### A SYSTEM STATE TRANSITION SAMPLING METHOD FOR COMPOSITE SYSTEM RELIABILITY EVALUATION

R. Billinton
Power Systems Research Group
Department of Electrical Engineering
University of Saskatchewan
Saskatoon, Sask., Canada

W. Li Regional System Planning System Planning Division B.C. Hydro Vancouver, B.C., Canada

### ABSTRACT

This paper presents a new Monte Carlo simulation technique for composite system reliability evaluation. The method is based on a system state transition sampling approach which can be used to calculate an actual frequency index without requiring an additional enumeration procedure and the associated approximate assumptions. The approach presented in this paper does not require sampling the up and down component cycles and storing chronological information of system states. Generating unit derated states can also be easily included without additional calculations. The effectiveness of the method presented is demonstrated by application to the IEEE Reliability Test System. A comparison study between a state sampling based technique and the proposed method is presented using the IEEE Reliability Test System.

### INTRODUCTION

Considerable attention has been devoted to composite generation and transmission system reliability evaluation during the last 20 years [1-3]. There are two fundamental approaches to composite system reliability evaluation: analytical enumeration and Monte Carlo simulation. Monte Carlo methods are more flexible when complex operating conditions and system considerations ( such as reservoir operating rules, bus load uncertainty and correlation , regional weather effects, etc.) need to be incorporated and a number of Monte Carlo based approaches have been documented [4-12].

There are two basic techniques utilized when Monte Carlo methods are applied to power system reliability evaluation. These methods are known as the state duration sampling (sequential) [12] or state sampling (non-sequential) [5-11] techniques. In the state duration sampling technique, the up and down cycles of all components are simulated first and a system state operating cycle is then obtained by combining all the component cycles. This technique has many advantages, such as, chronological issues can be considered and distributions of reliability indices can be calculated, etc. The disadvantages of this technique are large memory storage and computational requirements. Its application to practical composite system analysis may also be limited due to required CPU time. In the state sampling technique, the states of all components are sampled and a non-chronological system state is obtained. This technique requires much

92 SM 426-7 PWRS A paper recommended and approved by the IEEE Power System Engineering Committee of the IEEE Power Engineering Society for presentation at the IEEE/PES 1992 Summer Meeting, Seattle, WA, July 12-16, 1992. Manuscript submitted June 6, 1992; made available for printing June 4, 1992. less memory and CPU time compared to state duration sampling. Most of the Monte Carlo methods proposed for composite system reliability evaluation can be designated as non-sequential state sampling techniques. A major disadvantage of the state sampling technique is the difficulties associated with attempting to calculate frequency and duration (F&D) related indices. This is due to the fact that the frequency calculation related to a failure state requires the recognition of all no-load-curtailment states which can be reached from the failure state in one transition. This can not be achieved using the state sampling technique by itself. It is also necessary to add an enumeration procedure. In other words, if a failure state is associated with n components, each of which is modeled by two states, n+1 adequacy evaluations including load flow calculations, corrective actions, etc. are required to update the frequency estimation. If generating unit derated states are considered, the number of additional evaluations increases dramatically. In the case of a large size composite system, this task is very difficult and in many cases is not computationally feasible.

The conventional approach in composite system adequacy evaluation is to calculate the Expected Number of Load Curtailments (ENLC) rather than the actual frequency index [13]. The index ENLC is the sum of the occurrences of load curtailment states and therefore is a upper bound on the frequency index. Unfortunately, in many cases, ENLC based F&D indices are considerably different from the actual frequency based F&D indices.

Reference 11 presents a method to decrease the number of adequacy evaluations in the enumeration procedure associated with the frequency calculation for a failure state. The method uses either a sensitivity-based prediction to calculate a better upper bound of the frequency index than ENLC or "filters" to reduce the number of transitions to be tested. This method does not avoid the additional enumeration but decreases the number of additional evaluations required in order to obtain the actual frequency index. The required computational effort still depends on the number of no-load-curtailment states which can be reached from a failure state in one transition. When generating unit derated states are considered, this number can increase considerably.

This paper presents a new Monte Carlo simulation method for composite system reliability evaluation. The method is based on a system state transition sampling technique which creates directly a system state transition sequence. It can therefore be used to calculate the actual frequency index without requiring an additional enumeration procedure. It does not require sampling component up and down cycles and storing chronological information on the system state. Generating unit derated states can be easily included without additional calculations. An important restriction in the method is the assumption of exponential distributions for all state residence

0885-8950/93\$03.00 © 1992 IEEE

times. The proposed method is, however, a useful tool for composite system adequacy assessment.

### SYSTEM STATE TRANSITION SAMPLING

This technique focuses on state transitions of whole system rather than on component states or component state durations. Assume that a system contains m components and that the state duration of each component follows an exponential distribution. The system can experience a system state transition sequence  $\{S^{(1)},\ldots,S^{(n)}\}=G$ , where G is the system state space. Suppose that the present system state is  $S^{(k)}$  and the transition rates of the components related to  $S^{(k)}$  are  $\lambda_i$  (i=1,...m). The state duration  $T_i$  of the ith component corresponding to system state  $S^{(k)}$  therefore has the probability density function:  $f_i(t)=\lambda_i$   $\exp(-\lambda_i t)$ . Transition of the system state depends randomly on the state duration of the component which departs earliest from its present state, i.e., the duration T of the system state  $S^{(k)}$  is a random variable which can be expressed by

$$T = \min_{i} \{ T_{i} \}$$
 (1)

It can be shown that since the state duration  $\mathbf{T}_i$  of each component follows an exponential distribution with parameter  $\lambda_i$ , the random variable  $\mathbf{T}$  also follows an exponential distribution with the parameter  $\lambda=\Sigma$   $\lambda_i$  i.e.,  $\mathbf{T}$  has the probability density function i=1

$$f(t) = \sum_{i=1}^{m} \lambda_{i} \exp(-\Sigma \lambda_{i} t)$$
 (2)

Assume that transition of the system state from  $s^{(k)}$  to  $s^{(k+1)}$  takes place at instant  $t_0$ . The probability that this transition is caused by departure of the jth component from its present state is the conditional probability:  $P_j = P(T_j = t_0/T = t_0)$ . According to the definition of conditional probability and Equation (1), it follows that

$$P_{j} = P(T_{j} = t_{0}/T = t_{0})$$

$$= P(T_{j} = t_{0}/T = t_{0})/P(T = t_{0})$$

$$= P(T_{j} = t_{0} \cap (T_{i} \ge t_{0}, i = 1, ..., m))/P(T = t_{0})$$

$$= P(T_{j} = t_{0}) \prod_{\substack{i=1 \ i \ne i}}^{m} P(T_{i} \ge t_{0})/P(T = t_{0})$$
(3)

Since both  $\textbf{T}_{\underline{\textbf{i}}}(\textbf{i=1},\ldots,\textbf{m})$  and T follow an exponential distribution,

$$P(T_i \geqslant t_0) = \int_{t_0}^{\infty} \lambda_i e^{-\lambda_i t} dt = e^{-\lambda_i t_0}$$
 (4)

$$P(T_{j} = t_{0}) = \lim_{\Delta t \to 0} \lambda_{j} e^{-\lambda_{j} t_{0}} \Delta t$$
 (5)

and

$$P(T = t_0) = \lim_{\Delta t \to 0} (\sum_{i=1}^{m} \lambda_i e^{-i\frac{\Sigma}{2} \lambda_i t_0}) \Delta t$$
 (6)

Substituting Equation (4), (5) and (6) into Equation (3) yields

$$P_{j} = P(T_{j} = t_{0}/T = t_{0}) = \lambda_{j}/i \sum_{i=1}^{m} \lambda_{i}$$
 (7)

State transition of any component in the system can lead to a system state transition. Consequently, starting from state  $\mathbf{S}^{(k)}$ , a system containing m components has m possible reached states. The probability that the system reaches one of these possible states is given by Equation (7) and obviously

$$\sum_{j=1}^{m} P_{j} = 1$$
(8)

The next system state can be determined by the following simple sampling. The probabilities of m possible reached states are successively placed in the interval  $\{0,1\}$  as shown in Figure 1. A uniformly distributed random number U between  $\{0,1\}$  can be generated. If U falls into the segment corresponding to  $P_j$ , the transition of the jth component leads to the next system state. A long system state transition sequence can be obtained by drawing a number of samples and the consequences of each system state can be evaluated using the minimization model given in the following section.

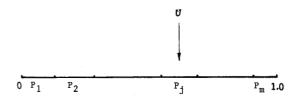


Figure 1 Explanation of the system state transition sampling

The system state transition sampling technique described above is quite general. It can be applied to a generation system, a composite system or to any basic engineering system reliability evaluation. The major advantage of this sampling technique is that it can be used to calculate an actual frequency index by creating a system state transition chain. It does not require sampling state duration distribution functions of all the components and the storage of chronological information as required in state duration sampling on an additional enumeration procedure as required in state sampling in order to calculate an actual frequency index.

METHODOLOGY OF COMPOSITE SYSTEM ADEQUACY ASSESSMENT

## 1. Basic Steps

Composite system adequacy assessment can be summarized in the following steps:

- The simulation process starts from the normal system state in which all generating units and transmission components are in the up state.
- (2) Let  $\lambda_i$  be the transition rate that one component departs its present state. If the component is in the up state,  $\lambda_i$  is its failure rate and if the component is in the down state,  $\lambda_i$  is its repair rate. If multistates of a component are considered (such as derated states of a generating unit), there

are several departure rates. The duration of the system state has a probability density function shown in Equation (2) and therefore the sampling value  $\mathbf{D}_{\mathbf{k}}$  of the duration of the present system state (numbered by k) can be obtained by [14]

$$D_{k} = -\ln \left( \frac{m}{\sum_{i=1}^{m} \lambda_{i}} \right)$$
 (9)

where m is the number of departure rates corresponding to the present system state k and  $\cup'$  is a uniformly distributed random number between [0,1].

- (3) If the present system state is a contingency state in which at least one component is in the outage or the derated state, the minimization model of load curtailment described later in this paper is used to evaluate the adequacy for this system state. If the present system state is the normal state in which all components are in up states, it is possible to proceed directly to Step (4) without utilizing the minimization model.
- (4) The adequacy indices are updated using the equations 16, 17 and 18 given later in this paper. If the coefficient of variation of the EDNS (Expected Demand Not Supplied) index is less than a given tolerant error or the specified number of system state samples is reached, the simulation process is terminated. If not, proceed to Step (5).
- (5) A uniform distribution random number U is generated to determine the next system state using the system state transition sampling technique described earlier. Return to Step (2).

### 2. Minimization Model Of Load Curtailment

For each contingency state selected using the system state transition sampling technique, the following minimization model of load curtailment is used to reschedule generation outputs in order to maintain generation—demand balance and alleviate line overloads and, at the same time, to avoid load curtailment if possible or to minimize total load curtailment if unavoidable:

$$\min_{i \in NC} C = \sum_{\substack{i \in NC}} (W_i \sum_{j=1}^{m_i} \beta_j C_{ij})$$
 (10)

s.t. 
$$PT_n = \sum_{k=1}^{N} A_{nk} (PG_k + \sum_{j=1}^{m_i} C_{kj} - PD_k)$$
 (11)

$$PG_i^{min} \leq PG_i \leq PG_i^{max}$$
 (i=1,...,NG) (13)

$$0 \le C_{ij} \le \alpha_j PD_i$$
 (iGNC; j=1,...,m) (14)

$$|PT_n| \leq PT_n^{max}$$
 (n=1,...,L) (15)

where  $C_{ij}$  is the jth load curtailment subvariable at

Bus i;  $PG_i$  and  $PD_i$  are the generation variable and the load demand at Bus i respectively;  $PT_n$  is the line flow on Line n;  $PG_i^{min}$ ,  $PG_i^{max}$  and  $PT_n^{max}$  are limit values, respectively, of  $PG_i$  and  $PT_n$ ;  $A_{nk}$  is the element of the relation matrix between line flows and power injections; NC and NG are the sets of all load buses and all generator buses respectively; L and N are the numbers of the lines and buses respectively;  $m_i$  is the number of load curtailment subvariables at Bus i;  $\alpha_j$  are the load percentages associated with each subvariable;  $\beta_j$  are the weighting factors corresponding to subvariables;  $W_i$  are weighting factors corresponding to each bus load.

Selection principle of  $\beta_j$  and  $W_i$  and detailed information on the minimization model can be found in Reference 7.

### 3. Evaluation Of Adequacy Indices

The total load curtailment C for the system state k can be obtained by solving the minimization model. If C equals zero, the system state is a no-load-curtailment state. If not, it is a failure state. Three basic system unreliability indices can be calculated using the following equations.

(1) Expected Demand Not Supplied (MW) --- EDNS

$$EDNS = \sum_{k=1}^{NK} C_k D_k / TD$$
 (16)

where  $C_k$  and  $D_k$  (hrs) are the system load curtailment and the duration for system state k; NK is the number of load curtailment system states; TD (hrs) is the sum of the durations of all system states in a long system state transition sequence.

(2) Expected Frequency of Load Curtailment (occ./year) —— EFLC

$$EFLC = NF \times 8760 / TD$$
 (17)

where NF is the number of occurrences of transition from a failure state to a no-load-curtailment state in the system state transition sequence.

(3) Probability of Load Curtailment --- PLC

$$PLC = \sum_{k=1}^{NK} D_k / TD$$
 (18)

Bus indices can be obtained using the similar equations since the minimization model also provides load curtailments at each bus for all drawn system states. Other unreliability indices can be calculated from these three basic indices. The following system indices are also given in the case studies presented in addition to the three basic indices.

EENS Expected Energy Not Supplied (MWh/year)

ADLC Average Duration of Load Curtailment (hrs/disturbance)

EDLC Expected Duration of Load Curtailment
 (hrs/year)

BPII Bulk Power Interruption Index (MW/MW-year)

BPECI	Bulk Power/Energy C (MWh/MW-year)	Curtailme	ent Index
BPACI	Bulk Power-supply Avera Index (MW/disturbance)	ge MW	Curtailment
MBECI	Modified Bulk/Energy Curta	ailment :	Index
SI	Severity Index (system mir	nites)	

A comprehensive interpretation and the formulae for these indices can be found in References 13 and 15.

#### CASE STUDIES

The application of the proposed method is illustrated using the IEEE Reliability Test System (IEEE RTS) [16]. The system has 24 buses, 38 lines/transformers and 32 generating units. The annual system peak load is 2850 MW and total installed capacity is 3045 MW. The single line diagram and all data including the annual load data (8736 hourly load points) are given in Reference 16. The studies presented in this paper include the base case and a generating unit derated case. In both cases, annualized and annual indices are given. Annualized indices are calculated at the peak load and expressed on a base of one year. In the case of annual indices, the annual load data is considered by modelling the 8736 hourly load points by a 70 step load level curve [7]. System state transition sequences containing 10000 state samples for each load level are created using the system state transition sampling method. In Monte Carlo simulation, a selected coefficient of variation is a useful indicator of the degree of convergence. Calculations show that frequency indices converge faster than the EDNS index. The coefficient of variation of the EDNS index ranges from 0.03 to 0.05 in all the study cases. All calculations are conducted on a VAX-6300 computer.

### 1. Base Case

No generating unit derated states are considered in this case. The annualized and annual system and bus indices are given in Tables 1 to 3. It can be seen that there are quite large differences between the annualized indices considering only the peak load and the annual indices considering the variable load levels throughout the year. It is interesting to note that the differences in the two per-disturbance frequency related indices (ADLC and BPACI) are much smaller than those of other indices. The calculation of annualized indices requires much less CPU time and these indices can prove useful in a wide range of studies such as comparing the relative adequacies of different systems or changes in system configuration or composition. Annual indices can be utilized for more comprehensive purposes, e.g., when reliability worth is considered in power system planning.

Table 1 The Annualized and Annual System Indices for the IEEE RTS in the Base Case

Index	Annualized	Annual
PLC EDLC EFLC ADLC	0.08434 736.77460 19.57152 37.64524	0.00119   10.36991   0.39417   26.30801
EDNS EENS BPII BPECI BPACI MBECI SI	14.44465 126188.45313 3.60198 44.27665 524.52002 0.00507 2656.59863	0.14673 1281.79480 0.04640 0.50578 296.39874 0.00006 30.34701
CPU time   (minutes)	1.51	39.73

Table 2 The Annualized Bus Indices for the IEEE RTS in the Base Case

No.Bus	PLC	EFLC	EDNS
1	0.00217	0.83179	0.04664
2	0.00228	1.02750	0.04338
3	0.00286	1.17429	0.08775
4	0.00387	1.46786	0.05557
5	0.00427	1.56572	0.05618
6	0.00529	2.00608	0.12514
7	0.00531	2.05501	0.01621
8	0.00854	2.78894	0.24391
9	0.01022	3.66966	0.31988
10	0.01239	4.20788	0.44179
13	0.01801	6.06717	0.78633
14	0.02905	7.63289	0.94747
15	0.04231	9.29647	2.42554
16	0.04531	10.07933	0.89705
18	0.05506	13.89578	3.37406
19	0.07450	17.66329	2.45133
20	0.08434	19.57152	2.12643

Table 3 The Annual Bus Indices for the IEEE RTS in the Base Case

No.Bus	PLC	EFLC	EDNS
1	0.00002	0.01518	0.00045
2	0.00003	0.01950	0.00052
3	0.00004	0.01851	0.00118
4	0.00006	0.02321	0.00068
5	0.00002	0.01069	0.00028
6	0.00003	0.01305	0.00071
7	0.00002	0.00972	0.00047
8	0.00003	0.01418	0.00086
9 .	0.00009	0.03730	0.00257
10	0.00012	0.04984	0.00381
13	0.00020	0.07477	0.00782
14	0.00027	0.10290	0.00821
15	0.00043	0.15914	0.01948
16	0.00049	0.17973	0.00821
18	0.00078	0.27737	0.03735
19	0.00101	0.34300	0.02894
20	0.00119	0.39461	0.02519

## 2. Generating Unit Derated Case

In this case, the two 400 MW and the 350 MW generating units in the IEEE-RTS have been given a derated state of 50% capacity using the derated state data given in Reference 17. The state probabilities of the derated models are such that the derating-adjusted two-state model data is identical to that used in the base case. The annualized and annual system and bus indices considering the generating unit derated states are given in Tables 4 to 6. It can be seen that utilization of derating-adjusted two-state models can leads to a pessimistic evaluation of system adequacy. The method presented does not require an increase in computing time when considering generating unit derated states.

### 3. Comparison Between The Two Monte Carlo Methods

Almost all the Monte Carlo methods proposed for composite system adequacy evaluation use the state sampling technique [2-11]. As noted earlier, this technique can not be used by itself to calculate an actual frequency index. The technique presented in this paper creates a system state transition sequence and can be used to calculate an actual frequency index. It is important to consider and to appreciate the differences in the results obtained using these two different sampling techniques. The annualized and

annual system indices for the base case IEEE RTS using the method presented in this paper are compared with those obtained using the method described in Reference 7 in Tables 7 and 8. The \* flag indicates the three frequency related indices. In the method of Reference

Table 4 The Annualized and Annual System Indices for the IEEE RTS in the Generating Unit Derated Case

Index	Annualized	Annual
PLC	0.06784	0.00080
EDLC	592.67310	6.97203
EFLC	17.73042	0.30638
ADLC	33.42691	22.75599
EDNS	10.31458	0.09089
EENS	90108.20313	794.00793
BPII	2.72068	0.03337
BPECI	31.61691	0.31221
BPACI	437.32452	274.59671
MBECI	0.00362	0.00004
SI	1897.01465	18.73249
CPU time   (minutes)	1.47	35.20

Table 5 The Annualized Bus Indices for the IEEE RTS in the Generating Unit Derated Case

No.Bus	PLC	EFLC	EDNS	
1 .	0.00138*	0.69011	0.02977	
2	0.00146	0.74319	0.02749	
3	0.00196	0.79628	0.05794	
4	0.00233	0.90245	0.03251	
5	0.00340	0.90245	0.03626	
6	0.00426	1.16787	0.10440	
7	0.00433	1.27404	0.02116	
8	0.00651	2.07032	0.19142	
9	0.00807	2.97276	0.24290	
10	0.00889	3.60979	0.33023	
13	0.01154	4.45915	0.53226	
14	0.01838	6.10478	0.58483	
15	0.02906	8.59978	1.56789	
16	0.03114	9.07755	0.62075	
18	0.04139	12.52808	2.37746	
19	0.05863	15.28850	1.86892	
20	0.06784	17.73042	1.68838	

Table 6 The Annual Bus Indices for the IEEE RTS in the Generating Unit Derated Case

No.Bus	PLC	EFLC	EDNS
1	0.00003	0.01807	0.00044
2	0.00003	0.02083	0.00049
3	0.00003	0.01870	0.00084
4	0.00004	0.02407	0.00053
5	0.00001	0.00660	0.00012
6	0.00001	0.00881	0.00028
7	0.00000	0.00127	0.00003
8	0.00001	0.00656	0.00028
9	0.00006	0.02918	0.00142
10	0.00008	0.04112	0.00228
13	0.00010	0.05261	0.00397
14	0.00016	0.07725	0.00471
15	0.00025	0.12079	0.01154
16	0.00030	0.13646	0.00486
18	0.00052	0.21417	0.02355
19	0.00066	0.26728	0.01903
20	0.00079	0.30851	0.01653

7, the Expected Number of Load Curtailment (ENLC), i.e., the sum of occurrences of load curtailment states, is calculated as a surrogate for the actual frequency index (EFLC) and therefore the ADLC and BPACI are also ENLC-based indices. In both methods, the number of samples at each load level is 10000. The same 70-step load level model is used in each case to calculate the annual indices.

Table 7 The Annualized System Indices for the IEEE RTS Using the Two Monte Carlo Methods

Index	Presented Method	Method in Ref. 7	Difference
PLC	0.08434	0.08000	5.1
EDLC	736.77460	698.88000	5.1
EFLC	19.57152	54.75342	179.7
		(ENLC)	i
ADLC	37.64524	12.76413	194.9
EDNS	14.44465	13.97045	3.3
EENS	126188.45313	122045.88281	3.3
BPII	3.60198	3.33652	7.4
BPECI	44.27665	42.82311	3.3
BPACI	524.52002	173.67123	202.0
MBECI	0.00507	0.00490	3.4
SI	2656.59863	2569.38672	3.3
i			
PU time	1.51	0.62	1
minutes)			
	PLC EDLC EFLC  ADLC EENS EENS BPII BPECI BPACI MBECI SI PU time	Method  PLC 0.08434 EDLC 736.77460 EFLC 19.57152  ADLC 37.64524 EDNS 14.44465 EENS 126188.45313 BPII 3.60198 BPECI 44.27665 BPACI 524.52002 MBECI 0.00507 SI 2656.59863  PU time 1.51	Method         Ref. 7           PLC         0.08434         0.08000           EDLC         736.77460         698.88000           EFLC         19.57152         54.75342           (ENLC)         37.64524         12.76413           EDNS         14.44465         13.97045           EENS         126188.45313         122045.88281           BPII         3.60198         3.33652           BPECI         44.27665         42.82311           BPACI         524.52002         173.67123           MBECI         0.00507         0.00490           SI         2656.59863         2569.38672           PU time         1.51         0.62

Table 8 The Annual System Indices for the IEEE RTS Using the Two Monte Carlo Methods

	Index	Presented   Method	Method in Ref. 7	Difference in %
	PLC	0.00119	0.00117	1.7
1	EDLC	10.36991	10.23695	1.3
i *	EFLC	0.39417 i	0.80854	105.1
i		,,,,,,	(ENLC)	
j⋆	ADLC I	26.30801	12.66103	108.2
i	EDNS	0.14673	0.13137	10.5
i	EENS	1281.79480	1147.61108	10.5
İ	BPII	0.04640	0.03574	22.9
i	BPECI	0.50578	0.45015	10.9
*	BPACI	296.39874	112.64307	163.1
i	MBECI	0.00006	0.00005	16.7
İ	SI	30.34701	27.00913	10.9
, -	CPU time   minutes)	39.73	15.97	

It can be seen that although the two methods utilize completely different sampling techniques, the inadequacy indices which are not associated with the frequency index are quite close. Additional calculations indicate that when the number of samples is increased, these indices become closer. On the other hand, however, the three frequency related indices (EFLC/ENLC, ADLC and BPACI) obtained using the two methods have very large differences. The results show that the utilization of ENLC as a surrogate for EFLC can lead to a considerable over estimation. The CPU time required by the method presented in this paper is larger than that required by the method in Reference 7. This is the price paid for calculating the actual frequency index. The method presented, however, requires less computing time than that required to incorporate an additional enumeration procedure for each drawn state to test possible transitions in the state sampling technique [11].

It is not possible to use a theoretical analytically exact method to compute the "true" frequency and duration indices of a composite generation and transmission system due to the

computational burden involved. The method presented in this paper is theoretically accurate given that enough samples are used. The accuracy can be estimated therefore by the coefficient of variation of the index in question.

It is difficult if not impossible to make a direct comparison of the CPU time required by the method presented in this paper and that prepared in Reference 11 due to the differences in computing systems, programming skills and the system applications. The method proposed in this paper involves sampling a system state while that of Reference 11 involves sampling a system state plus additional enumerations. A rough comparison can, however, be made as follows. Reference 11 gives CPU times of 3 min 47 sec (coherence assumption) and 19 min 50 sec (non-coherence assumption) on an IBM 4381 computer for the MRTS and a coefficient of variation of 0.03 for the LOLF index. In the method proposed in this paper, the CPU time was 1.51 min (Table 7) for the RTS on a VAX-6300 at an EDNS coefficient of variation of 0.03.

#### CONCLUSIONS

Most Monte Carlo methods proposed for composite system adequacy evaluation basically utilize the state sampling technique. This technique itself can not be used to calculate an actual frequency index.

This paper presents a new method for composite system adequacy evaluation. It creates a system state transition sequence and therefore can be used to calculate an actual frequency index. Compared to state duration sampling, this method does not require sampling component distribution functions and the storage of chronological information and therefore both required memory and CPU time can be decreased dramatically. Compared to the state sampling technique, this method does not require any conceptual approximation or an additional enumeration procedure.

Case studies with the IEEE RTS indicate that the method presented and the state sampling based method described in Reference 7 provide comparable values for those indices which are not frequency related. In the case of the frequency related indices, however, the ENLC, which is the sum of occurrences of load curtailment states, can be considered as an upper bound of the actual frequency index. Incorporation of generating unit derated states in the proposed method does not necessitate an increase in computing time. The method presented is a useful tool for frequency related reliability index evaluation of practical composite systems.

### REFERENCES

- [1] M.Th. Schilling, R. Billinton, A.M. Leite de Silva and M.A. El-Kady, "bibliography on Composite System Reliability (1964-1988)", IEEE Trans. on PS Vol.4, No.3, Aug. 1989
- [2] L. Salvaderi, R.N. Allan, R. Billinton, J. Endrenyi, D. Mcgills, M. Lauby, P. Manning and R.J. Ringlee, "State of the Art of Composite System Reliability Evaluation", 1990 CIGRE Meeting, WG38.03, Paris, Sept. 1990
- [3] IEEE tutorial Course Text, Reliability
  Assessment of Composite Generation and
  transmission Systems, IEEE Publishing services,
  90EH0311-1-PWR,1990
- [4] P. Noferi, L. Paris and L. Salvaderi, "Monte Carlo Methods for Power System Reliability Evaluation in transmission and Generation Planning", Proceedings 1975 Annual Reliability & Maintainability Symposium, Washington D.C., Tan. 1975, Paper 1294, p449-459

- [5] J.C. Dodu and A. Merlin, "An Application of Linear Programming to the Planning of Large Scale Power Systems: The MEXICO Model", Proceeding of the 5th PSCC, Paper 2219, Cambridge, England, Sept. 1975
- [6] EPRI Report, "Development of a Monte Carlo Composite System reliability Evaluation Programming, Report El-6926, Aug. 1990
- [7] R. Billinton and W. Li, "Hybrid Approach for Reliability Evaluation of Composite Generation and transmission Systems Using Monte Carlo Simulation and Enumeration Technique", IEE Proceeding-C, Vol. 138, No. 3, May, 1991, p233-241
- [8] W. Li and R. Billinton, "Effect of Bus Load Uncertainty and Correlation in Composite System Adequacy Evaluation", IEEE 1991 Winter Meeting, Paper 91WM175-0 PWRS, Feb. 1991, New York
- [9] R. Billinton and W. Li, "A Novel method for Incorporating Weather Effects in Composite System Adequacy Evaluation", IEEE Trans. on PS, Vol.6, No. 3, Aug. 1991, p1154-1160
- [10] M.V.F. Pereira and L.M.V.G. Pinto, "A New Computational Tool for Composite Reliability Evaluation", IEEE 1991 Sumer Meeting, Paper 91 SM443-2 FWRS, Aug. 1991, San Diego
- [11] A.C.G. Melo, M.V.F. Pereira and A.M. Leite de Silva, "Frequency and Duration Calculations in Compositev Generation and Transmission Reliability Evaluation", IEEE 1991 Sumer Meeting, Paper 91 SM440-8 FWRS, Aug. 1991, San Diego.
- [12] R. Billinton and L. Gan, "A Monte Carlo Model for Adequacy assessment of Multi-Area Generating Systems", Proceedings of the Third International Conference on Probabilistic methods Appied to Electric Power Systems, IEE, London, July, 1991
- [13] R. Billinton and R.N. Allan, Reliability Evaluation of Power Systems, Plenum Press, 1984.
- [14] R.Y. Rubinstein, Simulation and the Monte Carlo Method, John Wiley & Son Inc. 1981
- [15] IEEE Committee Report, "Reliability Indices for Use in Bulk Power Supply Adequacy Evaluation", IEEE Transaction on Power Apparatus and Systems, Vol. PAS-97, 1978, pp1097-1103
- [16] IEEE Committee Report, "IEEE Reliability Test System", IEEE Transaction on Power Apparatus and Systems, Vol. PAS-98, 1979, pp2047-2054.
- [17] R.N. Allan, R. Billinton and N.M.K. Abdel-Gawad, "The IEEE Reliability Test System---Extension to and Evaluation of the Generating System", IEEE Transaction on Power Systems, Vol.PWRS-1, Nov. 1986, pp1-7.

### **BIOGRAPHIES**

Roy Billinton (F, 1978) came to Canada from England in 1952. Obtained B.Sc. and M.Sc. Degrees from the University of Manitoba and Ph.D. and D.Sc. Degrees from the University of Saskatchewan. Worked for Manitoba Hydro in the System Planning and Production Divisions. Joined the University of Saskatchewan in 1964. Formerly Head of the Department of Electrical Engineering. Presently C.J. MacKenzie Professor of Engineering and Associate Dean, Graduate Studies, Research and Extension of the College of Engineering.

Author of papers on Power System Analysis, Stability, Economic System Operation and Reliability. Author or co-author of seven books on reliability. Fellow of the IEEE, the EIC and the Royal Society of Canada and a Professional Engineer in the Province of Saskatchewan.

Wenyuan Li (SM, 1989) was born in China. Graduate from QingHua University. Obtained M.Sc. and Ph.D. Degrees from Chongqing University. Worked for the North-East Power Company of China in the Equipment Division and at the Electricité de France as a visiting engineer. Professor of Electrical Engineering Department of Chongqing University. He is presently doing research work with Department of Electrical Engineering, University of Saskatchewan.

Author of papers on Secure and Economic Operation, Optimization Techniques, Power System Planning and Reliability. Author of a book on secure and economic operation of power systems. Senior member of the IEEE and member of the Chinese Society of Electrical Engineering and the Société des Electriciens et des Electroniciens de France.

## Discussion

M. G. Lauby (EPRI, Palo Alto, CA): The authors are congratulated for their paper exploring the calculation of system state transition rates when employing Monte-Carlo methods for composite system reliability analysis. This approach attacks a serious limitation with the use of Monte-Carlo methods which generally do not account for the transition into a study state.

The approach suggested by the authors is very important as it provides a way to calculate the transition rates and therefore, the frequency of a system state, not just the probability. However, the Monte-Carlo method being tested is based solely on probability. Therefore, those states with high frequencies and low probabilities are not necessarily visited. Have the authors thought about how this limitation could be eliminated?

One related concern is the confidence that adequate states have been visited so that a reasonable approximation of the transition rates is obtained. The authors struggled with this problem and indicate the number of samples should be increased. Is there a good analytical way to determine the needed sample to confidently predict both the probability and the transition rates?

Again, the authors have prepared an important paper which increases the value of Monte-Carlo methods for composite system reliability. Their comments on the above would be appreciated

Manuscript received July 24, 1992.

C. SINGH, T. PRAVIN CHANDER and JUN FENG (Texas A&M University): The discussors would like to complement the authors for this interesting paper. We have used [A1] the reverse of this approach for the modeling of multi-state generators. Our approach is that we first determine the state to which a multi-state device will transit. This is done by drawing a random number and using the transition probabilities given by equation(7). This is achieved by the proportionate allocation method[A2]. Once the next state to which a multi-state component will transit is known, then the time to this transition can be calculated by drawing a second random number. We have used this procedure for multi-state components because, once the next state of a multi-state component is known, it can be treated as a two state component and, therefore, the main simulation procedure can be constructed for two state components. The procedure can ofcourse, be used for the whole system since the system can be treated as one multi-state component. The advantage of doing so, however, is not clear to us and therefore we would like the authors to help us understand their approach

better by commenting on the following discussion and questions. To facilitate comparison, a brief description of the usual manner of the next event approach [A2] is described. Assume that the nth transition has just taken place at  $t_n$  and, the time to next transition of component i is given by  $T_i$ . Thus the vector of times to component transitions is given by  $\{T_i\}$  and the simulation now proceeds in the following steps.

1. Determine

$$T = \min\{T_i\} \tag{A.1}$$

If this T corresponds to  $T_p$ , i.e., pth component, then the next transition takes place by the change of state of this component.

2. Simulation time is now advanced

$$t_{n+1} = t_n + T \tag{A.2}$$

3. The residual times to component transitions are calculated by

$$T_i^r = T_i - T \tag{A.3}$$

where .

 $T_i^r$  = residual time to transition of component i.

- 4. It is clear that residual time for component p, causing system transition will be zero and therefore the time to next transition of this component is determined by drawing a random number.
- 5. Set the times

$$T_{i\ i\neq p} = T_i^{\ r} \tag{A.4}$$

and

$$T_{i \ i=p} = T_p \tag{A.5}$$

6. Return to step 1.

It can be seen from the above description that for every system transition, only one random number needs to be drawn and equations (A.1) and (A.3) need to be executed. The storage requirement is basically one vector which stores the times to transition of components. In the case of a Markovian system, some authors determine the times to component transition afresh at every system transition. This can avoid equation(A.3) but then a random number has to be drawn for each component. In our experience, the approach described above is more efficient.

In the approach described by the authors, for every system transition, two random numbers will need to be drawn. Also  $P_j$  would need to be determined after every system transition since the denominator of equation(7) is state dependent. In our judgment the two methods, the one described in the discussion, and the the one described in the paper would be almost equal in computational time. It is easy to see that the two methods are mathematically equivalent and will asymptotically yield the same results. In essence the authors are simulating the Markov process by a semi-Markov model[A2].

In the light of the above discussion, we would like the authors to comment why their approach would take less computation time or require less memory? We would again like to complement the authors for their interesting paper.

# References

[A1] C. Singh, T. Pravin chander, M. P. Bhavaraju and N. Nour, "Monte Carlo Approach for Estimating Contingency Statistics", Voll, Models and Methods, EPRI Report.

[A2] C. Singh and R. Billinton, System Reliability Modeling and Evaluation, Hutchinson Education, London, 1977.

Manuscript received August 6, 1992.

A.C.G. Melo (CEPEL, Rio de Janeiro, Brazil), A.M. Leite da Silva (PUC/RJ, Rio de Janeiro, Brazil), M.V.F. Pereira (PSRI, New York, NY), J.C.O. Mello (CEPEL, Rio de Janeiro, Brazil): We would like to congratulate the authors for their interesting application of the state transition Monte Carlo method to the reliability evaluation of composite power systems. Indeed this technique is very useful and has been used in other reliability fields [A]. Our comments are as follows:

We do not agree with the authors when they say that the non-sequential Monte Carlo sampling cannot calculate by itself

frequency and duration (F&D) indices. "True" F&D indices were calculated in Reference 11 of the paper within a non-sequential Monte Carlo scheme, using the Lagrange multipliers associated with the remedial actions model in a filtering process to test the component transitions. Also, Reference B describes another approach to compute F&D indices based on the concept of conditional probability; again, the "true" F&D indices were calculated in a non-sequential Monte Carlo sampling. Moreover, the algorithm proposed in Reference B requires the same computational effort as the estimation of LOLP and EPNS indices.

The F&D calculation requires the identification of all success states which can be reached from a failure state in one transition. It does not matter if these success states are all identified at once (as in Reference 11) or if they are reached during the simulation process (as in the authors' paper or in Reference B). Indeed the potential number of component transitions to be tested increases considerably with multi-state component representation [11]. However, the filtering process is very efficient in dealing with generator transitions. Therefore, we would say that the efficiency of the filtering process will not be deteriorated in this case. Also, in the case of multi-state load representation, the transitions between the load states work as a filter by themselves because a load state only transits to its neighbor load states which are usually just a few. Hence, even in these cases, the estimation of F&D indices through the approach of Reference 11 is computationally feasible.

We agree with the authors that it is a difficult task to make a direct comparison of CPU time required by the method presented in their paper and that developed in Reference 11 due to the differences in computing systems, programming skills and system applications. For these reasons and because the test systems used in both papers are different (IEEE-RTS in this paper and the Modified Reliability Test System (MRTS) in Reference 11) their results cannot be directly compared. The MRTS system is a modification of the IEEE-RTS with the objective of stressing the transmission network [C]. Therefore, each adequacy analysis performed with the MRTS will require, in average, more computational effort than with RTS.

In our opinion, a comparison between two simulation techniques cannot be made only in terms of CPU time, i.e., only in terms of the number of adequacy analysis, even if both approaches use the same adequacy model and test system. The efficiency of a Monte Carlo method should take into account both the number of adequacy analysis and the accuracy of the estimate because the gain in decreasing the relative uncertainty usually pays the additional adequacy analysis performed (see Table 8 of Reference B).

It is not clear in the paper if the EFLC index had took into account the transition rates between load levels. Observe that to simulate a 70 step load curve and to sample, independently, 10,000 states for each load level, it does not account for the rates of transitions among the load states. As shown in Reference B, the major contribution to the frequency of failure index may come from load transitions.

We would appreciate the authors' comments and once again we would like to congratulate the authors for their interesting paper.

### References

- [A] H. Kumamoto, K. Tanaka, K.Inoue, E.J. Henley, "State-Transition Monte Carlo for Evaluating Large, Repairable Systems", IEEE Transactions on Reliability, Vol. R-29, No. 5, pp. 376-380, December 1980.
- [B] A.C.G. Melo, M.V.F. Pereira, A.M. Leite da Silva, "A Conditional Probability Approach to the Calculation of Frequency and Duration Indices in Composite Reliability Evaluation", 1992 IEEE PES Summer Meeting, Paper 92 SM 425-9 PWRS, Seattle, WA, July 12-16, 1992.

 EPRI, Composite System Reliability Evaluation Methods, Report EL-5178, June 1987.

Manuscript received August 13, 1992.

R. Billinton, W. Li: We would like to thank the discussers for their comments and for their interest in our paper. In response to Mr. Lauby we would like to offer the following comments. The method presented creates directly a system state transition process by system state transition sampling. Based on this process, relevant indices including probability, frequency and energy related indices can be calculated. If the system state transition process is long enough, any system state which statistically makes an effective contribution to the total system adequacy indices can be visited. The question of confidence is one that is associated with any Monte Carlo application and involves the utilization of an appropriate convergence criterion. Calculations indicate that the EDNS index has a slower convergence than the frequency index and therefore the coefficient of variation for the EDNS index has been used as the stopping rule in the method presented.

In response to Dr. Singh and associates' comments regarding the procedure used in their modeling of multi-state generators, we find their proposed procedure to be an interesting variation of the basic sequential sampling technique and one that appears to provide computational advantages over the basic technique. The procedure described in our paper is, however, a fundamentally different procedure which originates from a drawn system state. The technique focuses on state transitions of the whole system rather than on component states or component state durations. The system state duration is directly sampled using Equation 9 which does not appear to be utilized by the discussers. It should also be noted that the procedure used by the discussers also involves the assumption that component transition rates are independent with respect to the system states. This assumption is not always valid in composite system adequacy evaluation where dependency may be an important consideration. In these cases, component transition rates can be system state dependent. This situation occurs, for example, when adverse weather considerations are included in the analysis [1]. We therefore cannot agree with the authors that the two methods are mathematically equivalent. We have no experience with their method and therefore cannot comment on the computer time required using this approach in composite system adequacy assessment. We do agree, however, that given that both methods are technically and mathematically sound that they should yield similar results.

In response to Dr. A.C.G. Melo and his associates we would like to make the following comments. Dr. Melo appears to have answered his own question regarding the inability of the nonsequential Monte Carlo procedure to provide frequency and duration indices without additional computations and procedures. Our comment was, as stated by Dr. Melo, that nonsequential Monte Carlo sampling cannot calculate by itself frequency and duration indices. Reference 11 uses a non-sequential procedure augmented by an additional enumeration procedure to calculate the frequency index. The technique described in Reference B by Dr. Melo also involves additional procedures above the basic non-sequential sampling approach and in addition utilizes the very restrictive assumption that the system is coherent. This is not an automatic assumption in a composite generation and transmission system. Practical experience with reliability evaluation in the B.C. Hydro South Metro System indicates that non-coherence has a considerable effect on the calculated adequacy and cannot be neglected.

We agree that a direct comparison of CPU times between two different methods is a difficult task. This is particularly difficult when the two test systems are not the same. An efficiency comparison of two Monte Carlo methods applied to the same problem can be conducted using the following equation [2]

Efficiency = 
$$\frac{t_1 \sigma_1^2}{t_2 \sigma_2^2}$$

where:  $t_1$  and  $t_2$  are CPU times and  $\sigma_1^2$  and  $\sigma_2^2$  are variances of appropriate reliability indices.

We did not incorporate load level transitions in the applications illustrated in our paper. Given a simple load level transition rate (or rates), it is relatively easy to incorporate it in this method. We are not convinced, however, that it is a simple matter to incorporate load level transitions in the process without some major simplifying assumptions, one of which is that the loads at all buses will transit at the same time. Unlike a conventional generating capacity application where all the system load is included in one system representation. A composite generation and transmission system will have multiple loads, each of which are dictated by the customer characteristics at that load point. It is not reasonable to expect that all loads will transit in unison.

In conclusion, we would like to thank the discussers for their comments and for their interest in our work.

- R. Billinton and W. Li, "A Novel Method For Incorporating Weather Effects in Composite System Adequacy Evaluation", IEEE Trans on Power Systems, Vol. 6, No. 3, August 1991.
- [2] R. Y. Rubinstein, "Simulation and Monte Carlo Method", John Wiley & Sons, New York, 1981.

Manuscript received October 8, 1992.