

## EXERCÍCIOS - SALA DE AULA

2º)  $X \rightarrow$  n.º de pontos obtidos  
no dado de Maria.

$Y \rightarrow$  n.º de pontos obtidos  
no dado de José.

$$X = \{1, 2, 3, 4, 5, 6\}$$

$$Y = \{1, 2, 3, 4, 5, 6\}$$

$$P(X=1) = P(X=2) = \dots = P(X=6) = \frac{1}{6}$$

$Y$  apresenta a mesma distribuição, ou  
seja:  $P(Y=1) = P(Y=2) = \dots = P(Y=6) = \frac{1}{6}$

$$a) H(X) = 6 \times \left[ \frac{1}{6} \cdot \log_2 \frac{1}{\frac{1}{6}} \right] = \boxed{2,58 \text{ bits}}$$

$$b) H(Y) = H(X) = \boxed{2,58 \text{ bits}}$$

c)  $H(Z)$ , em que  $Z = X + Y$

$Z \backslash Y$	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$P(Z=2) = P(Z=12) = \frac{1}{36}$$

$$P(Z=3) = P(Z=11) = \frac{2}{36}$$

$$P(Z=4) = P(Z=10) = \frac{3}{36}$$

$$P(Z=5) = P(Z=9) = \frac{4}{36}$$

$$P(Z=6) = P(Z=8) = \frac{5}{36}$$

$$P(Z=7) = \frac{6}{36}$$

logo:

$$H(Z) = 2 \times \left[ \frac{1}{36} \cdot \log_2 \frac{1}{\frac{1}{36}} \right] + 2 \times \left[ \frac{2}{36} \cdot \log_2 \frac{1}{\frac{2}{36}} \right] +$$

$$+ 2 \times \left[ \frac{3}{36} \cdot \log_2 \frac{1}{\frac{3}{36}} \right] + 2 \times \left[ \frac{4}{36} \cdot \log_2 \frac{1}{\frac{4}{36}} \right] +$$

$$+ 2 \times \left[ \frac{5}{36} \cdot \log_2 \frac{1}{\frac{5}{36}} \right] + \frac{6}{36} \cdot \log_2 \frac{1}{\frac{6}{36}}$$

$$\boxed{H(Z) = 3,274 \text{ bits}}$$

## EXERCÍCIOS - SALA DE AULA

2º) d)  $H(X, Y) = H(X) + H(Y/X)$

$X$  e  $Y$  são v.a. independentes, logo:

$$H(X, Y) = H(X) + H(Y)$$

$$H(X, Y) = 2,585 + 2,585 = \boxed{5,17 \text{ bits}}$$

e)  $H(X/Y) = H(X) = \boxed{2,585 \text{ bits}}$

f)  $I(X, Y) = H(X) - H(X/Y) = H(X) - H(X) = \boxed{0}$

3º)  $[X_1, X_2, X_3]$

$$[0, 0, 0] \rightarrow 1/4$$

$$[0, 1, 0] \rightarrow 1/4$$

$$[1, 0, 0] \rightarrow 1/4$$

$$[0, 0, 1] \rightarrow 1/4$$

a)  $H(X_1) = P(X_1=0) \cdot \log_2 \frac{1}{P(X_1=0)} +$

$$+ P(X_1=1) \cdot \log_2 \frac{1}{P(X_1=1)}$$

•  $P(X_1=1) = P(X_1=1, X_2=0, X_3=0) = \frac{1}{4}$

$$\hookrightarrow P(X_1=0) = 1 - \frac{1}{4} = \frac{3}{4}$$

logo:

$$H(X_1) = \frac{3}{4} \times \log_2 \frac{1}{\frac{3}{4}} + \frac{1}{4} \times \log_2 \frac{1}{\frac{1}{4}}$$

$$\boxed{H(X_1) = 0,811 \text{ bits}}$$

b)  $H(X_2) = ?$

•  $P(X_2=1) = P(X_1=0, X_2=1, X_3=0) = \frac{1}{4}$

$$\hookrightarrow P(X_2=0) = 1 - \frac{1}{4} = \frac{3}{4}$$

logo:

$$H(X_2) = H(X_1) = \boxed{0,811 \text{ bits}}$$

c)  $H(X_3) = ?$

•  $P(X_3=1) = P(X_1=0, X_2=0, X_3=1) = \frac{1}{4}$

$$\hookrightarrow P(X_3=0) = 1 - \frac{1}{4} = \frac{3}{4}$$

Logo:

$$H(X_3) = H(X_2) = H(X_1) = \boxed{0,811 \text{ bits}}$$