

A Linear Programming Methodology for the Optimization of Electric Power–Generation Schemes

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Abstract—A mathematical model, based upon the application of a linear-integer programming algorithm, is presented for the optimum selection of independent electric power–generation schemes in industrial power systems, taking reliability considerations into account. The problem is formulated as a mathematical programming problem—considering investment costs, fuel costs, operation and maintenance costs, power balance, maximum and minimum limits on the generated power of the units, along with reliability considerations, such as the unavailability of the generation scheme. These considerations include assumptions made and simplifications performed to obtain an accurate enough linear model. The problem is solved using a conventional branch and bound algorithm for linear-integer programming, yielding to the optimum number of units, as well as the corresponding size and type. Results are presented for the application of the proposed methodology to a real case of an industrial power system. The technique has proven to be a valuable tool for the planning engineer.

Index Terms—Optimization methods, power generation, power–generation economics.

NOMENCLATURE

Cap_i	Available capacity of the generators.
Cc_i	Fuel cost coefficient (dollars per kilowatt per year).
Cf_i	Operation and maintenance fixed cost coefficient (dollars per year).
C_{ij}	Investment cost coefficient (dollars per year).
Cun_i	Unavailability cost coefficient.
Cv_i	Operation and maintenance variable cost coefficient (dollars per kilowatt per year).
$C(n, i)$	Number of possible combinations of n in i , $0 < i < n$.
D	Total number of hours of each operation cycle.
Dav	Average system load.
D_{\max}	Maximum system load.
FOH	Total number of hours in forced outage, for the total time period in evaluation.
MMTS	Total number of hours (arithmetic mean) elapsed between two successive starts of the generating unit.
n	Maximum number of units that can be selected.
Nc	Number of available capacities.

Nu	Number of units.
P_{ij}	Real power generated by each unit.
SH	Total number of hours that the unit is in service, for the total time period in evaluation.
SOR	Scheduled outage rate.
USP	Average unserved power.
X_{ij}	Decision variable associated with the installation of unit number (j) with capacity index (i).
X_i	Decision variable associated with the installation of the generator array with capacity index (i).

I. INTRODUCTION

A GROWING demand for electric energy due to increasing production levels, and the need for higher reliability levels in the electric power supply, among other factors, have kept the independent power production as a valid choice for those industrial systems with processes that require an uninterrupted and reliable electric energy supply.

The selection of the optimal independent generation scheme must be based upon the following:

- the characteristics, types, and models of the generation units available in the market: capacity, investment costs, operation, and maintenance costs;
- the cost of the unserved energy, which is to be calculated taking into account the characteristics of the production process;
- the efficiency curve of the turbine-generator group (the article is centered in the case of a gas turbine, without loss of generality);
- the reliability of the units and an exhaustive enumeration of the possible operation states [1], [2];
- the operation costs of the equipment, according to international standards;
- the technical and operational constraints imposed by equipment capabilities.

In this paper, a new methodology that is based upon mathematical optimization is presented for the selection of independent electric power–generation schemes for industrial power systems. Particularly, it is assumed that loads and generators are connected to the same bus.

II. STATEMENT OF THE PROBLEM

The mathematical optimization model, described as follows, consists of an objective function to be minimized, subject to constraints.

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A. Objective Function

The objective is to minimize the sum of investment costs, fuel costs, operation and maintenance costs, and unavailability costs:

$$\begin{aligned} \text{Minimize} & [(\text{investment costs}) + (\text{fuel costs}) \\ & + (\text{operation and maintenance costs}) \\ & + (\text{unavailability costs})]. \end{aligned} \quad (1)$$

1) *Investment Costs*: The investment costs can be expressed as

$$\text{Reserve}_i = \sum_{j=1}^{Nu} \text{Cap}_i \cdot X_{ij} - D_{\max_i} \cdot X_i \quad (2)$$

where

Cap_i available capacity of the generators;

D_{av} average system load;

Nu number of units;

X_{ij} decision variable associated with the installation of unit number (j) with capacity index (i);

X_i decision variable associated with the installation of the generator array with capacity index (i).

2) *Fuel Costs*: The fuel costs are proportional to the average generated power and are separated from the operation and maintenance costs in the problem statement

$$\sum_{i=1}^{Nc} \sum_{j=1}^{Nu} Cc_i \cdot P_{ij} \quad (3)$$

where

Cc_i fuel cost coefficient (dollars per kilowatt per year);

Nc number of available capacities;

P_{ij} real power generated by each unit.

3) *Operation and Maintenance Costs*: The operation and maintenance costs include two terms—stating the fixed costs and variable costs:

$$\sum_{i=1}^{Nc} \sum_{j=1}^{Nu} (Cf_i X_{ij} + Cv_i \cdot P_{ij}) \quad (4)$$

where

Cf_i operation and maintenance fixed cost coefficient (dollars per year).

Cv_i operation and maintenance variable cost coefficient (dollars per kilowatt per year).

X_{ij} decision variable associated to the installation of unit number (j) with capacity index (i).

4) *Unavailability Costs*: The costs of unavailability of the units are assumed to be proportional to the average unserved power:

$$\sum_{i=1}^{Nc} \text{Cun}_i \cdot \text{USP}_i \quad (5)$$

where:

Cun_i : is the unavailability cost coefficient, and the unserved power (USP_i) is equal to

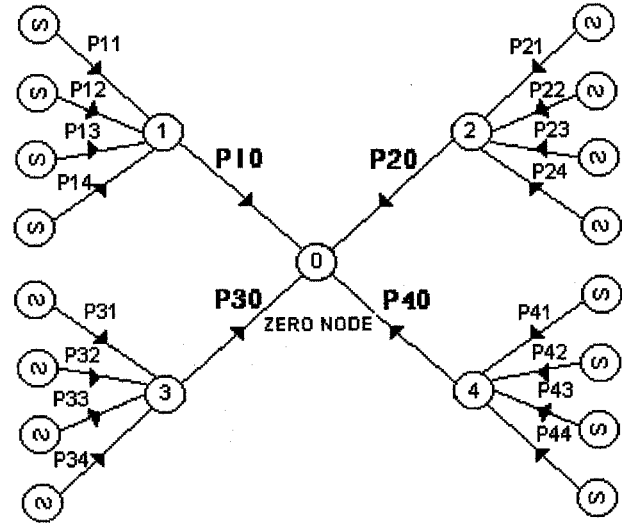


Fig. 1. Power balance in the load node.

$$\text{USP}_i = \text{Cap}_i \cdot X_i - \text{Reserve}_i. \quad (6)$$

The power reserve is defined as follows:

$$\text{Reserve}_i = \sum_{j=1}^{Nu} \text{Cap}_i \cdot X_{ij} - D_{\max_i} \cdot X_i. \quad (7)$$

The final expression for the average unserved power is

$$\begin{aligned} \text{USP}_i &= (\text{Cap}_i + D_{\max_i}) \cdot X_i - \sum_{j=1}^{Nu} \text{Cap}_i \cdot X_{ij} \\ i &= 1, 2, 3, \dots, Nc \end{aligned} \quad (8)$$

B. Constraints

The optimization is to be performed, enforcing a set of restrictions, which can be classified in the following groups:

1) *Power Balance at Node Zero*:

$$\sum_{j=1}^{Nc} P_{i0} = D_{av}. \quad (9)$$

2) *Power Balance in Fictitious Nodes*: Fictitious nodes are created to relate the capacity of the chosen array with the capacity its units. There are as many fictitious nodes as available capacities plus one more node, named the zero node or load node, where all of the arrows converge, as shown in Fig. 1.

The sum of the powers generated by each group of generation units must be equal to the power that flows from the correspondent fictitious node to the load node or zero node

$$\sum_{j=1}^{Nu} P_{ij} = P_{i0}; \quad i = 1, \dots, Nc. \quad (10)$$

3) *Maximum Power for Each Group of Generators*: The power supplied by one group of generators must be less than

the capacity of each generator times the maximum number of units in the array

$$P_{i0} \leq n \cdot \text{Cap}_i \cdot X_i; \quad i = 1, \dots, Nc. \quad (11)$$

4) *Minimum Capacity of the Generator Array*: The total generation capacity of the generation array must be greater than the maximum system load:

$$\sum_{i=1}^{Nc} \sum_{j=1}^{Nu} \text{Cap}_i \cdot X_{ij} \geq D_{\max_i}. \quad (12)$$

5) *Limits on the Power Generated by Each Machine*: The power supplied by each generator must be less than or equal to the capacity of the unit

$$P_{ij} \leq \text{Cap}_i \cdot X_{ij}; \quad i = 1, \dots, Nc; \quad j = 1, \dots, Nu. \quad (13)$$

6) *“Radial” Constraint at Node Zero (0)*: This artificial constraint allows the model to select only one group of machines of the correspondent capacity. The sum of the decisions variables associated to the installation of the generator arrays that reach the zero node is equal to 1, that is

$$\sum_{i=1}^{Nc} X_i = 1. \quad (14)$$

III. PROPOSED METHODOLOGY

To solve the mathematical model, the well-known “branch-and-bound” mixed-integer linear programming algorithm is proposed [8]. However, in order to formulate the problem as a linear program, the unavailability cost coefficients must be calculated as follows.

1) *Unavailability Cost Coefficients*: The well-known forced outage rate (FOR) is the fraction or portion of the time in which the unit is out of service due to a failure in any of its components, in relation to the total amount of time for which the same unit is available. Thus, assuming that the maintenance time is not considered.

$$\text{FOR} = (\text{FOH} \cdot 100\%) / (\text{FOH} + \text{SH}). \quad (15)$$

Some adjustments must be performed before using FOR values, to take into account that gas generating units are mainly used to supply demand peaks instead of base load in continuous operation mode.

To calculate the appropriate value of the FOR for a gas turbine under continuous operation, the following expressions are valid [5]:

$$\text{FOR} = \text{FOH}(D/\text{MTTS}) / (\text{FOH}(D/\text{MTTS}) + \text{SH}). \quad (16)$$

The probabilistic distribution of the generation availability for each array can be determined by considering the FOR, after the necessary adjustments are pointed out above, and the scheduled outage rate (SOR). The procedure to calculate the probabilistic distribution is presented in [4].

For an array of n equal units of m (in megawatts) generation capacity, each one with a FOR, assuming that there is no unit under maintenance (scheduled outage), there are $n + 1$ possible states.

In general, the probability associated to a generation availability of $(i \cdot m)$ (MW), $n > i > 0$ is given by the following expression [4]:

$$C(n, i) \cdot (1 - \text{FOR})^i \cdot (\text{FOR})^{n-i}. \quad (17)$$

If there is one unit out of service, then there are n possible states, and the probabilities change. The probability term associated to a generation of $i \cdot m$ (MW), $n > i > 0$, is given by the following expression [4]:

$$C(n-1, i) \cdot (1 - \text{FOR})^i \cdot (\text{FOR})^{(n-1)-i}. \quad (18)$$

If it is assumed that it is not possible to have more than one unit out of service at the same time, considering only those cases where only one generation unit is out of service, then the cases with $i = n-1, i = n-2$ cover all of the possibilities. It is clear that this approximation is only valid for generation arrays with a relatively small number of units, where the probability of having more than one unit out of service is neglectable, which is frequently the case for small-to-medium-size industrial systems.

The scheduled outage factor (SOF) is defined as the fraction or portion of time, relative to the total lapse of operation of the system (including the time in which the unit is in forced outage), where the unit is out of service because of scheduled inspection and/or maintenance.

If the maintenance program is considered, then the probability associated to a generation of $i \cdot m$ (in megawatts), $n > i > 0$ is given by [4]

$$C(n-1, i) \cdot (1 - \text{FOR})^i \cdot (\text{FOR})^{(n-1)-i} (1 - n \cdot \text{SOF}). \quad (19)$$

Note that it is assumed in the methodology that each unit is considered independently.

2) *Unavailable Time*: Unavailability calculations yield the total number of hours in a year where it is not possible to satisfy the demand, given the total possible amount of generation.

Nevertheless, in order to consider unavailability costs, a measurement for the interruption duration is required, so that in general, a mathematical model for the probability distribution of the service unavailability is needed.

It is possible to estimate the mean duration of the interruptions or average repair time, based on the historical data available about equipment forced and scheduled outages. It can be approximated from statistical data on forced outages for different types of turbogeneration [5].

In this paper, the following procedure was followed to estimate the average repair time for the calculation of unavailability.

- An array with only one unit is assumed, and its correspondent series block diagram is built. Then, the mean duration of interruptions is calculated.
- Once the mean duration of interruptions is obtained, the total number of interruptions is calculated.
- The total unavailability time is calculated as the product of the number of interruptions times the difference between

TABLE I
FREQUENCY DISTRIBUTION OF THE ELECTRIC POWER LOAD

Demand (MW)	Frequency	
	Absolute (hrs)	Relative (%)
7.2	51	14.0
11.9	4	1.1
14.3	250	68.5
15.4	6	1.6
19.0	42	11.5
20.2	2	0.5
22.5	10	26.4

TABLE II
ELECTRIC POWER LOAD

	Load (MW)
Minimum	7.2
Average	14.1
Maximum	22.5

the mean duration of the interruption (or average repair time) and the storage slackness for the specific case:

$$\text{Unavailability time} = (\# \text{interruptions}) \cdot (\text{average repair time} - \text{slackness}). \quad (20)$$

Unavailability cost coefficients are calculated as the product of the number of hours of unavailability times the cost of unserved energy, which is a process-dependent parameter. Finally, as expressed in (5), the cost of unavailability for each array is the result of the product of the unavailability cost coefficient for the type of unit considered times the unserved power for the array considering only single contingency—that is, the failure of only one unit.

IV. RESULTS

The methodology was applied in two real systems of the oil industry.

A. Test Case 1

The first test case consists of an oil (petroleum) extraction, transportation, and storage industrial system, comprising a network of oil wells, pipes, tanks, and electric pumps of several types, sizes, and operating cycles. Table I presents the frequency distribution of the demand, as calculated from the load curve of the system.

In Table II, the minimum, maximum, and mean values of the system load are presented.

1) *Cost Coefficients:* Estimated costs for each type of generating unit under consideration are shown in Table III.

The fuel cost coefficients depends are shown in Table IV. They depend upon the efficiency of the unit, and are given in dollars per kilowatt per year. The cost of the gas fuel was assumed to be that of the international market: 1.90 \$/MMBtu.

The average costs used in the test case for the operation and maintenance of the generating units are those given in [7]. The fixed operation and maintenance cost used was 1 \$/kW/year, and

TABLE III
INVESTMENT COST COEFFICIENTS

Generation Unit	Capacity (MW)	Cost (M\$)		Cost Coefficient (\$/year)
		FOB [\$]	FIV	
GT35	13.2	7.43	14.86	2,600,500
GT10	19.2	10.23	20.46	3,580,500
PG5371	20.5	7.98	15.96	2,793,000
PG6551	30.5	11.55	23.1	4,042,500

TABLE IV
FUEL COST COEFFICIENTS

Units	Heat Rate (Btu/kWh)	Coefficient (\$/kW/year)
GT35	10,670	177.6
GT10	9,970	165.9
PG5371	11,990	199.6
PG6551	10,730	178.6

TABLE V
OPERATION AND MAINTENANCE COST COEFFICIENTS

Unit	Fixed Cost Coefficient (\$/year)	Variable Cost Coefficient (\$/MW/year)
GT35	13,200	26.3
GT10	19,200	26.3
PG5371	20,500	26.3
PG6551	30,500	26.3

TABLE VI
UNAVAILABILITY COST COEFFICIENTS

Units	Hours of Unavailability	Coefficient (\$/MW/year)
GT35	494	231,098
GT10	435	203,310
PG5371	575	287,405
PG6551	575	270,408

the variable cost was assumed to be 3 \$/kW/year. Table V shows the operation and maintenance cost coefficients calculated using these factors, for a one-year time period (8760 h).

The slackness and the unserved energy (USE) are parameters given for the analysis. The application of the procedure results in the unavailability cost coefficients shown in Table VI.

The unavailability cost of a unit is equal to the correspondent cost coefficient (given in Table VIII in dollars per megawatt per year times the unserved power.

2) *Optimization Results for Test Case 1:* The methodology resulted in the selection of two units of 20.5 MW each, with a total yearly cost of \$5 630 186.00. The branch-and-bound algorithm converged after 27 iterations of linear programming problem solutions.

TABLE VII
CASE 2. LOAD CHARACTERISTICS

Demand (MW)	Frequency	
	Absolute (hrs)	Relative (%)
7.2	53	73.6
11.9	19	26.4

TABLE VIII
CASE 2. INVESTMENT COST COEFFICIENTS

	Load (MW)
Minimum	8.6
Average	9.3
Maximum	11.2

TABLE IX
CASE 2. FUEL COST COEFFICIENTS

Generation Unit	Capacity (MW)	Cost (M\$) Turbo-generator		Cost Coefficient (\$/year)
		FOB [5]	FIV	
PGT10	8.1	4.85	9.70	1,697,500
GT35	13.4	7.43	14.86	2,600,500
GT10	19.6	10.23	20.46	3,580,000
PG5371	20.9	7.98	15.95	2,793,000

These results match with those presented in [6], where the same case was analyzed using extensive evaluation of the alternatives. The resultant cost components are the following:

Investment costs	5586.000	(\$/year)
Fuel costs	2814	(\$/year)
Operation and maintenance costs	41 371	(\$/year)
Unavailability costs	0	(\$/year).

Note that the unavailability costs are equal to zero. This is because the correspondent cost coefficients have a relatively high value for the oil industry.

B. Test Case 2

Test Case 2 consists of the selection of an electric generation plant for an oil-storage and pumping station. The frequency distribution of the demand obtained from the load curve of the system, as well as the values of the mean and maximum load, are shown in Table VII.

1) *Cost Coefficients:* The investment cost coefficients are presented in Table VIII. The fuel cost coefficients, which are shown in Table IX, were those associated with the cost of the gas, which is assumed as that of the international market: 1.90 \$/MMBtu.

The operation and maintenance costs are those provided in [7]. Fixed and variable costs were assumed to be 1 dollar per kilowatt per year and 3 dollars per kilowatt per year, respectively. The results obtained using these factors are shown in Table X, for an operation time period of one year (8760 h).

For the calculation of the unavailability cost coefficients, the slackness as well as the unserved energy are given data. The resultant coefficients are shown in Table XI.

TABLE X
CASE 2. OPERATION AND MAINTENANCE COST COEFFICIENTS

Units	Heat Rate (BTU/kWh)	Fuel Cost Coefficient (\$/MW/year)
PGT10	11050	183.9
GT35	10670	177.6
GT10	9970	165.9
PG5371	11990	199.6

TABLE XI
CASE 2. UNAVAILABILITY COST COEFFICIENTS

Generation Units	Fixed Cost Coefficient (\$/year)	Variable Cost Coefficient (\$/MW/year)
PGT10	8,100	26.3
GT35	13,400	26.3
GT10	19,600	26.3
PG5371	20,900	26.3

2) *Optimization Results for Test Case 2:* The application of the methodology resulted in the selection of an array of three units of 8.1 MW, with a total yearly cost of \$5 118 756.00. The branch-and-bound algorithm converged after 69 iterations of linear programming problem solutions. The resultant cost components are as follows:

Investment costs	5586.000	(\$/year)
Fuel costs	2814	(\$/year)
Operation and maintenance costs	41 371	(\$/year)
Unavailability costs	0	(\$/year).

As in the previous case, the unavailability costs are set equal to zero by the optimization algorithm, due to the nature of the process and the characteristics of the oil industry.

V. CONCLUSIONS

The selection of electric power-generation schemes in industrial power systems can be stated as an optimization problem and successfully performed using conventional mixed-integer linear programming techniques, yielding to the optimum selection of the independent generation scheme, associated to minimum costs.

The proposed methodology was tested with two real cases, proving to be a fast and reliable tool to assist the engineer in the solution of the problem of optimum selection of independent generation and/or co-generation schemes.

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