Wave propagation:
Solid Mechanics

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Outline

Wave features

There are great variety of waves but all of them could experiment

- **Reflection**:Occurs when a wave find a new medium, that can not cross, change its direction.
- **Refraction**: Occurs when a wave change its direction when enter in a new medium with different propagation speed.
- **Doppler Effect**: Effect caused by the relative motion between the source and the receptor.
- **Interference**: Occurs when two or more waves coexist in the same place and are superimposed.
- **Diffraction**: Occurs when a wave find the border of an obstacle and change its *form* to round it.

Waves in a string

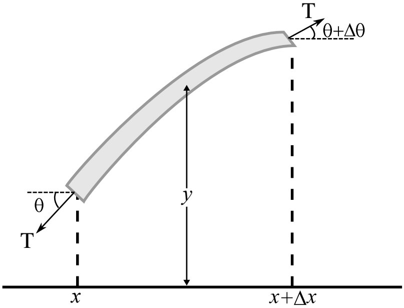

Figure: Forces diagram over an element of the string with length Δx.

For a string with length L, linear mass density λ and a tension T, let's take a small segment with small displacements in y. The force balance over the element showed in the Figure is:

$$F_y = T\sin(\theta + \Delta\theta) - T\sin(\theta)$$
$$F_x = T\cos(\theta + \Delta\theta) - T\cos(\theta) \ ,$$

Waves in a string

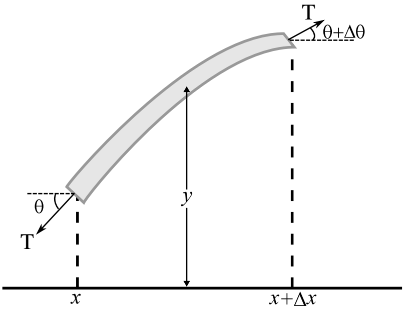

Figure: Forces diagram over an element of the string with length Δx.

Taking small displacements in the string, the angles are also small and then

$$
\begin{aligned}
F_y &\approx T(\theta + \Delta\theta) - T(\theta) = T\Delta\theta \\
F_x &\approx 0 \ .
\end{aligned}
$$

Waves in a string

From the second Newton's law we get

$$T \ \Delta\theta = \underbrace{(\lambda \ \Delta x)}_{\text{mass}} a_y \quad ,$$

if we take the limit $\Delta x \to dx$

$$T \ d\theta = (\lambda \ dx) a_y \quad . \tag{1}$$

And we now that $\tan\theta = \frac{\partial y}{\partial x}$, taking derivative respect x

$$\sec^2\theta \frac{d\theta}{dx} = \frac{\partial^2 y}{\partial x^2} \quad .$$

Due to small displacements $\sec^2\theta \approx 1$, hence

$$d\theta \approx \frac{\partial^2 y}{\partial x^2} dx \tag{2}$$

and replacing (2) in (1)

$$T \frac{\partial^2 y}{\partial x^2} dx = (\lambda \ dx) \frac{\partial^2 y}{\partial t^2} \quad ,$$

we finally get the 1D Wave Equation