3º)

d) $H(X_1, X_2, X_3) = ?$

Temos que:

$P(X_1=0, X_2=0, X_3=0) = \frac{1}{4}$

$P(X_1=0, X_2=1, X_3=0) = \frac{1}{4}$

$P(X_1=1, X_2=0, X_3=0) = \frac{1}{4}$

$P(X_1=0, X_2=0, X_3=1) = \frac{1}{4}$

Assim:

$H(X_1, X_2, X_3) = 4 \times \left[\frac{1}{4} \times \log_2 \frac{1}{\frac{1}{4}} \right] = 2 \text{ bits}$

e) $H(X_2/X_1=0) = P(X_2=0/X_1=0) \cdot \log_2 \frac{1}{P(X_2=0/X_1=0)}$

$\qquad + P(X_2=1/X_1=0) \cdot \log_2 \frac{1}{P(X_2=1/X_1=0)}$

$P(X_2=0/X_1=0) = \frac{P(X_1=0, X_2=0)}{P(X_1=0)}$

$(X_2=0/X_1=0) = \frac{P(X_1=0, X_2=0, X_3=0) + P(X_1=0, X_2=0, X_3=1)}{P(X_1=0)}$

$P(X_2=0/X_1=0) = \frac{\frac{1}{4} + \frac{1}{4}}{\frac{3}{4}} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$

logo: $P(X_2=1/X_1=0) = 1 - \frac{2}{3} = \frac{1}{3}$

Assim:

$H(X_2/X_1=0) = \frac{2}{3} \cdot \log_2 \frac{1}{\frac{2}{3}} + \frac{1}{3} \cdot \log_2 \frac{1}{\frac{1}{3}} = 0,916 \text{ bit}$

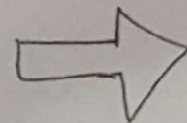f) $H(X_2/X_1=1) = P(X_2=0/X_1=1) \cdot \log_2 \frac{1}{P(X_2=0/X_1=1)} +$

$\qquad + P(X_2=1/X_1=1) \cdot \log_2 \frac{1}{P(X_2=1/X_1=1)}$

$P(X_2=0/X_1=1) = 1 \longrightarrow P(X_2=1/X_1=1) = 0$

Assim: $\boxed{H(X_2/X_1=1) = 0}$

g) $H(X_2/X_1) = P(X_1=0) H(X_2/X_1=0) + P(X_1=1) \cdot H(X_2/X_1=$

$H(X_2/X_1) = \frac{3}{4} \times 0,916 + 0 = 0,689 \text{ bit}$

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3º) h) $H(X_3/X_1X_2=00) = P(X_3=0/X_1X_2=00) \cdot \log_2 \dfrac{1}{P(X_3=0/X_1X_2=00)} + P(X_3=1/X_1X_2=00) \cdot \log_2 \dfrac{1}{P(X_3=1/X_1X_2=00)}$

• $P(X_3=0/X_1X_2=00) = \dfrac{P(X_1=0, X_2=0, X_3=0)}{P(X_1=0, X_2=0)} = \dfrac{P(X_1=0, X_2=0, X_3=0)}{P(X_1=0,X_2=0,X_3=0) + P(X_1=0,X_2=0,X_3=1)} = \dfrac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{4}}$
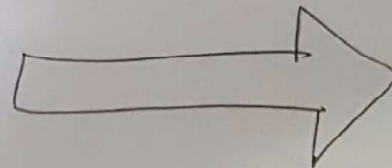
$P(X_3=0/X_1X_2=00) = \dfrac{\frac{1}{4}}{\frac{1}{2}} = \dfrac{1}{2} \text{//} \longrightarrow P(X_3=1/X_1X_2=00) = 1 - \dfrac{1}{2} = \dfrac{1}{2} \text{//}$

Assim: $H(X_3/X_1X_2=00) = 2 \times \left[\dfrac{1}{2} \cdot \log_2 \dfrac{1}{\frac{1}{2}} \right] = \underline{1 \text{ bit } \text{//}}$

i) $H(X_3/X_1X_2=01) = P(X_3=0/X_1X_2=01) \cdot \log_2 \dfrac{1}{P(X_3=0/X_1X_2=01)} + P(X_3=1/X_1X_2=01) \cdot \log_2 \dfrac{1}{P(X_3=1/X_1X_2=01)}$

• $P(X_3=0/X_1X_2=01) = 1$, pois X_3 fica univocamente determinado quando $X_1=0$ e $X_2=1$.

Logo: $\boxed{H(X_3/X_1X_2=01) = 0}$

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3º) j) $H(X_3/X_1X_2) = P(X_1X_2=00) \cdot H(X_3/X_1X_2=00) + P(X_1X_2=01) \cdot H(X_3/X_1X_2=01) +$

$+ P(X_1X_2=10) \cdot H(X_3/X_1X_2=10) + P(X_1X_2=11) \cdot H(X_3/X_1X_2=11)$

$\longrightarrow 0$ "NÃO EXISTE!"
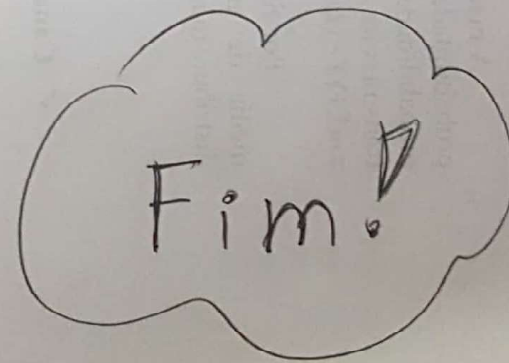
Temos que:

. $H(X_3/X_1X_2=00) = 1$ bit (letra "h")

. $H(X_3/X_1X_2=01) = 0$ bit (letra "i").

. $H(X_3/X_1X_2=10) = P(X_3=0/X_1X_2=10) \cdot \log_2 \dfrac{1}{P(X_3=0/X_1X_2=10)} + P(X_3=1/X_1X_2=10) \cdot \log_2 \dfrac{1}{P(X_3=1/X_1X_2=00)}$

$\hookrightarrow P(X_3=0/X_1X_2=10) = 1$, pois X_3 fica univocamente determinado quando

$X_1 = 1$ e $X_2 = 0$. Assim: $\boxed{H(X_3/X_1X_2=10) = 0 \text{ bit}}$

Logo:

$H(X_3/X_1X_2) = \dfrac{1}{2} \cdot 1 = \dfrac{1}{2}$ bit

Fim!

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