Project 1
INF5620

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Description

In this project, we will develop a general solver for the vertical motion of a body with quadratic air drag. The solver will be verified and applied to a skydiver performing a parachute jump.

Differential Equation

The falling skydiver is subject to two forces: gravity and air drag. Using a quadratic model for the air drag, and applying Newton's 2. law of motion gives the differential equation

$$m\dot{v} = mg - \frac{1}{2}C_D \rho A|v|v,$$

where C_D is the body's drag coefficient, ρ is the density of the air and A is the cross-sectional area of the body perpendicular to the motion. All of these parameters will in reality vary during a skydive. The drag coefficient and cross-sectional area due to the skydiver changing posture and the density of the air as a function of height. Starting of, we will consider all three parameters as constant throughout the skydive.

To have a tider notation, we introduce the parameter $a \equiv C_D\rho A/2$, so our ODE can be written

$$m\dot{v} = mg - a|v|v,$$

Numerical Scheme

The time domain is now discretized uniformly with a step length of Δt, and the ODE sampled between two mesh points, at the time $t_{n+1/2}$. Applying a Crank-Nicolson finite difference approximation to the time derivative of the velocity gives

$$\big[D_t v = g - \frac{a}{m}\overline{(|v|v)}^t\big]^{n+1/2}.$$

Writing this out, and using a geometric average for the for the square of the velocity gives

$$\frac{v^{n+1} - v^n}{\Delta t} = g - \frac{a}{m}|v^n|v^{n+1}.$$

Solving for the unknown, v^{n+1}, gives the numerical scheme

$$v^{n+1} = \frac{v^n + g\Delta t}{1 + a\Delta t|v^n|/m}.$$

Method of Manufactured Solution

To apply the MMS, we must first include a source term in our ODE and numerical scheme. Naming the source term $f(t)$, we get the ODE

$$m\dot{v} = mg - a|v|v + f(t).$$

To test our solver, we will attempt to reproduce an exact solution on a linear form

$$v(t) = bt + c,$$

inserting this exact solution into the ODE gives

$$mb = mg - a|bt + c|(bt + c) + f(t).$$

We can now solve to find the source term, which yields

$$f(t) = a|bt + c|(bt + c) + m(b - g).$$

The source term also needs to be included in the numerical scheme, which now looks as follows

$$v^{n+1} = \frac{v^n + g\Delta t + \Delta t f(t_{n+1/2})/m}{1 + a\Delta t|v^n|/m}.$$