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A new formulation for power system reliability assessment with AC constraints

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ABSTRACT

Recent investigations have revealed the significant role of reactive power in blackout events. The associated disturbances frequently emerge in the form of voltage instability and collapse. Due to the computational complexity of modeling and analysis of enormous contingencies, DC approximation of the power flow, without accounting the role of reactive power, is usually used to evaluate the contingencies and to mitigate the associated probable violations. This paper, at first, presents a linear power flow model based on an approximated version of AC power flow formulation. The proposed model is then used to develop an efficient reliability assessment approach which is capable of taking both active and reactive powers into account. The analysis technique is based on the linear programming format which leads to an optimal solution within a short computation time. Voltage and reactive power violations as well as transmission system overloads are alleviated by generation rescheduling or load shedding as the last resort. Numerical tests on the IEEE-RTS and the Iranian power grid show the acceptable accuracy of the results along with a significant reduction in the computational effort. Various sensitivity analyses are also investigated to reveal the robustness and performance of the proposed model.

1. Introduction

Reliability evaluation of large power systems, specifically when generation and transmission facilities are involved together, is a complicated and computationally expensive problem [1]. Reliability assessment at this level, which is generally known as composite system reliability evaluation, consists of three major steps including state selection, state evaluation, and reliability indices calculation. Large number of system states, which exponentially increases with respect to the system dimension, along with the complex and time consuming solving process of each state are the two major reasons for excessive complexity of the composite system reliability studies [2]. This complexity is high enough to force power system engineers to apply several simplifying assumptions to perform the analysis.

Remarkable efforts have been devoted in the literatures to the acceleration of composite system reliability evaluation [3]. Some techniques attempted to reduce the number of states; while, others sought efficient ways to analyze system states more quickly. A brief review is provided in the following to clarify the contribution of the work presented in this paper.

Artificial intelligence techniques have attracted much attention in dealing with the computational complexity of composite system reliability evaluation [4–6]. These algorithms are used as a search tool to truncate the huge state space by tracing failure states, i.e., states resulting in load curtailments. Referring to the conducted simulations, their application leads to satisfactorily outcomes; however, the performance of these approaches strongly depends on the parameter tuning and operator experiences. The efficiency of the self organizing map in the state selection of the Monte Carlo simulation (MCS) technique was investigated in [7]. The role of intelligent methods consisting of ant colony system, artificial immune system, binary particle swarm optimization, and genetic algorithm in the composite system reliability evaluation has been comprehensively explored and compared in [8].

In [9], the variance reduction approach was applied to the MCS method in order to speed up the study. In [10–13], at first, the set of probable system states was decomposed into three categories: acceptable states, unclassified states, and loss of load states. Then, by eliminating the first two categories, reliability indices were calculated by analyzing the states located in the third category. Although these approaches are attractive from the computational perspective, accuracy of the obtained results is not guaranteed or even measurable.

While the aforementioned researches focused on the acceleration of the state sampling procedure, distribution factors

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Nomenclature

Indices and sets

B	set of buses
G_i	set of units connected to bus i
i, j	indices of buses
k	index of generating units
l	index of branches
L	set of transmission lines
s	index of scenarios
S_c	set of scenarios with load curtailment

Parameters and constants

A_{il}	element of bus-line incidence matrix, 1 when bus i is the sending bus of line l, -1 when bus i is the receiving bus of line l, and 0 otherwise
A'_{il}	element of modified A_{il} , 1 when bus i is the sending bus of line l, and 0 otherwise
g_l, b_l	real and imaginary parts of admittance of line l
p_i^d, q_i^d	active and reactive power demands at bus i
$\overline{p_L}$	capacity of line l
$\overline{p_k^g}, \underline{p_k^g}$	maximum and minimum active power generations of unit k
pr_s	occurrence probability of scenario s
$\overline{q_k^g}, \underline{q_k^g}$	maximum and minimum reactive power generations of unit k
$\overline{q_i^{sh}}, \underline{q_i^{sh}}$	maximum and minimum reactive power generated by shunt compensators and line charging susceptances connected to bus i
r_l, x_l	real and imaginary parts of impedance of line l
$\overline{V_i}, \underline{V_i}$	maximum and minimum acceptable voltage levels at bus i
$voll_i$	value of lost load at bus i

Functions and variables

BPEC	bulk power energy curtailment
EDNS	expected demand not served
EENS	expected energy not supplied
LOLP	loss of load probability
p_k^g, q_k^g	active and reactive power generations of unit k
p_i^c, q_i^c	active and reactive load curtailments at bus i
p_l^s, q_l^s	active and reactive power flows at sending end of line l
p_l^r, q_l^r	active and reactive power flows at receiving end of line l
p_l^{loss}, q_l^{loss}	active and reactive power losses of line l
q_i^{sh}	reactive power generated by shunt compensators and line charging susceptances connected to bus i
V_i, δ_i	voltage magnitude and phase angle of bus i

were employed in [14] to detect transmission line overloads with a low computational effort. In [15], linear sensitivity coefficients along with pre-contingency transmission line flows were used to obtain post-contingency flows. A direct method has been devised in [16] to release line overloads by rescheduling and curtailing candidate generating units and load points, respectively. These units and loads are the most influencing ones and are identified based on the sensitivity between line flows and power injections. Although in this method reliability indices are calculated with a lower computational burden, they are not sufficiently accurate. Local load shedding optimization method in order to lessen the optimization problem dimension has been discussed in [17,18]. The key drawback associated with such local optimizations is that the obtained solution might be sub-optimal and the reliability indices calculated by these methods are usually greater than the actual values. Artificial neural network, expert system based methods, and radial basis function networks are other techniques utilized to enhance the system state analysis [19–21]. The post optimal analysis, as a well recognized technique to attack a set of similar optimization problems, has been successfully used to assess the reliability of composite systems in [22]. This method exploits the similarity of the system states to speed up the state evaluation procedure without sacrificing the accuracy of the results. In [23], the performance and practical feasibility of the post optimal analysis based technique for power system reliability evaluation are successfully tested using a real-world power grid. However, due to restrictions of the post optimal analysis technique, this method is not applicable in composite system reliability evaluation with non-linear AC constraints. Parallel processing approaches to balance the computation among high performance computers were proposed in [24–26] to analyze system states. Necessity of advanced technology and communication protocols is the main obstacle against general application of these approaches.

The above techniques have particular pros and cons. Some has trivial impacts on the reduction of computation burden and some may lead to approximations or insufficient accuracies, which in turn, may result in inappropriate decision based on the reliability outcomes. DC power flow model is a frequently used technique to overcome excessive computational burden of system analysis [6,27,28]. Non-linearity and its associated solving complexities in the AC model are the main driving forces for application of DC model. In the DC model, shunt elements, branch resistances, and reactive powers are all overlooked and bus voltage magnitudes are assumed to be 1.0 per unit. However, voltage profile of buses and reactive power reserves play a significant role in wide-area blackouts and disregarding their effects results on optimistic outcomes and could threaten the operation security. Beside the papers reviewed above, there are a few other works related to composite system reliability assessment. In [29], the impacts of failure of reactive power sources such as synchronous condensers and compensators were accommodated in the reliability evaluation. Two new reliability indices focusing on reactive power shortage were also proposed in [29]. Reactive power aspects in reliability evaluation of power systems were addressed in [30]. The paper has introduced a set of new reliability indices to measure the network deficiencies and operational constraint violations caused by reactive power shortages.

Based on the above discussions, developing a fast/accurate straightforward approach in composite system reliability assessment is still of interest. This could be much more crucial when it comes to large scale power systems. This paper, as the main contribution, extracts a new formulation which is fast enough as it is linear such as DC model and is sufficiently accurate as it incorporates both active and reactive quantities similar to the conventional AC model. The model state variables consist of bus voltage magnitudes and phase angles. The developed model is then used in composite system reliability evaluation procedure with the aim of acceleration of state analysis stage. The problem formulation for state analysis is based on the computationally efficient format of linear programming (LP). The performance of the proposed model is examined using the IEEE reliability test system (IEEE-RTS) and the Iranian power grid. The results are discussed thoroughly and compared with those of the existing approaches. A set of sensitivity analyses are fulfilled to validate the effectiveness of the new model in a variety of situations covering: diverse loading conditions, different acceptable voltage levels, and various orders of the enumerated simultaneous outages.

The rest of the paper is organized as follows. The new power flow model is extracted in Section 2. Section 3 presents the developed reliability evaluation framework. Numerical results are demonstrated in Section 4. Concluding remarks are discussed in Section 5.

2. The developed power flow model

The new power flow model adopts bus voltage magnitudes and phase angles as unknown state variables. This model is formulated in three sets of equations including power balance equations, branch equations, and loss equations.

2.1. Power balance equations

The active and reactive power balance equations should be satisfied at all voltage controlled and load buses [31]. Eqs. (1) and (2) express power balance equations at bus i.

$$\sum_{k\in G_i} p_k^g - \sum_{l\in L} A_{il}\cdot p_l^s - \sum_{l\in L} A'_{il}\cdot p_l^{loss} = p_i^d \tag{1}$$

$$\sum_{k\in G_i} q_k^g - \sum_{l\in L} A_{il}\cdot q_l^s - \sum_{l\in L} A'_{il}\cdot q_l^{loss} + q_i^{sh} = q_i^d \tag{2}$$

In the above equations, reactive power generation at voltage controlled buses, active and reactive power flows at all branches, and active and reactive power losses at all lines are unknown. It should be pointed out that the last term in the left hand side of (2) is considered to model shunt compensators and line charging susceptances.

2.2. Branch equations

In most power system studies, transmission lines are represented by a π-equivalent circuit, shown in Fig. 1. Using this representation, the equations of active and reactive flows at sending and receiving ends of line l, connecting bus i to bus j, are written as:

$$p_l^s = g_l V_i^2 - g_l V_i V_j \cos(\delta_i - \delta_j) - b_l V_i V_j \sin(\delta_i - \delta_j) \tag{3}$$

$$p_l^r = -g_l V_j^2 + g_l V_j V_i \cos(\delta_j - \delta_i) + b_l V_j V_i \sin(\delta_j - \delta_i) \tag{4}$$

$$q_l^s = -b_l V_i^2 - g_l V_i V_j \sin(\delta_i - \delta_j) + b_l V_i V_j \cos(\delta_i - \delta_j) \tag{5}$$

$$q_l^r = b_l V_j^2 + g_l V_j V_i \sin(\delta_j - \delta_i) - b_l V_j V_i \cos(\delta_j - \delta_i) \tag{6}$$

In the first step, let:

$$V_i V_j \sin(\delta_i - \delta_j) \cong \delta_i - \delta_j \tag{7}$$

which is a good approximation since $(\delta_i - \delta_j)$ is usually small and voltage magnitudes are near one per unit [32]. Note that the above approximation is used in the DC power flow model too. Substituting (7) into (3)–(6) yields

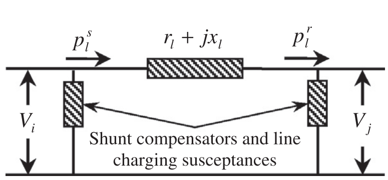

Fig. 1. π-Equivalent circuit of a transmission line.

$$p_l^s = g_l V_i^2 - g_l V_i V_j \cos(\delta_i - \delta_j) - b_l\delta_i + b_l\delta_j \tag{8}$$

$$p_l^r = -g_l V_j^2 + g_l V_j V_i \cos(\delta_j - \delta_i) + b_l\delta_j - b_l\delta_i \tag{9}$$

$$q_l^s = -b_l V_i^2 - g_l\delta_i + g_l\delta_j + b_l V_i V_j \cos(\delta_i - \delta_j) \tag{10}$$

$$q_l^r = b_l V_j^2 + g_l\delta_j - g_l\delta_i - b_l V_j V_i \cos(\delta_j - \delta_i) \tag{11}$$

According to the power exchange in a transmission line, one can derive active and reactive power losses in a transmission line using the following equations:

$$p_l^{loss} = p_l^s - p_l^r \tag{12}$$

$$q_l^{loss} = q_l^s - q_l^r \tag{13}$$

By replacing power flows from (8)–(11) into (12) and (13), active and reactive losses could be written as:

$$p_l^{loss} = g_l V_i^2 + g_l V_j^2 - 2g_l V_i V_j \cos(\delta_i - \delta_j) \tag{14}$$

$$q_l^{loss} = -b_l V_i^2 - b_l V_j^2 + 2b_l V_i V_j \cos(\delta_i - \delta_j) \tag{15}$$

The above equations can be rearranged to form the following equivalents for $V_iV_j \cos(\delta_i - \delta_j)$:

$$V_i V_j \cos(\delta_i - \delta_j) = \frac{1}{2}\left(V_i^2 + V_j^2\right) - \frac{1}{2g_l} p_l^{loss} \tag{16}$$

$$V_i V_j \cos(\delta_i - \delta_j) = \frac{1}{2}\left(V_i^2 + V_j^2\right) + \frac{1}{2b_l} q_l^{loss} \tag{17}$$

By substituting (16) into (8), (9), and (17) into (10) and (11), the equations of active and reactive flows are derived as:

$$p_l^s = \frac{1}{2}\left(g_l V_i^2 - g_l V_j^2\right) + \frac{1}{2} p_l^{loss} - b_l\delta_i + b_l\delta_j \tag{18}$$

$$p_l^r = -\frac{1}{2}\left(g_l V_j^2 - g_l V_i^2\right) - \frac{1}{2} p_l^{loss} + b_l\delta_j - b_l\delta_i \tag{19}$$

$$q_l^s = -\frac{1}{2}\left(b_l V_i^2 - b_l V_j^2\right) - g_l\delta_i + g_l\delta_j + \frac{1}{2} q_l^{loss} \tag{20}$$

$$q_l^r = \frac{1}{2}\left(b_l V_j^2 - b_l V_i^2\right) + g_l\delta_j - g_l\delta_i - \frac{1}{2} q_l^{loss} \tag{21}$$

The derived formulation is in terms of square bus voltage magnitudes. This parameter does not reflect the ultimately required practical knowledge of the network condition. Therefore, since voltage magnitude at all buses are always around the unity, any function in terms of V_i, e.g., $f(V_i) = V_i^2$, could be approximated by Taylor series:

$$f_{(V_i)} \cong f_{(1)} + \left.\frac{df}{dV_i}\right|_{(1)} \cdot (V_i - 1) \tag{22}$$

Accordingly,

$$V_i^2 \cong 2V_i - 1 \tag{23}$$

Then, Eqs. (18)–(21) can be expressed as follows:

$$p_l^s = g_l V_i - g_l V_j + \frac{1}{2} p_l^{loss} - b_l\delta_i + b_l\delta_j \tag{24}$$

$$p_l^r = -g_l V_j + g_l V_i - \frac{1}{2} p_l^{loss} + b_l\delta_j - b_l\delta_i \tag{25}$$

$$q_l^s = -b_l V_i + b_l V_j - g_l\delta_i + g_l\delta_j + \frac{1}{2} q_l^{loss} \tag{26}$$

$$q_l^r = b_l V_j - b_l V_i + g_l \delta_j - g_l \delta_i - \frac{1}{2} q_l^{loss} \tag{27}$$

In the above equations, voltage magnitudes and phases at all buses, active and reactive power flows at all branches, and active and reactive power losses at all lines are unknown.

2.3. Loss equations

Active and reactive power losses corresponding to line l, from bus i to bus j, are determined as:

$$p_l^{loss} = \frac{p_l^{s^2} + q_l^{s^2}}{V_i^2} r_l \tag{28}$$

$$q_l^{loss} = \frac{p_l^{s^2} + q_l^{s^2}}{V_i^2} x_l \tag{29}$$

Since active and reactive power losses, expressed in the above equations, are non-linear, a technique is required to linearize them. In this way, equations of active and reactive power losses are linearized around the system operating point. Using Taylor series, the linearized form of (28) and (29) are as:

$$p_l^{loss} = p_l^{loss}\Big|_{\substack{Operating\\point}} + r_l \times \left[\frac{2 \cdot p_l^s}{V_i^2} \quad \frac{2 \cdot q_l^s}{V_i^2} \quad -2\frac{p_l^{s^2} + q_l^{s^2}}{V_i^3}\right]\Bigg|_{\substack{Operating\\point}} \times \begin{bmatrix} p_l^s \\ q_l^s \\ V_i \end{bmatrix} \tag{30}$$

$$q_l^{loss} = q_l^{loss}\Big|_{\substack{Operating\\point}} + x_l \times \left[\frac{2 \cdot p_l^s}{V_i^2} \quad \frac{2 \cdot q_l^s}{V_i^2} \quad -2\frac{p_l^{s^2} + q_l^{s^2}}{V_i^3}\right]\Bigg|_{\substack{Operating\\point}} \times \begin{bmatrix} p_l^s \\ q_l^s \\ V_i \end{bmatrix} \tag{31}$$

Now, a set of equations equal to the number of state variables have been derived.

Consequently, a unique solution can be readily found. Ultimately, it is worthwhile to note that power balance Eqs. (1) and (2) in the developed model and the conventional AC power flow model are exactly the same. However, branch Eqs. (24)–(27) and loss Eqs. (30) and (31) are developed in this paper.

In the next section, the developed power flow model is applied to the composite system reliability evaluation. However, the linearity of the proposed formulation can be effective in a diverse range of power system applications such as state estimation [33] and complex reliability assessment issues [34].

3. The proposed framework for composite system reliability evaluation

This section provides a computationally efficient composite system reliability evaluation approach based on the approximated power flow model developed in the preceding section. Similar to all conventional reliability assessment methods, the proposed technique consists of three steps of state selection, state analysis, and reliability indices calculation. Each step is described in the following sub-sections. Then, the overall evaluation procedure is presented.

3.1. State selection

The first step of the reliability evaluation is the state selection of the system under study. The power system consists of a vast array of components with diverse functionalities. Also, some of the parameters affecting the system performance such as system load level and weather conditions vary with time. The combination of component operating or failure states, values of effective parameters, and system operation policies specifies system states. In this regard, the simplest way is to apply a complete enumeration scheme. However, the number of states exponentially increases with the system dimension and the number of components. The complete enumeration scheme is consequently infeasible approach in most real-world networks with a large number of elements.

To reduce the state space, variety of methods have been so far developed which can be classified into truncated enumeration and MCS categories [35]. The former category considers system states with the associated occurrence probability greater than a pre-determined value or those including outages up to a given order. In MCS techniques, system states are sampled randomly based on their occurrence probability to the point that the termination criteria are satisfied. Usually, either the convergence of reliability indices, or the total number of simulated states, or both is considered as the termination criterion [35].

Since system state selection step in the presented technique can be any of the above reviewed methods, both of the methods are employed in the simulations. The truncated enumeration method up to a specific outage order is used for the first case study that is a relatively small system, and the MCS method is used for the second case study which is a large-scale network.

3.2. State analysis

In the second step, each sampled state has to be judged whether the load can be fully or partially served owing to a set of pre-determined operation constraints. To alleviate possible violations, an optimization problem is usually solved in which the available remedial actions are triggered to lessen the amount of curtailed load. Remedial actions include adjusting network control variables such as generation rescheduling and reactive power support. In order to achieve practical outcomes, taking transmission network limitations into account in the optimization problem is indispensable. To this end, power flow equations are incorporated and the optimization problem is converted to an optimal power flow (OPF) model.

The power flow equations could be either AC or DC in the OPF problem and their associated attributes, advantages, and disadvantages are comprehensively discussed in the Introduction section. In short, AC model is accurate but time expensive; while, DC model is fast but inaccurate. Here, the approximated power flow developed in Section 2 is used. The objective function and constraints of the state evaluation problem are formulated in the following.

3.2.1. Objective function

In the event of constraint violation, load curtailment can be performed via two general philosophies: (1) load curtailment with minimum not-served demand and (2) load curtailment with minimum damage cost. In this paper, the second philosophy of load curtailment is adopted. Therefore, the total interruption cost is minimized as the objective function. The mathematical expression of the objective function is as follows,

$$\min \sum_{i \in B} voll_i \cdot p_i^c \tag{32}$$

In the above equation, $voll_i$ represents the average damage of customers connected to bus i if one megawatt of their demand is curtailed for 1 h. This parameter is considered here to incorporate load point priorities in the load curtailment procedure. The optimization process avoids any load curtailment, if possible; otherwise, alleviates violations via load curtailments with the minimum interruption cost. Evidently, assuming $voll_i$ associated with all load buses to be the same, leads to the minimum load curtailment solution.

3.2.2. Constraints

Bus voltage limits: These limits ensure acceptable voltages at all buses.

$$\underline{V_i} \leqslant V_i \leqslant \overline{V_i} \tag{33}$$

Generating unit limits: These constraints guarantee that the active and reactive dispatch of each unit lies within its maximum and minimum bounds, i.e.,

$$\underline{p_k^g} \leqslant p_k^g \leqslant \overline{p_k^g} \tag{34}$$

$$\underline{q_k^g} \leqslant q_k^g \leqslant \overline{q_k^g} \tag{35}$$

Although the real and reactive powers of a generator have to be within the *P–Q* curve area, it is a very common practice in reliability studies to assume the maximum reactive power of a generator to be constant. This simplifying assumption is reasonable due to the fact that reliability assessment is among long-term studies which are exposed to various sources of uncertainty such as load forecasts.

Shunt compensator capacity limit: This constraint ensures that the reactive power output of each compensator does not violate its maximum and minimum limits.

$$\underline{q_i^{sh}} \leqslant q_i^{sh} \leqslant \overline{q_i^{sh}} \tag{36}$$

Transmission line flow limit: The flow of each transmission line is capped with a capacity as

$$-\overline{p_l} \leqslant p_l^s \leqslant \overline{p_l} \tag{37}$$

Maximum active power curtailment at load points: Logically, the active power curtailment at load buses cannot be negative, nor greater than the actual load at that bus. That is,

$$0 \leqslant p_i^c \leqslant p_i^d \tag{38}$$

Constant power factor load curtailment: Usually disconnecting an electricity consumer lessens both active and reactive powers of a given load point. This situation is referred to as the constant power factor load curtailment and expressed by

$$q_i^d \cdot p_i^c - p_i^d \cdot q_i^c = 0 \tag{39}$$

Network equations: The operating point should satisfy both active and reactive power flow equations. All equations of the developed power flow model except the power balance equations are used here without any changes. The power balance equations are modified as follows to incorporate the load curtailment impact:

$$\sum_{k \in G_i} p_k^g - \sum_{l \in L} A_{il} \cdot p_l^s - \sum_{l \in L} A'_{il} \cdot p_l^{loss} = p_i^d - p_i^c \tag{40}$$

$$\sum_{k \in G_i} q_k^g - \sum_{l \in L} A_{il} \cdot q_l^s - \sum_{l \in L} A'_{il} \cdot q_l^{loss} + q_i^{sh} = q_i^d - q_i^c \tag{41}$$

It has to be noted that in the above formulation, decision variables are active and reactive power outputs of generating units, reactive power output of shunt compensators, and load curtailments at load point buses. Dependent variables are bus voltage magnitudes and phase angles. The above optimization problem is in the LP format which can be easily solved by matured algorithms such as Simplex [36–37] or commercial solvers.

3.3. Index calculation

The last step is to calculate the reliability indices based on the results obtained in the previous step and over the states selected in the first step. In the following, only some of the most popular indices, which are considered in this paper, and relevant formulas are introduced in brief and interested readers are referred to [38] for detailed explanation. These indices can be calculated as follows

$$\mathrm{LOLP} = \sum_{s \in S_c} pr_s \tag{42}$$

$$\mathrm{EDNS} = \sum_{s \in S_c} \sum_{i} pr_s \cdot p_i^c \tag{43}$$

$$\mathrm{EENS} = \mathrm{EDNS} \cdot 8760 \tag{44}$$

$$\mathrm{BPEC} = \frac{\mathrm{EENS}}{\text{system active peak demand}} \cdot 60 \tag{45}$$

The above formulas are known as system indices; while, individual load point indices can be calculated using the same formulas considering load curtailments in a given load point.

3.4. Overall Assessment Procedure

Here, the procedure of reliability assessment and implementation of the three aforementioned steps are described. The corresponding, shown in Fig. 2, is as follows:

- Step (1) At first, the system operating point, including generating unit outputs, bus voltage magnitudes and phase angles, and line flows, is determined. This point can be found based on economical aspects and operational constraints. In this paper, the system operating point is determined using an economical optimal power flow analysis. Clearly, linearizing the model around the operating point may lead to some levels of inaccuracy which would change by altering the system operating point. However, it cannot be a concern since, as it was proven in literature [32], the inaccuracy is within an acceptable range.
- Step (2) Power flow equations are linearized around the initial operating point of the system. The approximate version of power flow model and its linearized form are developed in Section 2.
- Step (3) Adopt a system state for which some of the components are on outage and/or system load level is changed.
- Step (4) Perform power flow calculation to determine if any constraint has been violated. Transmission network overloads and severe voltage drops constitute the best-known violations. This power flow study is a pre-analysis study to increase the speed of the reliability assessment. The power flow study is done to determine whether calculating the amount of load curtailment needs solving an optimization problem or not. Doing so, the computational burden required for calculating load curtailment for some system states is saved.
- Step (5) If any violation is observed or the power flow problem is not converged, further investigations are required. If the operational constraints are violated, the operator should take appropriate control actions while the load shedding is the last remedy. This procedure is fulfilled with an OPF problem (Section 3.2). Note that the possibility of demand interruption impedes any infeasible case in the optimization problem and a specific load curtailment solution associated with each state is necessarily obtained. On the contrary, if no violation is reported by the pre-analysis study, no optimization problem is needed to be solved and the amount of load curtailment is zero.

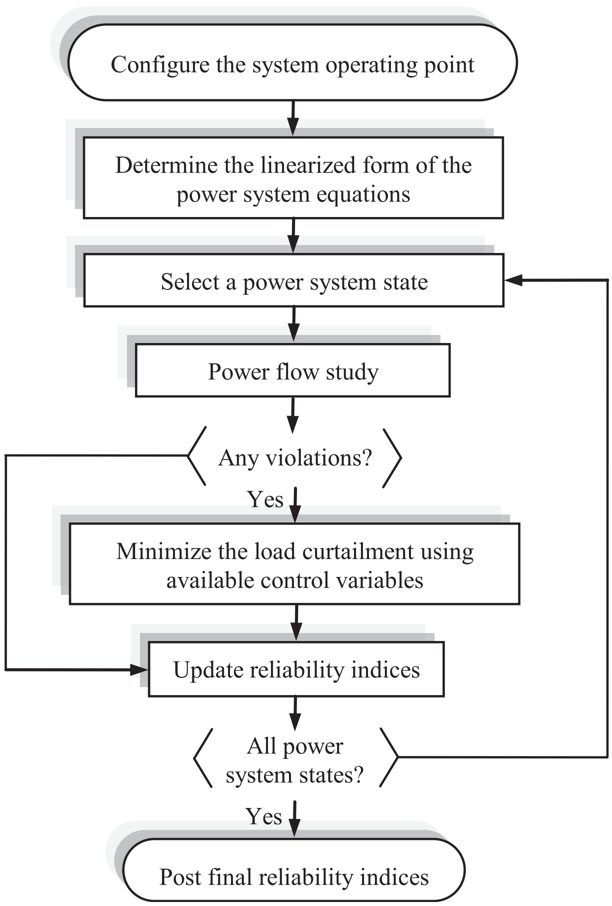

Fig. 2. Flowchart for the reliability evaluation of power system.

Step (6) After analyzing the selected state, the reliability indices are updated on the basis of curtailed load, if any, and the associated state probability. The final value of reliability indices would be corresponding to their last update.

Step (7) Finally, the termination criterion is checked. If it is met, the reliability evaluation procedure ends; otherwise, a new system state is examined. As discussed earlier, the termination criterion is different in enumeration and MCS methods.

4. Numerical analysis

In this section, reliability evaluation of the IEEE-RTS and the Iranian power grid is conducted to demonstrate the effectiveness of the proposed formulation. The minimum acceptable voltage level is assumed to be equal to 0.95 per unit. For the sake of verification, results obtained by the developed model are compared with those obtained using the simple DC and non-linear AC power flow models. Note that the results obtained by AC model are taken as the benchmark for comparisons. Additionally, different acceptable voltage levels, various order of simultaneous outages, and diverse loading conditions for the IEEE-RTS network are considered to reveal the performance of the new methodology in different situations. All cases are conducted in GAMS environment implemented on an Intel(R) Core 2 Duo 2.20 GHz processor with 2 GB RAM.

4.1. The IEEE-RTS

4.1.1. Network data

This test system has 24 buses, 32 generating units, and 38 transmission lines. The total installed capacity of the IEEE-RTS is 3405 MW and the system annual peak demand is 2850 MW. The system data were taken from [39].

4.1.2. Simulation results

The IEEE-RTS is a popular test-bed in power system reliability studies and the corresponding indices have been calculated in the literatures. However, reliability assessment solutions depend on several assumptions which are not explicitly being pointed out in the literature context. These assumptions consist of type of state selection methodology, employed termination criteria, order of simultaneous outages, and the minimum acceptable voltage level, just to name a few. These assumptions make the comparison of the results obtained by the developed technique with the existing literature impossible. Accordingly, the authors try to simulate the conventional methods to achieve appropriate comparison benchmarks.

In the simulations, up to third order outages are considered. Moreover, the system states with the associated occurrence probability lower than 10^{-6} are neglected to lessen the computational burden to a tractable level.

Composite system reliability assessment is performed using the three mentioned methods and the results are presented in Table 1. The evaluations are on annual basis in which the system demand remains constant equal to the system peak load level for the entire year.

Referring to Table 1, the results obtained by the proposed and conventional AC methods are almost identical; accordingly, the accuracy of the proposed method is verified. Note that the induced inaccuracies, which are the expense of reaching to a much more computational efficiency, lie within a small and negligible range. However, the results obtained from DC model are much smaller than the actual values. The reason goes back to the negligence of load curtailments at severe voltage drops. The load point indices including EDNS and EENS are given in Table 2. Also, LOLP associated with different load points are depicted in Fig. 3.

Similar to system indices, the accuracy of the load point indices verifies the acceptable performance of the proposed methodology. In the opposite point, the load point indices obtained by the DC model are rather misleading although these indices are key indicators in the system design and reinforcement. Both non-linear AC and the proposed linear AC models, in which both active and reactive power shortages are considered, recognize buses 3, 4, and 14 as the most unreliable load points. However, DC model identifies buses 13 and 18 as the worst load points from the reliability aspect. Therefore, the performance of DC model about the load point indices is even worse comparing to the system indices.

Table 3 compares the overall performance of the proposed and the conventional methods giving the total error of indices and time measures. As expected, the lowest computational time belongs to the DC model. However, inaccurate results of this method restrict its application. On the contrary, the significant reduction in the computational time of reliability assessment procedure along with

Table 1
System indices for the IEEE-RTS.

Index	AC model	Proposed model	DC model
LOLP	0.082358	0.080331	0.056143
EDNS (MW/yr)	10.4464	10.4026	7.8972
EENS (MW h/yr)	91510.76	91127.03	69179.10
BPEC (min/yr)	1926.54	1918.46	1456.40

Table 2
load point indices for the IEEE-RTS.

Load point	EDNS (MW/yr)			EENS (MW h/yr)		
	AC model	Proposed model	DC model	AC model	Proposed model	DC model
1	0	0	0.8154	0	0	7143.06
2	0	0	0.4388	0	0	3844.2
3	3.6028	3.5620	0.1319	31560.8	31203.1	1155.8
4	1.7205	1.6930	0.7172	15072.0	14830.7	6282.5
5	0.1579	0.0797	0.0175	1383.3	698.2	152.9
6	0.5050	0.6330	0.0451	4423.4	5545.1	394.7
7	0.0212	0.0152	0.0155	185.7	133.2	136.2
8	0.9336	1.0753	0.8188	8178.0	9419.6	7172.5
9	0.4198	0.3876	0.0833	3677.5	3395.4	729.4
10	0.0726	0.0649	0.2746	636.3	568.5	2405.1
13	0.0068	0	2.1579	59.3	0	18903.2
14	2.2336	1.9656	0.0005	19566.5	17218.7	4.2
15	0.0241	0.0180	0.5691	211.1	157.7	4985.7
16	0	0	0.0065	0	0	57.0
18	0.7313	0.8886	1.5912	6405.9	7784.1	13939.3
19	0.0172	0.0197	0.1468	150.6	172.6	1286.1
20	0	0	0.0670	0	0	587.2

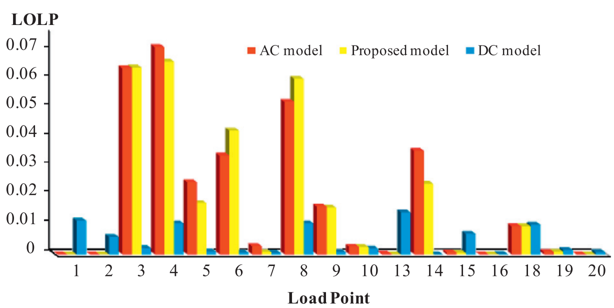

Fig. 3. Comparison of the load point reliability index (LOLP) calculated by different models for the IEEE-RTS.

Table 3
Comparison of the overall performance of the proposed and conventional methods for the IEEE-RTS.

Parameter	Power flow technique		
	AC model	Proposed model	DC model
OPF model	Non-linear	Linear	Linear
Error in LOLP	–	2.5%	32.6%
Error in EENS	–	0.4%	24.4%
Error in EDNS	–	0.4%	24.4%
Run time	312.3 s	190.4 s	118.6 s
Time improvement	–	39%	62%

the acceptable accuracy of the results verify the effectiveness of the proposed methodology. According to the obtained results, it is clear that the presented technique establishes an effective tool to compromise between accuracy of final indices and computational burden.

4.1.3. Impact of key parameters

(I) *Different acceptable voltage drops:* Power system reliability indices are extremely sensitive to the value of acceptable voltage level. In the simulations conducted in the previous section, minimum allowable voltage level was assumed as 0.95 per unit. In this sub-section, system indices are recalculated for different acceptable voltage levels and the results are listed in Table 4. As expected, results obtained by DC model are insensitive to the value of acceptable voltage level. However, the reliability indices resulted from either conventional AC or the proposed linear model become greater with the increase of acceptable voltage level and vice versa. Based on the results, the approximation of the proposed model is trivial and is not trouble maker.

(II) *Different loading conditions:* To reveal the effective performance of the proposed method, system indices are recalculated at various loading conditions including 70%, 80%, 90%, 100%, and 110%. Table 5 outlines the obtained results associated with all the models. It can be deduced that by changing the system loading condition, the accuracy of the results obtained using the new formulation is not degraded.

(III) *Different order of simultaneous outages:* In the simulations conducted in the previous sub-sections, only first, second and third order outages were considered. The inclusion of higher order of outages has an incremental effect on the indices but is conducted for the sake of examination of the new method. Table 6 illustrates the system indices when different orders of simultaneous outages are considered. Owing to the obtained indices, application of the proposed model leads to reasonable accurate results in all scenarios. The run time statistics associated with different orders of simultaneous out-

Table 4
Comparison of the results obtained by the proposed and conventional models with various under voltage limits.

Parameter	Minimum acceptable voltage level								
	0.94 per unit			0.95 per unit			0.96 per unit		
	AC model	Proposed model	DC model	AC model	Proposed model	DC model	AC model	Proposed model	DC model
LOLP	0.079730	0.079294	0.056143	0.082358	0.080331	0.056143	0.083113	0.082555	0.056143
EDNS	10.3971	10.3484	7.8972	10.4464	10.4026	7.8972	10.5445	10.5015	7.8972
EENS	91078.47	90652.10	69179.09	91510.76	91127.03	69179.09	92369.83	91993.12	69179.09
BPEC	1917.44	1908.47	1456.402	1926.54	1918.46	1456.402	1944.63	1936.70	1456.402

Table 5
Comparison of the results obtained by the proposed and conventional models in various loading conditions.

Power flow technique	System loading condition									
	70% loading		80% loading		90% loading		100% loading		110% loading	
	LOLP	BPEC	LOLP	BPEC	LOLP	BPEC	LOLP	BPEC	LOLP	BPEC
AC model	0	0	0.000373	5.02	0.012541	142.24	0.082358	1926.54	0.301974	12107.07
Proposed model	0	0	0.000373	5.21	0.013236	152.64	0.080331	1918.46	0.307834	12411.63
DC model	0	0	0.000373	3.44	0.005367	103.58	0.056143	1456.40	0.288705	10516.23

Table 6
Comparison of the results obtained by the proposed and conventional models with different order of simultaneous outages.

Power flow technique	Order of simultaneous outages									
	Up to first order		Up to second order		Up to third order		Up to forth order		Up to fifth order	
	LOLP	BPEC	LOLP	BPEC	LOLP	BPEC	LOLP	BPEC	LOLP	BPEC
AC model	0.001008	10.75	0.037028	698.81	0.082358	1926.54	0.101313	2735.13	0.102188	2800.31
Proposed model	0.001008	10.53	0.035828	695.33	0.080331	1918.46	0.099209	2724.55	0.100084	2789.58
DC model	0	0	0.019696	467.12	0.056143	1456.40	0.072952	2162.81	0.073827	2223.46

Table 7
Run time statistics of the proposed and conventional methods for different order of simultaneous outages.

Order of simultaneous outages	Power flow technique		
	AC model	Proposed model	DC model
Up to first order	2.9 s –	2.1 s 28% imp.	1.2 s 59% imp.
Up to second order	67.9 s –	48.3 s 29% imp.	27 s 60% imp.
Up to third order	312.3 s –	190.4 s 39% imp.	118.6 s 62% imp.
Up to forth order	669.6 s –	467.4 s 30% imp.	268.8 s 60% imp.
Up to fifth order	775.8 s –	557.2 s 28% imp.	364.4 s 53% imp.

imp.: run time improvement.

ages are shown in Table 7. Given the statistics provided, the proposed formulation ability in the computational complexity alleviation is remarkable.

4.2. The Iranian power grid

4.2.1. Network data

The Iranian power grid comprises of three voltage levels consisting of 135, 230, and 400 kV. Technical data used in this paper are based on the Iranian 400-kV power grid corresponding to the configuration in 2011 peak condition. Single-line diagram of the system is depicted in Fig. 4. The 400-kV network has 121 buses and 185 transmission lines. The system demand is served through 82 buses and the load of 18,712 MW is distributed among 82 buses. Total amount of installed capacity of the 400-kV network of the Iranian power grid is equal to 26,000 MW. The technical data for the Iranian power grid are available at http://www.igmc.ir/Default.aspx?tabid=477&EntryId=282462.

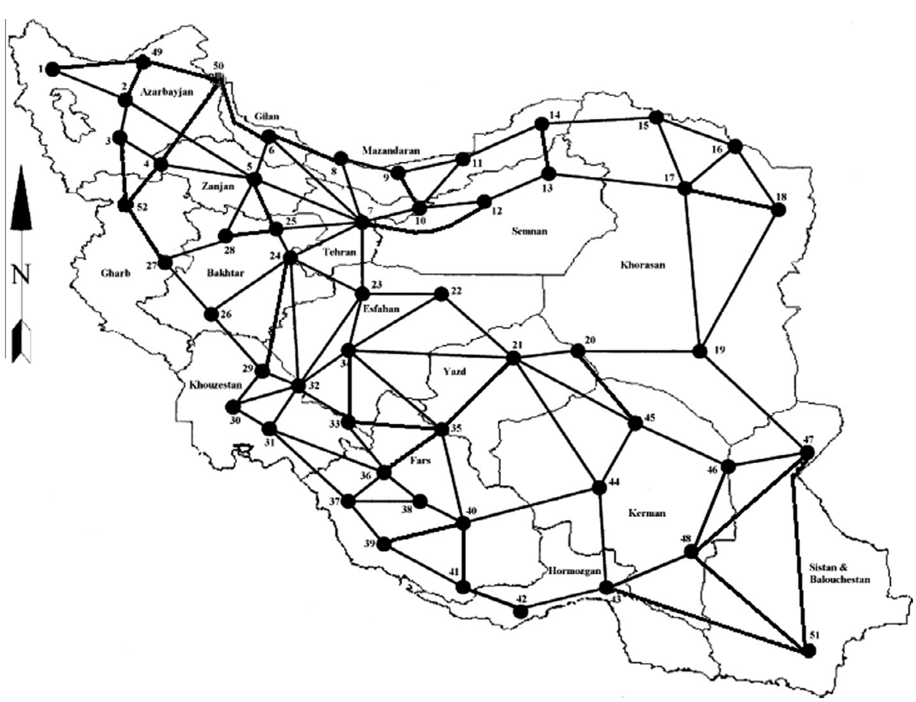

Fig. 4. Single-line diagram of the Iranian 400-kV power grid corresponding to the configuration in 2007.

Table 8
System indices for the Iranian power grid.

Index	AC model	Proposed model	DC model
LOLP	0.1206	0.118	0.09848
EDNS (MW/yr)	390.1056	385.674	22.23942
EENS (MW h/yr)	3417325	3378504	194817.3
BPEC (min/yr)	10957.65	10833.18	624.6818
Run time (min)	112.86	61.95	10.58

4.2.2. Simulation results

In the simulation, MCS technique is employed as the state sampling method. The state sampling terminates once the number of sampled states reaches 10,000 samples.

Composite system reliability assessment is performed using the three mentioned methods and the results are presented in Table 8. The reliability indices are calculated for the system hourly peak load and they are therefore designated as annual indices.

Referring to Table 8, the results obtained by the proposed and conventional AC methods are almost identical; accordingly, the accuracy of the proposed method is verified. However, the results obtained from DC model are much smaller than the actual values mainly due to the ignorance of load curtailments at severe voltage drops. Owing to the results given in Table 8, significant improving effect of the proposed formulation in the execution time is observed. The computation time for the reliability assessment has been reduced about 1.8 times by the presented technique to that of the conventional AC method. Again, it is demonstrated that the new formulation provides an effective tool for compromising between computational burden and accuracy of the final indices.

5. Conclusion

A new formulation for evaluating the reliability of composite power systems considering reactive power aspects was presented in this paper. The presented technique is established based on an approximated power flow model and therefore, offers a system of linear equations wherein the cumbersome non-linearity of conventional AC model is avoided. The proposed methodology allows operators to compute different reliability indices arising from active and reactive power shortages as well as transmission network overloads. Also, the developed approach is applicable in both sorts of truncated enumeration and MCS state selection methods. The approach is more attractive from the computational complexity perspective. A comparative study with conventional AC and DC formulations was conducted in this paper and validated the effectiveness of the proposed technique in the execution time saving while not jeopardizing the accuracy. Actually, it was shown that the new formulation provides an effective tool to compromise between accuracy of final indices and computational burden. Also, a set of sensitivity analyses demonstrated the acceptable accuracy of the reliability indices in the case of new method implementation.

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