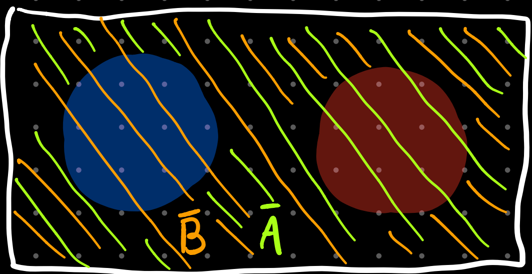
Probability:

$$P(B|A)=P(B),\quad P(A|B)=P(A)$$

a) given $P(A,B)=P(A)\cdot P(B)$ [independant]

Show → $\bar{A}$ & $B$, $A$ & $\bar{B}$, and $\bar{A}$ & $\bar{B}$ are also independent.




$$P(A|B)=P(A)$$

$$\cancel{1}+P(\bar{A}|B)=\cancel{1}+P(\bar{A})$$ total prob

$$P(\bar{A}|B)=P(\bar{A})$$ ind implication

$\therefore$ $\bar{A}$ & $B$ are independent ✓

use same logic for $A$ & $\bar{B}$ then

→ for $\bar{A}$ & $\bar{B}$:

$$P(A|\bar{B})=P(A)$$ given

$$\cancel{1}+P(\bar{A}|\bar{B})=\cancel{1}+P(\bar{A})$$

$$P(\bar{A}|\bar{B})=P(\bar{A})$$ ✓

b) 0.3 → A, 0.7 → B.

0.05 defective. 0.04 defective

$P(\overset{D}{\text{is defective}}, \text{sent to } \overset{A}{A}) = 0.015$

$P(\bar{D}, A) = 0.285$

$P(D, \text{sen to } B) = 0.28$

$P(\bar{D}, B) = 0.672$

this is low since the chances of being sent to A is already pretty low. (like the disease example)

c) we know: $P(A|B)=\dfrac{P(B|A)P(A)}{P(B)}$

$P(A|B) > P(A)$ implies dependence.

$$P(A|B)=\frac{P(B|A)\cancel{P(A)}}{\cancel{P(B)}} > \cancel{P(A)}P(B) \to P(B|A) > P(B) \quad \square$$