost, the angelic number. No contemporary reader could have missed this graceful indication of the mold in which Beatrice was cast." Then he continues to reconcile this with Dante's own explanation and sums up: "To put it baldly, Beatrice and nine are both earthly mirrors of the First Cause, and her nature is directly cognate to the angelic" (138-139).

6. Agrippa later states this symbolism for the number 7 even more explicitly: "it [7] contains body and soul, for the body consists of four Elements, and is endowed with four qualities: Also the number three respects the soul, by reason of its threefold power, viz. rationall, irascible, and concupiscale. The number seven therefore, because it consists of three, and four, joyns the soul to the body . . . (II.193).
7. Indeed, the previously quoted passage from Honorius culminates a longer section which explains in detail the justification for this 3/7 division on the Commandment tablets. Honorius connects the seven social directives of the second tablet to the seven which symbolize the human person.

Three Dances of Three: The *Imago Trinitatis* in Dante's *Commedia*

John Secor

Dante, the consummate craftsman, puts order into his universal poem, the *Divina Commedia*, by means of numbers. The poem, as a reflection of God's Creation, must of necessity reflect its numerical perfection. God's Creation is a work of 7 days, but fundamentally based on a binary system, hence heaven and earth, darkness and light, Night and Day, the waters and firmament, grass yielding seed and trees yielding fruit, Sun and Moon, creatures of the sea and creatures of the air, mammals and reptiles, and finally Man, male and female (*Genesis* 1:1-27). Old Testament thinking is likewise binary in many ways, for example, the Chosen People of Israel and the Gentiles, the Promised Land and the land of exile, the circumcized and the uncircumcized. From the 4th century of our era onward, however, (i.e., the Nicene Creed), the doctrine of the Trinity, of God being Father, Son, and Holy Spirit, has permeated all of Western civilization, and has exerted an especially strong influence on any works dealing with organized religion.

John Guzzardo, in his dissertation on "Christian Medieval Number Symbolism and Dante," distinguishes between God's numbers and the poet's numbers in the *Commedia*. The natural division of the work into 3: Hell, Purgatory, and Heaven, is, according to Guzzardo, one of God's numbers; the writing of the poem in *terza rima*, however, is one of the poet's numbers. Likewise the number of holy women—Maria, Lucia, and Beatrice—or the number of guides—Vergil, Beatrice, and Bernard,—3 in each case, is a poet's number. The poet's use of the number is a conscious choice and used to place the numerologically "required" amount of people at a particular place. In view of the overall triple structure of the *Commedia*, it is not surprising to find many such groups of three.

Leaving aside the Trinity for a moment, the number three is one of God's numbers especially because Christ rose from the Dead on the 3rd day. Dante imitates the Resurrection himself by his 3-day pilgrimage, beginning in the Dark Wood of Confusion on Good Friday, going through Hell, and completing the ascent to the Mount of Purgatory, with Heaven in sight, on Easter morning. The insistence of the idea of "resurrection in three days" is facilitated by the fact that the pilgrims Dante and Vergil meet at each level of Purgatory six souls, three each to represent the cardinal virtue extolled or the capital sin purged at that level. In Purgatory we find, therefore, groups of three used in a binary way to represent good and evil; there is a balance between examples—3 on each side—but ultimately, being based on the Trinity (the number three), good will win.

Dante's constant use of the number three becomes a meditation on the idea of God's 3-in-1-ness, on his omnipotence, omniscience, and omnipresence as revealed by the *Commedia*. Hopper give many examples:

"Its unity" (i.e., the Mind's or God's) is figured by the very fact of the circular revolution. So revolving images and emblems of the Trinity are common throughout the ascent: the lights in the Sun, who circle 3 times around Dante and Beatrice (*Paradiso X*, 76-7), the 3 circles in the same / heaven, the second thrice singing of the One and Two and Three (*Paradiso XII*, 28), the 3 disciples joining in the circling dance in the Starry Heaven (*Paradiso XXV*, 106-8). These, together with constant references to the Trinity, diffuse the atmosphere of Supreme Perfection throughout the entire Realm.

Hell is also rigidly constructed according to Divine Plan and resembles Heaven for two reasons. In the first place, this realm was created by God, as the inscription on the gate of Hell testifies. In the second place, it is the exact opposite, the antithesis of Heaven....

By reason of the absolute opposition of Hell to Heaven, the many representations of the Trinity in Paradise are echoed in the subterranean vaults of the Inferno. Lucifer is 3-headed; so is Cerberus. Geryon is triple in nature. In Lucifer's 3 mouths are clamped 3 archtraitors, Brutus, Cassius, Judas. There are 3 furies, 3 centaurs, 3 giants, 3 rivers flowing into the frozen lake

of Cocytus (actually one continuous stream), 3 popes mentioned in the 8th circle, 3 signal lights before the city of Dis.

The unique quality of these infernal triads is their distortion, by which they appear as perverted images or even parodies of the triplicities of Paradise. In the same way, the opening of canto XXXIV is an ironic premonition of the pageant of the Church. Tricephalic Lucifer is an anti-Trinity in himself, and his 6 wings, sending 3 winds, make him an antitype of the seraphim. The black cherubim of the 8th infernal circle correspond to the white cherubim of the 8th sphere of Paradise. The circling dance of the 3 heavenly lights, Peter, James, and John, is parodied by the 3 who wheel, heads backwards, in the 3rd round of the 7th circle (*Inferno XVI*, 150-2).

Yet of the examples of triplicity found in the *Inferno* almost all are parodies of the Trinity, such as the three heads of Cerberus, or the three heads of Lucifer, who holds in his three mouths the three "archtraitors" Brutus, Cassius, and Judas. Do these parodies in any way undermine the trinitarian scheme of the poem as a whole, and in particular its structure? No, on the contrary, these triple figures in Hell underline God's influence on Hell. Hell is Heaven topsy-turvy, but it is nevertheless part of Creation, and must exhibit the form symbolic of the triune Godhead.¹

It is the nature of the *Commedia* to be a mixture of high and low styles, thus the rhythmically joyful dance of Peter, James, and John in *Paradiso XXV* is the true image of the Trinity: its reflection is a false image of a dance, the three Florentines running backwards in their wretched circle (*Inferno XVI*). The true image is identified and recognized as good, in opposition to a false or bad image. This parallelism is typical of Dante, and serves to unify the *Commedia*.²

The number three takes the geometric form of the triangle. The equilateral triangle is the most perfect of geometric forms, having three sides of equal lengths, and thus three angles, each of 60 degrees. The equilateral triangle has therefore three equal parts, and is the perfect figure to represent the Trinity, which is also made up of three equal parts—Father, Son, and Holy Spirit,—which together make up the unity of one Triune God.³

The equilateral triangle is a figure that may not be bent or pushed out of shape, as a square may to become a rhombus or parallelogram with equal sides. And to further set out the triangle, it may be

inscribed in a circle. Each point of the triangle is now equi-distant from the center of the circle. Thus a more refined figure to describe the Triune God whose power encompasses the whole, round Universe. The figure of the circle predominates in the *Commedia*. All the various levels of Hell, Purgatory, and Heaven are circular, enclosed, and self-contained. They are part of the great unity of Creation. God encompasses all.

Therefore both the triangle and the circle stand for God.⁴ In this study I shall concentrate on the figure of the triangle inscribed in a circle as it appears in the *Commedia*, on what this image means in the context of its canto, and on what the symbolic value of the triangle-inscribed-in-a-circle is in the larger context of the whole *Commedia*. As noted before, there are three of these scenes in all, one in each *cantica*. It would serve our purpose to recall these now.

In the first scene, Dante and Vergil come to the edge of the 7th level of Hell (*Inferno* XVI)—sins of violence,—where they are seen by three sinners who are obliged to run around perpetually beneath a rain of fire. The fire that falls on them punishes their sin of sodomy (4-6). These three Florentines recognize Dante as a fellow citizen, and beg him to stay a moment with them (7-9). As Dante and Vergil wait, the three resume their hellish jogging, and make a wheel ("Fenno una rota di sè tutti e re" 21) as wrestlers used to do, circling to find the advantage (22-4). Of course, here, there are three men, not 2 as there would be in a normal wrestling match. Be that as it may, the Florentine trio continue in their wheel, each with one eye on each opponent, when suddenly, they all turn their heads to face Dante (25-7). From the image of wrestlers facing each other and revolving about a fixed center, that is, the exact figure of a triangle inscribed in a circle, we now have a circle broken, whose center is, as it were, outside the circle. This is a distorted, perverted wheel now, parodying the joyful wheel of Peter, James, and John (Hopper, 152). There can be no happy dancing in Hell.

When the Florentines speak it is in harsh, bitter tones: "ciascuna gridava...(7)" and "A le lor grida (13)." There is tension built into the metaphor of the naked and oiled wrestlers, waiting to seize an advantage before the adversary does the same: "Avvisando lor presa e lor vantaggio,/primo che sien tra lor battuti e punti" (23-4).⁵ There are the metaphorical blows about to rain down upon each other, in addition to the ever-present literal rain of fire that beats down and

scorches the Florentines' backs. Lastly, the twisting of the necks to face Dante: "...ciascuno il visaggio/drizzava a me (25-6)" implies a certain pain in wrenching the neck around. (To wrench the neck around is unnatural in the same way that their crimes against Nature are unnatural.)⁶

Later on, at the very bottom of Hell, in the last canto of the *Inferno*, we encounter the terrifying, triple-headed Satan, in whose mouths are the three greatest sinners of all, Cassius, Brutus, and Judas. This is one of a number of examples of the "infernal triads" (Hopper's term, p. 152). The infernal triads are characterized by the torment of their *contrappassi*. In the case of Cassius, Brutus, and Judas, the sin is treachery against a benefactor, and the punishment is to be stuck in Satan's mouths at Hell's lowest spot (Cassius and Brutus are even stuck head first). In the case of the three Florentines, their crime against nature is sodomy, and they are punished by subjection to the rain of fire, an unnatural rain. As sodomy is a perversion of nature and of the act of procreation, so the rain of fire is a perversion of the watery precipitation that brings regeneration to the earth and enhances the growth of new things. In both cases, the triangular image (i.e., of perfection) is distorted by the three contorted people that form the triangle. The ultimate analogy is with the scene at Calvary, where Christ and the two thieves are the three points of a triangle, each point the pathetic contorted figure of a human being.

Thus the first dance of three people in Hell is an unnatural one. A dance is supposed to be a pleasurable exercise of skill, where two, three, or more people are in rhythmic harmony with each other and with the music. In Hell there is no joy, so the fact that the three Florentines appear to be dancing is quite ironic; the medieval aesthetic that found public torture an amusing spectacle might even find this scene comic! This Infernal dance certainly mocks the later one in Paradise. But there is also an intermediate step, where the nearness of Heaven's joy is felt at the outer bounds of Purgatory.

The crux of my argument is that the dance of the three ladies in Purgatory is essential to the plan of the poem, and to the underlying meditation on the Trinity. In a work whose very structure is based on the number three, one might well assume that the poet would want to develop the figure of three times three. This Dante does by the inclusion of a dance of three people in the *Purgatorio*.⁷ The dances of the Florentines in the *Inferno* and of the Disciples in the *Paradiso*

account for the realms of sin and virtue; the realm of penance is the necessary complement to these two, and here follows an account of its dance.

In Purgatory the three characters who make up the triangle (or circular dance) are ladies, nondescript except for color, whereas the other two sets of dancers, the Florentines and the Disciples, are all called by name, and are also men. These ladies are unnamed because they have allegorical significance in the procession at the end of the *Purgatorio*. As allegorical personifications, the three ladies are of striking appearance, yet they lack completely the poignant humanity of all the souls pictured throughout the *Commedia*.

The central figure in the procession in *Purgatorio* XXIX-XXX is of course Beatrice, riding in the chariot like some female Apollo.⁸ However, for the purposes of this study, our attention is focused on the three ladies who form a circle around the right wheel of the griffin-drawn chariot in *Purgatorio* XXIX, 121-9. About the left wheel are four ladies in purple.⁹ The place of the three ladies in this allegorical procession that represents the books of the Old and New Testaments, the Patriarchs, et al., is that of three theological virtues: Faith, Hope, and Charity. To each of these is assigned a color; thus Faith is white, Hope, green, and Charity, red.

As Dante watches this stupendous semiotic parade, he sees the three ladies come forward dancing in a circle (121-2). The order of the three is novel and noteworthy: first Charity, then Hope, and lastly Faith. The entrance of the three Disciples is exactly the reverse: Peter, James, and John appear in order, standing respectively for (and questioning Dante about) Faith, Hope, and Charity. Taken together, the 2 sets of three form a nice chiasmus—Charity, Hope, Faith; Faith, Hope, Charity; Charity surrounds everything, and Faith lies at the center.

In Dante's description of the three ladies, there are different levels of intensity. What strikes Dante first is the brightness of Charity, described as fire or, rather, so red that she would blend in were she in the midst of flames (122-3). In this *cortège* of strange and wonderful creatures, surely such a flamboyant creature would stand out. And so it should be. I Cor. xiii, 13 states that Charity is the greatest of these three cardinal virtues; its fire illuminates Faith and Hope.¹⁰

Next is the lady who stands for Hope, who looks as if her flesh and bones were made of emerald (124-5). Here is an image of

intensity secondary to that of fire, but emerald, the precious gem, has its own brightness and beauty. Emerald flashes while fire burns. And the third lady is white as new-fallen snow (126). What is stressed in the white color of Faith is not her sparkling brightness, but her purity and freshness.¹¹

As these colorful creatures continue their dance, then, it is interesting to note that the dance changes direction, and so it appears that Charity leads one way, while Faith leads the other (127-8). The speed of the dance changes as well, quickening or slowing, yet the three keep in perfect harmony, step for step (129). It is Faith and Hope's due responsibility to follow the pacesetter, Lady Charity (128).

The dance of the three dames of different hues also previews Dante's final vision in *Paradiso* XXXIII, 115-20, where Father, Son, and Holy Ghost are described as three circles (116)—white, green, and red—of one "dimension" (?) (117), such that the first two (i.e., Father and Son) are reflections of each other, like double rainbow, and the third (Holy Ghost) is derived equally from both (120).¹² Here we have a more complex geometric pattern. The Trinity again takes the shape of a triangle, but at each angle instead of just a point there is contact with a *giro*, a circle which encompasses the triangle, touching each apex. This is a common figure from Christian iconography, showing three equal parts making up one whole. As Dante would have it, the first two parts are mutual reflections, and the third part derives equally from both.

The reflection implied in the first two Persons of the Trinity recalls Dante's earlier description of Peter and James (*Paradiso* XXV, 19-24) which constitutes the *Commedia*'s third dance:

Sì come quando il colombo si pone
Presso al compagno, l'uno all'altro pande,
Girando e mormorando, l'affezione,
Così vid'io l'uno dall'altro grande
Principe glorioso essere accolto,
Laudando il cibo che là su li prande.
[In the same way as, when a dove alights,
Next to its companion, one pours out to the other,
Circling about and cooing, its affection,
So I saw one great and glorious
Prince welcomed by the other,
Praising the food that feeds them up there.]

Peter and James are indistinguishable reflections of each other. And they stand for Faith and Hope, the first two cardinal virtues,¹³ corresponding to the first two Persons of the Trinity, Father and Son. With this in mind, can it be a coincidence that Dante compares Peter and James to cooing doves? In medieval iconography, the dove is the standard representation of the Holy Spirit being visited upon Mary at the time of the Annunciation. Dante thus prepares a way for the coming of a third party to the dovecote.

The third party is John, who stands for Charity. The way in which Dante introduces this Disciple is a most joyful one, and it puts us immediately in mind of the three ladies left lately dancing (*Purgatorio XXIX*), for John, somewhat strangely described as a "vergine lieta" (*Paradiso XXV*, 104), enters the dance not for vanity's sake, but for the sake of honor (104-5), and joins his fellow Disciples, who are likewise filled with Charity, to honor Lady Beatrice outside (110). Charity fills them all.

Here is another reversal. In the *Inferno*, the three Florentines twisted their necks to stare at Dante. In this place, "...la mia Donna in lor tenea l'aspetto,/Pur come sposa tacita ed immota" (110-1). Joyfully, Peter, James, and John dance in Beatrice's honor. She smiles benevolently upon them, no doubt. The fact that the three Florentines stared at Dante while here Beatrice stares at the three Disciples is perhaps a subtle link between Beatrice and Dante. He calls her his "Lady"; does her being like a "bride" make him a "bridegroom"?

Another reversal making this a true dance is that the three Disciples keep their attention inward, on the elegance of the dance itself, as the three Virtues swing merrily around. The dance of Florentines is a false dance because it is decidedly unjoyful, and because the three participants neither join hands nor keep their eyes within the circle of the dance.

John, the favored of the Disciples, is described as "lo schiarato splendore" when he arrives. (*Paradiso XXV*, 106). Already Peter and James are "wheeling" (*volgièno*, 107) in a dance, so inspired by Love are they (108). They burn with Love, in fact. John, the favored one, joins the others in the singing and the dance: "Misesse li nel canto e nella rota" (109), remembering that in Heaven there is a great deal of music pouring forth out of joy. John completes the triangle-inscribed-in-a-circle that stands for the Trinity.

It is interesting that Dante chooses the word meaning "wheel" to mean the dance. This wheel, ever turning now, is in sharp contrast by reason of its movement to the quiet, stationary joy of Beatrice, who gazes approvingly on them: "sposa tacita ed immota" (111). Indeed the atmosphere of this scene is reminiscent of a wedding dance (*ballo*, 103), where the three Disciples whirl about as young maidens, while Beatrice, in a rather matronly pose, stands off to one side, perhaps slightly envious and ready to abandon her cool, and to join their uninhibited exuberance.

With this third and final Heavenly dance is completed the triad of dances found in the *Commedia*. The figure of the dance of 3—the triangle-inscribed-in-a-circle—occurs once in each *cantica*, so that the dances can be overlaid on a 3-dimensional scale. This is further perfection of the Trinity, as three times three is a perfectioning of the number three. The 3-dimensional representation of this trebled dance is something not to be seen but imagined for they are not simultaneous but consecutive dances, as seen through the poet's eyes. By constantly giving us triads and triplicities, and by furnishing us with a sort of mnemonic device—the dance of three persons—does Dante tie meanings together through the three parts of the *Commedia*, and make us fundamentally aware of the three-in-oneness of Creation, of the unity implied by the workings of the Trinity, Father, Son, and Holy Spirit, over the vast reaches of Hell, Purgatory, and Heaven.

Notes

1. Hopper has drawn up an elaborate chart to show some of the many triads in the *Commedia* (164-5). Most of the examples serve to demonstrate parallelism between the *Inferno* and the *Paradiso*, eg., 1) the 3 faces of Satan in Hell correspond to (read "mock") the Father, Son, and Holy Ghost in Heaven (whose face we never actually see), 2) Impotence (or incontinence), Ignorance, and Malice are the anti-types of Power, Wisdom, and Love, and 3) the damned are placed respectively in Circles 2-5, Limbo, and the City of Dis, while the corresponding blessed go to the first 3, second 3, and third 3 Heavens. There are more examples.
2. Lewis Carroll explores in an enchanting way the theme of true and false images, or objects and their reflections, and the necessity of the existence of both, in *Alice in Wonderland* and *Through the Looking Glass*. This binary concept appeals as much to the

- modern psyche as it did to the medieval one and as it did to the ancient Hebrews.
3. Marie-Madeleine Davy has this to say about the representation of the Trinity in human form: ...A l'époque romane, nous voyons quelquefois la Trinité avec les traits de trois personnes humaines; Abélard les fait sculpter sur son abbaye du Paraclet. Dans un manuscrit d'Herrade, trois personnages de forme et de visage identiques symbolisent la Trinité. On utilise aussi le triangle qui apparaît une des images les plus caractéristiques des trois personnes en un seul Dieu (pp. 222-3). In our three examples of people representing the Trinity, the first troika of Florentines is identified by name (*Inferno* XVI), as are the three Disciples (*Paradiso* XXIII-V); while the three ladies (*Purgatorio* XXIX) are distinguished by color alone. The three Florentines would presumably be known by sight by many of Dante's contemporaries.
 4. A more common medieval circular figure is the Wheel of Fortune, with four, not three, points on it, making spokes joined to a central hub. The pagan goddess Fortuna governs this wheel, symbolic of the rise and fall of men's destinies. The four spokes of this wheel also stand for the non-pagan Cross, which itself symbolizes Christ's own fall into death, and rising into life again.
 5. This is a cynical transposition—"Do unto others before they do the same unto you"—of Christ's second Commandment (Mark xii, 31), and entirely appropriate to the scene in *Inferno* XVI, all the more so when these three men are sodomites. It is also worth noting how appropriately Dante associates the naked, oiled bodies of wrestlers with those of these pederasts.
 6. To return for a moment to the idea of the circle broken, it is possible to relate the circle of three Florentines who suddenly twist their necks to look at a point outside (away from the focus and strength of the circle) to the scene of the belt that Dante wears (*Inferno* XVI, 106-8). With this belt he hoped to defeat Fraud ("la pelle dipinta" 108). Strengthened instead by Reason now, Dante hands over the belt to Vergil, who hurls it into the pit to summon Geryon, the tripartite monster.

Commentators are uncertain how to interpret this scene. Some relate the cord to Dante's possible affiliations; Grandgent (p. 143) says only that in the Bible the belt symbolized strength. I

- myself see a connection between two circular images, the "rota" of the Florentines, and the "corda" circling Dante's waist. Grandgent (p. 143f) points out that in *Paradiso* XXVIII, 12, Dante uses the word "corda" in the sense of a trap or snare: "a pigliarmi fece Amor la corda." This is a question not easily resolved. The point is that at this juncture in the *Inferno* the Godly strength of the circle is ruptured. In this first instance, the three Florentines look outside the circle they are running, and in the second instance Dante does not need the belt any longer as Reason will protect him now through this region of Fraud. (Perhaps, under the guidance of Vergil, Dante feels better protected by the sophistication of Reason than by the simplicity of Faith.)
7. There are linguistic connections between the three dances, such as the use of the word "rota" (*Inferno* XVI, 21, *Purgatorio* XXIX, 121, and *Paradiso* XXV, 109). Furthermore, the rhyme-words are consistent from the 2nd to the 3rd dances, viz. *rota* and *nota* (*Purgatorio* XXIX, 121, 123), and "nota" and "rota" (*Paradiso* XXV, 107, 109).
 8. The three colors that Beatrice wears in *Purgatorio* XXX, 31-3 (white, green, and red) suggest the three Theological virtues (see Grandgent's note, p. 605), and also anticipate the likening of Father, Son, and Holy Ghost to three circles of the same colors in *Paradiso* XXXIII, 115-20 (see below).
 9. These represent the four cardinal virtues: Prudence, Temperance, Justice, and Fortitude. Grandgent (601) explains their purple garb as being "...in the Middle Ages nearly identical with red, to indicate that they depend for their existence on Love..." Personally, I am more inclined to agree with Sinclair (vol. 2, *Purgatory*, p. 389), who identifies purple as the color of the Empire; the four cardinal or "moral" virtues being more appropriate to the Empire (Marcus Aurelius) than to the Church. The three theological virtues fall naturally into the Church's domain. Given the tense political atmosphere of Dante's era, the struggle for power between Guld (the Pope) and Ghibelline (the Emperor), it is highly understandable that Dante establish this duality by clearly separating Church from State in the *Commedia*.

It is possible to extend this dualism further to include *Paradiso* XII, 106-11, where Saints Francis and Dominic are described as

the two wheels ("rote") with which the Church defends herself. Francis and Dominic stand respectively for the powers of Love and Knowledge, or the theological and cardinal virtues. One must not go so far as to associate Saint Dominic with the (Holy Roman) Empire, but it is not wrong to recall another binary principle—Faith and Reason—to which Dante makes reference in *Inferno* XVI (see above).

10. Cf. *Purgatorio* VIII, 89-90, where Dante sees three "torches" or stars in the evening to represent Faith, Hope, and Charity (Grandgent's note). Fire also stands for the Holy Spirit at Pentecost.
11. Lines 121-6: Tre donne ingiro dalla destra rota
venian danzando: l'una tanto rossa
ch'a pena fora dentro al foco nota;
l'altr'era come se le carni e l'ossa
fossero state di smeraldo fatte;
la terza pareva neve testè mossa;
12. See Grandgent, pp. 970-1.
13. See Grandgent, p. 888, quoted by Singleton in his edition, p. 402: *Paradiso* XXV, 31-3—fa risona... carezza:

Three of the Disciples (Peter, James, John) were chosen by Jesus to be present, and to receive the clearest revelation of his character, on three different occasions: at the Transfiguration (Mat. xvii, 1-8), in the Garden of Gethsemane (Mat. xxvi, 36-8), and at the raising of the daughter of Jairus (Luke viii, 50-6). On these three occasions Peter, James, and John stand respectively for Faith, Hope, and Love.

Numbers as Symbols and Numeric Inflation in the Grail Family

Anne Berthelot

Usually the Grail family, that is to say, the people who have been entrusted with the care of the holy thing, or are in some way related to it, tends to reproduce in a quite transparent manner the main structure of Christian numeric symbolism: that is to say, the ternary system which is heralded by the Holy Trinity. The figure "three" appears quite often in a number of texts dealing with the Grail, indeed it becomes some kind of an obsession. These occurrences, however, do not constitute a very elaborate attempt to use numeric symbolism: on the contrary, the model is quite easy to follow, so easy that one may suspect that other, more important things are hidden behind this apparent simplicity. However, as far as theology is concerned, and with the possible exception of the *Quest del saint Graal*,¹ the French novels of the XIIth and XIIIth centuries are not known for their subtlety. Since the story of the Grail, at least at the beginning, is not exactly considered as a true part of the Scriptures (although it begs for that consecration) the use of so simple Christian references may be intended to flaunt the orthodoxy of the holy thing, even at the price of verisimilitude.

For instance, in the *Roman de l'Estoire dou Graal*,² as well as in the prose *Perceval* supposedly written by Robert de Boron, the temporal gap between the Crucifixion and Arthur's reign, or Perceval's adventures, is filled with three generations only, which is *per se* a kind of minor miracle! This tripartition is reinforced by the mention of three tables: the first one is the table of the Last Supper, where the Grail first appears; the second is the table around which Joseph of Arimathea's true followers take place, to eat the fishes provided by the first "Fisher King" and multiplied by the Grail. And the third one is, of course, the Round Table, which Merlin finds with the explicit purpose of "completing the image of the Trinity":

Et se vos me voulez croire, nos establirons la tierce ou non de la Trinité, car la Trinitez senefie touz jorz par trois.
(*Merlin*, 48, 75-77)³

[And, if you agree to trust me, we shall establish the third one in the name of Trinity, for Trinity always signifies by threes.]

Precisely, Merlin, the devil's son, is always in search of his own legitimacy, which he achieves to a degree by writing the new Gospel of the Grail, intended to join the other four—and, as we may suppose, some apocryphal, too—and to constitute "une mesme chose" (the same thing) with Joseph's Book. His is an hopeless task, since Joseph of Arimathia is not much more respectable, from the point of view of the real Christian doctrine, than the prophet-magician. But it shows that the numeric symbolism of the Trinity may indeed be used quite deliberately to proclaim the orthodoxy of a character, or of a story.

The same ternary structure reappears inside the family of the Grail, that is to say, in the older novels, Perceval's family. Perceval himself is the third son of a departed knight:

Et vos, qui petiz esteiez,
.II.⁴ molt biaux freres aveiez.

(*Conte du Graal*, vv.427-428)⁵

[And you, who were quite small,
you had two fine brothers.]

In later works, this fact is not mentioned anymore: it is more interesting to develop his sister's story, who will indeed play an important part in the *Queste*—and the related texts—as Galahad's mystical bride. But, whenever Perceval's brothers do exist, there must be two of them, unless the meaning of the complete tale has undergone a considerable change, as we will see. In the same way, the Trinity structure is wont to be present also in the preceding generation, that is the generation of Perceval's parents. This is one of the most controversial points where in this of the Grail story: Chrétien de Troyes' account of it is not so clear that his followers may have an easy time with it. Basically, as far as we can perceive, there are just two brothers in the *Conte du Graal*: the king whom the Grail serves and the hermit who hears Perceval's confession. In this version, Perceval belongs to the Grail family by his mother. However, things are growing more complicated than it seems at first, since we have to fit somewhere into this genealogical construction the figure of another king, the Fisher King: Chrétien's text says

clearly that he is the Grail king's son, but does not elaborate further on his relationship to Perceval, for instance:

Cil cui l'an en sert est mes frere,

Ma suer et soe fu ta mere,

Et del Riche Pescheor croi

Que il est filz a celui roi

Qui del Graal servir se fet.

(*Conte du Graal*, vv. 6341-6345)

[The one who is served from it (the Grail) is my brother,

My sister and his was your mother,

And as for the Fisher King, believe

That he is the son of this king

Who gets served from the Grail.]

And so, for all practical purposes, we have three paternal figures hovering over Perceval, which strongly mimic the Holy Trinity, the more so since one of them is at once presented as brother and father. However, this is not a very clear situation, and other novels try to improve this pattern. Even those which keep the notion of Perceval's maternal ascendancy with regard to the Grail suggest a more classical Trinity, even if it means introducing completely new characters in the story. The *Perlesvaus*,⁶ as always, presents a very special version of the facts: there are indeed three brothers, but of one them has nothing to do with the Holy Trinity; on the contrary, the King of Chastel Mortel is the embodiment of the devil.

Icil mauvais roi fu ses oncles, e cil mauvais roi fu frere lo Roi Pescheor, e frere lo bon roi Pelles, qui guerpi avoit sa terre por Deu servir, e frere a la Veve Dame, la mere Perlesvaus. (*Perlesvaus*, l.6226-6229)

[This bad king was his uncle, and this bad king was brother to the Fisher King, and brother to the good king Pelles, who had left his land to serve God, and brother to the Widowed Lady, Perlesvaus' mother.]

We may notice, as usual when the Grail family comes under discussion, that the sentence is rather convoluted, as if the text did not know exactly what it intends to say. It seems always that there are more characters than would be normal, because one or the other is presented twice, under two different designations: here the King of Chastel Mortel is said to be Perceval's uncle, and Pelle's and the Fisher King's brother.

However, the main idea is clear: the "lignage du Graal" should be adequately represented by three brothers, who would embody the different persons of the divine Trinity. The onomastical tradition comes to help this assumption: where the Fisher King is called Pellès, his two brothers are usually named Pellinor and Pellehan: the relationship is quite unmistakable! Furthermore, Perceval's mother, as long as the hero's ties to the Grail are on the maternal side, is assimilated to the allegorical figure of Church, considered as the mystical bride of the Trinity—or as His sister, according to the mixing principle which make all familial categories crash into one another inside the frame of mystical representations. Indeed, in the *Perlesvaus*, the name of Perceval's mother is given as Yglais, that is to say, quite clearly, "Eglise."

So we find the constant presence of the number three, or of any kind of tripartition and ternarity, in most of the texts dealing with the Grail and the Grail family. But, as a matter of fact, this may be considered as something quite natural: three is, of course, the numeric symbol of the Trinity, but it is also a figure which appears in many other circumstances, without any relation to Christianity - especially in fairy tales. And the *Conte du Graal*, for one, presents a number of sequences and of schemas pertaining to this kind of tale. Maybe it is for this reason that some novels try to replace this magical number by another, as magical, but perceived as more authentically Christian. This is, again, what happens in the *Perlesvaus*: beside the maternal line, where the ternary structure is quite obvious, as we have already seen, the hero's paternal line is also given in great detail. In accord with Robert de Boron's account of Perceval's family, his father Alain le Gros (Julain in the *Perlesvaus*) is said to have had eleven brothers, whose names are dutifully given. To complete the favourable impression such a presentation is apt to make, the text remarks that each of these lived as a knight for only twelve years:

E cil Julains ot .xi. freres, molt buens chevaliers autressi com il fu, e ne vesqui chascuns que .xii. ans chevaliers, e morurent tuit a armes [...] por avancier la loi qui renovelee estoit. (*Perlesvaus*, l. 44-46)

[And this Julains had eleven brothers, very good knights as well as he, and each of them lived only twelve years as a knight, and all died in arms (...) to advance the law which had been renewed.]

This time it is not the Trinity which is chosen as a model, but the Apostles. That being the case, there is always a kind of uncertainty: must the number be twelve, including the hero, or his father, or must it be thirteen, so that Alain le Gros appears to embody the figure of Christ, with his twelve brothers playing the part of the Apostles? Another difficulty arises from the sheer number of the characters. If a text should try to describe the adventures of each brother, it soon stops being a Grail novel, to become some kind of compilation of adventures. Twelve brothers are an interesting notion, as long as eleven of them are dealt with as a whole, generally to oppose their attitude to the last brother's behavior: this is what Robert de Boron does in the *Roman de l'Estoire de Joseph*, confronting the willingness to marry of Boron's sons and the vow of chastity of Alain le Gros. But, the situation becomes more awkward in the *Perlesvaus*, where these superfluous brothers, necessary only from the point of view of structure and numeric symbolism, are suddenly given names, and even, for two or three of them, the beginning of a story.

Quite frankly, anyway, the number twelve is not necessarily a Christian figure, either. However, the Christian connotation of this number, as well as that of the three, can be proved *a contrario* by some texts where the relationship of Perceval's family to the Grail is partly forgotten, and where concurrently the three and twelve numbers stop being the basic figures. The estrangement of Perceval regarding the Grail begins as early as the Cycle of the *Lancelot-Grail*.⁷ In the *Queste*, he is only the second behind Galahad; his sister, however, plays an important part, but there is no mention of his brothers. The subsequent novels, according to the tendencies of any prose novel in the XIIIth century, try to remediate to this lack of information. But these texts, especially the prose *Tristan*,⁸ are just marginally concerned with the Grail; they cannot avoid it completely, and they have to conform themselves to the narrative frame given by the *Queste*, but otherwise they do not deal in spiritual matters. In some other texts, there is indeed a Grail, but it is a peculiar one, the relation of which to Christian law is at best dubious: such is the case in the *Huth-Merlin*.⁹ While it is quite natural for Perceval's family, his father, his brothers, to be introduced at length in the narrative of Arthur's reign, it is also to be expected that this prehistory of the "vallet Gallois" be somewhat at odds with the minimal information given in Chrétien or Robert de Boron's works.

The main modification concerns the number of Perceval's brothers. But first of all, King Pellinor, who is supposed to be his father, is given his own story,¹⁰ which is not very commendable: far from being the pious king and true knight, as described by Chrétien, he is revealed as the father of an illegitimate son, Tor, "son of Arès"—whose mother he apparently raped during his wandering youth. Moreover, he is known for having killed King Lot of Oranie, although it is not clear whether he did kill him in an honourable fight, or whether he murdered him. Whatever be the case, he is destined to be killed in revenge by Gauvain, Lot's son. And indeed, the two families of Oranie and Listenois offer curious similitudes: where Lot d'Oranie has four sons (Gauvain, Agravain, Gaheries and Guerrehes), Pellinor has four, too: Perceval is the youngest, but he has three elder brothers, all of them still alive when he begins his knightly career: these are Lamorat (also called "li Amoras" de Galles),¹¹ Driant and Agloval. (One may observe that the parallelism between the two families includes the resemblance of two sons' names: there is a "doublet" Agloval-Perceval as there is a "doublet" Gaheries/Guerrehes.) Furthermore, both kings have a fifth son, whose status is somewhat peculiar: Lot of Oranie is supposedly Mordret's father, although Mordret is in fact King Arthur's incestuous son. And Pellinor is Tor's father, although Tor is supposedly the "villain" Arès' son.

These modifications take place in a novelistic complex which seems to have completely forgotten the Grail, or at least, to have erased it as much as possible. The prose *Tristan* is much more preoccupied with knightly deeds and courtly love than with any kind of religious quest. This Pellinor's family has a very secular behavior, and is presented as the counterpoint of Gauvain's family—Gauvain himself and his brothers being at the same time turned into villains and traitors. The last remnant of the relation of Perceval's family to the Grail is to be found in the superior honorability of his brothers compared to Gauvain and Agravain. But in fact this is but a very thin superiority: Lamorat falls a victim to Agravain and Gauvain's attack, but he has been surprised in the queen of Oranie's bed. Although King Lot is supposed to be dead at this time, this relationship is exposed as adulterous, and does not fit the dignity of a Grail-related family.

According to the medieval conceptions of numeric symbolism, three is indeed a divine figure, and four is supposed to symbolize the world, since it is the adjunction of one to three; from this point of

view, four might be a rather negative number, although it may also be considered in a positive light, as the *signe of completude*, of the perfection in the physical world: there are four Gospels, four rivers in Paradise, four cardinal points, etc... On the other hand, five is definitely the number of man, and of the material universe.¹² If it is possible to argue that four brothers, four sons, still constitute a Christian symbol, especially with regard to the four Evangelists, five is another matter entirely. The passage from four to five in both families, on one side by the adjunction of Mordret, the son of the incest, on the other side by the late discovery of Tor, the son of the adultery, emblematises the degradation of the once Grail-related family: the spiritual dimension of this family is altered almost beyond recognition. Apparently, the prose *Tristan*'s author(s) confirm(s)—whether deliberately or not is difficult to determine—the relation between numbers and signification: as long as Perceval's family is linked to the Grail, its predominant structure is a ternary one, reproducing the Holy Trinity. But when the family's secularisation is almost consummated, then the emblematic number is four (or, as a worse variant, five).

As a further step, the minimal meaning of all numbers tends to disappear completely: since the intention of the new prose novels is not to tell the holy story of the Grail, but to reproduce in writing the richness and complexity of the world, it is more important to have as many characters as possible, each of them with a story, so that the Arthurian universe can extend itself horizontally, if not vertically. Inflation of the numbers and confusion of the names are two principles, or two consequences, of this new aesthetic point of view. The *Livre d'Artus*¹³ for instance, boasts of two kings Pellinor, each of them being somewhat loosely related to the Grail, and to Perceval: the first one is indeed the hero's father, but it so happens that he is also gifted with fourteen other sons, plus Agloval, the only remaining character of the more classical *Tristan en prose!* As for the second Pellinor, who is supposed to be a distant cousin of the first (the relationship is not thoroughly explored, although he is also a Roi Méhaignié, a Maimed King, in a kind of crazy reduplication), he is generously given twelve sons, his wife being pregnant with a thirteenth one at the time when he receives the fatal injury. None of these sons has a name, however, and indeed, it would be difficult to name them, since they are obviously duplicates, *Doppelgänger*, of the main characters. At this level of the interweaving of narratives, the initial elements have been completely

forgotten: it seems that the genealogies are going on and on without making sense. The Grail family is undergoing a purposeless and monstrous expansion; one might think that this proliferation has a biblical connotation: the Grail kings are gifted with many sons, as patriarchs in the Old Testament. However, this tendency goes against the grain in the context of the Grail *parousia*, by producing an overload of inconsistent characters, deprived of any narrative function.

So the cautious use of numeric symbolism that is manifest in the first Grail novels vanishes utterly in later works: numbers do appear, but they don't have any meaning left; they are just rhetorical figures, each trying to be more impressive than those in another text, or another section of the same text. Besides, the fragile coherence which the *Queste de saint Graal*, for instance, maintains, has also gone mad. As long as there is a religious purpose in a novel, even a superficial one—and usually it is a superficial one, for novels are not treatises in theology—numbers are used to hammer in the reader's mind some of the underlying Christian truths. But when this eschatological orientation of the novels gets lost, numbers have no value anymore, they are but the symptom of narrative disintegration: a machine running wild, where symbol is replaced by exaggeration in a desperate inflationist attempt to produce still some "benefice," some meaning.

Notes

1. Cf. *La Queste del Saint Graal*. Ed. A. Pauphilet. Paris: Champion, 1980.
2. Cf. Robert de Boron, *Le Roman de l'Estoire dou Graal*. Ed. W. A. Nitze. Paris: Champion, 1971. This verse text is also called the *Joseph*. A prose version of it constitutes the first part of a prose trilogy attributed to Robert de Boron (probably wrongly), the other parts of which are the *Merlin* and the *Perceval*. This last text has long been called the *Didot-Perceval*, because it was mainly known by the manuscript of Didot. The whole trilogy has been published by Bernard Cerquiglini, *Le Roman dou Graal*. Paris, 1982.
3. Cf. Robert de Boron, *Merlin*. Ed. A. Micha. Genève: Droz, 1980.
4. Actually, there is a certain confusion in the manuscripts about this number: in some cases, it seems to be twelve instead of two, which of course would also make very good sense!

5. Cf. Chrétien de Troyes, *Le Conte du Graal*. Ed. W. Roach. Genève: Droz, 1974.
6. Cf. *Perlesvaus*, 2 vol. Ed. W. A. Nitze and T. A. Jenkins. Chicago: The Univ. of Chicago Press, 1972. This is a very original novel, which gives a quite peculiar account of the quest for the Grail and the bleeding lance. Critics disagree as to the date of this text, although it is surely one of the earliest prose novels of the XIIIth century.
7. This Cycle, composed around 1225-1230, consists of five novels: the prose *Estoire dou Graal*, the *Merlin* with a "Suite", the *Lancelot* proper, the *Quest del saint Graal*, and the *Mort le roi Artu*.
8. Supposedly written around 1230-1240 by two authors, Luces de Gast and Helye de Boron (both fictitious characters). It is difficult to speak of *one* prose *Tristan*, since there are many manuscripts of this novel, largely different from each other. However, certain episodes, such as those I am referring to, are quite the same in a number of manuscripts.
9. The *Huth-Merlin* (ed. G. Paris and J. Ulrich. Paris: SATF, 1886, 2 vol.) is the most original of the three known *Suites* to the *Roman de Merlin*, the second part of the *Lancelot-Grail* cycle.
10. Actually, the most detailed version of this story does not appear in the *Tristan* proper, but in the *Huth-Merlin*. But the main characters appear from time to time in the *Tristan*, and it is in this text that the matter comes to an end.
11. "Li Amoras" may be understood as "the lover," and be construed as an allusion to the adulterous relationship of the character with the Queen of Oranie.
12. Cf. Robert Delort, "Structures mentales et vie sociale," *La vie au Moyen Age*. Paris: Points, 1982. Pp. 82-3.
13. Cf. *Le Livre d'Artus* in *The Vulgate Version of the Arthurian Romances*, Vol. 7. Ed. O. Sommer. Washington: Carnegie Institute, 1913. This is the third, and least known, *Suite* to the *Merlin*.

Gold and Grace in Hartmann's *Gregorius*

Aaron E. Wright

Hartmann von Aue's *Gregorius* is a tale of grace and redemption, an exemplum illustrating God's saving love for even the most depraved of sinners. While this legend of the "holy sinner" is found across the entire European tradition,¹ Hartmann's late-twelfth-century verse novella stands out from competing redactions, both Latin and vernacular, both older and younger, as a literary masterpiece. The German poem's clarity of expression and narrative skill drew lavish praise from Hartmann's contemporaries, and certain memorable scenes, such as the dying words of Gregorius's grandfather and the moralizing speeches of the insular abbot, belong to the passages most frequently cited by later medieval poets, adapted or simply lifted *in extenso* for use in works ranging from the courtly romance to the burlesque *mære*.²

Hartmann is more delicate in his approach to this material. Early in the prologue, the narrator warns that the story he is about to relate is *vil starc ze horenne* (53),³ too harsh almost to be spoken aloud; if not for its uplifting lesson, the brutal tale might better be left untold. This warning clearly serves as much to titillate the curious as to caution the timorous; but disingenuous as it may be, the narrator's squeamishness is not entirely unjustified. The title figure is a man sunk so deep in sin, his own and others', as to seem beyond any hope of redemption. Gregorius is the fruit of an incestuous union between brother and sister; abandoned at birth, the luckless child grows up in a monastery, which he leaves, on learning of his sinful origins, to live the worldly life of a knight-errant. Brought unknowing and as if by chance into his own beleaguered native land, the young man valiantly frees it from a long and bitter siege, and wins first the gratitude and then the hand of its lovely sovereign—who is, of course, his own mother. Ultimately, though, both mother and son, aunt and nephew, husband and wife, find salvation; indeed, Gregorius, purged of his sins by long and difficult penance, succeeds to the throne of St. Peter. The story thus provides a reassuring argument *a fortiori*: if even these transgressions might be

forgiven, then how much greater the certainty that we, less extravagantly sinful, can be restored to the grace of God.

For Hartmann, this loss and subsequent recovery of divine favor is an essentially mercantile transaction, and his account of the process is strikingly and significantly dominated by figures of economy and exchange. The sinner, says Hartmann's narrator, plunges his soul into *armuot* (107). *Sælde*, though, is a commodity that must be bought (1706), and so salvation will remain out of reach for the spirit impoverished by sin unless it profit by the *wuocher der riuwe* (75), a productive investment in repentance.

Nowhere is this economic framework more fully developed, and nowhere is its full significance plainer, than in Hartmann's carefully constructed presentation of Gregorius's patrimony. The infant Gregorius is set adrift in an ark, in hopes that Christ will bring the child to good and caring Christian folk. Favorable winds of divine provenance drive the frail craft to two fishermen, who rescue it from the waves. On reaching shore, the fishermen encounter the kindly abbot, Gregorius, who opens the chest to find the child wrapped in precious silks, beside him an ivory tablet recounting the circumstances of his conception and birth and twenty marks of red gold. The abbot immediately gives two gold marks to the one fisherman, charging him with the baby's maintenance; to the other, he gives one mark as the price of his silence.

The foundling is christened with the abbot's own name, received into the fisherman's family, and educated at the convent. All goes well until one day, more than fifteen years after his arrival on the island, when Gregorius overhears his foster mother identify him as a parentless waif. The boy rushes to the convent, where the abbot not only reluctantly confirms the woman's story, but, by producing the ivory tablet, discloses the truth in its full enormity. Disregarding the abbot's advice to stay and do penance for his family's sins, the young Gregorius insists that he be allowed to pursue fame and honor as his noble birth commands, as a knight. The abbot at length acquiesces, and reveals to the boy his unexpected wealth: rather than the twenty marks mentioned on his mother's tablet, the abbot, thanks to the prudent investment of the seventeen originally left in capital, is able to present Gregorius with 150. And so, with the three of the original twenty that were disbursed immediately on the child's discovery, the island's economy has been enriched by a total of 153 marks of gold.

This number 153, at first regard so incongruous, is a number of great significance both mathematically and symbolically.⁴ Its fascination is traceable at least as far as the geometric number representations of the Pythagoreans; 153 in such a system is a "triangular" number, its units forming an equilateral triangle with each side comprising seventeen. Modern mathematics can express such a number as the sum of the series $1+2+\dots+n$, or, more compactly, as half the product of n and the next greatest integer; thus, where n is 17, the result is 153.⁵

In the Middle Ages, such numbers were more than mere mathematical curiosities. They were the symbols of a higher order, instituted by God and revealed in sacred Scripture. Indeed, the most significant of Biblical figures, from the number of the Trinity to the number of the Beast, are triangular. The number 153, too, plays a prominent role in John's Gospel: in chapter 21, the risen Christ commands His disciples to cast their empty net again, and it is this time filled with fish, 153 fish so large that Simon Peter can hardly haul it ashore.

Hartmann's text shares more than just the number with this latter Biblical passage. Gregorius, after all, is raised by a fisher family, after having been drawn from the sea himself like a large fish; and, like Simon Peter, Gregorius ends his days as Pope.⁶ Such biographical parallels, however, do not exhaust the contribution made to the German text by the presence in it of so richly suggestive a number. Far more helpful than these merely narrative similarities is the exegetical tradition that identifies 153 as sign and symbol of the body of the elect. That such an interpretation should have been applied to the original Johannine passage is not surprising, even without knowledge of the mathematical background: for what else, after all, must a number of fish represent when they are drawn into a boat that is, as here, only too clearly *navicula Petri*, the ship of the Church? Even so, however, there is a broad medieval tradition, apparently originating in large part in Augustine, that derives the mystic signification of the number 153 not just from its narrative context in Scripture but from its mathematical properties, in particular its relation to its "triangular root," 17.⁷

Hartmann quite clearly exploits those exegetic tendencies in his Gregorius. One hundred fifty-three, in one strand of the Augustinian tradition, is reduced to 17, which in its turn is seen as composed of its addends 10 and 7. Ten, the number of the Commandments given on Sinai, represents the Law, and 7, the number of the gifts of the Holy Spirit, evokes Grace. Their sum, 17, and the triangular number it

generates, encapsulate both principles, which are likewise precisely the elements identified in Hartmann's prologue as the conditions of redemption, both of them necessary but neither alone sufficient. Thus, the gold marks of Gregorius's patrimony and their mystic increase reiterate the work's theme as proclaimed in the prologue, and affirm the narrator's promise that salvation will be possible after all.⁸

While its relevance to Gregorius's gold is clear, this most widespread and well-known explication does not exhaust the significance of the number 153 in Hartmann's poem. Equally pertinent is the alternative derivation set forth in Augustine's Epistle LV, *Ad Inquisitiones ianuarii*.⁹ Here, in the context of a discussion of the variable date of Easter, the saintly bishop affirms the validity of his calculations by showing that the numbers he has adduced for his demonstrations—three 50's and a three—add to 153, a number he triumphantly identifies as the sign of our new life in Christ. Once again, Augustine derives this meaning by the reduction of 153 to 17; for, he says, just as 153 grows mathematically from the lesser number, so too does the heavenly beatitude it symbolizes arise from the qualities figured in seventeen. Unlike the tradition sketched above, however, in this instance Augustine identifies the mystic sense of 10 as reward through grace, and the spiritual signification of 7 as purification by fire.

This less common and somewhat anomalous interpretation of the Epistle is quite as relevant to Hartmann's story as the more widespread identifications. While it would surely go too far to suggest that the German poet's *Gregorius* is immediately indebted to this one particular Augustinian document, both texts nevertheless share certain strikingly substantial details. Both Augustine and Hartmann derive 153 in two distinct processes, both as the triangular product of 17 and as the sum of 150 and 3. And, most significantly, both avail themselves in illustration of these procedures of pecuniary metaphor; where Hartmann has gold, Augustine's letter speaks of penance as refined silver and of grace as a golden denarius freely given:

tunc homo perfectus et quietus, purgatus in anima et corpore per eloquia domini casta, argentum igne examinatum terrae, purgatum septuplum accipiet mercedem denarium...¹⁰

[Then the righteous and peaceful man, purged in body and soul by the holy words of God, shall receive his reward, silver from the earth refined by fire, the denarius of mercy seven times purged.]

Here, too, Augustine's spiritual explication of the numbers 7, 10, 17, and 153 accords entirely with the thematic tendencies of the German text. For the principles identified in 7 and 10, purification and grace, are the very principles exemplified in *Gregorius*. By indirectly invoking their numerical representations at so portentous a moment in his poem, Hartmann both alerts his audience to the calamities to come—the second incest and Gregorius's suffering in the wilderness—and assures us that his hero will emerge from them purified, cleansed by the depth of his penance and deserving of the grace of God. That the 153 gold marks of his salvation have their origin in seventeen reiterates for the knowing listener the assertion in Hartmann's prologue, that sinful man to be sure of salvation can neither rely exclusively on the force of his own repentance, nor count solely on God's unconditional love; but rather that both elements, the grace and the spiritual cleansing mystically and simultaneously contained in the figure seventeen, are necessary to redemption.

That in the German text it is money that undergoes this mystical increase is not without a significance of its own. Hartmann was, after all, under no obligation to make gold the medium of his demonstration; indeed, given the maritime setting of his story and the trade practiced by several of its characters, the casual mention of an especially large catch of seafood would be neither unexpected nor unnatural. Still, though, Hartmann insists on gold, even at the risk of exposing his good abbot to suspicions of usury. Thus it is worldly economic dealings, of a type that plainly make the abbot uncomfortable, that make possible Gregorius's further career, from his fall into sin through his repentance and eventual glorious resurgence. In this fact, at first regard something of a paradox, there lurks an important and hitherto little regarded theme of Hartmann's *Gregorius*.

The original twenty marks of gold, it will be recollected, are most closely associated with Gregorius's mother, who laid them next to her infant son before committing him to the waves (713-15). This connection is explicitly recalled by the abbot, who, on turning the money over to his charge, assures the boy that he has acted as his mother—and not as his parents—requested (1763; cf. 741-45). Indeed, it is Gregorius's mother who stands at the center of every one of the text's discussions of wealth. The issue is first raised in the crucial interview between Gregorius's young parents and a trusted family friend, whom they call on for advice after discovering the girl to be

pregnant. The narrative significance of this famous and often imitated passage is highlighted by its form: the children's consultant concludes his sensible remarks with a long virtuoso display piece (607-23) whose only rhymes are morphological variations on *guot*, material wealth, and *muot*, moral propriety. And it is surely no coincidence that the children's adviser maintains this scheme for a total of seventeen lines, leaving it to Hartmann's narrator to fill the final couplet (624).

The counsel ultimately offered in this speech is that brother and sister should perform separate and different penance for the crime they have committed together. Gregorius's father, whom the narrator has identified as the active instrument of the devil, is urged to renounce his political power and to betake himself on a solitary pilgrimage to the holy sepulcher (571-73). The sister, on the other hand, Gregorius's mother, is admonished to keep her gold, to maintain her temporal authority, and to use both wisely, in effect to join *guot* with *muot* (603-605). Though this may seem an easier punishment, reflecting perhaps the frailty of the sister's present condition or her originally passive role in the incest, the narrator explains that hers is in fact the heavier penance, and he marvels that she, though a woman, survives it, while her brother dies on the road to the Holy Land (842-850). The charity of the heart is here set forth not only as an alternative but as an equivalent to the asceticism and poverty to be endured by Gregorius's father.¹¹

Precisely the same question is raised in a parallel scene later, where Gregorius and his wife find that they are in fact mother and son, doomed to repeat the very sin with which the family's troubles began. Cast, says the narrator, into a despair deeper even than that experienced by Judas (2623-26), the two wonder what penance could possibly reconcile them to God. Gregorius, echoing the remarks made by the narrator in the prologue, warns his mother against the sin of desperation (2695-98), and chooses for himself a punishment of extreme harshness: he strikes out for the wilderness, where he spends seventeen years chained to a rock, without food or human companionship. To his mother, however, he offers the same course as that proposed years before: she is to remain in her country and devote her wealth wisely to good works, enriching the poor and endowing churches (2707 ff.). Strikingly enough, the *muot*—*guot* rhyme reappears here (2115-116),¹² and again, the penance Gregorius prescribes for his mother is acknowledged as an exceptionally difficult one.

The conclusion twice suggested, therefore, is that such extravagant acts of penance as barefoot journeys and ascetic diets, effective as they may be, are not necessarily more pleasing to God than the more modest, and more common, gestures born of the impulses of a sincere charity. This heartening message, that gold need not be only the abstract sign of redemption but its instrument, is contained, too, in the account of the infant Gregorius's twenty marks. Wealth, in Hartmann's *Gregorius*, is not just symbolic, not just the refined silver of purity and the golden denarii of heavenly grace; Gregorius's gold is not only a figure for salvation but its means in the world, and just as the abbot's cautious investments paid their rich return, so too do our small acts of charity buy us a place in heaven.

From the modern point of view, such complacent reassurances seem oddly out of place. The clear call of the legend to acts of radical penance, still so compelling in Hartmann's presumed source, is obscured in the German text; where the legend might be understood to urge self-abasement and the utter renunciation of all that is worldly, Hartmann in his *Gregorius* takes a more measured view of such excesses, to admit of a second, perhaps more realistic possibility. This alternative, the responsible use of material advantage, is equally a guarantee of redemption; and so Hartmann can label both Gregorius and his mother *guote sündære*,¹³ sinners made holy each by a different, but equally effective brand of penance.

Disappointing as Hartmann's dilution of the legend's original strong medicine may seem, no message must have been more eagerly welcomed by the ministerials and nobles of the contemporary audience, as much concerned for their souls as they were mindful of their great temporal responsibilities. For them, Hartmann's Gregorius and the mystic transmutation of twenty bars of gold promised even the least heroic a share in the mystic increase of grace.

Notes

1. See the literature cited by Volker Mertens, "Gregorius," *Die deutsche Literatur des Mittelalters: Verfasserlexikon*, 2d ed (Berlin: De Gruyter, 1981) 3: col. 248.
2. Hartmann's claim to the poetic laurels is most famously upheld by Gottfried von Straßburg in his *Tristan*. Both that work and the *Helmbrecht* of Werner der Gartenære are well known for their extensive borrowings from Gregorius.

3. Line numbers in parentheses refer to Hartmann von Aue, *Gregorius*. Ed. Hermann Paul, 13th revised edition by Burghart Wachinger. Altdeutsche Textbibliothek 2. Tübingen: Niemeyer, 1984.
4. See the entry "153" in Heinz Meyer and Rudolf Suntrup. *Lexikon der mittelalterlichen Zahlenbedeutungen*. Münstersche Mittelalterschriften 56 (Munich: Fink, 1987) 814-815.
5. See G.W. Butterworth, "Triangular Numbers in the New Testament." *Journal of Theological Studies* 15 (1914): 67-76.
6. Compare the rather inadequate interpretation given in Fritz Tschirch, *Spiegelungen: Untersuchungen zum Grenzraum zwischen Germanistik und Theologie* (Berlin: Schmidt, 1966) 276.
7. Meyer and Suntrup (814-815) cite the most important patristic texts.
8. Compare Ute Schwab, *Lex et Gratia: Der literarische Exkurs Gottfrieds von Strassburg und Hartmanns Gregorius*, Pubblicazioni dell'istituto di lingue e letterature straniere 1 (Messina: Università degli studi, 1967) 22-25, 73-74. Schwab's study was available to the author only after the completion of this essay.
9. *Epistulae*, ed. Al. Goldbacher. CSEL 34 (Prague: Tempsky, 1895).
10. Ed. Goldbacher, 205-206.
11. Compare Eva-Maria Carne, *Die Frauengestalten bei Hartmann von Aue: Ihre Bedeutung im Aufbau und Gehalt der Epen*, Marburger Beiträge zur Germanistik 31 (Marburg: Elwert, 1970) 109, who explains the different modes of penance as simply those most appropriate to each character's sex.
12. Compare also lines 3945-946, where it is noted that she has done all that was required of her in penance "mit libe und mit guote,/ mit beitendem muote."
13. This epithet is used in the text only for Gregorius, but compare line 3960: this is the tale of "disen sündæren," that is to say, of both.

Two, Three and Eleven: Disharmony, Disorder, Disarray Emblematizing Social and Ideological Change in Thirteenth Century Iconography and Poetic Documents

Konrad Haderlein

Twelve highwaymen came to an ogre's house in the woods,
Who promptly devoured eleven of them without encountering
resistance.

As he was done with these and made ready to finish the lot,
The last put up a fight, pretending to be a hero.
The ogre said, "No struggle will benefit you now!
When there were twelve of you,

You should have defended yourself" (Schröder III, 59, 121-7).¹

Do these numbers mean something? We have the poet's oblique answer when he elucidates his own allegory at the end of the poem. A clan who is set upon by its feudal laird is like that, explains Conrad von Würzburg; it must fight back in concord when he presses because, he concludes, if its members permit themselves to be subjugated one by one, the laird will cut the clan to pieces until it is ultimately destroyed.

No numbers here. However, the author's own interpretation and moral of his story reflects the political reality of a society where the individual lives without the security and expectation of legal recourse at a higher instance, in an empire without an emperor. But whether eleven was significant as a number or whether it was merely "kontextgebunden" as Hellgardt describes the purported allegorical arbitrariness of numbers (Hellgardt 9), can only be established by further corroboration. Hellgardt, incidentally, asserts, in criticising further the assignment of so-called "Proprietäten" to designated numbers, that it is impossible to derive from such numbers qualities which themselves beget particular meaning, but rather that as numbers they acquire particular meaning only through the relationship in which they are put with other things:

Bei den Zahlen ist es nun immer wieder so, dass eine sinnstiftende Eigenschaft sich aus ihnen selbst gar nicht entwickeln lässt, sondern dass sie ihnen erst aus dem Zusammenhang, in dem sie mit anderen Dingen stehen, beigelegt wird (12).

[When considering numbers as such, it is quite impossible to derive from them qualities which themselves beget particular meaning, but rather they acquire meaning only through the relationship in which they are put with other things.]

It is never quite clear in Mayer-Suntrup's *Lexikon* whose concepts are here the subject of Hellgardt's criticism, whether "Proprietäten" are properties in the Hegelian sense of "Eigenschaften," that is to say, innate characteristics; or whether indeed they are properties, in the sense in which it is proper for the number "n" to bear the connotation of quality "x" but improper to mean "z" or "y." The whole concept of *Kontextgebundenheit*, i.e. the dependency of a numeric symbol upon the content of a literary message remains an issue of major contention. My own position is that in 13th century literary tradition, certain specific connotations of the number "n" are common knowledge, much as the meaning of an heraldic detail "h" in a 13th century blazon is common knowledge, and that, in the process of composition the author, for the purpose of reinforcing the allegorical message, "tags" an event or iteration with a numerical configuration which is recognized by the discerning.

This paper will address the issue only to demonstrate that the categorical rejection of "properties" and proprieties of the number "n" outside of the literary context, specifically in the case of vernacular literature of the 13th century, would be erroneous. On the contrary, specific connotative features of the number "n" must be assumed to have been applied outside of the literary text, in much of 13th century tradition. In the case of eleven, for example, failure, folly, disarray and impending disaster have been attributes which the number would bring to a given literary or iconographic event rather than borrowing them therefrom, or assuming them therein.

In a telling example from a twelfth-century poem, Wilfried Schouwink had gone so far as to observe exactly eleven instances where eleven, as a number or a factor, carries apocalyptic connotation. Three of these (items 1, 7 and 10 in Schouwink's list) deal with the irony of success and failure:

- a. as the wolf Ysemgrimus vainly tries to free himself from the ice 11 times;
- b. the rumour that the wolf has become a monk attracts 11 abbots to his monastery; and
- c. in the wolf's investiture as a bishop he is knocked down then times by the broken bell; on his eleventh attempt to rise, he remains floored (Schouwink 514-5).

Dietz-Rüdiger Moser, although dealing with medieval and modern aspects of 11 as a number of fools, came to similar conclusions about the apocalyptic connotation of eleven. He quotes several *Fastnacht-Carnival* songs in use today in which the appearance of the Antichrist is connected with the use of eleven as the number of fools. In two of these, the phonetic stage of Antichrist as "*Entechrist*" and "*Entchrist*" (not: *Anti-Christ*) attracts attention. Moser might well have pointed out that the phonetic characteristic of Entechrist and/or Entchrist (for "Antichrist") as employed in these songs could date them or their precursors as early as the eleventh century, because of the evident sound shift (*Sekundärumlaut*).

Moser must be credited for having drawn attention to the affinity of the traditional Carnival-Folly of *Fastnacht* with the "End-of-the-World-Theme," or "Endzeitthematik," as he calls it. In the same context, Moser also drew attention to the triplism of eleven in traditional iconography, as the number is plotted to convey its allegorical, tropological, and anagogical dimensions (Moser 362).

If number configurations employing eleven in 13th century vernacular literature are scarce, theoretical discussion, specifically of an allegorical employment of 11 in literary composition is even more rare. Ulrich Ernst has more recently drawn attention to the treatment of 9 and 11 by Pietro Bongo in his *numerorum mysteria* of 1599. With Bongo, an important factor of interpretation is the location of the number in the algebraic progression, relative to its neighbouring numbers and their asserted significance, as well as the location of a number relative to values of the decimal system (Ernst 295). In 13th century thinking the duality of the medieval bias toward certain numbers, as they are viewed relative to their location in the numerical progression, where "n" can be seen at once as *transgressio* ($n=n_1+n_x$) or as a *numerus deficiens* ($n=n_2-n_x$) is often reflected by the way in which "n" is employed in the fable. For example, let there be a connotation for the number "n" by which it displays certain signals in

harmony with the intent of the fable, and let "n" furthermore emblemize properties that are congruent with the just outcome of the fable, then surplus or deficiency of "n" as, say, $(n-1)$ or $(n+2)$ will surface as an emblem of imbalance or disruption. The case of eleven is a significant paradigm for such duplicate interpretation because the number is at once transgression and deficiency, each by the sum of one.

Seemingly correct numbers employed in a literary context have much in common with these qualities of eleven when they are in fact the result of excess and deficiency, for instance an appearance of three in the place of one (*transgressio*) two in the place of three (*numerus deficiens*) or an incorrect four instead of three (*transgressio*), as several examples will later demonstrate; discordance and corruption (two instead of three: *deficiens*) havoc and upheaval (*transgressio*: three instead of the correct one) are typically the result. As the numeral four is frequently seen to function as an agent of retribution and reinstatement, its double appearance, as sum together with three in the aggregate of eleven is most interesting to note ($4+3+4 = 11$).

We return to Conrad von Würzburg whose voluminous work, meticulously recorded and prolifically transmitted, might afford us some hint of corroboration in the poet's employment of number as a significant emblem. In the case of eleven, Conrad's works yield two further examples in addition to the poem about the ogre and the highwaymen from which we quoted at the beginning of this paper. The first is from his epic entitled *Silvester* dated 1243 and first recorded in 1279.² In it, one hundred and twenty Jewish priests, on orders of the emperor's mother, rival Silvester in an attempt at converting Constantine to their faith. Out of their midst come twelve, reports Conrad, who excel in wisdom and knowledge. How will Silvester cope in the face of the formidable opponents, to prevent Constantine's conversion to the Jewish faith, and win him over to Christianity?³ The episode extends over several thousand lines of verse, but at the beginning of this section the author hints at the outcome. He introduces the twelve sages one by one, naming each by name and number, extolling their virtues and scholarly skills; but coming to the twelfth he concludes:

He was so extraordinarily gifted
With wisdom of the scriptures
That all their confidence
Rested in him alone. (2772-5)

Conrad von Würzburg elaborates (2776-87) that the eleven sages took solace in imagining that should they fail in debate with Pope Silvester it would be Zambri, their learned friend, on whose scholarly and rhetorical skills they could ultimately rely to win the victory. At the same time, however, Conrad implies that as far as their own efforts were concerned, the eleven priests had already given up:

ob si gesigen solten
daz müeste an sine helfe stan
[whether they would be victorious
would depend on his assistance]

that is to say, their victory over Silvester would depend entirely on Zambri. We are not pursuing the details of Conrad's story further than this, but note that for the discerning listener such a demonstrative split of twelve into eleven and one bodes failure.

The second example is more elaborate, albeit from a relatively brief poetic document of just over 250 lines. I gave it the English title, "Statement of Claim by Mistress Art."⁴ In it Art, before a conglomerate assembly of virtues, all female and presided over by Justice on the high bench, launches an accusation against Charity whom she charges thrice with falsehood. By the poet's description, Mistress Art is destitute; and her appearance in rags and tatters before the splendidly attired assembly clearly marks her as ostracised from the echelon. While once more the outcome of the parable is not relevant to this discussion, the poet's arrangement of virtues in the echelon by numbers and ranks, is. The essentials of Conrad's elaborate seating arrangement are stalls in a chancel upon which are seated four, three, and again four virtues in descending order. Justice, throned above these stalls of eleven received, almost as an afterthought, the company of two associates, Truth and Forthright Love, reminiscent of the exclusion of *philosophia*, in the company of Plato and Aristotle from the circle of the seven *artes liberales*.

As he did in *Silvester* with the eleven scholars, Conrad von Würzburg calls each virtue by name but numbers them in groups, naming the numbers each time. Thus the top bench of the perceived bleachers is occupied "by four women bejeweled with rich crowns," namely, Mercy, Loyalty, Steadfastness and Humility (60-74). Beneath these are seated "three most worthy women whom I would never tire to gaze at," namely, Goodness wearing the four-pronged crown,

Charity and Honor (76-83). Finally, on a third rank are seated the four women Modesty, Temperance, Propriety and Chastity (84-99).

For the observer of Conrad's von Würzburg's reflection on social and ideological order there is nothing surprising in this configuration in fours and threes and nothing surprising indeed in the central placement of *milte*, representing Charity-Generosity-Magnanimity, and flanked in its triad by Goodness on the left and Honor on the right. *Ere* representing the honor of both wealth and office; and *güete* representing both moral rectitude and compassion, are thus both instruments and functions of *milte* occupying the centre of all virtues. Also typical for Conrad that *erbarnehertekeit* and *triuwe*, Mercy and Loyalty, are ranked first and highest among virtues and are placed in a foursome together with *staete* an *bescheidenheit*, Steadfastness and Humility in the top rank.

Worrisome is the absence of *saelde* in this configuration, representing bliss and fulfilment in chivalric life. The word is spoken but the virtue as such is not part of the echelon of personified allegories in the poem. Worrisome, too, is the *numerus deficiens* resulting from the evident expulsion of Art out of the assembly. This fact ultimately leads to the accusation launched by Mistress Art, in which Charity is charged thrice with falsehood. When Art's "Statement of Claim" is acknowledged as justified, the carefully erected scaffold comes tumbling down.

Wilfried Schouwink demonstrated how, contrary to earlier interpretations, the death of the wolf Ysengrimus under the ferocious fangs of the sow Salaura and her band is an emblematic representation of End-of-the-World-Theme, not by the appearance of a herd of apocalyptic swine, but by their number, 66. Schouwink allows that the number 666 of the terrestrial beast in *Apoc.* 13,18 had been earlier noted by others in this connection; but he points out that unlike the terrestrial beast, in the number of Salaura's band the factor of eleven looms large. In fact, the author of the poem quotes Salaura's number of swine not as *sexaginta sex*, but as *undecis seni*. (513)

Late 13th century and 14th century iconography reflects by use of tangible emblem how Charity manifests itself as a triad, in company with Goodness and Honor. Typically, where treachery was involved and legal charges were laid in threefold issue as was the case in Mistress Art's accusation of Charity, restitution is made in a corresponding emblematic manner, employing an allegorical personification of Charity and the symbol of three.

To peruse yet another case, two of four window panels completed in 1320, in the second South-side window of the Freiburg Cathedral depict episodes from the life of St. Nikolas.⁵ In one of them Nikolas, in full episcopal attire and wearing mitre and staff, offers golden apples to three young women. Later French and Latin literary sources explain how a poor man had agonized over his decision to send his three daughters into the street as harlots, to prevent family destitution. Nikolas, who had heard about the man's impending decision, proffers in three consecutive nights to each of the three virgins a lump of gold. Corroboration of the fable is illustrated in a painting dated at around 1370 by the Florentine painter Agnolo Gaddi. The three desolate virgins are here seated around a room into which, thrown by Nikolas through a window, a golden ball or apple has just dropped. The Saint is outside, about to cast a second lump of gold through the window with his right hand, while the left hand holds a third.

The other Freiburg panel in which the holy bishop is seen blessing three naked youths rising out of a meat barrel has two basic versions of literary background. According to one of these a father had butchered his sons during a famine; according to another a greedy innkeeper had murdered three rich students lodging at his establishment, cut them to pieces and salted and pickled them. The panel shows how Nikolas brings the murdered and pickled boys back to life. Because Nikolas is venerated for his exemplary representation of *milte*, that is to say, Charity, the two Freiburg panels are exemplary demonstrations of the vanity of *minne* ("courtship," panel of the three virgins) and *ere* ("power, wealth," panel of the three youths).

Late 13th century and 14th century iconography also depicts triads emblematising the excess of one. A paradigm of such a *transgressio* appears in the manuscript Sig. 11. 2. Aug. 4^{to} at Wolfenbüttel in a scene showing the assassination of Julius Caesar. In a miniature on the top of column b, 140^{recto} Julius is seen. He wears purple tights and a purple gown and is seated on a golden throne with green covers. There is a golden crown on his golden hair. Cassius, dressed in red (left front), has run his sword through Caesar, who looks out of the picture in surprise. A blond, bare-headed assailant, dressed in green, stands behind him raising the curved sword that cuts across the picture. Brutus (at right) stands behind the throne; he wears a blue gown, red tights and black shoes. Holding his raised sword at the hilt with both hands, he is ramming it home. . . . In a blue sky above, *three suns*,

large and yellow, with wavering rays like tassels, and sad human faces.
The caption on 140^{recto} reads,

"Tod des Kaisers Julio;" on column b, below the caption:

Do Julius funf yar
Kaiser was gewesen in rom
Do must er in das leben lan.
[When Julius had been Emperor in Rome for five years
they came and took his life]

A legend under the miniature comments on the phenomenon of the three suns:

D daz also was geschehen
do wurden drei sunne gesehen
Die sach man an dem himmel sten
do wurden si zu ein andr gen
daz ein schein dar auz ward tan etc.

[And after that had happened there appeared three suns they
could be seen shining in the sky but then they merged so
that they became again one single light]

Finally, in the parable of the *Judge and the Devil* by the thirteenth century poet *Der Stricker*, no allusion to numbers or their meaning is made in the concluding "moral of the story," and the direct mention of numbers is studiously avoided elsewhere in the tale, save in one characteristic instance proclaiming the Judge's fame: "he was well-known for two things,"⁶ explains *Der Stricker*, "for being both the wealthiest and the most iniquitous man alive". (line 4) On the other hand, significant acts and iterations in the story are all repeated by the principals in significant numbers of times. I am choosing two examples to show how, in the transgression of three by the sum of one, evil is perpetrated and retribution occurs, by the same measure; just as, in the same context, the deficiency of three by the sum of one reflects evil as the duplicity of iniquity and wealth.

In the first example, the Judge attempts to compel the Devil to disclose information he refuses to give, with an incantation using the name of God. But the Judge, in his zeal to strafe the Devil, invokes the name of God four times instead of the correct three; "woe to me," taunts the Devil, "that I was ever born! You have bound me in such fearful fetters, that I have never been in greater trouble!"⁷ Part of the bargain seemingly compels the reluctant Devil to let the Judge witness

his mission: he has come to the city to accept as a gift whatsoever or whosoever this day is offered to him in earnest.

The two encounter three instances in each of which an angry woman proffers her pig, her heifer and, indeed, her child to the Devil, with the Judge each time urging the Devil on to take what was rightfully his, but the Devil each time refusing to accept, arguing that the proposition was not made in earnest. But when in a fourth encounter an accusing woman damns the Judge to hell, body and soul, the Devil allows that she is serious and drags the Judge off by his hair, in a gesture of expediting a just verdict.

Some years ago Heinz Meyer drew attention to eleven as both *transgressio* and *numerus deficiens*. Meyer furnished a list of medieval authors who used eleven in a significant manner and quoted two passages from Beda which are repeated here in an abbreviated form.

Undecim quippe qui denarium transeunt neque adduodenarium, hoc est apostolicum, numerum pervenient transgressionem decalogi legis significant (De Tabernaculo, CCL 119A, 51).

[Eleven indeed exceeds ten but does not reach twelve, the number of the apostles; it thus signifies the transgression of the commandments.]

Undecim autem transgressionem denarii significant.... Manifestum est quod transgressio denarii peccatum significat per superbiam plus aliquid habere cupientis. (In luc. expositio, CCL 120, 91)

[Eleven, however, signifies the transgression of ten . . . And it is evident that this transgression of ten, by its arrogance of coveting excess, signifies sin.]

A thirteenth century wall painting in Hartberg, Styria depicts the apocalyptic boar as a mount of the Holy Roman Emperor; out of the boar's head, on projecting rods, jut ten crowns, while the rider, holding the imperial regalia (sceptre and orb), himself wears the eleventh crown.

An early 14th century parchment at Wolfenbüttel contains the narrative of Charlemagne's Death, following the famous thirteenth century biography of the emperor by the poet *Der Stricker* which is also recorded there.⁸ According to the story, the emperor fell ill in January, sent for his son Louis and called eleven bishops to his bedside. Shortly afterwards his condition worsened and he died. I quote the salient lines in the original:

Dar nach nicht lanck

wart der keyser alz kranck
 Daz er davon muost sterben
 und an dem leib verderben.
 [Not long thereafter the Emperor became so ill that his body
 decayed and he had to die]

When I discovered this passage it seemed clear to me that the episcopal entourage summoned, by virtue of its peculiar numeric configuration would prove most ineffective. Perhaps it is no coincidence that according to the count of the Imperial Roman Calendar January was the eleventh month.

Notes

1. Zwelf schacher zeines türsen hus in einem walde quamen
 der fraz ir einlif sunder wer die schiere ein ende namen
 sit begunde er ramen daz si alle würden gar verzert
 do werte sich der zwelfte und wolte alsam ein helt gebaren
 do sprach der türse du enmacht nu keiner wer gevaren
 do din zwelwe waren
 do soltestu dich han gewert
 (Translation of this and the following German, Middle-High-German and Latin texts by the author.)
2. The last extant critical edition by Paul Gereke in: *Konrad von Würzburg. Die Legenden*. Halle: Niemeyer, 1925; can no longer be considered reliable.
 For dating the poem see Haderlein, K. "Die Handschriften der Gedichte Konrads von Würzburg in Österreich. Ein Beitrag zur Vita des Dichters und der Chronologie seiner Werke." *Prisma 14*, i (1989), 66-101. Separate Reprint Regina: University of Regina, 1990, 37 pp.
 For dating the MS, see Betty C. Bushey (unpubl., working title): "Die deutschen und niederländischen Handschriften bis 1600, bearbeitet von Betty C. Bushey." New Series to be entitled, *Die Handschriften der Stadtbibliothek Trier ...* Wiesbaden: Harrassowitz, 199__?
3. For Konrad's sources see Jackson, Timothy R. *The Legends of Konrad von Würzburg. Form, Content, Function*. Erlangen: Palm & Enke, 1983, as well as the excellent contribution by Wilhelm Pohlkamp (*infra*).
4. "Meister Conrades von Wurtzburg getichte von unmiltickeit gein künstrichen leuten." For details see WORKS CITED, *infra*.

5. *Nicolas of Myra*. The legend of the three virgins and the three golden apples has been placed in 12th-century Normandy and/or Lorraine. For the iconographic history of this motif, excepting examples cited in this paper, see Petzold, L. in *Lexikon der Christlichen Ikonographie*, 1976, 46-58 who cites a late 14th-century painting by Gentile da Fabriano (Vatican Pinacoteca) as the earliest iconographic occurrence as individual work. Illustration #6. p. 56.

Recent reproductions and discussion of the Freiburg glass panels in Konrad Kunze (*Himmel in Stein* 1990, 72-3). Thirteenth-Century church glass panels containing both scenes: Clermont-Bressanone, Bourges, Tours, Bari (stone relief) and Chartres (only the golden apples scene).

Painting by Agnolo Gaddi: München. Alte Pinakothek. Illustration in Cornelia Syre, *Frühe Italienische Gemälde* 1990, 62 which see below.

6. Kay Slocum argues that in contemporary music the binary system is understood to reflect the corruptible and that two, as an exemplary case is understood to reflect evil and corruption. Its numerical proportions are *not* harmonious, thus its employment causes discordance. Slocum draws attention to Aristotle's remark that the binary system was imperfect. For the 13th-century evaluation of two in the present context the parallel is obvious: The number is both *transgressio* and *numerus deficiens* even as the paradigmatic eleven, each by the sum of one.

7. Lines 76-79:

du hast mich in so
 starkiu bant
 beidiu gevangen und gebunden
 daz ich ze manigen stunden
 so groze not nie gewan

8. Cod. Guelf. 1.5.2. Aug. fol. 220^{recto}

John Donne, the Number 23 and the Tradition of Spiritual Autobiography

Kate Frost

For the past two decades I have been engrossed with the study of John Donne's *Devotions Upon Emergent Occasions*, both in preparing an edition of the text and in studying the arithmetical program which underlies it. A rather strange work, composed in 1623 on what Donne then thought his deathbed, the *Devotions* consists of twenty-three three-part essays on the human condition in all its miseries. Its critical history indicates that no one seems to have had any cogent idea of just what Donne was trying to accomplish in this curious book: it is autobiographical, mystical, political, scientific (if your science runs to basilisks and the application of pigeon carcasses to one's feet to assuage a fever). No critic, including the great T. S. Eliot, seemed comfortable with it. Perhaps a key to Donne's purpose may be found in an attempt to answer the following question: Why are there *twenty-three* Devotions? The process of finding an answer will take the critical reader into an area of rather arcane scholarship—into literary and numerological traditions shared by Donne and, indeed, many authors long before his own time.

The present essay is an attempt to explore the complex ramifications of the number twenty-three, particularly in its association with the rich tradition of medieval spiritual autobiography, a tradition with which Donne was fully cognizant. I shall begin by identifying the early associations of the number with the hexaemeral tradition, with Divine Justice, and, concomitantly, with the Second Coming and the Advent liturgy. Then I will demonstrate its association with the vertical structure of the universe, the tradition of the Harrowing of Hell, and with the solar ecliptic—emphasizing its connection with the winter solstice and the theme of moral conversion. Next to be addressed are the implications of the number when it is associated with autobiography, its use by Dante, Henry Suso, and Donne (with some side

excursions into Ficino and Milton). Finally, I refer to the recent discovery of a twenty-three-part, fourteenth-century illustrated manuscript autobiography of Opicinus de Canistris which demonstrates the ongoing tradition of spiritual autobiography that lasted at least from the thirteenth through the seventeenth centuries.

The association of the number twenty-three with the hexaemeral tradition is ancient, in fact as early as Philo: "When, then Moses says, 'He finished His work on the sixth day,' we must understand him to be adducing not a quantity of days, but a perfect number, namely six, since it is the first that is equal to the sum of its own fractions $1/2$, $1/3$, and $1/6$, and is produced by the multiplication of two unequal factors 2×3 " (Philo 1: 249). Philo contemplates, in their permutations, the integers which comprise the number: interpreted as 2×3 the hexad was associated with the days of Creation and, particularly with the sixth and last age which will end in the Last Judgment. Augustine, in his second discourse on Psalm 29, improved on this, allying the six ages of the world with the ages of man. The scheme was common, receiving its definitive statement in the twenty-third chapter of Dante's *Convivio*. There Adolescence, Manhood, Age, and Decrepitude are likened to the four horses who draw the chariot of the sun through the course of the year. Entering through Cancer, "the gate of good life," they proceed through the seasons with their corresponding humors, from hot, moist Adolescence to cold, dry Decrepitude (Dante, *Convivio*, 274). This journey Dante compared to an arch, its highest point between the years thirty and forty, its final point at seventy, the Biblical limit to man's life—a number which, in its association with the Babylonian captivity, had come to represent, as Christopher Butler notes, "man's enslavement to the world of sense—a captivity terminated only by the redemptive action of Christ, who releases the soul of man from the fetters of the flesh, permitting him to ascend above the sphere of time" (Røstvig, 88). Considered in this sense, 2×3 is associated with patterns of spiritual and moral ascent that will manifest themselves in the discussion of spiritual autobiography later in this essay.

When the number twenty-three is considered as the sum of its integers, that is $2 + 3$, the resulting number is traditionally associated with the Justice, and ultimately with the hexad and the Final Judgement. Most of the arithmetical commentaries refer to I Corinthians 10:8. Peter Bongo, who gave us one of the few extant treatises on numbers, represented twenty-three as a number signifying God's

judgment of sinners, coupled with his eventual mercy. "According to Moses," he says:

[It] signifies Divine Vengeance over sinners. The twenty-three thousand constructing the calf were ordered slain down to the last one by Moses. Paul, writing to the Corinthians, refers to the twenty-three thousand dying on a day who defiled themselves with the Midianite women. It is possible by this number to represent the fullness of human salvation, and especially that perfection of righteous belief which is maintained by good works (sig. E3r).

The connections of twenty-three with the hexad and the pentad are clear and workable. Our forebears, however, did not keep things simple for our modern convenience, and, when one begins to contemplate the astronomical/astrological and cosmic associations of the number, matters grow complex. In midwinter, when the earth is at the diametrically opposite side of its orbit around the sun, the North Pole points twenty-three degrees away from the solar direction. To the naked eye, the sun has reached the southernmost point in its journey along the zodiac, slowing as it nears the apogee in the apparent retrograde motion characteristic of the solstice. This occurs during the liturgical season of Advent, when Christmas and the rising of the *sol iustitiae* are imminent and when the Church sings the *O Sapientia* antiphons, calling for Christ the Physician and announcing his coming, in the words of Jacobus de Voragine, as "our Teacher, our Redeemer, our Liberator, our Guide, our Enlightener, and our Savior"(3). As the sun turns to begin its annual climb, bringing life and light and health, so must we turn to meet the rising *sol iustitiae*.

The analogy between the conversion of the sun at the ecliptic and the individual spiritual life was an easy one. Sacrobosco defined the sign of Capricorn as "a turning point. For this is the winter solstice, and from thence the sun is turned to us"(sig. Biir). Hence, again according to the *Legenda Aurea*:

The whole of this fugitive life is divided into four periods: the period of erring, or wandering from the way; the period of renewal, or returning to the right way, the period of reconciliation, and the period of pilgrimage The period of renewal began with Moses and lasted until the birth of Christ, for this is the period during which man was renewed and called back to the faith through the mouth of the Prophets. These four divisions of the spiritual life are comparable

to the four parts of the natural year, so that the first is Winter, the second the Spring, the third the Summer, and the fourth the Autumn (1-3).

The *Legenda* goes on to associate the winter solstice with the historical period of the pre-Covenant, with night as one of the four times of day, with the flesh, and so forth. In this context, it is not hard to see twenty-three as a number of conversion, a turning back along the ecliptic course of one's life.

The Christian typology of conversion was established in the New Testament by Paul (Acts 11, 1-22), itself reflecting the Old Testament story of King Hezekiah (Isaias 38): a casting down, a period of blindness and abstinence, cure and raising by a specially appointed messenger of God, and immediate preaching of the good news by the recovered sinner. Thus, in the twenty-third chapter of the *Vita nuova*, Dante is prostrated with fever; in a dream he confronts first the passing of Beatrice ("Vero e che morta giace la nostra donna") and then his own imminent ("Tu pur morrai") and actual ("Tu se morte") death. There is a vision of angels and, after his cure, the recounting of the experience in song ("Donna pietosa e di novella etate")—a song which by its title, moreover, indicates that a conversion of some sort has taken place. As Charles Singleton has pointed out, this chapter twenty-three falls at midpoint in the *Vita Nuova* (7).

A strikingly similar pattern can be observed in Henry Suso's *Life of the Servant*, which follows the *Vita Nuova* by some fifty years and which forms the first section of his autobiographical four-part *Exemplar*. The twenty-third chapter of Suso's *Vita* begins with a series of worldly injuries and then concentrates on an illness—a burning fever caused by an abscess—which supposedly has brought Suso near to death. In his suffering, the Dominican mystic meditates on Christ's agony and is rewarded by angelic messengers who comfort him and request that he join with them in a song of praise. To this he replies: "The only kind of music I feel like accompanying is a funeral dirge, because my life's clock points to half past eleven." Quoting Psalm 30, the angels urge: "Take courage and be stouthearted. Cheer up, act like a man. Death is still a hundred leagues from your door. You have many earthly days ahead of you in which you will praise God and console your weary fellow men with the carols of eternity" (I, 64). Thereupon the abscess bursts and Suso is cured. His *vita* forms his thanksgiving. Although many of the parallels between the *Vita Nuova*

and Suso's *vita* may be explained by their participation in a general tradition of spiritual autobiography, certain of these similarities are striking, particularly in that the twenty-third chapter to each, which presents much the same conventions, forms the numerical center of its whole.

That the use of the number in association with spiritual autobiography persisted well into the seventeenth century is attested by Donne's structural employment of it in his *Devotions Upon Emergent Occasions*. In late November 1623 Donne fell ill with epidemic typhus (Frost, "Donne's *Devotions*"). The progress of the illness was swift and, although the poet nearly died, by mid-December he was out of danger. Despite being denied access to his books through fear of damaging his sight, he spent the initial month of his recuperation in an invalid chair where he readied the book for the printer. In Donne's words: "I have used this leisure to put the meditations had in my sickness into some such order as may minister some holy delight" (*Letters*, 249). In his Epistle Dedicatory, Donne draws explicit attention to the likeness of his case to that of King Hezekiah: "Examples of Good Kings are Commandments; And Ezechiah writh the Mediations of his Sicknesse, after his Sicknesse" (*Devotions*, sig. A5r).

The progress of the illness, its cure, and Donne's recovery all fulfill the conditions of the typological model, as well as provide discursive linear structure to the book. Like Hezekiah, he was struck down at the winter solstice. Lying near death, he was treated, as was Hezekiah and Suso, by a poultice on the breast (Frost, "Prescription and Devotion", 414). Unable to attend services at the adjoining church he turned his face, like Hezekiah, toward the temple and called on God to witness his faith. As Isaiah visited Hezekiah and witnessed his miraculous cure, so the personally delegated physician of James I attended the illness of John Donne; the sign of the backward moving shadow on the sundial of Ahaz signalled the extension of Hezekiah's life, Donne's extension was indicated by another kind of shadow: a cloudy urine which signified a return to health. Hezekiah, Dante, and Suso all sing in praise and thanksgiving; Donne's *Devotions* forms his own hymn of recognition.

If the linear progression of the *Devotions* echoes the conventions of medieval spiritual autobiography, its internal arithmetical structure shows even stronger parallels. In its twenty-three part structure, the *Devotions* presents a complex, numerologically structured artifact which

mirrors the structure of the hexaemeral week, the solstitial day, and a year-by-year progress through the ecliptic of his own life. Indeed, the latter seems directly derived from Dante's architectural arch of the ages of man in the *Convivio* (Frost, *Holy Delight*, 144-46).

Here in particular I draw attention to Devotions XVI, XVII, and XVIII. These digress from the linear movement of the text, which otherwise presents the progress of the illness. Instead, they provide a fulsome meditation on the death of an unknown neighbor, whose passing and funeral bells sound in Donne's sickroom. The titles of these meditations echo remarkably the language of the twenty-third chapter of the *Vita Nuova*: (XVI) "From the bels of the church adioyning, I am daily remembred of my buriall in the funeralls of others" ["*Vero e che morta giace la nostra donna*"]; (XVII) "Now, this Bell tolling softly for another, saies to me: Thou must die" ["*Tu pur morrai*]; XVIII "The bell rings out, and tells me in him, that I am dead" ["*Tu se morte*"]. Moreover, when one compares them to the *Devotions'* autobiographical structure based on Donne's own life years, it is evident that they follow the year 1617, in which Anne Donne died (Frost, *Holy Delight*, 148-49).

Surely, we ought to recognize more than merely an intelligent reinterpretation of Dante. We know Donne owned a copy of the *Vita Nuova*. (There is thus far no evidence that he read Suso.) But he was not allowed his books during recuperation; whatever internal structure he imparted to the *Devotions* was drawn from a model in his memory. Was he then perhaps participating in an established tradition, one which constructed mnemonic personal life calendars? If so, how did this come to him?

There is evidence that the tradition, or a variant of it, was known by some of Donne's near contemporaries. In his study of *Paradise Lost*, Gunnar Qvarnström connects the number twenty-three with patterns of ascent, calling to mind the joint ascensions of Christ and the Virgin in Canto 23 of the *Paradiso*. Milton, he maintains, has arranged the central book of *Paradise Lost* to depend on the number. There, immediately after two sets of twenty-three-line speeches, Satan is driven to hell, pursued by Christ the bringer of Divine Justice who then returns to Paradise rising through the twenty-three levels of the created universe—the same twenty-three levels he was again to traverse from his Harrowing of Hell. Hence Milton has conflated the ascension of Divine Love with Justice (Qvarnström, 98-103). Ficino also, in his

commentary on *Phaedrus*, associated the number with cosmic ascent as well as with conversion, for in Summa 23 of his commentary, he halts the progress of the soul/charioteer and begins a return from the contemplation of the divine to the concerns of the mundane (Allen, 158-61). Donne's pattern in the *Devotions* is analogous: the twenty-three units form an arch of his own life through which he ascends and descends to the door of life, expecting final justice, only to be given the mercy, like Hezekiah, of an extended life.

The temptation now is to study the number twenty-three in its connection with patterns of ascent and conversion, and such a study will, it seems to me, prove fruitful. But even more fruitful, given the strong association of the number with autobiography, would be the investigation of individual medieval *vitae*. These will, I think, demonstrate evidence of underlying, numerologically based schema—what Philip Sidney called in *The Defense of Poesie* a "foreconceit" (Sidney, 111). Moreover, they should also demonstrate the existence of spatial planning very likely based on mnemonic systems. These, in turn, despite the almost nonexistent evidence of rough drafts from the period, may be based on extant charts and diagrams.

The existence of such a planning document has not heretofore been demonstrated, and its discovery would be the capstone of any argument for the existence of numerical structure in a literary artifact (Fowler, 1-2). Recently, however, such a document has come to light, and it is one which has strong bearing on Donne's *Devotions* and the genre of spiritual autobiography.

In 1927, Faustino Gianani came across a series of strange illustrations filed in the Vatican's map collections which he identified as coming from the hand of Opicus de Canistris (1296-ca. 1350), scriptor at Avignon under John XXII (7-14). Although the fifty-four parchment skins of Codex Vaticanus Palatinus 1993 provoked some interest at first among geographers then seeking the map sources of Christopher Columbus, further study was soon abandoned when its map-like illustrations proved to have no connection. However, the parchments were edited under the auspices of the Warburg by Richard Salomon, who found that what had at first appeared to be maps were rather charts which furnished the foundation for autobiographical illustration (*Opicus de Canistris*, I A & B). In 1952 Salomon announced the discovery of a companion work to which the "maps" of Opicus seemed intended as illustration: a twenty-three-part autobiog-

raphy documenting the author's prostration by illness, cure by the Virgin, and rising to proclaim his thanks by means of the autobiography ("A Newly Discovered Manuscript"). The illustrations give strong evidence of a highly developed personal memory system. Among them, in particular, I found a year-by-year structured chart of the forty years of Opicinus' life.

I am at present embarked on an edition of the autobiography and a study of its relation to the separate illustrations. In the course of my preliminary investigations, interesting things have come to light. Donne, for example, had access to the illustrations, including the life chart, for Codex Vaticanus Palatinus 1993 was among the manuscripts which comprised the famous "occult" collection of the Palatine Library at Heidelberg, seized by the Vatican in 1623. In the course of his travels, Donne had occasion to visit the city at least four times and, given his scholarly bent, the possibility of his examining the collection is real. But, although the illustrations (including the life chart) were housed in Heidelberg, the twenty-three-part spiritual autobiography, Codex Vaticanus Latinus 6435, lay at Avignon until 1594, whence it made its way to Rome. According to his most recent biographers, Donne could not have visited Italy after 1591, and the chances of his seeing it are slim. The question is still open, then, as to how the tradition made its way from the fourteenth to the seventeenth century. But that there is such a tradition and that the number twenty-three is central to our understanding of it seems now established. As we examine more premodern autobiographies with an eye to their spatial and arithmetical substructures, my estimate is that the tradition will reveal itself in force and thus impart new insight into medieval self-revelation and into the linkages between centuries.

Remnants of Medieval Symbolic Number Usage in Northern Europe

Jens Ulff-Møller

Throughout the history of mankind, many different counting systems have been used, from the simplest—one, two, many count—to the decimal calculation used today. The decimal calculation is a system based on exponentiation of the base (10, 100, 1000, etc.). When Arabic numerals came to Europe late in the Middle Ages, they replaced the Roman numerals, which were mainly used by the church.

How ordinary people calculated in early medieval Europe is virtually unknown, and therefore some scholars assume that the Germanic peoples did not possess techniques for calculating (Mahoney 205, Murray 143-4). The Germanic peoples did not leave much evidence of their practical calculations, but some vestiges can be found in archaeological materials, and especially in linguistics and in medieval texts.

The Germanic languages belong to the group of Indo-European languages, which is dated to the third millennium B.C. In all Indo-European languages decimal calculation predominates, but duodecimal calculation is equally significant. So, even if all Mediterranean cultures have purely decimal systems of calculation, weights and measures often reveal the employment of duodecimal calculation (Alberti 34ff), and it is possible to find traces of nondecimal calculation in all Indo-European cultures.

The purpose of this study is to examine the historical evidence pertaining to the duodecimal system—in particular the problem of the Germanic "long hundred" (in German *Großhundert*), in which the word "hundred" had the numerical value "hundred twenty." The "long hundred" has been thought to bear only on counting special merchandise; e.g. fish and planks (*OED* V, 456), just as we use the dozen and the score today, but as the decade numbers 110 and 120 have specific names, we have to consider the "long hundred" as a part of a specific system of calculation. The system is not an exponentiation of the base but a hybrid construction using multiple bases (10 times 12).

Numerals have been studied intensely in comparative philological research, which has demonstrated that the Germanic number words from one to hundred originate from Indo-European. Even though the decimal system prevails, traces of a duodecimal count can be found: a twelve and a twenty count, a rupture in the sequence of numbers after sixty, as well as the prolongation of the decades to 120 in the Germanic "long hundred."

For example, in Old English two types of hundred existed: a hund teontig, and a hund twelftig; i.e. a short (100) and a long (120) hundred.¹ Further, in the Domesday Book, the long hundred is mentioned as "Numero Anglice" (I, 336a, col. 1 and 2). In Old Norse studies scholars have assumed that the hundred mentioned is the long hundred, except in ecclesiastical matters.² The problem is that long hundreds are mixed with short hundreds; in medieval texts the "hundred" and even the Roman numeral "C" is often unqualified,³ so that one cannot easily decide which hundred is used.

The philological study of the "long hundred" has a long history. It was first studied by Jacob Grimm in 1819 (Grimm 238ff). Then about a hundred years ago, there was considerable attention drawn to the "long hundred." W. H. Stevenson thought that it was the original hundred of the Teutonic tribes, and that it throws considerable light upon that early history of European nations that is gradually being revealed by the study of comparative philology (Stevenson 313-27).

In a well-known treatise, Johannes Schmidt stated that the break between 60 and 70, as well as the "long hundred" was a proof of an influence from the Babylonian sexagesimal count, and that consequently, the homeland of the Indo-Europeans should be found in Asia. This article actually led the debate astray: instead of describing the counting system, the debate became pro or con the Babylonian theory.

In 1950 Ferdinand Sommer tried to prove that there were no signs of any Babylonian influence, as he belittled the importance of the "long hundred": why should the Germanic peoples choose an imperfect system of calculation by combining a ten count with a twelve count, instead of using the perfect systems—ten times ten, or twelve times twelve? Later H.-Fr. Rosenfeld and Karl Menninger came to the same conclusions. Since then the "long hundred" has been treated almost as a nonexistent phenomenon, e.g. by Frings and Szemerényi, while the work of O. S. Reuter has gone unnoticed in the main debate. More recently, the debate of the Germanic decades has been reopened by

Gernot Schmidt, Rosemarie Lühr, Carol Justus, Voyles and Ulff-Møller, but without creating further debate.

But let us look closer at the problems discussed by the philologists. In the Germanic languages, they have found a break in the numeral sequence of words after 12, and after 60, and a continuation of the decades to 120:

The Germanic Numbers Eleven, Twelve, Thirteen

<u>Gothic</u>	Old <u>Norse</u>	Old <u>English</u>	Old <u>Saxon</u>	Old High <u>German</u>
ainlif	ellifu	anleofan	elleban	einlif
twalif	tolf	twelf	twelif	zwalif
*threis-taihun	threttán	thri-tene	thri-tehan	drizehan

The Germanic Decades: 50, 60, 70, 80, 90, 100, 110, 120

<u>Gothic</u>	Old <u>Norse</u>	Old <u>English</u>	Old <u>Saxon</u>	Old High <u>German</u>
fimf tigjus	fimm tøgr	fiftig	fif-tig	finf-zug
saihs tigjus	sex tøgr	sixtig	*sehstig	sehs-zug
sibun-te-hund	sjau-roor	sund-seofontig	antsibunta	sibun-zo
ahtau-te-hund	átt-roor	hund-eahatig	antahtoda	ahto-zo
niun-te-hund	nír øor	hund-nigon-tig	antnigonda	*niun-zo
taihun-te-hund	tíreor	hund-teontig	hund	zehan-zo
taihun-taihund	tíu tøgr			
	hundrao-tirett			
?	ellifu tigir	hund-	?	?
		endleofan-tig		
hunda (?)	hundrao	hund	Chunn-tualepti	?
	(trolfrott)	(twelftig)	(Lex Salica)	

The breaks in the sequence of number words between twelve and thirteen, sixty and seventy, can easily be observed. The prolongation of decades to 120 might have disappeared in some Germanic languages, as the "long hundred" is mainly attested in Old English and Old Norse. The "long hundred" is only indirectly attested in the Gothic bible (300 A.D.), as "hundred" is expressed as the tenth ten. The Salic Law (507-11 A.D.) mentions the sum of Chunn-Tualepti denarii (120 pennies = 3 solidi), and multiples of 120 are frequently mentioned in the Law, but besides that there is not much evidence of "long hundreds" except within metrology.

However, the comparative philological method has serious limitations, as mainly phonological implications are considered, whereas arithmetical considerations concerning the general system of calculations are ignored.

The history of mathematics primarily explains the transfer of mathematics and numerals from the Orient, India and ancient Greece to Western Europe. Early European arithmetic skills have been more or less ignored by historians of mathematics, who are unaware of the problems of the "long hundred," and very little has been written on the difference between ancient systems of calculation and the modern decimal system of numerals.

The basic human computer is the hand. On two hands we have 8 fingers and 2 thumbs (or ten in all), and on one hand we have four fingers, each with three joints—leaving us with some basic counting systems to have 8, 10 or 12 as bases (Fomin 3-6). These bases also have certain arithmetic qualities: eight is a number in the dyadic system of doubling: i.e. 2, 4, 8, 16, 32, etc. Twelve is easily divisible, as 2, 3, 4, and 6 are divisors, whereas ten has only 2 and 5 as such.

In popular calculations, which are concrete, the reckoning of fractions presents a problem, as fractions must be considered as parts of the unit. The Germanic peoples liked to use unit fractions. In a twelve-count system, the half, third, quarter, sixth and twelfth parts are integers. Therefore, a duodecimal system is more workable than a simple decimal system, where only the half and the fifth are integers. It is important that ordinary fractions be integers in order to avoid recurring decimals, as when 10 is divided by 3 for example.

There are substantial discrepancies between the mathematical and the "everyday" approach to number systems. In decimal calculation the numbers 0, 10, 100, 1000, etc. constitute the beginnings of new number series, whereas in ancient and "linguistic" calculations those num-

bers are ends of number sequences. Accordingly, we have to consider the number series as having no definite beginning, but having definite limits.

The main possibilities for creating the higher bases is usually to exponentiate the base: $10 \times 10 \times 10$, etc. The "long hundred" is a hybrid construction combining a 12-count with a 10-count, and thereby combining the divisors of both systems:

A Mathematical Analysis of
 10×10 (100), 12×10 (120), and 12×12 (144)

<u>100:</u>	<u>120:</u>	<u>144:</u>
2×50	2×60	2×72
(3)	3×40	3×48
4×25	4×30	4×36
5×20	5×24	(5)
6	6×20	6×24
(7)	(7)	(7)
(8)	8×15	8×18
(9)	(9)	9×16
10×10	10×12	(10)
(11)	(11)	(11)
(12)	12×10	12×12

The analysis of these three systems will show which numbers will be round numbers in each system. As each system has specific subdivisions, it can easily be seen that in the normal hundred (100), a 3/12 count does not fit, because one cannot multiply anything by 3 to get a hundred, nor can one for 6, 7 or 9. In the "hundred" of 144 a 5/10 count does not fit. Clearly, the "handiest" counting system is the hybrid construction consisting of both systems—the long hundred of 120.

To continue, the higher powers of the long hundred are constructed according to the mathematical formula 12×10^n which gives the basic powers 12, 120, 1,200, 12,000, and 120,000. The system is built in two parts: one for addition, and one for division. In the following illustration, the point of origin is double-underlined. In the divisional mode the quarter and the half is underlined. Within the basic number series from 1 to 12 the two systems are identical. In the upper system addition takes place from the low point of origin in the number series

(e.g. 120 + 120 is 240, and so on). In the lower system, division takes place from the top end of the number series:

Round Numbers in the 120 Number System

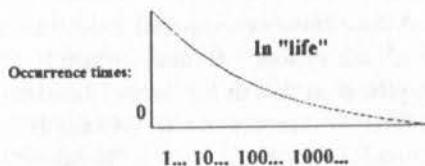
-	1	2	3	4	5	6	7	8	9	10	11	12
-	12	24	36	48	60	72	84	96	108	120		
-	10	20	30	40	50	60	70	80	90	100	110	120
-	120	240	360	480	600	720	840	960	1080	1200		
-	100	200	300	400	500	600	700	800	900	1000	1100	1200
-	1200	2400	3600	4800	6000	7200	8400	9600	10800	12000		
-	1000	2000	3000	4000	5000	6000	7000	8000	9000	10000	11000	12000
-	12000	24000	36000	48000	60000	72000	84000	96000	108000	120000		
-	10000	20000	30000	40000	50000	60000	70000	80000	90000	100000	110000	120000

(Hundred thousand?)

One notices that with the numbers between 10 and 120, the lower system is normally used because of the popularity of the linguistic 10-count system. The passage from ones to tens is therefore not constructed in a regular way. The fractions of a "long thousand" of 1200 are short hundreds, so the long hundred existed together with the short hundred. Having established some basic mathematical procedures within the long hundred, another numerical problem can also be presented.

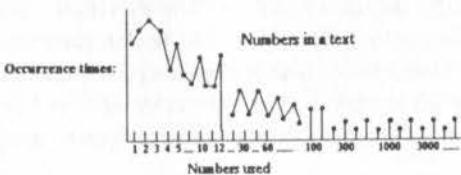
Most numbers are likely to derive, not from exact counting, but from estimates, and an estimated number will be constructed according to the number system used. In "real life," every number is likely to occur, but smaller numbers will occur more often than larger numbers:

Occurrence of Numbers in "Life"



In a text, the round numbers will occur more often than others, and with regular intervals:

Occurrence of Numbers in a Text



The more the numbers occurring look like the first graph, the more the text reflects a witnessed actuality, and the more authentic is the information in the text. A statistical method might be developed and used on all numbers occurring, and thereby one may estimate the authenticity of number usage in a given text.

It is very difficult to reveal how the "long hundred" system of calculation functioned. The proof can only be found in arithmetic problems, where the exact result is known—but such information is rarely found in medieval literature. The best examples can be found in Icelandic Sagas. The following examples reveal how addition and multiplication was performed in early Scandinavian society:

"They had a hundred men, and went to Jomsborg; 80 men were sworn, and 40 went away" (Islenzk 9).

The 365 days of the year are explained as three hundred and five days (Islenzk 12, 57, 65, 76, 143, 156, 175).

The 532 years of the Easter cycle are 440 and 12 (Islenzk 530).

King Olaf the Holy came with four hundred men from the Swedish king; his brother met him with six hundred men—a total of one thousand two hundred men is mentioned (Sturlason 441). This only makes sense if the hundreds are "long hundreds", whereas the "thousand" consists of 12 "short hundreds," i.e. $480 + 720 = 1,200$.

Berntsen explains the counting of fish in Iceland: 1 load of fish is 1,000 fist. "100" fish are 6 score or 120. Three commercial "vaet" of fish are "100"; as 1 vaet is 40 fish, 3 vaet is 3×40 or 120 fist. When "100" is 120, then "1,000" must be 1,200 even if this is not explained (Berntsen 530).

Other examples show quite a different construction - division:

In one case it was reported that six navies had been seen each with one king, every one of them contained five thousand ships, and on each were three hundred men. Even so it was indicated that every thousand of the total was held together in four wings, and with the "thousand" was understood "thousand two hundred," as each wing comprised three hundred men in number.⁴

The Venerable Bede (731) has a similar example: "Wight has twelve hundred hides, the bishop was given three hundred hides, which was a quarter of the total (Bede 382-3).

Berntsen, in another example, explains that in Jutland the whiting fish is counted: 15 score is one quarter; 4 quarters is "1,000" or 1,200 (Berntsen 552).

These examples provide the following equations: One thousand = $1,200 : 4 = 300$, or $4 \times 300 = 1,200$. The thousand mentioned is a "long thousand," but it is divided into "small hundreds" of "100," which obviously seems to contradict the extensive evidence of a "hundred" being "120," and also its multiples are commonly found in medieval texts. But, from the examples, it can be seen that the long and the short hundred interact. In Old Norse, the two kinds of "hundred" are often indicated as "hundrad tirott" or "hundrad tolfrött," (or "hundred of the ten-count" and "hundred of the twelve-count"), but often the hundred mentioned is unqualified so that it is impossible to observe which hundred was meant—except from perhaps looking at the context (Reuter 112).

construction is not the principle of the "long hundred," and the "long thousand" should not be $12 \times 120 (= 1,440)$, even if a given example of such a construction is found (Islenz 84).

The variety of number constructions may reveal a disintegration of the older counting system when being confronted with the Roman numerals and its different rules of calculation.

In Saxo's *Gesta Danorum* numbers are written about 500 times, but they only represent about 60 different numbers. The numbers below twelve comprise 80% of all numbers mentioned. The main round numbers cited are:

2	3	4	6	7	12
20	30	40	60	-	100 (= 120)
-	300	-	-	-	1,200
-	3,000	-	6,000	-	-
-	30,000	-	60,000	-	-

We can observe that the round numbers group neatly in a long hundred module, with the quarter and the half represented in the ones, tens, hundreds, thousands and ten-thousands. In Saxo the number system is easily observed, which means that the historical authenticity of the text may be generally low.

The following examples from the Norwegian *History of the Kings* also show that the long and the short hundred existed together: "King Olaf the Holy came with 400 men from the Swedish king, and from Norway came his brother with 600 men; in all they were 1,200 men" (Snorre 441). The arithmetic problem only makes sense if the hundreds are long hundreds, and the 12-hundred refers to small hundreds. Some early editors have thought the the 12-hundred also

should be long hundreds, i.e. 1,440, but this no longer seems a tenable position.

In Snorre numbers occur about 700 times, and they represent about 70 different numbers. The numbers below 12 comprise about 56% of all numbers. The main round numbers are shown below:

2	3	4	5	-	7	10	12
15	20	30	-	-	60	-	-
	2H	3H	4H	-	6H	-	-
15H	20H	30H	40H	-	60H	-	-

(H = hundred)

The round numbers do not easily fit into a system because there is a great variety of numbers. Accordingly, the historical authenticity of the text should be good, and much better than that of Saxo.

When turning to the English material, numerous examples of the usage of the "long hundred" can be found:

The *Tribal Hidage* (Birch, Maitland) is one of the earliest estimates of English land measurements. The "hide" is supposed to be the "land of one family." The text/listing on the following page shows the particular divisions of a region:

<u>The Tribal Hidage</u>				
Land	Number of hides	= 100s	= 1,000s	
1. Myrcna	thrittig thousand		30.000	
2. Vocen	syfan thousand		7.000	
3. Vesterna	syfan thousand		7.000	
4. Pecsæta	twelf hund	1.200		
5. Elmed	syx hund	600		
6. Lindes farma..	syfan thousand		7.000	
7. Suth gyrwa	syx hund	600		
8. North gyrwa	syx hund	600		
9. East wixna	thryu hund	300		
10. Vest wixna	syx hund	600		
11. Spalda	syx hund	600		
12. Vigesta	nygan hund	900		
13. Herefinna	twelf hund	1.200		
14. Sweordora	thryu hund	300		
15. Gifla	thryu hund	300		
16. Hicca	thryu hund	300		
17. Vihtgara (Wright)	syx hund	600		
18. Noxgaga	fif thousand		5.000	
19. Ohtgagga	twa thousand		2.000	
20. Hwinca	syfan thousand		7.000	
21. Ciltern sætna	feower thousand		4.000	
22. Hendrica	thryu thousand (+ 500)		3.000	
23. Unecung	twelf hund	1.200		
24. Aro sætna	syx hund	600		
25. Færwinga	threo hund	300		
26. Bilmiga	syx hund	600		
27. Viderigga	eac swa	600		
28. East willa	syx hund	600		
29. Vest willa	syx hund	600		
30. East engle	thrittig thousand		30.000	
31. East sexena	syfon thousand	7.000		
32. Cantwarena	fiftene thousand	15.000		
33. Suth sexena	syufan thousand		7.000	
34. Vest sexena	hund thousand		100.000	

In the text a series of hundreds, and a series of thousands can be observed. The hundreds are 300, 600, 900 and at the top-end, 12-

hundred. Furthermore, one could add two examples (nos. 7 and 8, or 28 and 29) of two-times 600 being 12-hundred. So, the 600 may be regarded as a half-thousand or 12-hundred. The small thousands are 2, 3, 4, 5 and 7-thousand, which are not commensurate to a one-thousand being 12-hundred. However, 2 and 5-thousand, and 4 and 3-thousand could be added to each other respectively so that only 7,000 would be represented. We then only have the following numbers in the *Tribal Hidage*:

300	600	900	<u>1200</u>
7,000	15,000	30,000	- - - <u>Hundred thousand</u> .

The 7-thousand, as they stand, fall outside all numeral systems, whereas the 30 and the 15-thousands could be regarded as 1/4 and 1/8 of a long hundred-thousand, being 120,000.

It may be concluded that all the numbers of the *Tribal Hidage* are round numbers. Thus, is it easy to find the number system, whereas the information brought to us is not very reliable, and the correspondence with reality low.

Another example of lot distribution in "hides" is provided in Bede's *History*; again, it shows division mode of the "long hundred" (Bede, tr. Sherley-Price 6a, 114, 146, 184, 185, 187, 208, 227, 229, 233, 308):

Hides	Lot
5	Iona
10	Acquired later at Stamford
30	Monastery at Ripon
40	at Ripon
50	given at Lindsey
87	Grant
120	12 grants of land, 6 in Deira, 6 in Bernicia, each of 10 hides in extent.
300	Man
600	Thanet
960	Anglesey
1,200	to Wright. The bishop was given 1/4 of the land = 300 hides.
7,000	to the South Saxons.

Finally, there was an allotment of 12,000 "hides" to Mercia: 5,000 to the South Mercians and 7,000 to the North Mercians.

The low number of hides tells us that the unit was 5 or 10 hides. The grant of 120 hides reveals that a hundred is constructed of 12 groups of 10 hides. The 1,200 hides of Wright shows the next unit, of which a quarter is 3 short hundreds. The next unit is 12-thousand to Mercia, being 5,000 plus 7,000: in this context, the numbers are short thousands. The 960 hides of Anglesey is constructed of 8 long hundreds.

At present, these examples will have to suffice for England. The long hundred can also be observed in the *Anglo-Saxon Chronicle*; as an example, the Andredes forest is 30 miles broad and 120 miles long: 120 men are killed in a battle at Wright: 120 ships perished, and 840 men died—or 7 long hundreds (Whitelock, cf. years c. 850-900).

This study will not consider continental material: it seems that the short hundred is mainly used, but long hundreds do occur, especially in metrological calculations. In the "Lex Salica" (A.D. 507-11), in which fines are stipulated according to the duodecimal system, the long hundred is mentioned as *unum tualepti sund denarii CXX* (Drew 52).

When moving back through time to antiquity, an absolute ten-count was predominating all over the Mediterranean world, which is well known from the Roman numerals, as well as from Greek and Egyptian counting systems, which all have a ten/hundred base.

But despite the predominance of the decimal count, duodecimal calculation can be found in texts and in metrology. For example, the Roman army was grouped in 1,200 "hastates," 1,200 "principes," and 600 "triarias." A "maniple" consisted of 12 lines, 10 men deep (=120). Duodecimal systems can be found in temple construction, water pipes, and the computing diagrams for the use of ballistic missiles.

The long hundred principle is also found in the Roman system of field measurement. An ideal field "actus" was 120 square feet, and they were combined into a "jugerum," being a square of 120 x 240 feet. When found necessary, the "jugerum" was subsequently subdivided into duodecimal fractions. This was probably to retain the length (240 feet), while dividing the breadth into 12 units, each of 10 feet. Again, we witness the combination of a 10-base and a 12-base system for practical purposes.

Further elucidation of the usage of the "long hundred" is beyond the scope of this brief study, but further research in hand will consider the Mycenian (ca. 1500 B.C.) duodecimal system in weights and measures (Chadwick) and the proto-Elamite (4th millennium B.C.)

numerical sign-system, which combined a decimal system with a sexagesimal system (Damerow, Friberg and Green).

In conclusion, it may be said that in Scandinavian and Germanic societies it is possible to find a hybrid number system, in which the word "hundred" may have the double meaning "100" or "120." An arithmetical explanation of the system would be a 12 times 10 construction—not a 10 x 10, nor a 12 x 12 construction.

"Long hundred" constructions can also be found in antique metrology, particularly in the measurement of spaces. The oldest example found to date is a use of the proto-Elamite bisexagesimal counting system.

At the present time, it is impossible to prove if there may be any connection between Mesopotamian calculation and the early Germanic. Should such a proven connection be found, there would still be the question of its being inherent to Indo-European calculation, or of its transmission through the Ancient Greek and Roman civilizations.

Notes

1. Cf *An Anglo-Saxon Dictionary*: Based on the Manuscript Collections of the Late Joseph Bosworth. Ed. and enl. by T. Northcote Toller. Oxford, 1898 (1972), p. 566; Supplement, 1921 (1972), p. 571-2. Also, *The Middle English Dictionary*. Ed. Sherman M. Kuhn. Ann Arbor, 1963, Vol. G-H, p. 1033.
2. Cf *Kulturhistorisk Leksikon for Nordisk Middelalder*. Copenhagen, 1981. Vol. 7, pp. 83-7; and Reuter, Otto Sigfrid, "Zur Bedeutungsgeschichte des hundred im Altwestnordischen," *Arkiv for nordisk filologi*, Vol. 49. Lund, 1933, pp. 36-67.
3. Cf *Dictionary of Medieval Latin from British Sources*. Ed. R. E. Latham. 1975 and 1981. Vide "Anglice," and "centum."
4. "Sex classium senos reges, earumque quamlibet quina navium milia complectentem vidiisse se retulit, quarum unamquamque trecentorum remigum capacem esse constaret. Quamlibet vero tocius summe millenarium quaternis alis contineri dicebat, volebat autem millenarium mille ac ducentorum capacem intelligi, cum ala omnis recentorum numero compleatur." *Saxo. Gesta Danorum*, Bk 5. Ed. Alfred Holder, 1886, p. 155.