Phys 512 Problem Set 5

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1. With the sample parameters already included in `planck_likelihood.py`, the χ^2 value is about 15267.9 for 2501 degrees of freedom. For the suggested parameters [69, 0.022, 0.12, 0.06, 2.1e-9, 0.95], the χ^2 value is rather 3272.20 for the same number of degrees of freedom. For 2501 DoFs, we would expect a mean χ^2 of 2501 with a standard deviation of $\sqrt{2 \times 2501} \sim 50\sqrt{2} \sim 70$. Clearly, neither values give an acceptable fit (the closer 3272 is still around 11σ away from the mean).

2. To find the best fit parameters using Newton's method, we proceed exactly as we did for problem set 4, using the same `num_derivs` function. By trial and error, it seems that the best way to make it work is by decreasing the derivative step for τ by 10^{-5}. Other values for $d\tau$ would either yield a singular matrix for the left-hand side (which gave poor results even when using SVD), or end up converging on a negative value of τ, which is unphysical. After that, we can get away with taking 10 steps since we're going to end up using a MCMC in the end. We then write the best-fit parameters to `planck_fit_params.txt`. The contents of the file are :
```
Best fit parameters are :
H0 = 68.24735666652363 +/- 1.187218256305278
ombh2 = 0.022364409137958098 +/- 0.00022925788292985978
omch2 = 0.11765632819493432 +/- 0.0026551966320841895
tau = 0.0854208365758012 +/- 0.034173883954142954
As = 2.2192121552240766e-09 +/- 1.432973144133891e-10
ns = 0.9730736805867614 +/- 0.006570253043921322
```
Fortunately, the planck data provided us with noise estimates for each data point, which makes it trivial to get our N^{-1} matrix and our covariance matrix that will be handy later.

3. Once more, this part is similar to the last problem set. We use the

covariance matrix, feed it into `np.random.multivariate_normal` to get our next MCMC step (with a chosen step scaling of 0.9 based on the comments in the PS4 solution, and it turns out well). The results we end up getting after 15000 steps and taking the mean/standard deviation for each parameter are :

```
H0 is 68.209098319072 +/- 1.0969598780845116
ombh2 is 0.022343330247673262 +/- 0.00022349827389843885
omch2 is 0.1177070999039286 +/- 0.0024403933668329667
tau is 0.07847908663303418 +/- 0.032831387884617695
As is 2.1932017286826e-09 +/- 1.3741459806891084e-10
ns is 0.9730677375283865 +/- 0.00603489817435084
```

and the results are summarized in the following corner plot. It can also be noted that the error we get on τ is relatively large : it is around half the value. This may be due to the way the model depends on τ, and could be related to why we had to fine-tune our $d\tau$ so much as compared to the other parameters. Nonetheless, the chain appears converged, as can be seen when we take the Fourier transform of each parameter's chain.

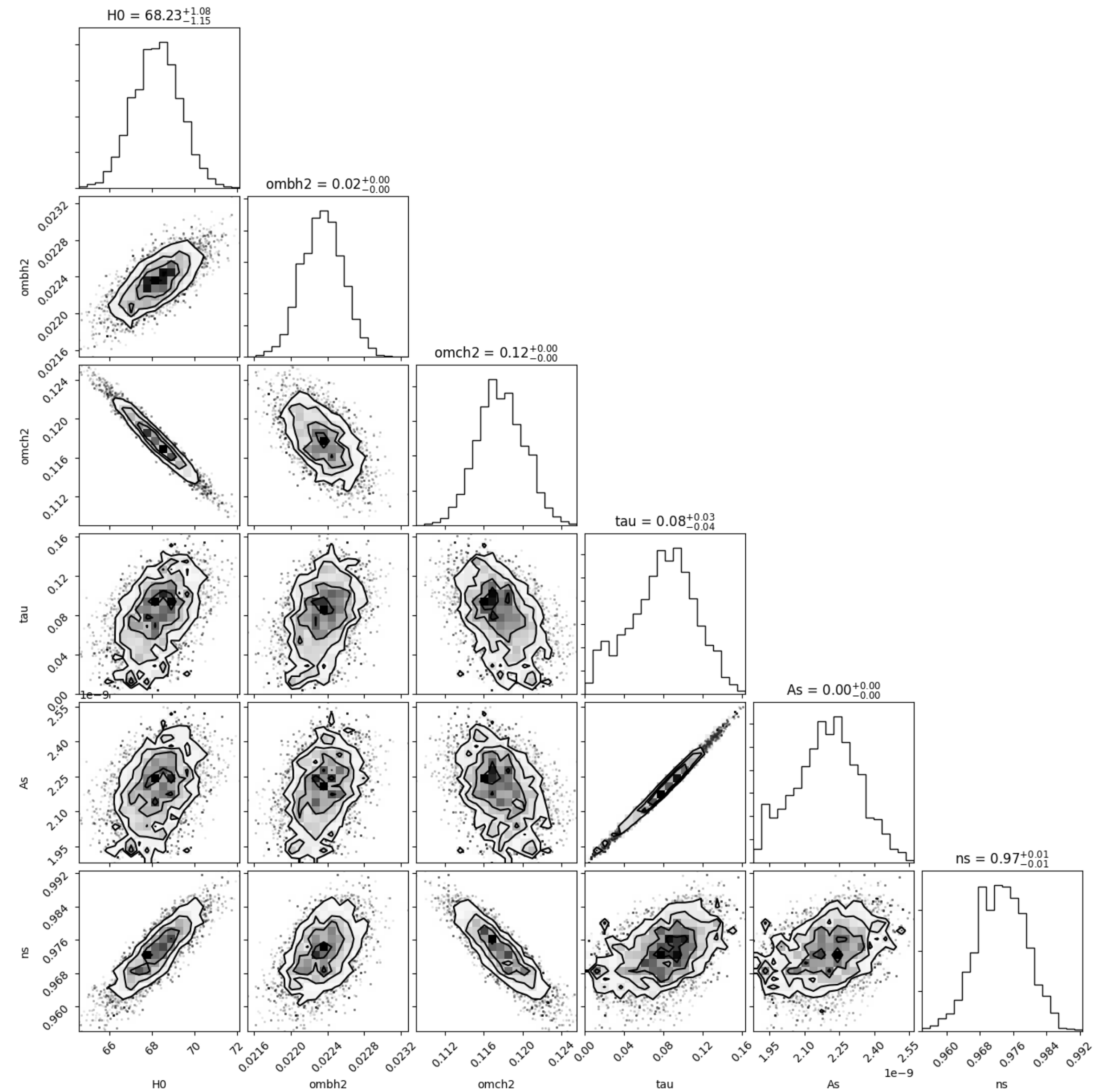

Figure 1: Corner plot showing the results after 15000 steps of the MCMC. As we can see, each parameter's binned PDF resemble a Gaussian. However, that works better on some parameters than others, and some contour plots look pretty scrambled. Two features appear clearly however, τ and A_s show a very strong correlation, as well as H_0 and Ω_b. Some other pairs show correlation, but in some cases it is harder to tell.

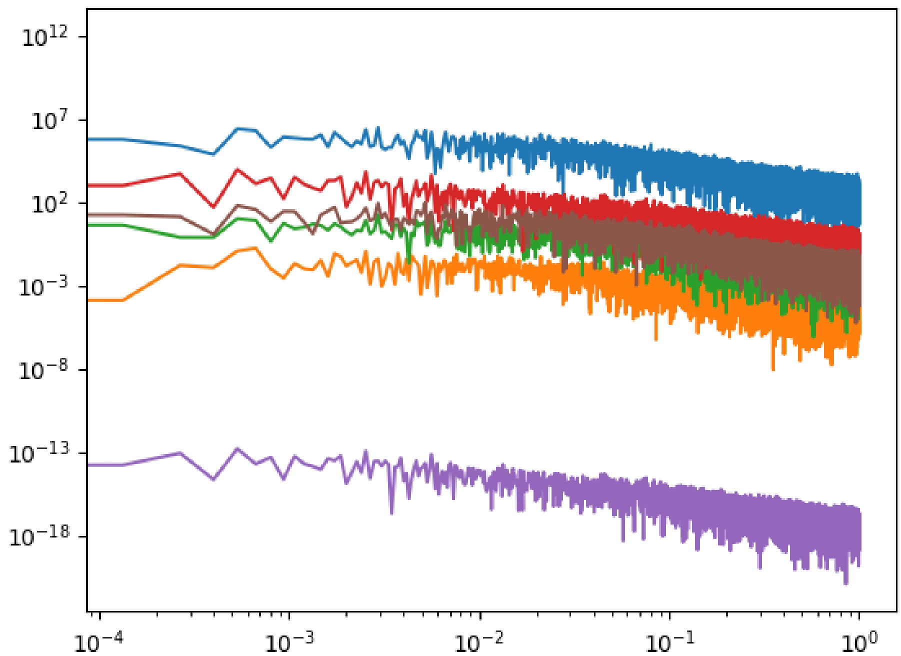

Figure 2: Fourier transform of each parameter's chain. As can be seen, we have a flat portion on the left for each parameter, showing that the chain converged for every single parameter.

We have:

$$\begin{aligned}
\Omega_b + \Omega_c + \Omega_\Lambda &= 1 \\
\Omega_\Lambda &= 1 - (\Omega_b + \Omega_c) \\
&= 1 - \left(\frac{100}{H_0}\right)^2 \times (\Omega_b h^2 + \Omega_c h^2) \\
&= 1 - \left(\frac{100}{68.21}\right)^2 \times (0.02234 + 0.1177) \\
&\simeq 1 - 0.3010 \\
&\simeq 0.6990
\end{aligned}$$

For error propagation, we look at a table and compare it with the formula we have to get :

$$\begin{aligned}
\sigma_\Lambda &= \Omega_\Lambda \sqrt{(2H_0\sigma_{H0})^2 + \frac{\sigma^2_{ombh2} + \sigma^2_{omch2}}{(\Omega_b h^2 + \Omega_c h^2)^2}} \\
&= 0.6990\sqrt{(2 \times 68.21 \times 1.10)^2 + \frac{0.00022^2 + 0.0024^2}{(0.02234 + 0.1177)^2}} \\
&\simeq 0.0225
\end{aligned}$$

So our full estimate for Ω_Λ is something like :

$$\Omega_\Lambda = 0.70 \pm 0.02$$

4. To perform importance sampling, we take the chain from the previous question and perform a weighted mean. The weights are the new τ prior, which if it's gaussian is just $w_i = \exp\frac{(\tau_i-\mu)^2}{2\sigma^2}$ where $\mu = 0.0540$ and $\sigma = 0.0074$. There's no need to normalize it, since we end up normalizing when taking the mean :

$$\text{importance sampled value} = \frac{\sum_i \text{parameter value} \times w_i}{\sum_i w_i}$$

where the sums are over the whole chain. The results are then :

```
With importance sampling :
H0 is 67.7403731297042 +/- 0.020274892832357744
ombh2 is 0.022274065400428623 +/- 4.130874466822276e-06
omch2 is 0.1187537138680934 +/- 4.510530874450376e-05
tau is 0.055149308565345785 +/- 0.0006068160597273086
As is 2.0950602205885474e-09 +/- 2.5398068836516166e-12
ns is 0.9706573408893923 +/- 0.00011154183136835576
```

For the re-run chain, I changed the chi-squared function to include the new prior, which increased the chi-squared value the further τ was from the polarization data mean of 0.0540. I simply added a $\frac{(\tau-\mu)^2}{2\sigma}$ term. I didn't update the covariance matrix, as I wasn't sure how I would go about that. The results are still somewhat sensible :

```
H0 is 67.73012885362449 +/- 0.9814056768008849
```

```
ombh2 is 0.022289579083605626 +/- 0.00021449986072748714
omch2 is 0.11882800973672844 +/- 0.00225219035214407
tau is 0.05666032395656838 +/- 0.010062060034104621
As is 2.1025784516399777e-09 +/- 4.271102539909518e-11
ns is 0.9701085744591954 +/- 0.005404365735317027
```

So the values from the two methods are sensibly the same. I think I messed up something in how I got the errors in the importance sampling method, as a mere weighted mean shouldn't reduce the errors by such a large factor.