

General formula for on-axis sun-tracking system and its application in improving tracking accuracy of solar collector

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Abstract

Azimuth-elevation and tilt-roll tracking mechanism are among the most commonly used sun-tracking methods for aiming the solar collector towards the sun at all times. It has been many decades that each of these two sun-tracking methods has its own specific sun-tracking formula and they are not interrelated. In this paper, the most general form of sun-tracking formula that embraces all the possible on-axis tracking methods is presented. The general sun-tracking formula not only can provide a general mathematical solution, but more significantly it can improve the sun-tracking accuracy by tackling the installation error of the solar collector.
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1. Introduction

Sun-tracking system plays an important role to ensure that the solar collector can receive maximum solar irradiation at all times. More importantly, for either imaging or non-imaging solar concentrator, inaccurate sun-tracking will directly deteriorate the quality of solar flux distribution at the receiver and thus affect the performance of the whole system. Generally, a good tracking mechanism must be reliable and able to track the sun at the right angle even in the periods of cloud cover. Sun-tracking systems are available as either a passive tracking system using open-loop approach or an active tracking system using closed-loop approach. For the passive tracking system, the tracker will perform calculation to identify the sun's position and to determine the rotational angles of the two tracking axes using a specific sun-tracking formula in order to drive the solar collector towards the sun. On the other hand, for the active tracking system, the sun tracker normally will

sense the direct solar radiation falling on a photo-sensor as a feedback signal to ensure that the solar collector is tracking the sun all the time. Instead of the above options, some authors have also designed a hybrid system that contains both the active and passive tracking system to achieve a good tracking accuracy (Luque and Andreev, 2007; Poulek and Libra, 1998).

Two most commonly used configurations in two-axis sun-tracking system are azimuth-elevation and tilt-roll (or polar) tracking system. Inspired by an ordinary optical mirror mount, azimuth-elevation system is among the most popular sun-tracking system employed in various solar energy applications (Beltran et al., 2007; Georgiev et al., 2004; Luque and Andreev, 2007). In the azimuth-elevation tracking, the collector must be free to rotate about the zenith-axis and the axis parallel to the surface of the earth. The tracking angle about the zenith-axis is the solar azimuth angle and the tracking angle about the horizontal axis is the solar elevation angle (Stine and Harrigan, 1985). Therefore, the accuracy of the azimuth-elevation tracking system highly relied on how well the azimuth-axis is aligned to be parallel with the zenith-axis.

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Alternatively, tilt-roll (or polar) tracking system adopts an idea of driving the collector to follow the sun-rising in the east and sun-setting in the west from morning to evening as well as changing the tilting angle of the collector due to the yearly change of sun path (Nuwayhid et al., 2001; Sharan and Prateek, 2006). Hence, for the tilt-roll tracking system, one axis of rotation is aligned parallel with the earth's polar-axis that is aimed towards the star Polaris. This gives it a tilt from the horizon equal to the local latitude angle. The other axis of rotation is perpendicular to this polar-axis (Poulek and Libra, 1998, 2000). The tracking angle about the polar-axis is equal to the sun's hour angle and the tracking angle about the perpendicular axis is dependent on the declination angle. The advantage of tilt-roll tracking is that the tracking velocity is almost constant at 15 degrees per hour and therefore the control system is easy to be designed (Stine and Harrigan, 1985). The accuracy of the tilt-roll tracking system relies strongly upon how well the roll-axis can be aligned in parallel with the polar-axis, which is also latitude dependant.

In this paper, we will derive a general formula for on-axis sun-tracking system that consists of two orthogonal driving axes with any arbitrary orientation to tackle the problem of installation defect. Chen et al. (2006) was the pioneer group to derive a general sun-tracking formula for heliostats with arbitrarily oriented axes. The newly derived general formula by Chen et al. is limited to the case of off-axis sun tracker (heliostat) where the target is fixed on the earth surface and hence a heliostat normal vector must always bisect the angle between a sun vector and a target vector. As a complimentary to Chen's work, we derive the general formula for the case of on-axis sun tracker where the target is fixed along the optical axis of the reflector and therefore the reflector normal vector must be always parallel with the sun vector. With this complete mathematical solution, the use of azimuth-elevation tracking formula and tilt-roll tracking formula are the special case of it. In this context, the precision of foundation alignment during the installation of solar collector becomes more tolerable because any imprecise alignment in the tracking axes can be easily compensated by changing the parameters' values in the general sun-tracking formulas.

In our study, the solar collector can exist in any form, which is either an immediate solar receiver or a reflector that directs the sunlight to a target (Kalogirou, 2004). For the immediate solar receiver, it can be photovoltaic module, Fresnel lens that focuses the sunlight to concentrator photovoltaic cells or solar heat absorber etc. In addition, the reflector can be made of either imaging reflector such as parabolic dish or non-imaging reflector such as compound parabolic concentrator.

2. Derivation of general formula

Prior to mathematical derivation, it is worth while to state that the task of the on-axis sun-tracking system is to aim a solar collector towards the sun by turning it about

two perpendicular axes so that the sunray is always normal relative to the collector surface (Blanco-Muriel et al., 2001; Reda and Andreas, 2004; Sproul, 2007). Under this circumstance, the angles that are required to move the solar collector to this orientation from its initial orientation are known as sun-tracking angles. In the derivation of sun-tracking formulas, it is necessary to describe the sun's position vector and the collector's normal vector in the same coordinate reference frame, which is the collector-centre frame. Nevertheless, the unit vector of the sun's position is usually described in the earth-centre frame due to the sun's daily and yearly rotational movements relative to the earth. Thus, to derive the sun-tracking formula, it would be convenient to use the coordinate transformation method to transform the sun's position vector from earth-centre frame to earth-surface frame and then to collector-centre frame. By describing the sun's position vector in the collector-centre frame, we can resolve it into solar azimuth and solar altitude angles relative to the solar collector and subsequently the amount of angles needed to move the solar collector can be determined easily.

According to Stine and Harrigan (1985), the sun's position vector relative to the earth-centre frame can be defined as shown in Fig. 1, where CM, CE and CP represent three orthogonal axes from the centre of earth pointing towards the meridian, east and Polaris, respectively. The unified vector for the sun position S in the earth-centre frame can be written in the form of direction cosines as follow:

$$S = \begin{bmatrix} S_M \\ S_E \\ S_P \end{bmatrix} = \begin{bmatrix} \cos \delta \cos \omega \\ -\cos \delta \sin \omega \\ \sin \delta \end{bmatrix},$$

where δ is the declination angle and ω is hour angle.

Fig. 2 depicts the coordinate system in the earth-surface frame that comprises of OZ, OE and ON axes, in which

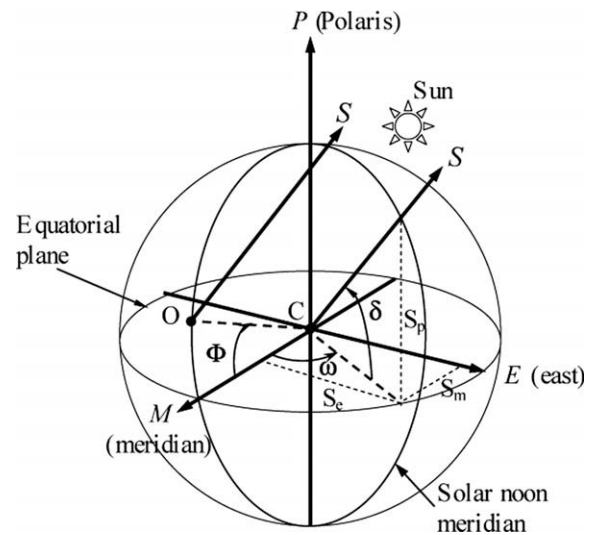


Fig. 1. The sun's position vector relative to the earth-centre frame. In the earth-centre frame, CM, CE and CP represent three orthogonal axes from the centre of the earth pointing towards meridian, east and Polaris, respectively.

they point towards zenith, east and north, respectively. The detail of coordinate transformation for the vector S from earth-centre frame to earth-surface frame was presented by Stine and Harrigan (1985) and the transformation matrix needed for the above coordinate transformation can be expressed as

$$[\Phi] = \begin{bmatrix} \cos \Phi & 0 & \sin \Phi \\ 0 & 1 & 0 \\ -\sin \Phi & 0 & \cos \Phi \end{bmatrix},$$

where Φ is the latitude angle.

Now, let us consider a new coordinate system that is defined by three orthogonal coordinate axes in the collector-centre frame as shown in Fig. 3. For the collector-centre frame, the origin O is defined at the centre of the collector surface and it also coincides with the origin of earth-surface frame. OV is defined as vertical axis in this coordinate system and it is also parallel with first rotational axis of the solar collector. Meanwhile, OR is named as reference axis in which one of the tracking angle β is defined relative to this axis. The third orthogonal axis, OH , is named as horizontal axis and it is parallel with the initial position of the second rotational axis. The OR and OH axes form the level plane where the collector surface is driven relative to this plane.

Fig. 3 also reveals the simplest structure of solar collector that can be driven in two rotational axes: the first rotational axis that is parallel with OV and the second rotational axis that is known as EE' dotted line (it can rotate around the first axis during the sun-tracking but must always be perpendicular with the first axis). From Fig. 3, θ is the amount of rotational angle about EE' axis measured from OV axis, whereas β is the amount of rotational angle about OV axis measured from OR axis. Fur-

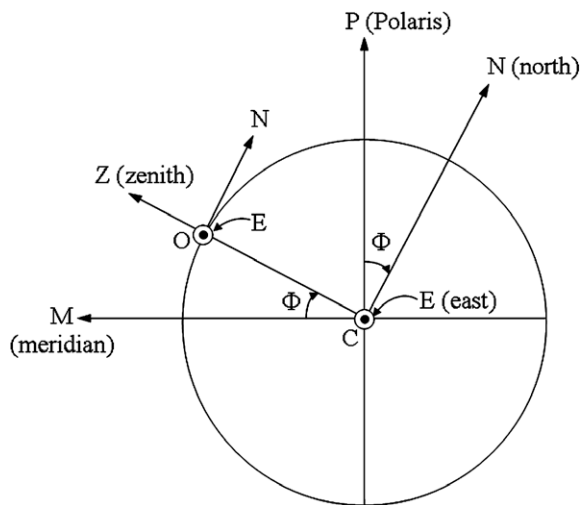


Fig. 2. The coordinate system in the earth-surface frame that consists of OZ , OE and ON axes, in which they point towards zenith, east and north respectively. The transformation of the vector S from earth-centre frame to earth-surface frame can be obtained through a rotation angle that is equivalent to the latitude angle, Φ .

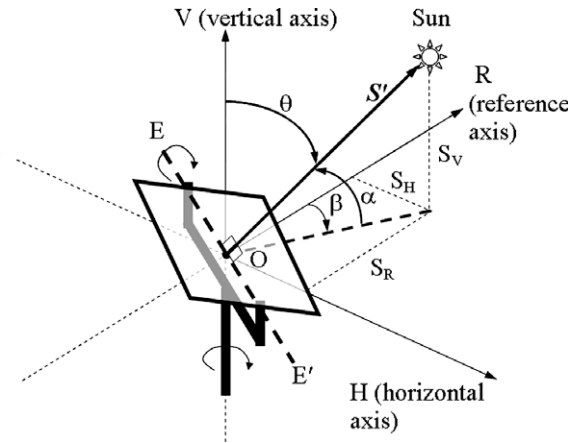


Fig. 3. In the collector-centre frame, the origin O is defined at the centre of the collector surface and it also coincides with the origin of earth-surface frame. OV is defined as vertical axis in this coordinate system and it is also parallel with first rotational axis of the solar collector. Meanwhile, OR is named as reference axis and the third orthogonal axis, OH , is named as horizontal axis. The OR and OH axes form the level plane where the collector surface is driven relative to this plane. The simplest structure of solar collector that can be driven in two rotational axes: the first rotational axis that is parallel with OV and the second rotational axis that is known as EE' dotted line (it can rotate around the first axis during the sun-tracking but must always remain perpendicular with the first axis). From the diagram, θ is the amount of rotational angle about EE' axis measured from OV axis, whereas β is the amount of rotational angle about OV axis measured from OR axis. Furthermore, α is solar altitude angle in the collector-centre frame, which is expressed as $\pi/2 - \theta$.

thermore, α is solar altitude angle in the collector-centre frame, which is equal to $\pi/2 - \theta$. In the collector-centre frame, the sun position S' can be written in the form of direction cosines as follow:

$$S' = \begin{bmatrix} S_V \\ S_H \\ S_R \end{bmatrix} = \begin{bmatrix} \sin \alpha \\ \cos \alpha \sin \beta \\ \cos \alpha \cos \beta \end{bmatrix}.$$

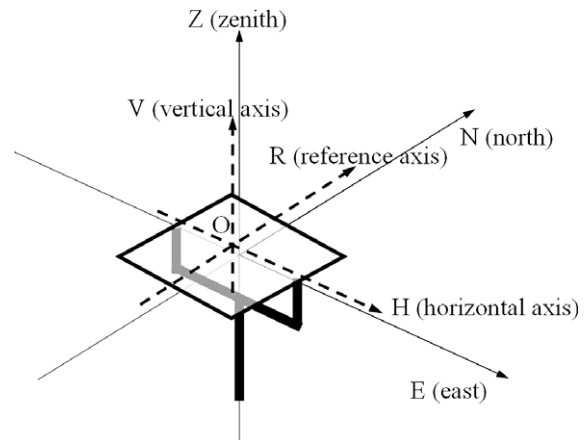


Fig. 4. In an ideal azimuth-elevation system, OV , OH and OR axes of the collector-centre frame are parallel with OZ , OE and ON axes of the earth-surface frame accordingly.

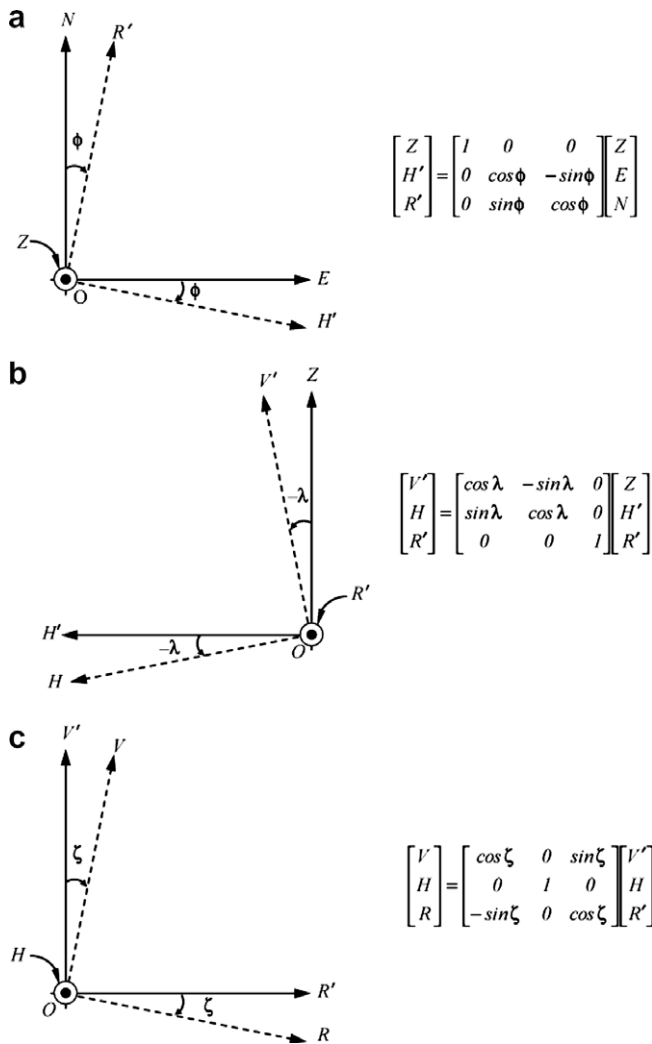


Fig. 5. The diagram shows the process of how the collector-centre frame is tilted step-by-step relative to the earth-surface frame, where OV' , OH' and OR' axes represent the intermediate position for OV , OH and OR axes, respectively. (a) The first tilted angle, $+\phi$, is a rotational angle about OZ -axis in clockwise direction in the first step of coordinate transformation from earth-surface frame to collector-centre frame. (b) The second tilted angle, $-\lambda$, is a rotational angle about OR' axis in counter-clockwise direction in the second step of coordinate transformation from earth-surface frame to collector-centre frame. (c) The third tilted angle, ζ , is a rotational angle about OH axis in clockwise direction in the third step of coordinate transformation from earth-surface frame to collector-centre frame.

In an ideal azimuth-elevation system, OV , OH and OR axes of the collector-centre frame are parallel with OZ , OE and ON axes of the earth-surface frame accordingly as shown in Fig. 4. To generalize the mathematical formula from the specific azimuth-elevation system to any arbitrarily oriented sun-tracking system, the orientations of OV , OH and OR axes will be described by three tilted angles relative to the earth-surface frame. Three tilting angles have been introduced here because the two-axis mechanical drive can be arbitrarily oriented about any of the three principal axes of earth-surface frame: ϕ is the rotational angle about zenith-axis if the other two angles are null, λ

is the rotational angle about north-axis if the other two angles are null and ζ is the rotational angle about east-axis if the other two angles are null. On top of that, the combination of the above mentioned three angles can further generate more unrepeatable orientations of the two tracking axes in earth-surface frame, which is very important in later consideration for improving sun-tracking accuracy of solar collector.

Fig. 5a–c show the process of how the collector-centre frame is tilted step-by-step relative to the earth-surface frame, where OV' , OH' and OR' axes represent the intermediate position for OV , OH and OR axes, respectively. In Fig. 5a, the first tilted angle, $+\phi$, is a rotational angle about the OZ axis in clockwise direction. In Fig. 5b, the second tilted angle, $-\lambda$, is a rotational angle about OR' axis in counter-clockwise direction. Lastly, in Fig. 5c, the third tilted angle, ζ , is a rotational angle about OH axis in clockwise direction. Fig. 6 shows the combination of the above three rotations in 3D view for the collector-centre frame relative to the earth-surface frame, where the change of coordinate system for each axis follows the order: $Z \rightarrow V' \rightarrow V$, $E \rightarrow H' \rightarrow H$ and $N \rightarrow R' \rightarrow R$. Similar to the latitude angle, in the direction representation of the three tilting angles, we define positive sign to the angles, i.e. ϕ , λ , ζ , for the rotation in the clockwise direction. In other words, clockwise and counter-clockwise rotations can also be named as positive and negative rotations, respectively. As shown in Fig. 5a–c, the transformation matrices correspond to the three tilting angles ϕ , λ and ζ can be obtained accordingly as follow:

$$[\phi] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix},$$

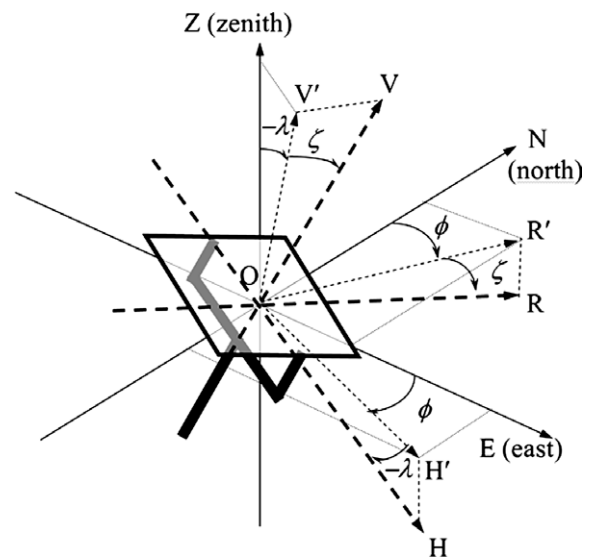


Fig. 6. The combination of the three rotations in 3D view from collector-centre frame to the earth-surface frame, where the change of coordinate system for each axis follows the order: $Z \rightarrow V' \rightarrow V$, $E \rightarrow H' \rightarrow H$ and $N \rightarrow R' \rightarrow R$.

$$[\lambda] = \begin{bmatrix} \cos \lambda & -\sin \lambda & 0 \\ \sin \lambda & \cos \lambda & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$[\zeta] = \begin{bmatrix} \cos \zeta & 0 & \sin \zeta \\ 0 & 1 & 0 \\ -\sin \zeta & 0 & \cos \zeta \end{bmatrix}.$$

The new set of coordinates S' can be interrelated with the earth-centre frame based coordinate S through the process of four successive coordinate transformations. It will be first transformed from earth-centre frame to earth-surface frame through transformation matrix $[\Phi]$, then from earth-surface frame to collector-centre frame through three subsequent coordinate transformation matrices that are $[\phi]$, $[\lambda]$ and $[\zeta]$. In mathematical expression, S' can be obtained through multiplication of four successive rotational transformation matrices with S and it is written as

$$\begin{bmatrix} S_V \\ S_H \\ S_R \end{bmatrix} = [\zeta][\lambda][\phi][\Phi] \begin{bmatrix} \cos \delta \cos \omega \\ -\cos \delta \sin \omega \\ \sin \delta \end{bmatrix},$$

$$\begin{bmatrix} \sin \alpha \\ \cos \alpha \sin \beta \\ \cos \alpha \cos \beta \end{bmatrix} = \begin{bmatrix} \cos \zeta & 0 & \sin \zeta \\ 0 & 1 & 0 \\ -\sin \zeta & 0 & \cos \zeta \end{bmatrix} \times \begin{bmatrix} \cos \lambda & -\sin \lambda & 0 \\ \sin \lambda & \cos \lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \times \begin{bmatrix} \cos \Phi & 0 & \sin \Phi \\ 0 & 1 & 0 \\ -\sin \Phi & 0 & \cos \Phi \end{bmatrix} \times \begin{bmatrix} \cos \delta \cos \omega \\ -\cos \delta \sin \omega \\ \sin \delta \end{bmatrix}.$$

Solving the above matrix equation for the solar altitude angle in the collector-centre frame, α , we have

$$\alpha = \arcsin \left[\frac{\cos \delta \cos \omega (\cos \zeta \cos \lambda \cos \Phi - \cos \zeta \sin \lambda \sin \Phi \sin \phi - \sin \zeta \cos \phi \sin \Phi) - \cos \delta \sin \omega (\sin \zeta \sin \phi - \cos \zeta \sin \lambda \cos \phi) + \sin \delta (\cos \zeta \cos \lambda \sin \Phi + \cos \zeta \sin \lambda \sin \phi \cos \Phi + \sin \zeta \cos \phi \cos \Phi)}{\cos \alpha} \right].$$

Thus, the first tracking angle along EE' axis

$$\theta = \frac{\pi}{2} - \arcsin \left[\frac{\cos \delta \cos \omega (\cos \zeta \cos \lambda \cos \Phi - \cos \zeta \sin \lambda \sin \Phi \sin \phi - \sin \zeta \cos \phi \sin \Phi) - \cos \delta \sin \omega (\sin \zeta \sin \phi - \cos \zeta \sin \lambda \cos \phi) + \sin \delta (\cos \zeta \cos \lambda \sin \Phi + \cos \zeta \sin \lambda \sin \phi \cos \Phi + \sin \zeta \cos \phi \cos \Phi)}{\cos \alpha} \right].$$

Similarly, the other two remaining equations that can be extracted from the above matrix equation in the cosine format are as follow:

$$\sin \beta = \frac{\cos \delta \cos \omega (\sin \lambda \cos \Phi + \cos \lambda \sin \phi \sin \Phi) - \cos \delta \sin \omega \cos \lambda \cos \phi + \sin \delta (\sin \lambda \sin \Phi - \cos \lambda \sin \phi \cos \Phi)}{\cos \alpha},$$

$$\cos \beta = \frac{\cos \delta \cos \omega (-\sin \zeta \cos \lambda \cos \Phi + \sin \zeta \sin \lambda \sin \phi \sin \Phi - \cos \zeta \cos \phi \sin \Phi) - \cos \delta \sin \omega (\sin \zeta \sin \lambda \cos \phi + \cos \zeta \sin \phi) + \sin \delta (-\sin \zeta \cos \lambda \sin \Phi - \sin \zeta \sin \lambda \sin \phi \cos \Phi + \cos \zeta \cos \phi \cos \Phi)}{\cos \alpha}.$$

In fact, the second tracking angle along OV axis, β , can be in any of the four trigonometric quadrants depending on location, time of day and the season. Since the arc-sine and arc-cosine functions have two possible quadrants for their result, both equations of $\sin \beta$ and $\cos \beta$ require a test to ascertain a correct quadrant. Consequently, we have either

$$\beta = \arcsin \left[\frac{\cos \delta \cos \omega (\sin \lambda \cos \Phi + \cos \lambda \sin \phi \sin \Phi) - \cos \delta \sin \omega \cos \lambda \cos \phi + \sin \delta (\sin \lambda \sin \Phi - \cos \lambda \sin \phi \cos \Phi)}{\cos \alpha} \right],$$

when $\cos \beta \geq 0$
or

$$\beta = \pi - \arcsin \left[\frac{\cos \delta \cos \omega (\sin \lambda \cos \Phi + \cos \lambda \sin \phi \sin \Phi) - \cos \delta \sin \omega \cos \lambda \cos \phi + \sin \delta (\sin \lambda \sin \Phi - \cos \lambda \sin \phi \cos \Phi)}{\cos \alpha} \right],$$

when $\cos \beta < 0$.

3. Special cases of the general tracking formula

The derived general sun-tracking formula is the most general form of solution for various kinds of arbitrarily oriented on-axis sun tracker. In general, all the sun-tracking systems can be classified into two major groups: (i) full tracking system and (ii) partial tracking system (Kalogirou, 2004; Solanki and Sangani, 2007). In a full tracking system, the solar collector tracks the sun in two axes such that the sun vector is normal to the aperture as to achieve 100% energy collection efficiency. In contrast, a partial tracking system only tracks the sun in a single-axis such that the plane of the sun's motion is normal to the aperture as to reduce the cosine loss (Hein et al., 2003). For the case of latitude-tilted tracking axis, the energy collection efficiency will be $(\cos \delta \times 100)\%$ where δ is the declination angle of the sun that varies from -23.45° to 23.45° .

For the full tracking system, such as azimuth-elevation and tilt-roll tracking system, their tracking formulas can be derived from the general formula by setting different values to the parameters, such as ϕ , λ and ζ . In the case of elevation-azimuth tracking system, the tracking formula can be obtained by setting the angles $\phi = 0$, $\lambda = 0$ and $\zeta = 0$ in the general formula. From that, the general formula can be simplified to

$$\theta = \frac{\pi}{2} - \arcsin[\sin \delta \sin \Phi + \cos \delta \cos \omega \cos \Phi] \quad \text{and}$$

$$\beta = \arcsin \left[-\frac{\cos \delta \sin \omega}{\cos \alpha} \right], \quad \text{when } \cos \beta \geq 0$$

$$\text{or } \beta = \pi - \arcsin \left[-\frac{\cos \delta \sin \omega}{\cos \alpha} \right], \quad \text{when } \cos \beta < 0.$$

On the other hand, tilt-roll tracking formula can also be obtained by setting the angles $\phi = \pi$, $\lambda = 0$ and $\zeta = \Phi - \pi/2$ in the general formula. For this case, the general tracking formula can be then simplified to

$$\theta = \pi/2 - \delta,$$

$$\beta = \omega, \text{ when } -\pi/2 < \omega < \pi/2.$$

For the partial tracking system, a single-axis tracking formula can be obtained from the full tracking formula by setting one of its tracking angles, which is either θ or β , as a constant value. For example, one of the most widely used partial tracking systems is to track the sun in latitude-tilted tracking axis (Stine and Harrigan, 1985). It is also called an equatorial mounting where the tracking axis is tilted from the horizon by the latitude angle towards south and it is parallel to the earth's rotational pole. The tracking formula of latitude-tilted tracking axis can be derived from tilt-roll tracking formula with the angle θ to be set as $\pi/2$ and the solar collector only tracks the sun with the angle $\beta = \omega$.

4. Application in improving tracking accuracy

In practice, most of the solar power plants over the world use a large area of solar collector in favour of saving the manufacturing cost. However, this has indirectly made the alignment work of the azimuth rotational shaft with the zenith-axis more complicated when the azimuth-elevation sun tracker is constructed. In this case, the alignment of the azimuth-axis normally involves an extensive amount of civil work due to the requirement of thick azimuth shaft for supporting the movement of a large solar collector, which normally has total collective area in the range of several tens of square meters to nearly hundred square meters. Under such a tough condition, a very precise alignment is greatly demanded for the azimuth shaft because a slight

misalignment of the azimuth shaft with the zenith-axis will incur significant sun-tracking errors.

To study the performance of the sun tracker owing to the misalignment of azimuth-axis, we consider a parabolic reflector attached to the azimuth-elevation tracking system with a target mounted along the optical axis of the parabolic reflector. In this case study, the parabolic reflector system, which is located on the earth surface with latitude of 3.2° and longitude of 101.7° , is programmed to track the sun using azimuth-elevation formula. Ray-tracing method is performed to trace the sunray from the reflector to the target and the pointing error of solar image on the target plane is simulated throughout the day. Figs. 7–9 show the deflection of solar image from the origin of target plane in the milli-radians (m rad) starting from 9.30 a.m. to 4.30 p.m. on 25th January provided that there is a misalignment of azimuth-axis relative to the zenith-axis for three different cases. In Figs. 7–9, the origin of the target plane is located at the coordinate (0, 0) that also intersects with the optical axis, vertical-axis of the target is referred to the axis that is parallel with the vertical direction of the parabolic reflector, while horizontal-axis of target is referred to the axis that is parallel with the horizontal direction of the parabolic reflector. Fig. 7 shows the tracking error of the sun tracker for the case of its azimuth-axis that deviated from the zenith-axis with the angle $\lambda = 0.4^\circ$. In this case, the pointing error is almost constant for daily sun-tracking ranges from 6.45 m rad (or 0.370°) to 6.52 m rad (or 0.374°). Similarly, Fig. 8 is plotted to show the tracking result for the case of azimuth-axis with a misaligned angle $\zeta = 0.4^\circ$ relative to the zenith-axis. For this case, its tracking error ranges from 4.12 m rad (or 0.236°) to 6.98 m rad

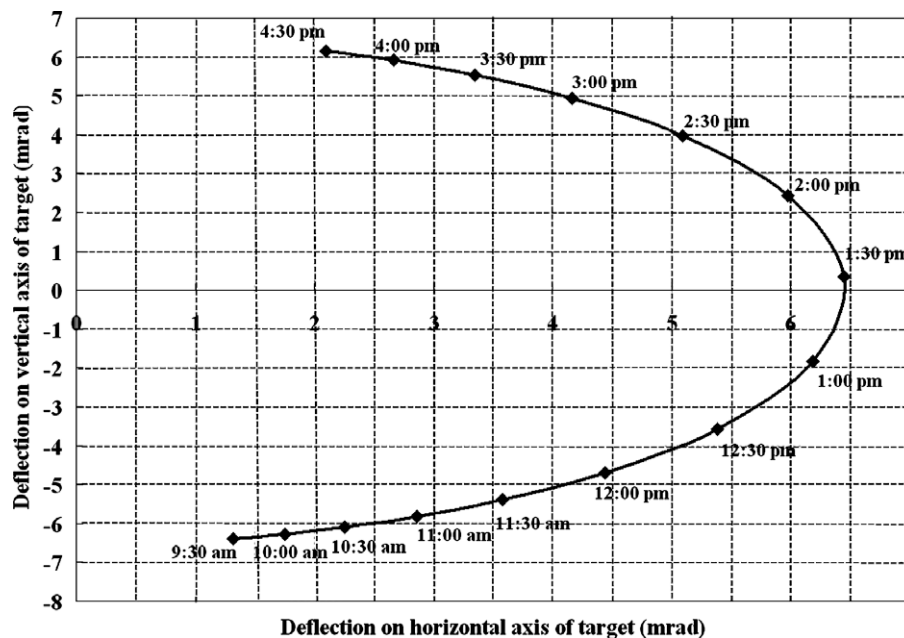


Fig. 7. The tracking error result on the target plane of the sun tracker from 9.30 a.m. to 4.30 p.m. on 25th January that is configured to track the sun along azimuth and elevation axes but its azimuth-axis is deviated from the zenith-axis with the angle $\phi = 0^\circ$, $\lambda = 0.4^\circ$ and $\zeta = 0^\circ$. In this case, the pointing error is almost constant for daily sun-tracking ranges from 6.45 m rad (or 0.370°) to 6.52 m rad (or 0.374°).

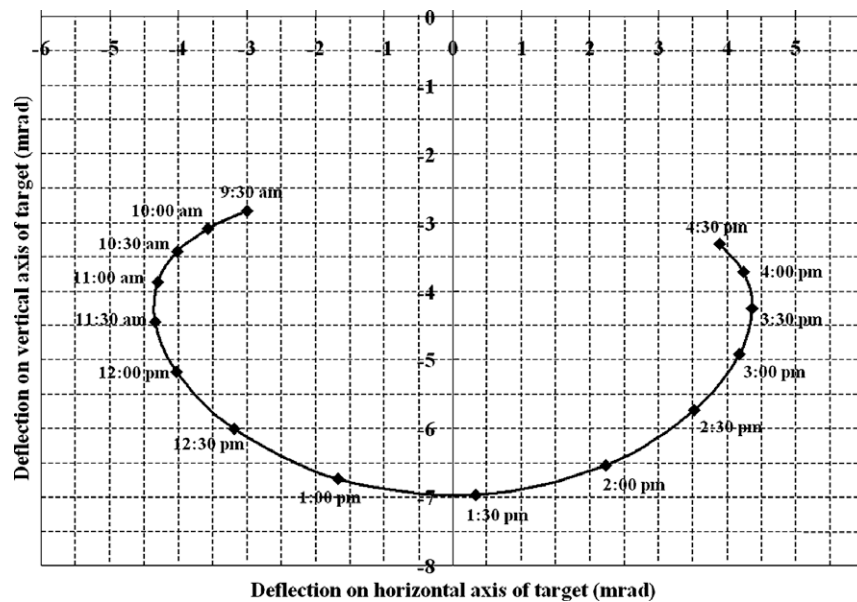


Fig. 8. The tracking error result on the target plane of the sun tracker from 9.30 a.m. to 4.30 p.m. on 25th January that is configured to track the sun along azimuth and elevation axes but its azimuth-axis is deviated from the zenith-axis with the angle $\phi = 0^\circ$, $\lambda = 0^\circ$ and $\zeta = 0.4^\circ$. For this case, its tracking error ranges from 4.12 m rad (or 0.236°) to 6.98 m rad (or 0.400°) with the maximum error occurred at around 1.30 p.m.

(or 0.400°) with the maximum error occurred at around 1.30 p.m. Finally, Fig. 9 shows the tracking error in the sun tracker with the misalignment of azimuth-axis for the case of $\phi = \lambda = \zeta = 0.4^\circ$. The combination of these three misaligned angles causes the worst tracking result, which ranges from 10.11 m rad (0.579°) to 12.10 m rad (0.693°). For the above three cases, the solar image moves in a partial elliptical path around the target in the counter-clockwise direction from 9.30 a.m. to 4.30 p.m. From the

simulation result, even though the misalignment of azimuth-axis is only 0.4° , the resultant tracking error is significant especially for the application in high concentration ratio module of solar collector with the nominal acceptance angle in the range of $\pm 0.6^\circ$ that requires a 0.34° minimum tracking accuracy (This angle is calculated from subtracting the acceptance angle 0.6° by the approximately 0.26° subtended half angle of the sun) (Luque and Andreiev, 2007.)

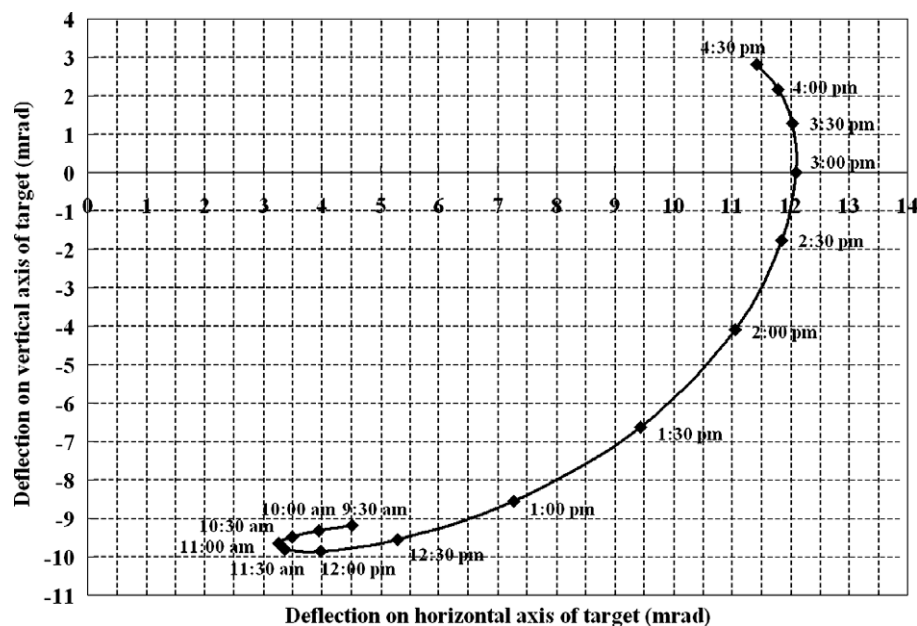


Fig. 9. The tracking error result on the target plane of the sun tracker from 9.30 a.m. to 4.30 p.m. on 25th January that is configured to track the sun along azimuth and elevation axes but its azimuth-axis is deviated from the zenith-axis with the angle $\phi = \lambda = \zeta = 0.4^\circ$. The combination of these three misaligned angles causes the worst tracking result, which ranges from 10.11 m rad (0.579°) to 12.10 m rad (0.693°).

As a matter of fact, there are many solutions of improving the tracking accuracy on account of this installation error such as adding a closed-loop feedback system to the controller (Luque and Andreev, 2007), designing a flexible mechanical platform capable of two-degree-of-freedom for fine adjustment of azimuth shaft (Chen et al., 2001), etc. Nonetheless, all these solutions require a more complicated engineering design to the sun tracker, which is also time consuming and costly. Contrarily, the above general sun-tracking formula can provide a much simpler solution if the formula is associated in the sun-tracking programme. With the new tracking formula, any installation error in the sun tracker can be corrected with the change of value in the parameters such as ϕ , λ and ζ in the sun-tracking programme.

5. Conclusion

General formula for on-axis sun-tracking system has been derived using coordinate transformation method. The newly derived sun-tracking formula is the most general form of mathematical solution for various kinds of arbitrarily oriented on-axis sun tracker, where azimuth-elevation and tilt-roll tracking formulas are specific cases. The application of the general formula is to improve the sun-tracking accuracy because the misalignment of solar collector from an ideal azimuth-elevation or tilt-roll tracking during the installation can be corrected by a straightforward application of the general formula. In our case study, 0.4° installation error of azimuth-elevation sun tracker has given a significant effect to the performance of high concentration solar modules.

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