# Reliability and Sensitivity Analysis of Composite Power Systems Considering Voltage and Reactive Power Constraints

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***Abstract*—This work investigates the effects of voltage and reactive power constraints on composite system reliability indices. Four indices have been used to assess the sensitivity of load curtailments to voltage and reactive power constraints. Further, expressions to evaluate the sensitivity of the severity indices with respect to the voltage and reactive power constraints have been developed. Extensive simulations on the IEEE Reliability Test System (RTS) based on the AC power flow model were carried out to correlate the violations of these limits and the loading level of the system. A non-linear, AC power flow based model has been used to accurately represent system redispatch. A state space reduction technique has been utilised to reduce the computation time. A comparison between redispatch models using AC and DC power flow has been provided to illustrate the benefits of the more accurate AC flow model and to investigate the effects of the voltage and reactive power constraints on reliability indices.**

***Index Terms*—Key words: Reliability, voltage and reactive power constraints, AC power flow, DC power flow.**

## I. Introduction

There is a growing need to include the effects of voltage and reactive power constraints in power system reliability evaluation. Most of the present methods that evaluate the reliability of composite systems represent the network using the DC power flow model which ignores the effects of the voltage and reactive power constraints on the reliability indices. Due to the integration of the renewable energy resources, the voltage and reactive power limits are expected to have more effect on the system reliability. The time and computation effort to evaluate the reliability indices and their sensitivities with respect to the operating conditions using models that incorporate voltage and reactive power limits such as AC power flow model are of concern in both planning and operation prospectives.

Traditionally, composite system reliability methods have used transportation model or DC power flow models to represent system flows, largely due to the complexity of AC flow-based redispatch algorithms. Evaluation of the effects of the voltage and reactive power limits has been introduced in [1]. The method in [1] is based on using the DC power flow model in two steps and linearising the relationship between the reactive power injections and the voltages at the nodes. Several methods have been recently introduced in the literature of power system reliability to investigate the effects of lack of reactive power support on the reliability indices. For example, using the AC power flow model in *two steps* to study the aspects of the reactive power on the composite system reliability has been introduced in [2]–[5].

Analytical as well as simulation techniques have been introduced and used in evaluating power system reliability indices. Composite system reliability indices include, but are not limited to, *Loss of Load Probability*, *Loss of Load Frequency*, *Loss of Load Duration* and *Severity Index*. Although these indices are of a great importance in both planning and operational power system reliability evaluation, they lack the ability of identifying the influence of the operation constraints on the system reliability.

Methodologies for calculating the sensitivity of some reliability indices with respect to the variations in component's parameters and system operating limits have been introduced in [6]–[10]. One of the advantages of using sensitivity analysis is that it allows planners to enhance the overall system reliability by improving the reliability parameters and available capacity of each component in a separate manner. The study of the sensitivity of these indices can be viewed as measures of deficiency in both generation and transmission subsystems and, hence, provide sense to the areas that to be reinforced.

Several methods have been introduced to decrease the time and computational effort in evaluating power system reliability indices. A significant number of research papers in the literature of power system reliability have introduced techniques to reduce the search space and the computational effort. The concept of the state space pruning has been introduced in [11], [12]. The concept of state space pruning using population-based intelligent search (PIS) methods has been introduced in [13]–[18]. The genetic algorithm (GA) and the modified GA have been used in [13]–[15] to prune the state space and use Monte Carlo simulation for the remaining part of the state space. Binary particle swarm optimisation technique has been utilised in [17] to prune the state space. The application of artificial immune systems (AIS) optimisation technique in pruning the state space has been proposed in [18]. A comparative study of using PIS methods specifically genetic algorithms, repulsive binary particle swarm optimisation and binary ant colony optimisation has been introduced in [16]. A search space reduction using the particle swarm optimisation

(PSO) technique was introduced in [19], [20].

This paper investigates the effects of the voltage and reactive power constraints on composite system reliability indices through performing sensitivity analyses of the severity index with respect to the voltage and reactive power constraints. Expressions have been developed to evaluate the sensitivity of the severity index with respect to these constraints. Further, four indices have been used to evaluate the contributions of the minimum and maximum voltage limits at the nodes and minimum and maximum reactive power capabilities in the loss of load probability index. The full, non-linear AC power flow model has been used to incorporate these constraints in the reliability evaluation. A state space pruning technique has been utilised to reduce the computation time. Also, in this work, composite system reliability indices using AC power flow model are provided to serve as a benchmark for future work involving power flow models. A comparison between the use of AC and DC power flow models is also provided.

## II. Network Modelling

In composite system reliability studies, power flow analyses are usually carried out in solving optimisation problems for minimum load curtailment. In this paper, the AC power flow model has been used. This section describes the formulation and incorporation of the objective function of minimum load curtailment in the nonlinear programming problem. This objective function is subject to equality and inequality constraints of the power system operation limits. The equality constraints include the power balance at each bus and the inequality constraints are the capacity limits of generating units, power carrying capabilities of transmission lines, voltage limits at the nodes and reactive power capability limits. The minimisation problem is formulated as follows [21],

$$\text{Loss of Load} = \min\left(\sum_{i=1}^{N_b} C_i\right) \tag{1}$$

Subject to

$$\begin{gathered}
P(V,\delta) - P_D + C = 0\\
Q(V,\delta) - Q_D = 0\\
P_G^{min} \leq P(V,\delta) \leq P_G^{max}\\
Q_G^{min} \leq Q(V,\delta) \leq Q_G^{max}\\
V^{min} \leq V \leq V^{max}\\
S(V,\delta) \leq S^{max}\\
0 \leq C \leq P_D\\
\delta \quad \text{unrestricted.}
\end{gathered} \tag{2}$$

In (1) and (2), $C_i$ is the load curtailment at bus $i$, $C$ is the vector of load curtailments $(N_d \times 1)$, $V$ is the vector of bus voltage magnitudes $(N_b \times 1)$, $\delta$ is the vector of bus voltage angles $(N_b \times 1)$, $P_D$ and $Q_D$ are the vectors of real and reactive power loads $(N_d \times 1)$, $P_G^{min}$, $P_G^{max}$, $Q_G^{min}$ and $Q_G^{max}$ are the vectors of real and reactive power limits of the generators $(N_g \times 1)$, $V^{max}$ and $V^{min}$ are the vectors of maximum and minimum allowed voltage magnitudes $(N_b \times 1)$, $S(V,\delta)$ is the vector of power flows in the lines $(N_t \times 1)$, $S^{max}$ is the vector of power rating limits of the transmission lines $(N_t \times 1)$ and $P(V,\delta)$ and $Q(V,\delta)$ are the vectors of real and reactive power injections $(N_b \times 1)$. Also, $N_b$ is the number of buses, $N_d$ is the number of load buses, $N_t$ is the number of transmission lines and $N_g$ is the number of generators.

In the standard minimisation problem given by (1) and (2), all generation and network constraints have been taken into consideration. Details about the constraints of (2) are given the appendix. Also, it has been assumed that one of the bus angles is zero in the constraints (2) to work as a reference bus.

## III. State Space Pruning

In this work, the state space pruning technique which has been introduced in [11] was adapted to reduce the search space. This technique is based on pruning out the success (no load-curtailment) subspace and performing Monte Carlo state sampling on the remaining (unclassified) subspaces. The pruning technique in [11] is intended to be used with the DC power flow with assuming that the system is coherent (failure or degradation of a component cannot improve system performance; likewise, the improvement or restoration of a component cannot deteriorate system performance) for generation capacity changes in pruning phase. Coherency cannot be assumed under transmission capacity variations [12], [22]. This is because transmission capacity changes are accompanied by changes in line impedances, causing the power flows to be redistributed.

Using the AC power flow model, the coherency condition cannot be assumed for both generation capacity changes and transmission capacity variations. This is due to the following reasons:(a) changes in the generation capacities are accompanied with changes in the reactive power resources, and (b) changes in the transmission capacities are accompanied by changes in line impedances, causing the power flows to be redistributed and over/under voltage violations may occur at some nodes in the system. In this work, only the first stage of state space pruning [11] was used; all system components (generation and transmission) are assumed in the working states. Using this assumption, the algorithm solves for minimum load curtailment. The vectors of real and reactive power generation for the minimum load curtailment at the buses are used as a minimum threshold for the system to meet the demand for the states that have all the transmission lines are in the up state. This threshold is used as a pruning device and can be justified in the following points:

- If the real and reactive power generation at all the buses are larger than the threshold, system performance cannot be deteriorated,
- Due to the fact that power system components are very reliable, this pruned subspace has a high probability in comparison with the other subspaces. Therefore, pruning this subspace will reduce the computational time significantly.

## IV. Calculation of Reliability Indices

In the literature, both analytical and simulation-based approaches have been used for composite system reliability evaluation. In this work, the Monte Carlo next event method [23], which is similar to the Monte Carlo state transition method which was proposed in [24], has been used for the following reasons: (a) the analytical methods are not practical due to the complexity of the composite power system reliability modelling and the computation speed and burden, (b) Monte Carlo state duration technique (sequential) requires large memory storage and computational burden, (c) Monte Carlo state sampling technique (non-sequential) has a disadvantage of difficulty associated with calculating frequency and duration indices due to the requirement that the system should be coherent which cannot be assumed in case of using AC power flow model.

In this work, the well-known composite power system reliability indices were evaluated which are loss of load probability index ($q$), loss of load frequency index ($\phi$), loss of load duration index ($\tau$) and Severity Index ($\rho$);

### A. Calculation of Probability Indices

Failure probability indices evaluate the probability of failure of the system to meet the demand. Through the simulation process, if the state under consideration is a failure state, the probability of this state is added to the failure probability index, $q$. The probability of system failure to meet the demand is given by,

$$q = \sum_{i=1}^{n_f} p\left\{x_i : x_i \in X_f\right\} \tag{3}$$

where $X$ is the set of all states, $X_f$ is the set of failure states ($X_f \subset X$), $x_i$ is the system state $i$, $p\left\{\cdot\right\}$ is the probability of the state and $n_f$ is the number of failure states.

The estimated loss of load probability index ($\hat{q}$) can be calculated using Monte Carlo state next event approach as follows,

$$\hat{q} = E\left[q\right] \tag{4}$$

where $E\left[\cdot\right]$ is the expectation operator and $q$ can be evaluated as follows,

$$q = \frac{1}{T}\sum_{i=1}^{N} \vartheta_i \tag{5}$$

where $N$ is the number of samples, $T$ is the sum of the durations of all sampled system states and $\vartheta_i$ is an indicator function that can be expressed as,

$$\vartheta_i = \begin{cases} \tau_i, & \text{If } x_i \in X_f \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

where $\tau_i$ is the duration of system state $i$.

### B. Calculation of the Severity Index

Severity index is one of the well-known reliability indices that measures the expected demand not supplied. Let $\rho$ denote the severity index. For every tested state, if the state is a failure state, the product of the probability of this state and the amount of load curtailment is added to the $\rho$ index. The Severity Index is given by,

$$\rho = \sum_{i=1}^{n_f} p\left\{x_i : x_i \in X_f\right\} \times C\left\{x_i : x_i \in X_f\right\} \tag{7}$$

where $C$ is the amount of load curtailment of state $x_i$.

The estimated Severity Index, $\hat{\rho}$, can be calculated using Monte Carlo state next event sampling approach as follows,

$$\hat{\rho} = E\left[\rho\right] \tag{8}$$

where $\rho$ can be evaluated as follows,

$$\rho = \frac{1}{T}\sum_{i=1}^{N} \psi_i \tag{9}$$

and $\psi_i$ is an indicator function for the state curtailment that can be expressed as,

$$\psi_i = \begin{cases} \tau_i \times C_i, & \text{If } x_i \in X_f \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

### C. Calculation of Frequency and Duration Indices

Calculation of frequency and duration indices is generally more difficult than calculation of probability and severity indices. An approach based on Monte Carlo state sampling approach has been developed in [12], [25]–[27] which requires that the system is coherent. In this work, the Monte Carlo state next event method has been used to calculate frequency and duration indices which does not necessitate the coherency condition. Also, the Monte Carlo state next event method is comparable with the Monte Carlo state sampling approach in terms of memory storage requirements and speed of computation.

The estimated loss of load frequency index ($\hat{\phi}$) can be calculated using Monte Carlo state next event method as follows,

$$\hat{\phi} = E\left[\phi\right] \tag{11}$$

where $\phi$ can be evaluated as follows,

$$\phi = \frac{1}{T}\sum_{i=1}^{N} \varphi_i \times 8760 \tag{12}$$

and $\varphi_i$ is an indicator function for the state frequency that can be expressed as,

$$\varphi_i = \begin{cases} 1, & \text{If } x_i \in X_s \text{ and } x_{i-1} \in X_f \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

where $X_s$ is the set of success states ($X_s \subset X$).

The estimated loss of load duration index ($\hat{\tau}$) can be calculated as follows,

$$\hat{\tau} = \frac{\hat{q}}{\hat{\phi}}. \tag{14}$$

### D. Indices of Voltage and Reactive Power Limits

In this paper, four indices were used to represent the contributions of violations of the voltage and reactive power constraints on the loss of load probability index. These indices are defined as follows: (a) $\upsilon i_{min}$ represents the contributions of the minimum voltage level constraints, (b) $\upsilon i_{max}$ represents the contributions of the maximum voltage level constraints, (c) $\varrho i_{min}$ represents the contributions of the minimum reactive power level constraints and (d) $\varrho i_{max}$ represents the contributions of the maximum reactive power level constraints. For every sampled state, if any of these indices is involved in the load curtailment, this index is updated. These indices are related to the failure subspace not to the system state space; these indices reflect the contributions of the violations of the voltage and reactive power constraints on the loss of load probability index. Therefore, the state space of these indices is $X_f$.

The estimated values of these indices can be calculated using Monte Carlo state next event method as follows,

$$f_k = E\left[q_k\right] \tag{15}$$

where $f_k$ is the index under consideration, ($f_1$ is for the $\upsilon i_{min}$, $f_2$ is for the $\upsilon i_{max}$, $f_3$ is for the $\varrho i_{min}$ and $f_4$ is for the $\varrho i_{max}$), and $q_k$ can be evaluated as follows,

$$q_k = \frac{1}{T_f}\sum_{i=1}^{N}\xi_{ki} \tag{16}$$

where $T_f$ is the sum of the durations of all sampled *failure* states and $\xi_{ki}$ is an indicator function for state violation that can be expressed as,

$$\xi_{ki} = \begin{cases} \tau_i, & \text{If } x_i \in X_k \\ 0, & \text{otherwise} \end{cases} \tag{17}$$

where $X_k$ is the set in which the index $k$ has a contribution in the load curtailment and $X_k \subset X_f$.

### E. A Stopping Criterion

Although in power system reliability analysis using Monte Carlo simulation it was found to be that energy indices are the slowest indices from convergence perspective [28], in this work, the stopping criterion was applied on three power system reliability indices which are the expected probability of system failure, $\hat{q}$, the expected frequency of system failure, $\hat{\Phi}$ and the expected system failure severity index, $\hat{\rho}$. The indices of voltage and reactive power limits are directly related to these three indices. Therefore, if these three indices converge to the specified tolerance, then the voltage and reactive power limit indices will converge too. The stopping criterion considering all the three indices can be expressed as,

$$\sigma = \max(\sigma_1, \sigma_2, \sigma_3) \tag{18}$$

where $\sigma$ is a pre-specified tolerances,

$$\sigma_i = \frac{\sqrt{V(f_i(x))}}{f_i(x)} \tag{19}$$

where $V(\cdot)$ is the variance function, $f_i(x)$ is a reliability index and $i$ is an index, i.e., $i = 1$ is for probability of system failure index, $i = 2$ is for frequency of system failure index and $i = 3$ is for the Severity Index.

During the simulation, if $\sigma$ is less than or equal to the specified tolerance, the algorithm is terminated; otherwise, the simulation will continue.

## V. Sensitivity Analysis

Sensitivity analyses of composite system reliability indices with respect to system parameters have been extensively addressed in the literature. Sensitivity analyses of reliability the indices done so far in the literature were based on calculating the amount of change of these indices with respect to component parameters such as availability/unavailability, capacity, failure rate and repair rate. However, sensitivity of these indices with respect to the voltage and reactive power limits has not been given much attention.

These analyses were conducted either by examining every single state of the system or by simulation procedures. The analysis has been performed by solving a optimisation problem of minim load curtailment or maxim load supplying capability. DC power flow with linear programming optimisation problem with the objective of minimum load curtailments is the most commonly model used in sensitivity analysis of the reliability indices of composite power systems. For instance, [6]–[8], [29], [30] have introduced methodologies for calculating the sensitivity of some reliability indices with respect to the variations in component availability, unavailability, capacity and failure and repair rates. Also, in a previous work [10], the authors have ranked system components from the risk point of view based on their weighted shadow prices. Sensitivity analysis of the composite system reliability indices considering emission allowance constraints has been proposed in [9]. In performing sensitivity analysis, Lagrange multipliers are usually required and the only data that can be accessible from the linear programming and DC power flow model are the Lagrange multipliers of the demand, generation and transmission. To calculate the sensitivity with respect to the voltage and reactive power limits; these limits need to be added to the optimisation problem.

The sensitivity analyses of the reliability indices with respect to the voltage and reactive power constraints are important in identifying the impacts of these constraints on power system reliability and help in hardening power systems against catastrophic failures such as voltage collapse. In this paper, the AC power flow model is used to include such constraints in the optimisation problem. Expressions for calculating the sensitivity of the Severity Index with respect to voltage and reactive power limits are developed. These expressions were developed from the Lagrange multipliers of the voltage and reactive power constraints.

### A. Sensitivity of the Severity Index with Respect to Voltage Limit Constraints

The derivation of the sensitivity of the Severity Index with respect to voltage and reactive power limits starts with utilising the expression of (7). The sensitivity of the Severity Index with respect to voltage limits allowed at bus $k$ can be expressed as follows,

$$\partial\rho/\partial\upsilon_k = \sum_{x\in X_f} p(x) \times (\partial C(x)/\partial\upsilon_k) \tag{20}$$

where $\partial C(x)/\partial\upsilon_k$ is the Lagrange multiplier, $\pi_{\upsilon k}$, of the voltage constraint at bus $k$ while the system is residing at state $x$.

The sensitivity of the Severity Index with respect to voltage limit constraints can be evaluated using Monte Carlo state next event method as follows,

$$\partial\hat{\rho}/\partial\upsilon_k = \frac{1}{T}\sum_{i=1}^{N} \varphi_i \times \pi_{\upsilon k}. \tag{21}$$

This expression can be used for both maximum and minimum voltages allowed at the buses.

### *B. Sensitivity of the Severity Index with Respect to Reactive Power Limit Constraints*

In the same manner, the sensitivity of the Severity Index with respect to the reactive power available at bus $k$ can be expressed as follows,

$$\partial\rho/\partial\varrho_k = \sum_{x\in X_f} p(x) \times (\partial C(x)/\partial\varrho_k) \tag{22}$$

where $\partial C(x)/\partial\varrho_k$ is the Lagrange multiplier, $\pi_{\varrho k}$, of the reactive power available at bus $k$ while the system is residing at state $x$.

Similarly, the sensitivity of the Severity Index with respect to reactive power constraints at bus $k$ can be evaluated using Monte Carlo state next event method as follows,

$$\partial\hat{\rho}/\partial\varrho_k = \frac{1}{T}\sum_{i=1}^{N} \varphi_i \times \pi_{\varrho k}. \tag{23}$$

Also, this expression can be used for both maximum and minimum reactive power limits at the buses.

## VI. Solution Algorithm

A flowchart describing the process of evaluating the reliability indices and their sensitivities with respect to the component parameters and the operating limits is shown in Fig. 1. The algorithm starts with applying the state space reduction technique as explained in section III and then performs Monte Carlo simulations on the unclassified subspace. For each sampled state, the algorithm solves for minimum load curtailment as described in section II. If the load curtailment is unavoidable, then the algorithm determines the amount of load curtailment according to (1) and updates the reliability indices as described in section IV. Also, the algorithm determines the Lagrange multipliers from (2) and updates the sensitivity indices as described in section V. If the stopping criterion is met as given in section IV, the algorithm stops; otherwise, it continues.

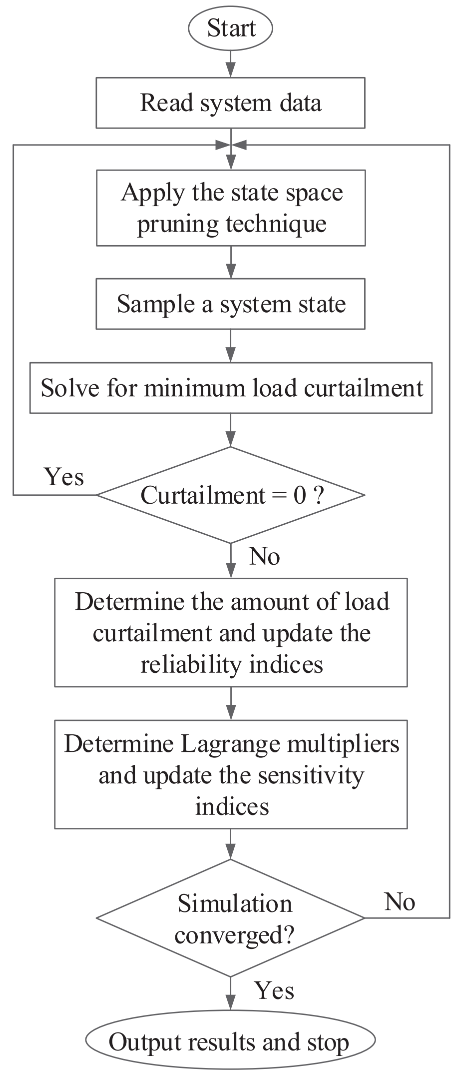


Fig. 1. System loss of load probability (LOLP) index profile against the change in the system loading level

## VII. Case Studies

The proposed formulation was applied on the IEEE RTS. The detailed data such as the capacities of generating units, the line carrying capabilities of the transmission lines, the failure and repair rates of system components and load profile are given in [31]. The single line diagram of the IEEE RTS is shown in Fig. 2 which was reprinted from [31]. The IEEE RTS system has been extensively tested for power system reliability analysis. IEEE RTS consists of 24 buses, 38 transmission lines/transformers (33 transmission lines and 5 transformers) and 32 generating units on 10 buses. The total generation of this system is 3405 MW and total peak load is 2850 MW.

### *A. Reliability Assessment*

Reliability assessments using both the AC and DC power flow models have been performed on the IEEE RTS system at the peak load (Annualised Indices). As a comparison, the

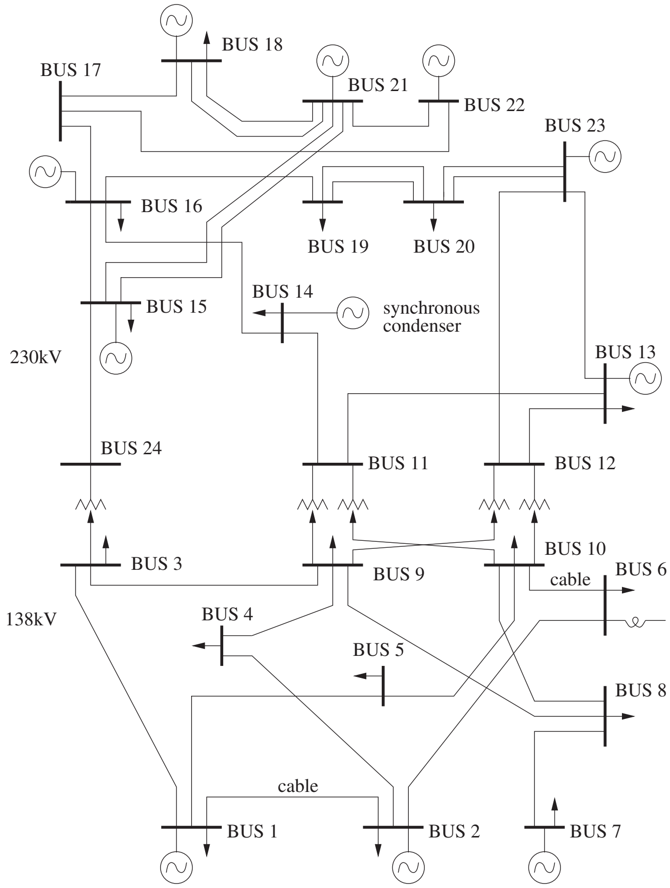


Fig. 2. Single line diagram of the IEEE RTS – reprinted from [31].

results of both models for the IEEE RTS are shown in Table I. As it is obvious from Table I, estimates from the DC power flow model produce optimistic results in comparison with the AC power flow model counterpart.

TABLE I
SYSTEM ANNUALISED INDICES OF THE IEEE RTS

| Power Flow Model | $\hat{q}$ | $\hat{\rho}$ MW | $\hat{\phi}$ occ./yr | $\hat{\tau}$ hour |
|---|---|---|---|---|
| AC | 0.10427 | 17.37479 | 22.76247 | 0.00458 |
| DC | 0.08455 | 14.72996 | 19.76867 | 0.00428 |

It is well-known that the transmission lines of the IEEE RTS are very reliable with respect to the generation. Also, the power carrying capability limits of the transmission lines are much higher than the normal loading level even in the case of the peak load. Therefore, the contributions of the transmission lines on the system reliability of this test system are very small and can be ignored. For this reason, several studies have suggested to use the modified version of it which is the same as the original system except that the generation is multiplied by 2 and the load at the buses is multiplied by 1.8. The results of both power flow models for the Modified IEEE RTS are shown in Table II. Again, from Table II, the DC power flow model gives *very* optimistic results in comparison with the AC power flow model counterpart.

TABLE II
SYSTEM ANNUALISED INDICES OF THE MODIFIED IEEE RTS

| Power Flow Model | $\hat{q}$ | $\hat{\rho}$ MW | $\hat{\phi}$ occ./yr | $\hat{\tau}$ hour |
|---|---|---|---|---|
| AC | 0.44337 | 20.13860 | 63.82885 | 0.00695 |
| DC | 0.07141 | 10.52347 | 17.76162 | 0.00402 |

Since there are significant differences in the values of the indices for different power flow models, two case studies have been performed by gradually stress the transmission lines to investigate the effects of the voltage and reactive power limits on the reliability indices and to compare the results of the AC and DC power flow models.

*1) Case Study I: Gradually Increasing the Generation and Load Levels:* In this case study, the generation and load levels are increased gradually to track the behaviour of system indices. The annualised reliability indices are shown in Table III. The first two columns of Table III show the factors by which the load and generation are increased respectively.

TABLE III
SYSTEM ANNUALISED INDICES FOR CASE I: EFFECT OF GRADUALLY INCREASING THE GENERATION AND LOAD LEVELS

| Loading Factor | Generation Factor | $\hat{q}$ | $\hat{\rho}$ MW | $\hat{\phi}$ occ./yr | $\hat{\tau}$ hour |
|---|---|---|---|---|---|
| 1.000 | 1.000 | 0.10427 | 17.37479 | 22.76247 | 0.00458 |
| 1.080 | 1.100 | 0.09245 | 15.73231 | 20.50596 | 0.00451 |
| 1.160 | 1.200 | 0.07459 | 13.91987 | 17.03678 | 0.00438 |
| 1.240 | 1.300 | 0.06243 | 13.07068 | 16.30765 | 0.00383 |
| 1.320 | 1.400 | 0.05668 | 12.99306 | 14.51361 | 0.00391 |
| 1.400 | 1.500 | 0.05551 | 12.74350 | 14.54252 | 0.00382 |
| 1.408 | 1.600 | 0.06897 | 12.51458 | 18.99008 | 0.00363 |
| 1.560 | 1.700 | 0.11062 | 13.18529 | 25.88540 | 0.00427 |
| 1.640 | 1.800 | 0.13719 | 14.96639 | 30.11179 | 0.00456 |
| 1.680 | 1.850 | 0.15788 | 15.37179 | 36.89292 | 0.00428 |
| 1.720 | 1.900 | 0.17134 | 16.72898 | 43.63865 | 0.00438 |
| 1.740 | 1.925 | 0.19511 | 16.66352 | 45.35073 | 0.00430 |
| 1.760 | 1.950 | 0.23595 | 17.61181 | 51.25988 | 0.00460 |
| 1.780 | 1.975 | 0.35426 | 18.56049 | 59.52425 | 0.00595 |
| 1.800 | 2.000 | 0.44337 | 20.13860 | 63.82885 | 0.00695 |

From Table III, not much change in the system Severity Index between the IEEE RST and Modified IEEE RTS. System loss of load probability and loss of load frequency indices are smoothly increasing with the increase in the loading and generation levels until the load level reaches around 172% of the peak load. After this point, these indices show sharp increase. This sudden increase can be attributed to the voltage and reactive power limits as it is explained in section VII-C.

*2) Case Study II: Gradually Increasing the Load Level and Keeping the Generation Level Constant:* This case study is similar to the case study I except that the generation level is kept constant by factor of 2. The reason of performing this case study is that in Case Study I, both generation and load levels were increasing and the point at which the indices start sharply increasing due to load effect is difficult to detect. The annualised reliability indices are shown in Table IV.

TABLE IV
System Annualised Indices for Case II: Effect of Gradually Increasing the Load Levels and Fixing the Generation

| Loading Factor | Generation Factor | $\hat{q}$ | $\hat{\rho}$ MW | $\hat{\phi}$ occ./yr | $\hat{\tau}$ hour |
|---|---|---|---|---|---|
| 1.000 | 2.000 | 0.00139 | 0.00800 | 0.37248 | 0.00373 |
| 1.080 | 2.000 | 0.00171 | 0.02570 | 0.68971 | 0.00248 |
| 1.160 | 2.000 | 0.00185 | 0.04468 | 0.85359 | 0.00217 |
| 1.240 | 2.000 | 0.00202 | 0.07438 | 0.85251 | 0.00237 |
| 1.320 | 2.000 | 0.00201 | 0.09948 | 0.82788 | 0.00243 |
| 1.400 | 2.000 | 0.00272 | 0.20431 | 1.23875 | 0.00220 |
| 1.480 | 2.000 | 0.00557 | 0.40744 | 2.55331 | 0.00218 |
| 1.560 | 2.000 | 0.01041 | 1.05772 | 4.68652 | 0.00222 |
| 1.640 | 2.000 | 0.04629 | 2.97801 | 16.07442 | 0.00288 |
| 1.680 | 2.000 | 0.08679 | 4.68413 | 23.47966 | 0.00412 |
| 1.720 | 2.000 | 0.12261 | 7.91592 | 27.23764 | 0.00450 |
| 1.740 | 2.000 | 0.14321 | 10.02282 | 33.99955 | 0.00421 |
| 1.760 | 2.000 | 0.18729 | 12.58832 | 42.44476 | 0.00441 |
| 1.780 | 2.000 | 0.22284 | 15.67655 | 49.43979 | 0.00451 |
| 1.800 | 2.000 | 0.44337 | 20.13860 | 63.82885 | 0.00695 |

From Table IV, the system Severity Index smoothly increases and system loss of load probability and loss of load frequency indices are smoothly increasing with the load levels increase until the load level reaches around 148% of the peak load. After this point, these indices increase sharply. Also, a very sharp increase after around 172% of the peak load. Again, this sudden increase can be attributed to the voltage and reactive power limits as it is explained in subsection VII-C.

### *B. Comparison with the DC Power Flow Model*

To compare the results of the reliability indices, the analyses of the above case studies have been repeated using the DC power flow model. To better visualise the differences in these values, the results of the system loss of load probability index are depicted in Fig. 3.

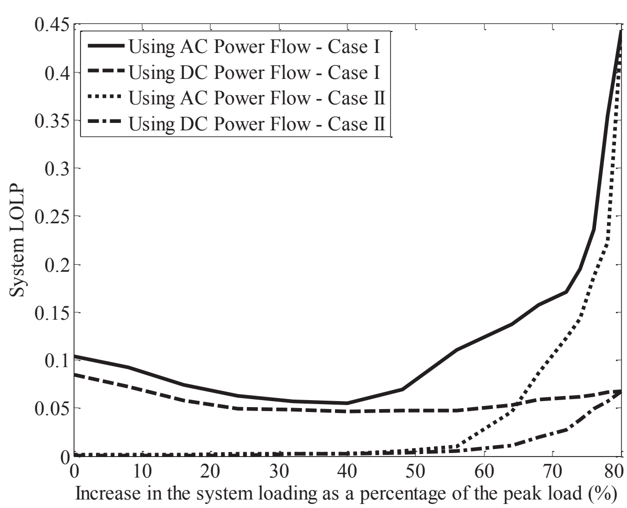


Fig. 3. System loss of load probability (LOLP) index profile against the change in the system loading level

From Fig. 3 it is obvious that the DC power flow model closely depicts the AC power flow model, although optimistic, until the load reaches around 148% of the base peak load level. After this point, the differences between the results of using the AC and DC power flow models are significant. Using the DC power flow model, the LOLP index is much smaller than using the AC power flow model. Since the AC power flow model is more accurate than the DC power flow model, the results of the DC power flow are optimistic. In other words, while some states are recognized as failure states by using the AC power flow model, they are recognized by the DC power flow model as success states. These failure states usually caused by voltage and/or reactive power limit violations which would not be captured if the DC power flow model were used.

### *C. Effects of Voltage and Reactive Power Limits on the Reliability Indices*

From the above results, after a certain loading level, the two power flow models produce significantly different results. Several factors may cause this difference. The effects of the voltage and reactive power constraints on the loss of load probability index are investigated through tracking the contributions of these constraints and the estimation of the proposed indices of the voltage and reactive power limits.

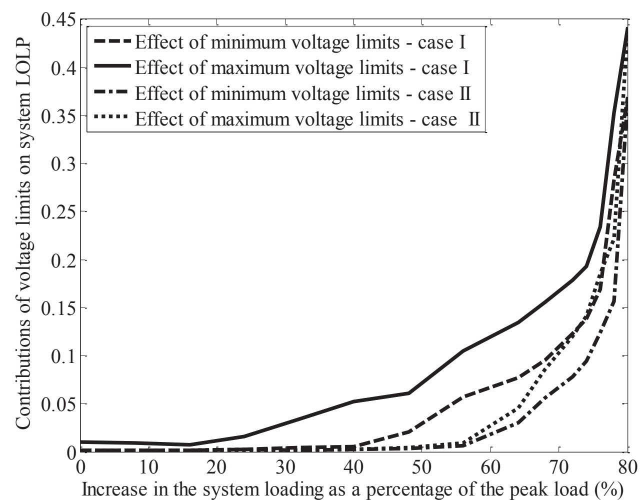


Fig. 4. Contributions of the voltage limit violations on the system loss of load probability (LOLP) index against the change in the system loading level

*1) Contributions of the Voltage and Reactive Power Constraints:* The contributions of the voltage limit violations on the system loss of load probability index against the change in the system loading level are shown in Fig. 4 and the contributions of the reactive power limit violations on the system loss of load probability index against the change in the system loading level are shown in Fig. 5.

The results totally agree with the observations of the sharp increase in the loss of load probability and loss of load frequency indices using AC power flow model as it is shown in the subsections VII-A1 and VII-A2. The contributions of the voltage and reactive power violations are very small until

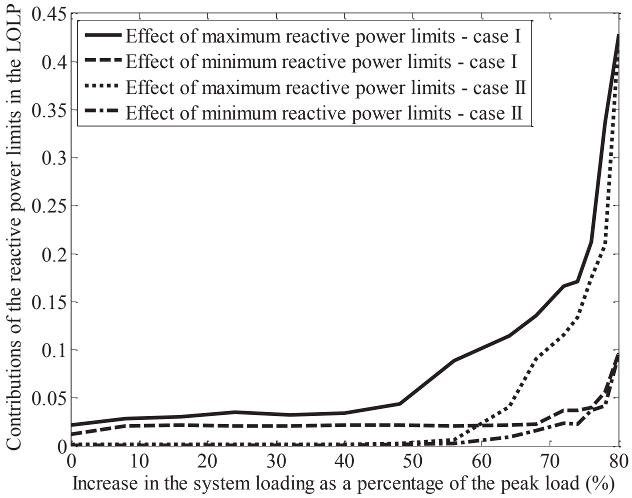


Fig. 5. Contributions of the reactive power limit violations on the system loss of load probability (LOLP) index against the change in the system loading level

the load level reaches about 148% of the original load level for the Case Study I and 172% for the Case Study II. Also, if the contributions of the voltage and reactive power violations are subtracted from the loss of load probability index, both models would produce similar results.

*2) Indices of the Voltage and Reactive Power Limits:* During the simulation, if the voltage or reactive power limits contributes in the load curtailment, the related index is updated according to (16). The results of these indices of the Modified IEEE RTS are shown in Table V. From Table V, it is clear that the maximum voltage limit at bus 2 has contributed in almost all the failure states (the probability of the contribution of the maximum voltage limit at bus 2 is 0.99643 or 99.64%). Maximum voltage limits at buses 7, 13, 21 and 23 have contributed in around 85.30%, 78.75%, 82.10% and 82.51% of the failure states, respectively. Minimum voltage limit at bus 6 has contributed in around 83.35% of the failure states. Maximum reactive power limit at buses 2, 3 and 15 have contributed in around 84.20%, 96.75% and 91.01% of the failure states, respectively. The minimum reactive power limit at bus 6 has contributed in around 15.43% of the failure states.

One of the possible solutions to avoid minimum voltage limit and maximum reactive power limit violations is to install condensers at these buses. These condensers will reactive power to the system so that voltages at the buses will be maintained within the specified limits. Also, one possible solution to maximum voltage limit and minimum reactive power limit violations is absorb the excess reactive power by means of reactors.

*3) Relaxing Voltage Constraints:* In this part, the voltage limit constraints are relaxed to investigate the effects of these constraints on the reliability indices of the Modified IEEE RTS. Fig. 6 shows the profile of the loss of load probability index with relaxing the voltage constraints. The results of the system indices with relaxing voltage constraints are given in Table VI. From Fig. 6 and Table VI it is clear that relaxing voltage limit constraints significantly reduces system indices. However, even with relaxing these constraints, the values of these indices still high in comparison with DC power flow counterpart. This can be related to the fact that the DC power flow model ignores the losses in the lines and in the case of the AC power flow, voltage constraints have been relaxed without increasing the reactive power support.

TABLE V
CONTRIBUTIONS OF VOLTAGE AND REACTIVE POWER CONSTRAINTS

| Bus No | $vi_{max}$ | $vi_{min}$ | $\varrho i_{max}$ | $\varrho i_{min}$ |
|---|---|---|---|---|
| 1 | 0.15438 | 0.00000 | 0.00684 | 0.00004 |
| 2 | 0.99643 | 0.00000 | 0.84201 | 0.00000 |
| 3 | 0.00059 | 0.00941 | 0.96751 | 0.00000 |
| 4 | 0.00002 | 0.00016 | – | – |
| 5 | 0.00002 | 0.00007 | – | – |
| 6 | 0.00011 | 0.83354 | 0.00309 | 0.15427 |
| 7 | 0.85299 | 0.00000 | 0.03379 | 0.00000 |
| 8 | 0.00000 | 0.04804 | – | – |
| 9 | 0.00011 | 0.00787 | – | – |
| 10 | 0.04770 | 0.00000 | – | – |
| 11 | 0.00000 | 0.00034 | – | – |
| 12 | 0.00000 | 0.00598 | – | – |
| 13 | 0.78751 | 0.00000 | 0.12792 | 0.00000 |
| 14 | 0.00259 | 0.00435 | 0.03030 | 0.00002 |
| 15 | 0.18674 | 0.00000 | 0.91008 | 0.00015 |
| 16 | 0.07582 | 0.00050 | 0.19863 | 0.08940 |
| 17 | 0.00011 | 0.00000 | – | – |
| 18 | 0.20134 | 0.00000 | 0.08019 | 0.00000 |
| 19 | 0.00018 | 0.00000 | – | – |
| 20 | 0.00050 | 0.00000 | – | – |
| 21 | 0.82095 | 0.00000 | 0.32305 | 0.00000 |
| 22 | 0.29347 | 0.00007 | 0.00003 | 0.00002 |
| 23 | 0.82510 | 0.00402 | 0.02226 | 0.00007 |
| 24 | 0.00029 | 0.04100 | – | – |

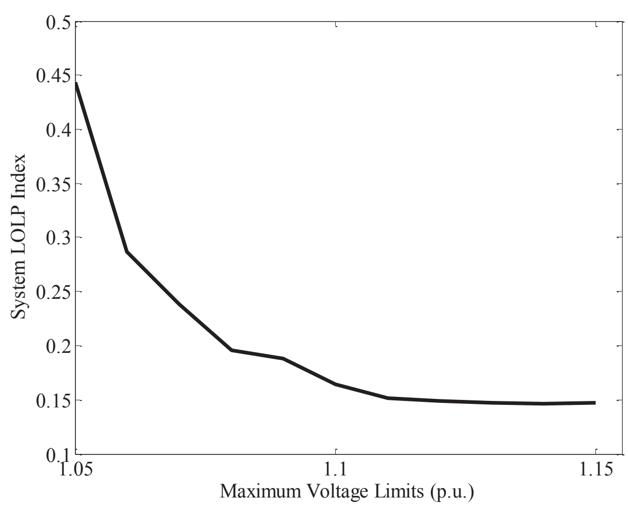


Fig. 6. The profile of system loss of load probability (LOLP) index with relaxing voltage limit constraints

TABLE VI
RELIABILITY INDICES WITH RELAXING VOLTAGE LIMIT CONSTRAINTS

| Voltage Max. | Voltage Min. | $\hat{q}$ | $\hat{\rho}$ MW | $\hat{\phi}$ occ./yr | $\hat{\tau}$ hour |
|---|---|---|---|---|---|
| 1.05 | 0.95 | 0.44341 | 19.63547 | 62.98836 | 0.00704 |
| 1.06 | 0.94 | 0.28647 | 18.52728 | 58.25106 | 0.00492 |
| 1.07 | 0.93 | 0.23826 | 18.10026 | 45.03072 | 0.00440 |
| 1.08 | 0.92 | 0.19594 | 17.13641 | 44.27397 | 0.00443 |
| 1.09 | 0.91 | 0.18810 | 16.55377 | 42.44361 | 0.00443 |
| 1.10 | 0.90 | 0.16416 | 16.71187 | 36.94868 | 0.00444 |
| 1.11 | 0.89 | 0.15108 | 16.40434 | 33.24840 | 0.00454 |
| 1.12 | 0.88 | 0.14926 | 16.21602 | 33.35913 | 0.00447 |
| 1.13 | 0.87 | 0.14719 | 15.83632 | 32.79869 | 0.00449 |
| 1.14 | 0.86 | 0.14642 | 15.60547 | 32.05549 | 0.00457 |
| 1.15 | 0.85 | 0.14687 | 15.68066 | 32.06075 | 0.00458 |

### D. *Sensitivity Analysis*

The results of the sensitivity analyses of the Severity Index with respect to the voltage and reactive power constraints are shown in Table VII. The following points can be observed from the sensitivity analyses: (a) bus 2 has the highest effect in terms of maximum voltage limit, therefore, the planner may consider installing a condenser at bus 2, (b) the minimum voltage limit at bus 6 has the highest effect on the Severity Index, therefore, the planner may consider installing a reactor, (c) bus 3 has the highest effect in terms of maximum reactive power limit, and (d) bus 6 has the highest effect in terms of minimum reactive power limit. It should be noted that the voltage sensitivity analyses are based on the p.u. values.

## VIII. CONCLUSION

In this paper, we have investigated the effects of the voltage and reactive power constraints on power system reliability through developing expressions and performing sensitivity analyses of the severity index with respect to these constraints. Also, four indices have been proposed to address the contributions of the voltage and reactive power constraints on system load curtailments. Extensive studies on the IEEE RTS and the Modified IEEE RTS have been conducted to investigate the effects of the voltage and reactive power constraints on the power system reliability indices. Several case studies have been performed through gradually stressing the transmission lines to track the profile of the reliability indices with increasing system loading level. A comparison between the use of the AC and DC power flow models in the composite system reliability studies are provided to test the effects of ignoring the voltage and reactive power constraints. The results show that, for the cases where the system is stressed, the accuracy of the DC power flow in evaluating the reliability indices is deteriorated. Also, a state space pruning technique has been used to reduce the computation time and burden.

## APPENDIX
## THE CONSTRAINTS OF THE OPTIMISATION PROBLEM

The constraints of the optimisation problem of (1) are: the power balance equations, generation capacity limits, transmission lines power carrying capabilities, allowable voltage limits and generation reactive power limits. The explanations of incorporating theses constraints to the optimisation problem are as follows,

TABLE VII
SENSITIVITY ANALYSES OF THE SEVERITY INDEX WITH RESPECT TO THE VOLTAGE AND REACTIVE POWER CONSTRAINTS

| Bus No | Sensitivity of the Severity Index with respect to: | | | |
|---|---|---|---|---|
| | $V_{max}$ | $V_{min}$ | $Q_{max}$ | $Q_{min}$ |
| 1 | 0.32604 | 0.00000 | 0.00003 | 0.00001 |
| 2 | 1.35209 | 0.00000 | 0.01412 | 0.00000 |
| 3 | 0.00002 | 0.17834 | 0.14772 | 0.00000 |
| 4 | 0.00000 | 0.00431 | – | – |
| 5 | 0.00000 | 0.00102 | – | – |
| 6 | 0.00005 | 10.3881 | 0.02302 | 0.16863 |
| 7 | 0.11922 | 0.00000 | 0.00088 | 0.00000 |
| 8 | 0.00000 | 0.10789 | – | – |
| 9 | 0.00000 | 0.02155 | – | – |
| 10 | 0.02656 | 0.00000 | – | – |
| 11 | 0.00000 | 0.00126 | – | – |
| 12 | 0.00000 | 0.03430 | – | – |
| 13 | 0.31398 | 0.00000 | 0.00579 | 0.00004 |
| 14 | 0.00745 | 0.00453 | 0.00120 | 0.00001 |
| 15 | 0.09631 | 0.00000 | 0.02306 | 0.00000 |
| 16 | 0.06129 | 0.00095 | 0.00106 | 0.00164 |
| 17 | 0.00000 | 0.00000 | – | – |
| 18 | 0.06080 | 0.00000 | 0.00071 | 0.00047 |
| 19 | 0.00000 | 0.00000 | – | – |
| 20 | 0.00000 | 0.00000 | – | – |
| 21 | 0.81786 | 0.00000 | 0.00460 | 0.00000 |
| 22 | 0.14240 | 0.00000 | 0.00000 | 0.00000 |
| 23 | 0.51735 | 0.00125 | 0.00000 | 0.00005 |
| 24 | 0.00056 | 0.25576 | – | – |

### POWER BALANCE CONSTRAINTS

The power balance equation is an equality equation represents the sum of the complex power at a bus. The injected real and reactive power at each bus can be expressed as,

$$P_i = V_i \sum_{j \in i} V_j \left( G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij} \right)$$

$$Q_i = V_i \sum_{j \in i} V_j \left( G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij} \right)$$

where $j \in i$ means the set of the buses connected to bus $i$, $P_i$ and $Q_i$ are the real and reactive power injected at bus $i$, $V_i$ and $V_j$ are the voltage magnitudes at buses $i$ and $j$, $G_{ij}$ is the conductance between buses $i$ and $j$, $B_{ij}$ is the susceptance between buses $i$ and $j$ and $\delta_{ij}$ is the angle difference between voltages of buses $i$ and $j$ ($\delta_{ij} = \delta_i - \delta_j$).

At each bus, the amount of injected power should equal to the load at that bus. If the amount of the injected power is less than the load, then load curtailment should be performed. A fictitious generator is added at each load bus which represent the amount of load curtailment. Since $P$ and $Q$ are functions of $V$ and $\delta$, then the balance equations with the fictitious generators are as follows as given in the first two constraints of (2).

$$P(V,\delta) - P_D + C = 0$$

$$Q(V,\delta) - Q_D = 0$$

## Real and Reactive Power Constraints of the Generators

Real power limits of the generators are bounded by maximum available capacity and minimum power where the later is considered zero. Reactive power limits are the maximum reactive power provided by a generator and the minimum reactive power assigned for each generator. Therefore, the constraints of real and reactive power of the generators can be expressed as,

$$\begin{aligned} 0 \leq P_G \leq P_G^{max} \\ Q_G^{min} \leq Q_G \leq Q_G^{max} \end{aligned} \tag{24}$$

where $P_G^{max}$ is the maximum available capacity for each generator, $Q_G^{min}$ is minimum reactive power can be absorbed by a generator and $Q_G^{max}$ is the maximum reactive power can be produced by a generator.

## Voltage Limit Constraints

Voltage constraints are limited according to the allowed voltage fluctuations. The maximum voltage limit, $V^{max}$, and the minimum voltage limit, $V^{min}$, which are assumed throughout this work as 1.05 p.u. and 0.95 p.u. respectively, are expressed as follows,

$$V^{min} \leq V \leq V^{max}. \tag{25}$$

## Line capacity Limits

Power flow through a transmission line connecting buses $i$ and $j$ can be calculated as follows,

$$P_{ij} = \frac{R_{ij}}{Z_{ij}^2} V_i^2 - V_j V_m \left( \frac{R_{ij}}{Z_{ij}^2} \cos\delta_{ij} - \frac{X_{ij}}{Z_{ij}^2} \sin\delta_{ij} \right)$$

$$Q_{ij} = -V_i^2 \left( \frac{B}{2} - \frac{X_{ij}}{Z_{ij}^2} \right) - V_i V_j \left( \frac{X_{ij}}{Z_{ij}^2} \cos\delta_{ij} + \frac{R_{ij}}{Z_{ij}^2} \sin\delta_{ij} \right)$$

where $R_{ij}$, $X_{ij}$, $Z_{ij}$ and $B$ are the resistance, reactance, impedance and shunt admittance of the line respectively.

The expression of transmission line power flow constraint is,

$$|S_{ij}| = \sqrt{P_{ij}^2 + Q_{ij}^2} \leq |S_{ij}^{max}|.$$

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