Aiyagari (1994) - Uninsured Idiosyncratic Risk and Aggregate Saving

This replication orginally appeared as part of Kirkby (2019).

In the model of Aiyagari (1994), infinitely lived households face a stochastic income—due to exogenous stochastic labor supply—and make consumption-savings decisions; given an interest rate. Therefore, the exogenous shock (z) is the labor supply h, the endogenous state (x) is the capital holdings k, and the decision variable (y) is the next periods capital k'. The state of a household is their current capital holding and their exogenous labor supply shock, (k, h). Individual household capital holdings, given an interest rate, aggregate to give aggregate capital holdings. The market clearance condition is that the interest rate will be determined by perfect competition in the goods market together with a representative firm with Cobb-Douglas production function. A general equilibrium is the condition when an interest rate determines household capital holdings, which in turn determine an interest rate by the market clearance condition, and when this latter interest rate is the same as the first. In short,

Definition 1. *A Competitive Equilibrium is an agents value function $V(k, h)$; agents policy function $k' = g(k, h)$; an interest rate r and wage w; aggregate capital K and labor H; and a measure of agents $\mu(k, h)$; such that*

1. *Given prices r & w, the agents value function $V(k, h)$ and policy function $k' = g(k, h)$ solve the agents problem:*

$$\begin{aligned} V(k,h) = & \max_{k'} \left\{ u(c) + \beta \int V(k', h') Q(h, dh') \right\} \\ & s.t. \quad c + k' = wh + (1+r)k \\ & \qquad\;\; c \geq 0,\ k' \geq \underline{k} \end{aligned}$$

2. *The aggregates are determined by individual actions: $K = \int k d\mu(k, h)$, and $H = \int h d\mu(k, h)$*
3. *Markets clear (in terms of prices): $r - (\alpha K^{\alpha-1} H^{1-\alpha} - \delta) = 0$.*
4. *The measure of agents is invariant:*

$$\mu(k,h) = \int \int \left[\int 1_{k=g(\hat{k},h)} \mu(\hat{k}, h) Q(h, dh') \right] d\hat{k} dh \tag{1}$$

where h is the labor supply shock, which takes values in $Z = \{h_1, ..., h_{n_h}\}$ and evolves according to the Markov transition function $Q(h, h')$. Note that the wage is residually determined by r.[1] The market clearance condition is more commonly expressed as $r = \alpha K^{\alpha-1} H^{1-\alpha} - \delta$, that the interest equals the marginal product of capital (minus the depreciation rate). Since $H = E(h) = 1$, the Cobb-Douglas production function is really only based on the aggregate capital (in the sense that H is a fixed constant).

We start with the results for the model of Aiyagari (1994). The functional forms and calibrated parameter values are as follows. The utility function is parameterized as $u(c) = \frac{c^{1-\mu}}{1-\mu}$. The shock

[1] The wage, which is given by the derivative of the Cobb-Douglas production with respect to labor, can be rewritten as a function of the interest rate and the parameters of the production function.

Table 1: Accuracy of the Tauchen Method in Aiyagari (1994)
Markov Chain Approximation to the Labour Endowment Shock
Markov Chain σ/Markov Chain ρ

σ/ρ	0.00	0.30	0.60	0.90
0.2	0.20/0.00	0.20/0.30	0.20/0.60	0.20/0.90
0.4	0.40/-0.00	0.40/0.30	0.40/0.60	0.40/0.90

Replication of Table 1 of Aiyagari (1994) using grid sizes $n_k = 1024$, $n_z = 27$, $n_p = 451$

Table 2: Original Version of Tauchen Method in Aiyagari (1994)
Markov Chain Approximation to the Labour Endowment Shock
Markov Chain σ/Markov Chain ρ

σ/ρ	0.00	0.30	0.60	0.90
0.2	0.21/0.00	0.21/0.30	0.21/0.59	0.24/0.90
0.4	0.43/0.00	0.43/0.28	0.44/0.58	0.49/0.89

Original Table 1 of Aiyagari (1994).

Table 3: General Equilibrium Interest Rates in Aiyagari (1994)

A. Net Return to Capital in %/Aggregate savings rate in % ($\sigma = 0.2$)			
ρ/μ	1	3	5
0.0	4.1528/26.13	4.0972/23.17	4.0278/23.16
0.3	4.1250/22.19	4.0417/24.08	3.9306/24.34
0.6	4.0972/23.31	3.9167/23.92	3.6944/24.81
0.9	4.0139/24.37	3.5833/24.74	3.0417/25.97
B. Net Return to Capital in %/Aggregate savings rate in % ($\sigma = 0.4$)			
ρ/μ	1	3	5
0.0	4.0694/23.64	3.8333/24.53	3.5278/25.04
0.3	4.0000/24.28	3.5556/24.89	3.0556/26.12
0.6	3.8472/24.28	3.0556/26.00	2.2222/28.16
0.9	3.5833/24.81	2.0972/28.53	0.6806/33.19

Replication of Table 2 of Aiyagari (1994) using grid sizes $n_k = 1024$, $n_z = 27$, $n_p = 451$

Table 4: Original Version of General Equilibrium Interest Rates in Aiyagari (1994)

A. Net Return to Capital in %/Aggregate savings rate in % ($\sigma = 0.2$)			
ρ/μ	1	3	5
0.0	4.1666/23.67	4.1456/23.71	4.0858/23.83
0.3	4.1365/23.73	4.0432/23.91	3.9054/24.19
0.6	4.0912/23.82	3.8767/24.25	3.5857/24.86
0.9	3.9305/24.12	3.2903/25.51	2.5260/27.32
B. Net Return to Capital in %/Aggregate savings rate in % ($\sigma = 0.4$)			
ρ/μ	1	3	5
0.0	4.0649/23.87	3.7816/24.44	3.4177/25.22
0.3	3.9554/24.09	3.4188/25.22	2.8032/26.66
0.6	3.7567/24.50	2.7835/26.71	1.8070/29.37
0.9	3.3054/25.47	1.2894/28.64	-0.3456/37.63

Original Table 2 of Aiyagari (1994).

process is $z' = \rho z + \epsilon$, where $\epsilon \sim N(0, \sigma^2)$. The discount rate is $\beta = 0.96$, capital share of production is $\alpha = 0.36$, and the depreciation rate is $\delta = 0.08$. Following Aiyagari (1994), we consider varying the parameters $\mu \in \{1, 3, 5\}$, $\rho \in \{0, 0.3, 0.6, 0.9\}$ and $\sigma \in \{0.2, 0.4\}$. The grid on the exogenous shocks is given by the Tauchen method with $n_z = 21$. The grid on the assets is $n_k = 512$ points, one-third evenly spaced on the interval $[0, K_{ss}]$, one-third evenly spaced on the interval $(K_{ss}, 3K_{ss}]$, and the final third evenly spaced on the interval $(3K_{ss}, 15K_{ss}]$; where $K_{ss} = (\frac{r_{ss}+\delta}{\alpha})^{\frac{1}{\alpha-1}}$ and $r_{ss} = 1/\beta - 1$ are the steady-state capital stock and interest rate of the corresponding complete markets representative agent economy. The grid on the prices is $n_p = 251$ points, one-third evenly spaced on the interval $[-\delta, 0)$, and the remaining two-thirds evenly spaced on the interval $[0, r_{ss}]$.

Table 2 shows that at least as measured by the first-order autocorrelation and variance of the process, the Tauchen method was accurate in discretizing the exogenous process. Although in my experience, the weakness of the Tauchen method tends to be related to the choice of the parameter q, an issue not addressed by this Table.Thus, these results confirm those of Aiyagari (1994).

A comparison of Table 3 with the corresponding Table 2 of Aiyagari (1994) (for readers convenience, the original version of Table 2 from Aiyagari (1994) is reproduced here as Table 4) shows that the original quantitative results that he gives for the equilibrium interest rates display the correct qualitative behavior; decreasing both in risk-aversion (μ) and in the riskiness of earnings (σ & ρ). However, quantitatively, the results of Aiyagari (1994) are quite inaccurate due to the roughness of the numerical approximations used. In particular, the degree of precautionary savings in the high-risk (high σ and/or ρ), high-risk-aversion (high μ) cases were substantially overestimated by Aiyagari (1994); for instance, compare $\mu = 5$, $\sigma = 0.2$, $\rho = 0.9$, or $\mu = 3, \sigma = 0.4, \rho = 0.9$.

References

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