A Robust Position Estimation Scheme Using Sun Sensor

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***Abstract*—A robust estimation scheme to locate accurately the position of the sun is proposed using a sun senor equipped to produce four output signals: each signal is proportional to the square of the distance of the focused light from one of the four corners of the sensor plane. Intermediate measurements, termed *pseudomeasurements,* are derived by computing the difference between outputs taken two at a time so that the sun position is linearly related to the pseudomeasurements. Both random noise and bias in the measurements are considered. A Kalman filter-based algorithm is used to compute an optimal estimate of the position of the focused light with respect to a reference frame. A model for the signal and for the noise are incorporated into the Kalman filter so that the signal is enhanced and the noise attenuated. The redundancy in the measurement is exploited to detect and isolate erroneous sensor measurements, which are then removed. The proposed scheme is evaluated on the simulated data as well as on the actual data obtained from the sun.**

I. Introduction

The SUN sensor is designed to determine the position of the sun in sensor elevation and azimuth coordinates. With the additional information of sensor roll and pitch and solar ephemeris data, the inertial heading of the sensor and hence the vehicle may be calculated. It consists of a lateral effects sensor (LES) located at the focal plane of the system of lenses that capture the incoming sun light rays. The LES functions as a two-dimensional photo diode designed to locate the position of focused light on the detector surface. The position of the focused light is a function of the elevation and the azimuth angles of the sun. The distances of the focused light from the four corners of the LES are measured, and the x and y position coordinates of the focused light in the LES reference frame is computed. The elevation and the azimuth angle of the sun with respect to the boresight or center line of the LES is obtained from the x and y position coordinates using a lookup table. The following issues are to be considered in the determination of the x–y position coordinates from the distance measurements.

- The relation between the x–y position coordinates of the focused light and the measured distances is nonlinear.
- The position of the sun and hence the position coordinates are not stationary.
- The x–y position determination accuracy may be reduced due to measurement noise: random and bias errors.

A novel approach to estimate the position coordinates of the focused light in the face of the above-mentioned problems is presented. To overcome the problem of nonlinearity, intermediate measurements, termed *pseudomeasurements*, are generated. The pseudomeasurements are some nonlinear function of the measured distances from the focused light where the nonlinear function is chosen such that the relation governing the position coordinates and the pseudomeasurements is linear. Fortunately, the measured signal of the pickoff lead is proportional to the square of the distance, and hence pseudomeasurements are obtained from a simple algebraic operation on the four measured outputs. The algebraic operation merely involves computing the differences between the outputs taken two at a time.

To overcome the problems associated with the continuous movement of position coordinates and bias in the measurement, a linear discrete-time state-space model is derived. The input to the state-space model is a zero-mean white-noise process, while the output of the model is the pseudomeasurements. The white-noise input driving the state space model characterizes an uncertainty associated with the derived model. The measurement inaccuracies resulting from dust contamination, fault, and sensor noise are included in the model by adding a zero-mean white-noise process to the true pseudomeasurements. Thus a linear state-space model with pseudomeasurement outputs and zero-mean white-noise inputs characterizes the evolution position coordinates in the face of nonstationary sun position, the measurement inaccuracies, and the nonlinearity. The Kalman filter to obtain the estimate of the position coordinates is a replica of the state-space model driven by the residual (error between the actual and the estimated pseudomeasurements). In view of the linear state-space model for the position coordinates, the Kalman filter is linear, and the position estimates are guaranteed to converge. Further, if the measurement noise is assumed to be a white Gaussian process, the estimates are optimal.

The proposed Kalman filter estimator is evaluated using extensive simulation as well as actual sun position data. The measurements of the focused light from the four corners are generated. The measurement errors are induced by simulating random and bias errors, and sensor faults and the estimation errors are analyzed. The proposed scheme is compared with the conventional scheme, which is based on estimating the position from an algebraic sum of the pseudomeasurements.

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II. Problem Formulation

A. Distance of the Focused Light from the Corners

Let the position coordinates of the focused light be $p = [x_R, y_R]^T$. Let (x_A, y_A), (x_B, y_B), (x_C, y_C), and (x_D, y_D) be the coordinates of the corners of the sensor plane A, B, C, and D, respectively; the coordinates are measured with respect to some sensor-fixed reference frame. Let the distance of the focused light source from the four corners A, B, C, and D of the sun sensor panel be r_{01}, r_{02}, r_{03}, and r_{04}, respectively (see Fig. 1). Expressing the distance in terms of the states (x_R, y_R) yields

$$\begin{bmatrix} r_{01}^2 \\ r_{02}^2 \\ r_{03}^2 \\ r_{04}^2 \end{bmatrix} = \begin{bmatrix} (x_R - x_A)^2 + (y_R - y_A)^2 \\ (x_R - x_B)^2 + (y_R - y_B)^2 \\ (x_R - x_C)^2 + (y_R - y_C)^2 \\ (x_R - x_D)^2 + (y_R - y_D)^2 \end{bmatrix}.$$

Expanding the expressions for the ranges and computing their differences, we get

$$\begin{aligned} r_{01}^2 - r_{02}^2 &= -2x_R(x_A - x_B) - 2y_R(y_A - y_B) + r_{\mathrm{AB}}^2 \\ r_{02}^2 - r_{03}^2 &= -2x_R(x_B - x_C) - 2y_R(y_B - y_C) + r_{\mathrm{BC}}^2 \\ r_{03}^2 - r_{04}^2 &= -2x_R(x_C - x_D) - 2y_R(y_C - y_D) + r_{\mathrm{CD}}^2 \\ r_{04}^2 - r_{01}^2 &= -2x_R(x_D - x_A) - 2y_R(y_D - y_A) + r_{\mathrm{DA}}^2 \\ r_{01}^2 - r_{03}^2 &= -2x_R(x_A - x_C) - 2y_R(y_A - y_C) + r_{\mathrm{AC}}^2 \\ r_{02}^2 - r_{04}^2 &= -2x_R(x_B - x_D) - 2y_R(y_B - y_D) + r_{\mathrm{BD}}^2 \end{aligned}$$

where r_{AB}, r_{BC}, r_{CA}, r_{CD}, r_{DA}, and r_{BD} are given by

$$\begin{aligned} r_{\mathrm{AB}}^2 &= (x_A^2 + y_A^2) - (x_B^2 + y_B^2) \\ r_{\mathrm{BC}}^2 &= (x_B^2 + y_B^2) - (x_C^2 + y_C^2) \\ r_{\mathrm{CA}}^2 &= (x_C^2 + y_C^2) - (x_A^2 + y_A^2) \\ r_{\mathrm{CD}}^2 &= (x_C^2 + y_C^2) - (x_D^2 + y_D^2) \\ r_{\mathrm{DA}}^2 &= (x_D^2 + y_D^2) - (x_A^2 + y_A^2) \\ r_{\mathrm{BD}}^2 &= (x_B^2 + y_B^2) - (x_D^2 + y_D^2). \end{aligned}$$

Without a loss of generality and with a view to simplifying the algebraic manipulations, the origin of the reference coordinate frame is chosen equidistant from the corners A, B, C, and D

$$(x_A^2 + y_A^2) = (x_A^B + y_B^2) = (x_C^2 + y_C^2) = (x_D^2 + y_D^2)$$

which implies

$$r_{\mathrm{AB}} = r_{\mathrm{BC}} = r_{\mathrm{CD}} = r_{\mathrm{DA}} = r_{\mathrm{AC}} = r_{\mathrm{BD}} = 0.$$

B. Pseudomeasurements

The pseudomeasurements are some nonlinear function of the outputs from the pickoff leads. Pseudomeasurements are chosen such that they are linear functions of the position coordinates of the focused light, $p = [x_R, y_R]^T$: pseudomeasurements are the difference between outputs taken two at a time. Define a true *pseudomeasurement* y_0 as

$$\begin{aligned} y_0 &= [y_{01} \quad y_{02} \quad y_{03} \quad y_{04} \quad y_{05} \quad y_{06}]^T \\ &= [r_{01}^2 - r_{02}^2 \quad r_{02}^2 - r_{03}^2 \quad r_{03}^2 - r_{04}^2 \quad r_{04}^2 - r_{01}^2 \\ &\qquad r_{01}^2 - r_{03}^2 \quad r_{02}^2 - r_{04}^2]^T. \end{aligned}$$

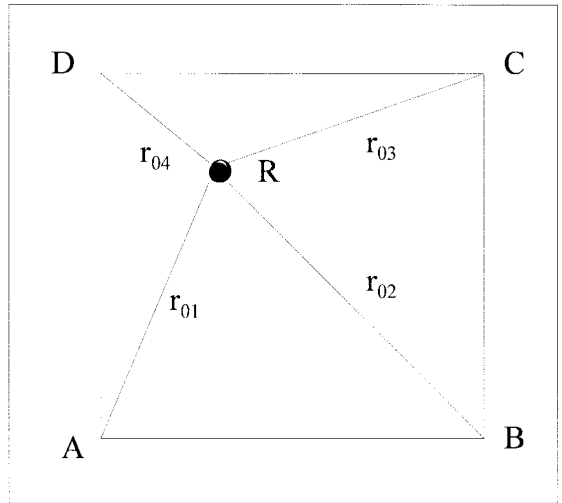

Fig. 1. Lateral effect sensor: R is the focused light, and A, B, C, and D are the four corners (pickoff leads).

The measurement model relating the true pseudomeasurement y_0 and the position p is linear and is given by

$$y_0 = Gp$$

where G is a (2 × 4) constant matrix given by

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{12} & G_{22} \\ G_{31} & G_{32} \\ G_{41} & G_{42} \\ G_{51} & G_{52} \\ G_{61} & G_{62} \end{bmatrix} = \begin{bmatrix} -2(x_A - x_B) & -2(y_A - y_B) \\ -2(x_B - x_C) & -2(y_B - y_C) \\ -2(x_C - x_D) & -2(y_C - y_D) \\ -2(x_D - x_A) & -2(y_D - y_A) \\ -2(x_A - x_C) & -2(y_A - y_C) \\ -2(x_B - x_D) & -2(y_B - y_D) \end{bmatrix}.$$

The actual pseudomeasurement y will be subject to errors. The discrepancy between the true and the actual pseudomeasurement is due to the atmospheric conditions such as moving clouds and other measurement errors. That is, y_0 and y are related by

$$y = y_0 + v$$

where v is the error

$$v = [v_1 \quad v_2 \quad v_3 \quad v_4 \quad v_5 \quad v_6]^T.$$

For example, v_1 is the error in the measurement y_1 associated with channels 1 and 2. v_2 is the error in the maturement y_2 associated with channels 2 and 3, and so on.

C. The Measurement Model

Thanks to the use of the true pseudomeasurement, y_0 and the unknown position p are linearly related by the measurement model given by

$$y_0 = Gp.$$

The measurement model relating the measured and the unknown quantities becomes

$$y = Gp + v.$$

The measurement error v accounts for various types of measurement inaccuracies including the bias and random errors.

III. Position Estimation Algorithm

The objective is to obtain a *best* estimate of the position p from the pseudomeasurements y using the linear measurement model $y = Gp + v$. There are essentially two approaches to estimate p:

1) using merely the measurements and no *a priori* knowledge about time evolution of the position and measurement errors; position is estimated using only the (algebraic) measurement model;
2) using the measurements as well as an *a priori* knowledge about the time evolution of position and the measurement errors: position estimation using a model formed of the algebraic measurement model and a dynamic model governing the time evolution of the position and the measurement errors.

A. Position Estimation Using Algebraic Model

The estimate of the position, $p = [x_R \quad y_R]^T$, is obtained by solving the linear least squares problem.

Let $\hat{p} = [\hat{x}_R \quad \hat{y}_R]^T$ be the estimate of p. The optimal estimate $\hat{p}$ is determined from

$$\min_{\hat{p}} \left\{ (y - G\hat{p})^T (y - G\hat{p}) \right\}.$$

The optimal $\hat{p}$ is a solution of the normal equation given by

$$\hat{p} = G^+ y$$

where G^+ is the pseudoinverse of G given by

$$G^+ = \left(G^T G\right)^{-1} G^T.$$

The expression for G^+ when the four corners A, B, C, and D form vertices of a square is

$$G^+ = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

and the optimal estimate $\hat{p}$ is given by

$$\hat{p} = \begin{bmatrix} y_2 + y_4 \\ y_1 + y_3 \end{bmatrix}.$$

B. Position Estimation Using a Dynamic Model

The estimate of the position p is obtained by solving the linear least squares problem with a constraint that the position of the focused light and the bias error in the measurement satisfy a dynamic equation.

C. Model for the Evolution of the Sun Position

The state-space model governing the evolution of the sun position takes the following form:

$$\begin{aligned} x_1(k+1) &= A_1 x_1(k) + B_1 w_1(k) \\ p(k) &= C_1 x_1(k) \end{aligned}$$

where x_1 is the state and $w_1(k)$ is a zero-mean white-noise process.

If the sun position remains essentially constant over the interval of observation, then the state $x_1 = p$ and $A_1 = I$ implies that the state—namely, the position of the focused light p—is essentially constant (mean of x is constant). If, however, the time interval of measurement is large, the sun position, and hence p, will vary; the state x_1 in general will include not only the position p but also its derivatives, $p^{(i)}$, $i = 1, 2, \cdots$, where $p^{(i)}$ is the ith derivative of p. Assuming a parabolic approximation to the trajectory, the state x will include the position p and the first and the second derivatives of p. The state x and the matrices A and B take the form

$$x_1 = \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix}, \qquad x_{11} = \begin{bmatrix} x_R \\ \dot{x}_R \\ \ddot{x}_R \end{bmatrix}, \qquad x_{21} = \begin{bmatrix} y_R \\ \dot{y}_R \\ \ddot{y}_R \end{bmatrix}$$

$$A = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{21} \end{bmatrix}, \qquad A_{11} = A_{21} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} B_{11} & 0 \\ 0 & B_{21} \end{bmatrix}, \qquad B_{11} = B_{21} = \begin{bmatrix} 0.1667 \\ 0.5 \\ 1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

If the plant noise w is absent—that is, if $w = 0$—then the above equation implies merely that the position and the state are deterministic.

D. Model for the Measurement Bias

The measurement error $v(k)$ may be expressed as a sum of two errors, namely, the bias error and the random error

$$v = v_b(k) + \xi(k)$$

where v_b is the bias error and ξ is the random error.

The state-space model governing the bias v_b takes the following form:

$$\begin{aligned} x_2(k+1) &= A_2 x_2(k) + B_2 w_2(k) \\ v_b(k) &= C_2 x_2(k) \end{aligned}$$

where x_2 is the state and $w_2(k)$ is a zero-mean white-noise process.

If the bias term is a constant over the interval of observation, then $A_2 = I$, implying that the measurement noise has a constant mean. The state-space model (A_2, B_2, C_2) in general will characterize a noise waveform having a rational spectrum.

Generally, the noise terms w_1 and w_2 are included even when the state is a deterministic process, to provide an additional parameter in the form of the covariances of the noise w_1 and w_2, to include a measure of uncertainty to the *a priori* knowledge about the model. Further, a better tradeoff is provided between the sensitivity of the estimate to noise and its ability to track the true estimate.

E. Position Estimation Using a Dynamic Model

The estimate of the position p is obtained by solving the linear least squares problem with a constraint that the position satisfies a dynamic equation. The dynamic equation governing the position may be expressed in a state-space form as follows:

$$x(k+1) = Ax(k) + Bw(k)$$
$$p(k) = Cx(k)$$

where x is the state

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \qquad A = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}$$
$$B = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}, \qquad C = \begin{bmatrix} C_1 \\ 0 \end{bmatrix}.$$

$w(k)$ is a zero-mean white-noise process. The matrices A, B, and C, along with the covariance of the noise w, determine the time evolution of the sun position p.

The state-space model includes the model for the sun position as well as that of the measurement bias. The state-space model relating the pseudomeasurement y and the state x may be derived from the above by merely including the measurement noise as follows:

$$x(k+1) = Ax(k) + Bw(k)$$
$$y(k) = Hx(k) + v(k)$$

where

$$H = GC$$

and v is the measurement error given by

$$v(k) = C_2 x_2 + \xi(k).$$

IV. Kalman Filter Algorithm

A Kalman filter is widely used to estimate the state variables recursively. To apply a Kalman filter algorithm, the state variables should satisfy the following linearity and observability conditions.

1) The state variable model, which is a discrete-time dynamic equation governing the state, must be linear.
2) The measurement model, which is a set of linear algebraic equations relating the measured variables and the state variables, must be linear.
3) The state and the measurement models must be observable, that is, given a set of measurements up to a finite time instant k, the initial state $x(0)$ can be uniquely determined.

A. Kalman Filter Estimator

Since the linearity and the observability conditions are met, a Kalman filter scheme may be used. The Kalman filter is designed based on the assumption that the plant noise $w(k)$ and the measurement noise $v(k)$ are independent and identically distributed Gaussian random processes with zero mean and known covariances

$$E\big[w(k)w(k)^T\big] = R_w$$
$$E[v(k)v(k)]^T = R_v$$

and the robustness of the resulting design is verified for various types of noise statistics. The Kalman filter equation is given by the following recursive equation:

$$\hat{x}(k+1) = \hat{x}(k) + K(y(k) - H\hat{x}(k))$$

where $\hat{x}$ is the estimate of x, k denotes the time instant $t = kT$, T is the sample period, and K is the Kalman gain. In many applications where the state and the measurements are stationary stochastic processes, only the steady-state solution of the Riccati equation is employed. The resulting Kalman gain will be constant, and the estimator equation will be time invariant. The Kalman gain K is obtained from the steady-state solution P_0

$$K = P_0 H^T [R_v + HP_0 H]^{-1}.$$

The $(n \times n)$ semidefinite matrix P_0, termed the error covariance matrix, is a steady-state solution to the following Riccati equation:

$$0 = P_0 H^T \big[R_v + HP_0 H^T\big]^{-1} HP_0 + R_w.$$

If the plant noise covariance $R_w = 0$, then the steady-state solution of the Riccati equation P_0 will be zero [that is, $P(k) \to 0$]. Hence R_w is chosen to be very small (but not zero) compared to R_v.

B. Optimal Estimation Error

We will first consider the case when the noise is a zero-mean, independent, and identically distributed Gaussian process with known covariance. The covariance of the estimation error, $\tilde{x} = x - \hat{x}$, is

$$\text{cov}(\tilde{x}) = E\big[(x - \hat{x})(x - \hat{x})^T\big].$$

The $\text{cov}(\hat{x})$ is given by the steady-state solution P_0 of the Riccati equation if the measurement noise v and the plant noise w are zero-mean Gaussian processes and have known covariances R_v and R_w, respectively

$$\text{cov}(\hat{x}) = P_0.$$

If the plant noise w has negligible variance compared to the measurement noise variance v—that is, $R_w \ll R_v$ (which is generally the case if the vehicle is motionless while the range measurements are taken)—then

$$\text{cov}(\hat{x}) = 0.$$

That is, if the plant noise is negligible and the measurement noise is a zero-mean Gaussian sequence, then the estimate $\hat{x}$ will converge to the true state x. The equation gives an exact expression for the covariance of the estimation error.

C. Bad Data Detection and Isolation

The bad data detection and isolation scheme is based on exploiting the redundancy in the pseudomeasurement: there are six measurements, $\{y_i i = 1, 2, \cdots, 6\}$, to estimate the two unknowns, $\{x_R \; y_R\}$. A χ^2 test using the residual of the Kalman filter is employed. Let the residual be $e(k)$

$$e(k) = y(k) - H\hat{x}(k).$$

The mean of the residual satisfies the following relation:

$$E[e(k)]\begin{cases} = 0, & \text{if there is no fault} \\ \neq 0, & \text{otherwise} \end{cases}$$

and the covariance of the residual R_e is given by

$$R_e = \text{cov}[e(k)] = HP_0H^T + R_v.$$

In the absence of fault, the residual will be a zero-mean white-noise process with covariance given by the expression above. Let us define the following test statistics $f(k)$:

$$f(k) = e^T(k)R_e^{-1}\,e(k).$$

Let Th be some threshold value chosen from the table of the χ^2 distribution. Then the test for fault *detection* is

$$f(k)\begin{cases} \leq Th, & \text{there is no fault} \\ > Th, & \text{faulty.} \end{cases}$$

The *fault isolation* is based on

$$F_{ii}(k) \geq Th, \text{ then } y_i \text{ is faulty}$$

where F_{ii} is the iith element of the matrix R_e^{-1}. The pseudomeasurement y_i is faulty if

$$F_{ii}(k) \geq F_{jj} \text{ for all } j \neq i.$$

V. Error Bounds on Position Estimates

There is no closed-form expression for the solution P_0, and the assumption that the noise is an independent and an identically distributed Gaussian process with *a priori* known statistics may not hold. Hence it is useful to obtain an analytical expression for a bound on the estimation error for various noise statistics with a view to design, test, and evaluate the performance of the sun sensor system. Further, an analytical expression on the bound will help determine the maximum allowable error in the measurements and hence an appropriate sensor system. To obtain a bound, we will assume a worst case scenario. Only one set of measurements of the ranges is available, which implies that an estimate is obtained directly from the measurement equation without recourse to the optimal Kalman filter scheme. However, the estimate thus obtained (although suboptimal) will have a closed-form expression. We will obtain error bounds for two types of measurement errors: one resulting from the bias and the other from the zero-mean random noise. The bias error can arise due to sensor bias and malfunction of the measurement system, while the random error results from the sensor and communication noise.

A. Estimation Error Under Bias Error

The measurement z suffers from bias error $\nu_i, i = 1, 2, 3, 4$. It is assumed that the bias error is unknown except that it has a known upper bound $\overline{v}$, that is

$$|\nu_i| \leq \overline{\nu}, \quad i = 1, 2, 3, 4.$$

Computing the position estimate $\hat{p}$ from the measurement equation yields

$$\hat{p} = G^+y$$

where H^+ denotes the pseudoinverse of H. Substituting for y, we get

$$\hat{p} = p + G^+v.$$

Using the expression for v in terms of x, we get

$$\hat{p} = \left(I + G^+\Lambda G\right)p.$$

The estimation error can be expressed as

$$|p - \hat{p}| = \left|G^+\Lambda Gp\right|$$

where the vector equality implies element-wise equalities.

B. Estimation Error Under Random Measurement Noise

The measurement noise $\nu_i,\ i = 1, 2, 3, 4$ is assumed to be an independent, identically distributed zero-mean random variable, and the standard deviation is bounded by $\overline{v}$

$$\begin{aligned} E(\nu_i) &= 0 \\ E(\nu_i\,\nu_j) &\leq \overline{\nu}^2 \quad \text{if } i = j \\ E(\nu_i\,\nu_j) &= 0 \quad\ \ \text{if } i \neq j. \end{aligned}$$

1) Error in Estimating the Location Vector: We will first obtain a bound on the estimation error of the location vector, namely, $(p - \hat{p})$. It takes the form

$$\|p - \hat{p}\| = \left\|G^+v\right\|$$

where $\|p\| = \sqrt{E[p^Tp]}$ is the standard deviation of p if p is a random variable and $\|p\| = \sqrt{p^Tp}$; $\|p\|$ is an ℓ_2 norm of p if p is deterministic. Substituting for v yields

$$\|p - \hat{p}\| = \left\|G^+\Lambda\,Gp\right\|.$$

2) Error in Estimating the Location Coordinates: An estimation error in the location vector does not give an indication of a corresponding bound in the estimation error of its coordinates x_R and y_R; a small per-unit error in locating the vector does not imply a small per-unit error in estimating each of the coordinates. A separate bound on each estimation error, $\|(x_R - \hat{x}_R)\|$ and $\|(y_R - \hat{y}_R)\|$, is required. Consider the covariance of the estimation error, $E[(p - \hat{p})(p - \hat{p})^T]$. Substituting for the estimation error yields

$$E\left[(p - \hat{p})(p - \hat{p})^T\right] = G^+E\left[vv^T\right]G^{+T}.$$

Substituting for v, we get

$$E\left[(p - \hat{p})(p - \hat{p})^T\right] = E\left[G^+\Lambda Gpp^TG^T\Lambda G^{+T}\right].$$

C. Estimation Error Bounds

The estimation error J under the bias error and the random error may be summarized as follows:

$$J = \begin{cases} |G^+\Lambda G|, & \text{if the measurement noise } v \text{ is a bias error} \\ \begin{cases} \|G^+\Lambda G\|, & \text{if } v \text{ is a random error considering location vector} \\ E\left[G^+\Lambda Gpp^TG^+\Lambda G^{+T}\right], & \text{if } v \text{ is a random error considering } \|x_R\| \text{ and } \|y_R\|. \end{cases} \end{cases}$$

TABLE I
ESTIMATION ERROR WITH RANDOM MEASUREMENT NOISE

std. of noise	e_x		e_y		e_p	
	proposed	conventional	proposed	conventional	proposed	conventional
0.01	0.0057	0.0071	0.005	0.0073	0.004	0.0052
0.05	0.0246	0.0251	0.0251	0.0367	0.0181	0.0264
0.1	0.0483	0.0728	0.0517	0.0713	0.0365	0.0516
0.2	0.0986	0.1425	0.0911	0.1363	0.0684	0.1018

TABLE II
ESTIMATION ERROR WITH BIAS ERROR

bias	e_x		e_y		e_p	
	proposed	conventional	proposed	conventional	proposed	conventional
bias in y_1	0.0184	0.3072	0.009	0	0.0112	0.1374
bias y_1, y_2	0.0184	0.3072	0.0184	0.3072	0.0184	0.3072
bias y_1, y_2, y_3	0.0113	0.6143	0.0184	0.3072	0.0172	0.3885
bias in all	0.0212	0.92	0.053	1.3822	0.0484	1.3

It is interesting to note that the estimation error is a function of $G^{+}\Lambda G$

VI. EVALUATION OF THE PROPOSED SCHEME

The proposed Kalman-filter-based algorithm to evaluate the position of the focused light was evaluated using:

1) simulation considering the following typical scenarios: the sun position is stationary and:
   a) the measurement noise is random;
   b) measurement error is a bias.
2) actual data.

A. Simulation Results

The error in the measurements of the position of the focused light in the face of cloud movement, electronics noise, etc., is modeled by a zero-mean Gaussian noise and a bias. The normalized error in locating (x, y) coordinates is computed as follows. The error in locating the x coordinates is given by

$$e_x = \frac{||(\hat{x}_R - x_R)||}{||x_R||}.$$

The error in locating the y coordinates is given by

$$e_y = \frac{||(\hat{y}_R - y_R)||}{||y_R||}.$$

The error in locating the (x, y) coordinates is given by

$$e_p = \frac{||(p - p)||}{||p||}.$$

Two cases are considered: one when the measurement error is random and the other when there is a bias.

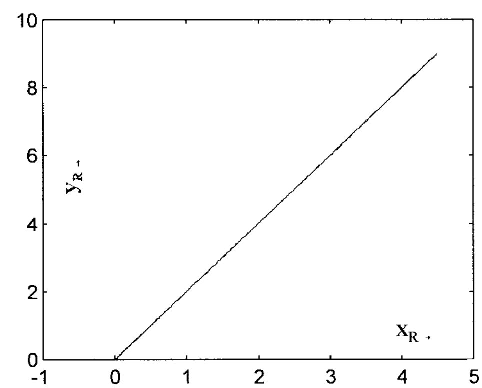

Fig. 2. The plot of the position y_R versus x_R using the proposed scheme with random measurement noise with a signal-to-noise ratio (SNR) $= 8$. The true and the estimated positions are recorded. The per-unit errors e_x, e_y, and e_p are, respectively, 0.0079, 0.0086, and 0.0084.

Case 1—Random Measurement Error: Table I gives the results when the measurement error is a zero-mean Gaussian white-noise process. From the table, it may be seen that the proposed scheme gives considerably lower error.

Case 2—Bias Error: The measurements were subject to bias. Table II gives the results.

The proposed scheme is unaffected by the bias error, while the conventional scheme is sensitive to bias. Figs. 2 and 3 give the true and the estimated trajectories of the sun position using the proposed and the conventional approach in the presence of random noise; Figs. 4 and 5 give the true and the estimated trajectories when there is a bias error; and Figs. 6–9 give the estimated and true trajectories when both bias and random noise errors are present.

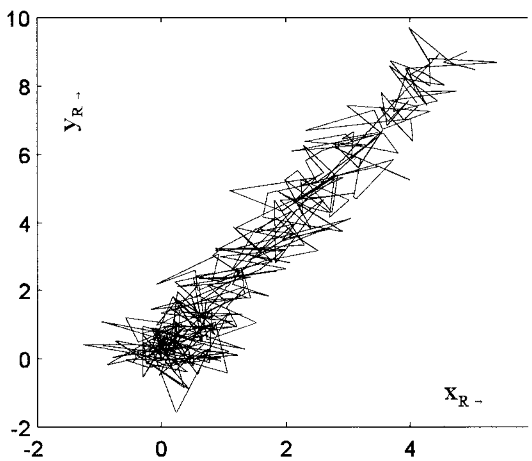

Fig. 3. The plot of the position y_R versus x_R using the conventional approach with random measurement noise with an SNR $= 8$. The true and the estimated positions are recorded. The per-unit errors e_x, e_y, and e_p are, respectively, 0.2576, 0.1160, and 0.1160.

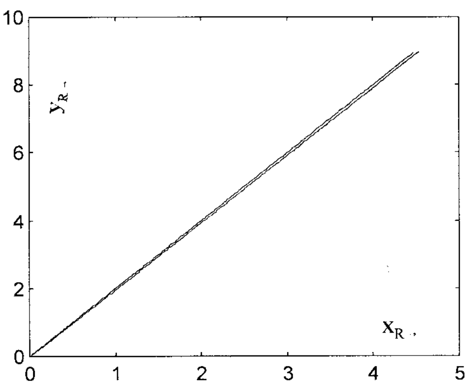

Fig. 4. The plot of the position y_R versus x_R using the proposed scheme with bias error of SNR $= 2$. The true and the estimated positions are recorded. The per-unit errors e_x, e_y, and e_p are, respectively, 0.0184, 0.0035, and 0.0088.

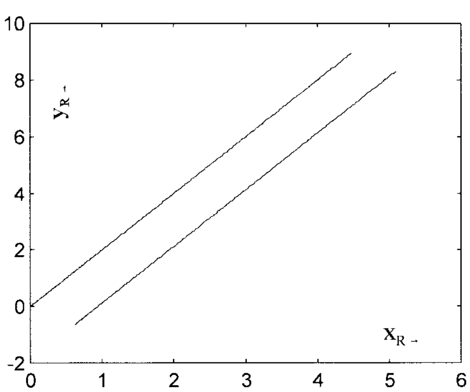

Fig. 5. The plot of the position y_R versus x_R using the conventional scheme with bias error of SNR $= 2$. The true and the estimated positions are recorded. The per-unit errors e_x, e_y, and e_p are, respectively, 0.3072, 0.1536, and 0.1943.

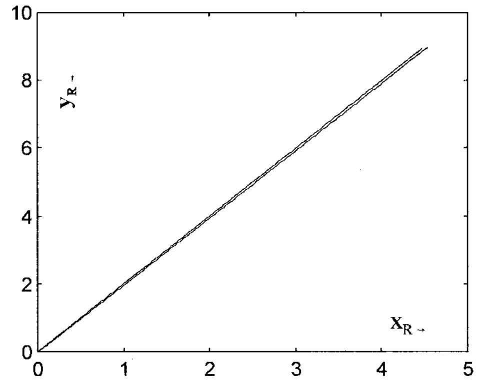

Fig. 6. The plot of the position y_R versus x_R using the proposed scheme with bias error of SNR $= 2$ and random noise of SNR $= 8$. The true and the estimated positions are recorded. The per-unit errors e_x, e_y, and e_p are, respectively, 0.0298, 0.0019, and 0.0134.

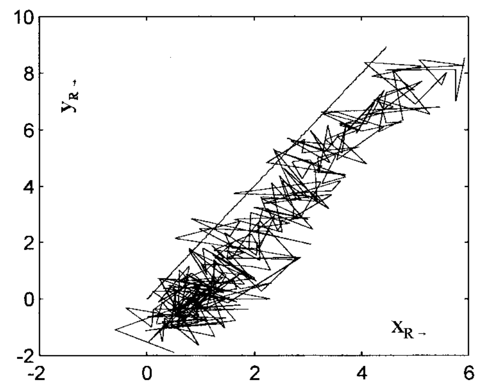

Fig. 7. The plot of the position y_R versus x_R using the conventional scheme with a bias error of SNR $= 2$ and random noise of SNR $= 8$. The true and the estimated positions are recorded. The per-unit errors e_x, e_y, and e_p are, respectively, 0.3886, 0.1850, and 0.2159.

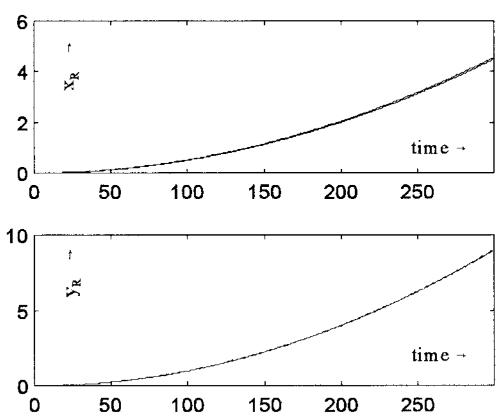

Fig. 8. The plot of the position y_R and x_R versus time using the proposed scheme with a bias error of SNR $= 2$ and a random noise with SNR $= 8$. The true and the estimated positions are recorded.

B. Fault Detection and Isolation

The pseudomeasurements were subject to bias errors to simulate faults. A bias in the pseudomeasurement y_i indicates that y_i is faulty. The inverse of the estimation error covariance F_{ii} was computed to isolate faults.

As can be seen from Table III, the proposed scheme is able to isolate the faults.

TABLE III
FAULT DETECTION AND ISOLATION WITH BIAS ERRORS

fault	F_{11}	F_{22}	F_{33}	F_{44}	F_{55}	F_{66}
1	2.7831	0.0154	0.06401	0.005	0.048	0.0002
1, 2	4.07	2.32	0.049	0.1237	0.063	0.0132
1,2 ,3	6.98	2.35	11.8	0.04	0.1	0
all	4.63	2.351	1.61	2.18	1.23	9.7

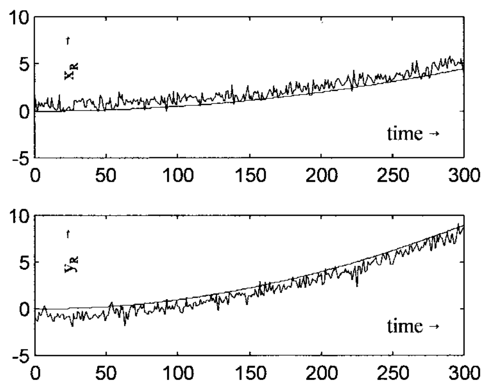

Fig. 9. The plot of the position y_R and x_R versus time using the conventional scheme with a bias error of SNR $= 2$ and a random noise with SNR $= 8$. The true and the estimated positions are recorded.

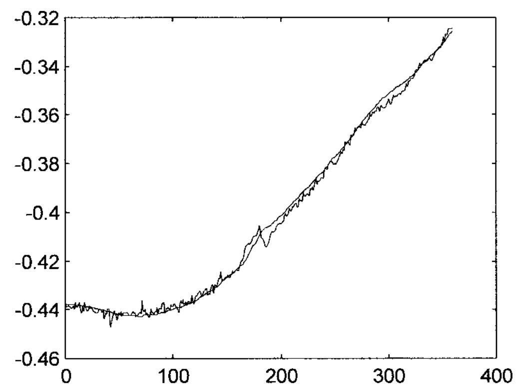

Fig. 11. The plot of the position x_R versus time using the proposed and the conventional schemes with actual data. A smooth trajectory is obtained with the proposed scheme.

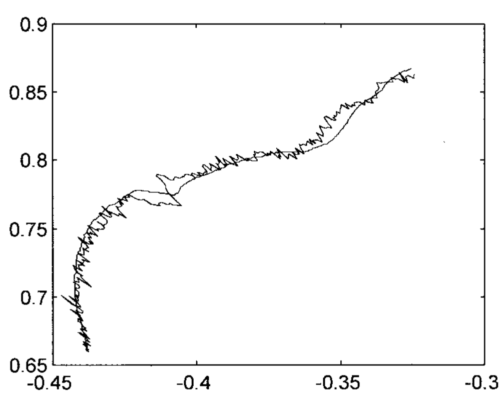

Fig. 10. The plot of the position y_R versus x_R using the proposed and the conventional schemes with actual data. A smooth trajectory is obtained with the proposed scheme.

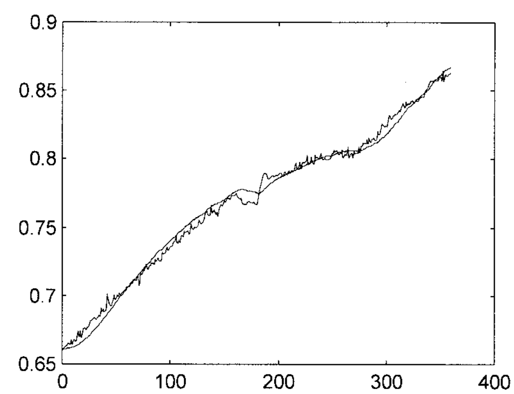

Fig. 12. The plot of the position y_R versus time using the proposed and the conventional schemes with actual data. A smooth trajectory is obtained with the proposed scheme.

C. Actual Test

Actual sets of data were collected by mounting the sun sensor and associated lab equipment and computers on a tower inside the Inertial Guidance Lab penthouse, which has large doors on the roof of the room. This arrangement provides an access to a larger portion of the sky. The sun sensor was fixed to the tower and a two-axes scan platform, thereby allowing precise alignment of the sensor to be maintained. By partially closing the roof doors, other sources of light may be introduced to simulate bias in the measurements. Figs. 10–12 give the estimated sun position using the proposed and the conventional approaches.

VII. CONCLUSIONS

The proposed Kalman-filter-based scheme is robust to measurement noise resulting from both the bias and the random errors and is globally convergent. Thanks to the inclusion of a dynamic equation governing the evolution of the position of the focused light and the model of the bias error, the proposed scheme can handle the movement of the sun position, the effects of atmospheric conditions such as the cloud movement, and partial observations of the sky and bias. Further, the presence of bad data may be detected and isolated. Extensive simulation and implementation on actual data confirms the superiority over the conventional static algorithms.

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