COMPUTER PROGRAMMING TO CALCULATE THE VARIATIONS OF CHARACTERISTIC ANGLES OF HELIOSTATS AS A FUNCTION OF TIME AND POSITION IN A CENTRAL RECEIVER SOLAR POWER PLANT

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ABSTRACT

The central receiver solar power plant is composed of a large number of individually stirred mirrors (heliostats), focusing the solar radiation onto a tower-mounted receiver. In this paper, an algorithm is developed based on vector geometry to pick an individual heliostat and calculate its characteristic angles at different times of the day and different days of the year. The algorithm then picks the other heliostats one by one and performs the same calculations as did for the first one. These data are used to control the orientation of heliostats for improving the performance of the field. This procedure is relatively straight-forward, and quite suitable for computer programming. The effect of major parameters such as shading and blocking on the performance of the heliostat field is also studied using this algorithm. The results of computer simulation are presented in three sections: (1) the characteristic angles of individual heliostats, (2) the incidence angle of the sun rays striking individual heliostats, and (3) the blocking and shading effect of each heliostat. The calculations and comparisons of results show that: (a) the average incidence angle in the northern hemisphere at the north side of the tower is less than that at its south side, (b) the cosine losses are reduced as the latitude is increased or the tower height is increased, (c) the blocking effect is more important in winter and its effect is much more noticeable than shading for large fields, (d) the height of the tower does not considerably affect shading; but significantly reduces the blocking effect, and (e) to have no blocking effect throughout the year, the field design should be performed for the winter solstice noon.

1. INTRODUCTION

The central receiver solar power plant is one of the most important thermal solar power plants to produce clean energy. It has been the focus of interest because of high temperatures and reasonable thermal efficiencies. In this power plant a large number of individually stirred mirrors (heliostats), focus the solar radiation onto a tower-mounted receiver. In order to control the heliostats, it is required to calculate the variations of characteristic angles of each heliostat as a function of time of day and day of year. Knowing that each heliostat has a different position with respect to the receiver, and the receiver is located in a specific geographical zone, makes the process of controlling the heliostats very complicated. Once the characteristic angles are known, it is also possible to study the effect of important parameters on the optical performance of the field. The heliostat field is the most expensive part of the power plant and therefore its optimum design can reduce the capital cost and improve the efficiency of the power plant. The purpose of this study is to predict the heliostats' characteristic angles which can be used for open loop control of the heliostat field. The incidence angle of the sun rays striking individual heliostats is determined, and the blocking and shading effect of each heliostat is investigated.

2. LITREATURE SURVEY

Riaz, 1976 [1] investigated the performance of large-area solar concentrators for central receiver power plants using a continuum field representation of ideal heliostat arrays that accounts for two governing factors: the law of reflection of light rays imposes steering constraints on mirror orientations; the proximity of mirrors creates shadow effects by blocking the incident and/or reflected solar radiation. The results of a steering analysis which develops the space-time characteristics of heliostats and of a shadow analysis which determines the local effectiveness of mirrors in reflecting solar energy to a central point are combined to obtain in closed analytical form the global characteristics of circular concentrators. These characteristics which appear as time profiles for mirror orientations, for effective concentration areas (i.e., reflected solar flux), and for concentration ratios, establish theoretical limits of performance against which actual or realistic solar power systems can be completed and assessed.

Lipps and Vant-Hull, 1978 [2] introduced a cell model for large central receiver systems, which establishes an array of representative heliostats for central receiver systems. They optimized the arrangement of heliostats in the collector field subject to the approximations of the cell model. Each cell contains an arbitrary regular two dimensional array of heliostats. Their study was limited to four categories of heliostats arrangements; namely radial cornfields, radial staggers, north-south cornfields, and north-south staggers.

Appelbaum and Bany, 1979 [3] analyzed the shadowing effect of vertical and inclined poles and collectors (the shadow components, height and area). The results of their analysis were used in an example of optimal deployment of collectors in a given area (which includes the tilt angle, collector size, spacing between collectors and the number of collector rows).

Budin and Budin, 1982 [4] developed a mathematical model for shading calculations. Their model led to closed form expressions for shadow position of an isolated point on a plane surface with arbitrary orientation. These expressions can be applied for shadow calculations of relatively complex objects using parallel projection methods. They also presented the shadow cast by the shading diagrams for some particular plane surface orientation. They also discussed the use of such diagrams for analysis on few illustrative examples.

Siala and Elayeb, 2001 [5] introduced a graphical method for a no-blocking radial staggered layout. It locates the heliostats in the field of a solar central receiver plant so that they provide no blocking losses over the year. In this method the field is divided into certain groups to increase the efficient use of land. The method is a simple one when compared to cell-wise procedures, making it more suitable for preliminary design of heliostat fields. At the same time the method can be represented by a set of mathematical equations, consequently facilitating its computer implementation.

3. MATHEMTICAL MODELLING

3.1 Vector Geometry

To determine the orientation of a plate in space, it is sufficient to obtain the components of the normal vector to that plate. Those three components in hand, we can easily calculate the tilt (β) and azimuth (γ). A Cartesian coordinate system is appropriate for this study, in which the origin is located at the base of the tower on the ground. The Z axis lies along the tower pointing upwards, while the axes X and Y are located on the ground level. The positive direction for X is from north to south and that for Y is from east to west. The main coordinate system is shown in Fig. 1.

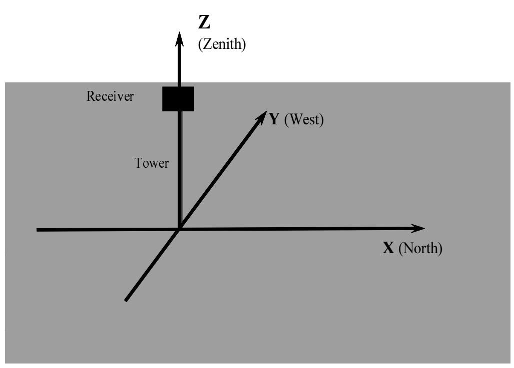

Fig.1: The main coordinate system.

The calculation of tilt (β) and azimuth (γ) is based on the laws of reflection:

1) The incident ray, the reflected ray and the normal to the surface at the point of incidence all lie in the same plane.

2) Angle of incidence is equal to the angle of reflection.

The unit vector along the reflected ray heading to the sun ($\vec{s}$), the unit vector along the reflected ray heading to the receiver ($\vec{r}$) and the unit vector along the reflected ray normal to the reflective plane are shown in Fig. 2.

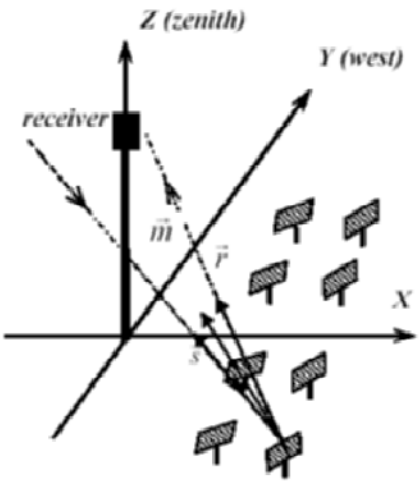

Fig. 2: Unit vectors $\vec{s}, \vec{m}$ *and* $\vec{r}$ in main coordinate system.

These unit vectors are expressed in terms of their components as follows:

$$\vec{s} = s_X\vec{i} + s_Y\vec{j} + s_Z\vec{k} \tag{1}$$

$$\vec{r} = r_X\vec{i} + r_Y\vec{j} + r_Z\vec{k} \tag{2}$$

$$\vec{m} = m_X\vec{i} + m_Y\vec{j} + m_Z\vec{k} \tag{3}$$

The unit vector $\vec{m}$ determines the orientation of the mirror plane with respect to the sun and should be found in such a way that the reflected ray strikes the receiver. The unit vector $\vec{r}$ is a function of heliostat position with respect to receiver. The components of $\vec{r}$ are:

$$\vec{r}_X = -\frac{X}{\sqrt{X^2 + Y^2 + (H - Z)^2}} \tag{4}$$

$$\vec{r}_Y = -\frac{Y}{\sqrt{X^2 + Y^2 + (H - Z)^2}} \tag{5}$$

$$\vec{r}_Z = \frac{H - Z}{\sqrt{X^2 + Y^2 + (H - Z)^2}} \tag{6}$$

X, Y and Z are the characteristics of mirror center and H is the vertical distance between receiver center and ground level (tower height).

The unit vector $\vec{s}$ can be defined in terms of three angles: latitude ϕ, solar hour angle ω, and solar declination angle δ. Declination angle is dependent on the day of year and can be expressed in degrees according to the following relationship:

$$\delta = 23.45\sin\left(360\times\frac{284+n}{365}\right) \tag{7}$$

where n is the day of the year. ω is a function of the day time and is expressed in degrees according to the following formula:

$$\omega = 15(hour - 12) \tag{8}$$

The vector $\vec{s}$ is related to these three angles as follows:

$$\begin{aligned}\vec{s} &= \left[-\cos(\omega)\sin(\phi)\cos(\delta) + \cos(\phi)\sin(\delta)\right]\vec{i} \\ &+ \left[\sin(\omega)\cos(\delta)\right]\vec{j} \\ &+ \left[\cos(\omega)\cos(\phi)\cos(\delta) + \sin(\phi)\sin(\delta)\right]\vec{k}\end{aligned} \tag{9}$$

The components of $\vec{s}$ in three directions are:

$$s_X = -\cos(\omega)\sin(\phi)\cos(\delta) + \cos(\phi)\sin(\delta) \tag{10}$$

$$s_Y = \sin(\omega)\cos(\delta) \tag{11}$$

$$s_Z = \cos(\omega)\cos(\phi)\cos(\delta) + \sin(\phi)\sin(\delta) \tag{12}$$

From the laws of reflection we have:

$$\vec{r}\times\vec{m} = \vec{m}\times\vec{s} \tag{13}$$

Using the above equation, the components of unit vector $\vec{m}$ are derived as follows:

$$m_X = \frac{\frac{|s_Z + r_Z|}{s_Z + r_Z}(s_X + r_X)}{\left[(s_X + r_X)^2 + (s_Y + r_Y)^2 + (s_Z + r_Z)^2\right]^{\frac{1}{2}}} \tag{14}$$

$$m_Y = \frac{\frac{|s_Z + r_Z|}{s_Z + r_Z}(s_Y + r_Y)}{\left[(s_X + r_X)^2 + (s_Y + r_Y)^2 + (s_Z + r_Z)^2\right]^{\frac{1}{2}}} \tag{15}$$

$$m_Z = \frac{\left|s_Z + r_Z\right|}{\left[\left(s_X + r_X\right)^2 + \left(s_Y + r_Y\right)^2 + \left(s_Z + r_Z\right)^2\right]^{\frac{1}{2}}} \quad (16)$$

3.2 Characteristic Angles

Knowing $\vec{m}$, the tilt (β) and azimuth (γ) can easily be calculated. From the vector algebra:

$$\vec{m} \cdot \vec{k} = \cos(\beta) \quad (17)$$

$$\cos(\beta) = m_Z \quad (18)$$

$$\beta = \cos^{-1}\left[\frac{\left|s_Z + r_Z\right|}{\left[\left(s_X + r_X\right)^2 + \left(s_Y + r_Y\right)^2 + \left(s_Z + r_Z\right)^2\right]^{\frac{1}{2}}}\right] \quad (19)$$

Based on definition of azimuth, this angle can be calculated as follows:

$$\gamma = \begin{cases} \pi - tg^{-1}\dfrac{\left|m_Y\right|}{\left|m_X\right|} & if\ (m_X > 0\ \&\ m_Y \geq 0) \\ tg^{-1}\dfrac{\left|m_Y\right|}{\left|m_X\right|} & if\ (m_X \leq 0\ \&\ m_Y > 0) \\ -tg^{-1}\dfrac{\left|m_Y\right|}{\left|m_X\right|} & if\ (m_X < 0\ \&\ m_Y \leq 0) \\ -\pi + tg^{-1}\dfrac{\left|m_Y\right|}{\left|m_X\right|} & if\ (m_X \geq 0\ \&\ m_Y < 0) \end{cases}$$

s_X, s_Y and s_Z are functions of time (based on equations 10, 11 and 12). r_X, r_Y and r_Z are functions of heliostat position relative to tower (according to equations 4, 5 and 6), so β and γ are computed in terms of time and position. Figure 3 shows the hourly variations of characteristic angles of a heliostat located on the X axes at the north side of the tower in three distinct days of the year. Fig. 4 and Fig. 5 shows the variations of β and γ with respect to time for various locations relative to the tower. In Fig. 4, Y is kept constant while in Fig. 5, X is invariant. The characteristic angles β and γ are calculated as a function of position (X and Y) at a given time. The two dimensional plots of β and γ with respect to X and Y may be illustrated in.

4. RESULTS AND ANALYSIS

The hourly variations of characteristic angles of a typical heliostat have been studied numerically using principles of vector geometry. The results are shown in Fig. 3 for 3 different days of the year. These days have freely been chosen to be December 21 (winter solstice), March 21 (vernal equinox) and June 21 (summer solstice). Such variations can be accomplished for any day of year using the numerical procedure developed in this research. The data based on which Fig. 3 is produced are related to the positions at the north side of the tower. The influence of

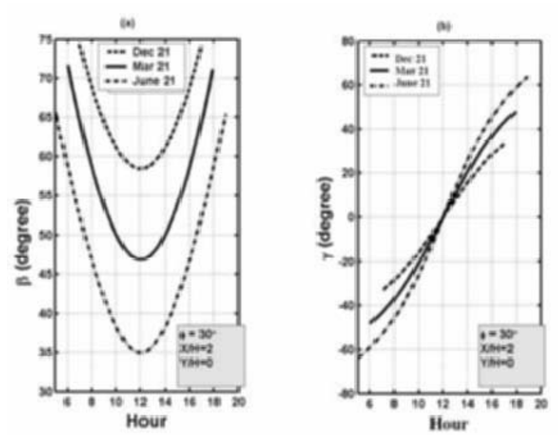

Fig. 3: Variation of characteristic angles.
(a): β and (b): γ with respect to time of day

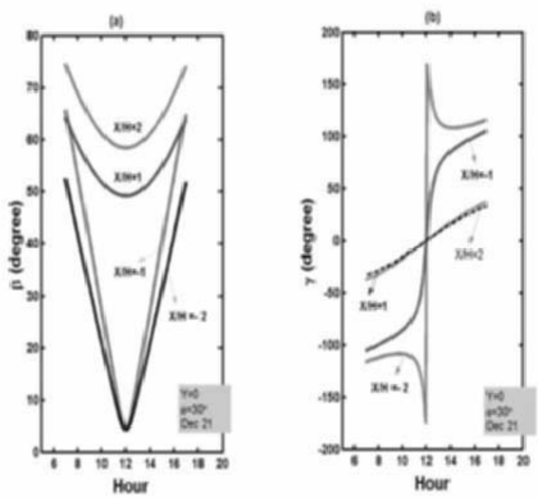

Fig. 4: Variations of characteristic angles.
(a): β and (b): γ with respect to time of day at afixed location on the west side of the tower.

heliostat position with respect to the tower on the characteristic angles is investigated. This investigation is performed when the heliostat is fixed on the west side (Fig. 4) as well as on the north side (Fig. 5) of the tower. Two dimensional plots of characteristic angles with respect to west-ward and north-ward positions of any heliostat can be produced at any instant of time.

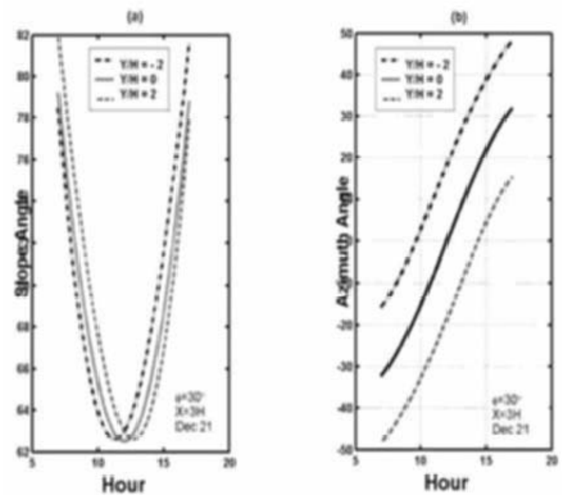

Fig. 5: Variations of characteristic angles.
(a): β and (b): γ with respect to time of day at a fixed location on the north side of the tower.

5. CONCLUSIONS

An algorithm is developed that is capable of calculating the characteristic angles of individual heliostats with respect to time and position. This algorithm may be used for open loop control of the heliostat field in a central receiver solar power plant. The simulation program, also, predicts the incidence angle of the sun rays striking individual heliostats, and the blocking and shading effect of each heliostat. However, because of page limitation in this paper, the results are not indicated.

6. NOMENCLATURE

$\vec{s}$	Unit vector heading to the sun
$\vec{r}$	Unit vector heading to the receiver
$\vec{m}$	Unit vector normal to the heliostat
X	Horizontal, north headed axis
Y	Horizontal, west headed axis
Z	Normal axis
H	Tower height
n	Number of day in Christian calendar
δ	Declination angle
ω	Solar hour angle
ϕ	Latitude
γ	Azimuth
β	Tilt

7. REFERENCES

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