Homework 4

[100 points]

This written homework focusses on Chapters 13 and 14. Your response should be well-thought-out, coherent, and concise. Quality of written expression will be a factor in the grading (please use full sentences when explaining something). Short, to-the-point answers are preferred.

(1) [25 points] Consider the following coalition game with four students. Each student happens to have a single shoe (not a pair of shoes), and the students are trying to form pairs of shoes. Specifically, student 1 has a left-shoe in red, student 2 a right-shoe in red, student 3 a left-shoe in blue, and student 4 a right-shoe in blue.

Once shoes are collected, they can sell a pair of shoes. If they collectively have a pair of blue shoes, the shoes are sold at \$30, a pair of red shoes at \$20, and a pair of red-blue or blue-red shoes (mixed-color shoes) at \$10. If there are multiple ways to sell the shoes, they will sell the highest-price pair of shoes first (that is, in order of blue, red, and mixed). For example, if all of them coordinate, then they would first sell the blue shoes at \$30, and then the red shoes at \$20 to make a total profit of \$50.

(a) [12 points] Formally describe this as a coalitional game. $N = \{1, 2, 3, 4\}$, and you are to describe the characteristic function $v : 2^N \rightarrow \{\$0, \$10, \$20, \$30, \$50\}$. Fill out the table below to specify v. Only specify values for $v(S)$ where $v(S) \neq \$0$ (to avoid tedious work). Use the table below as a reference.

$S \subseteq N$	$v(S)$
$\{1, 2\}$	\$20
$\{2, 3\}$	\$10
$\vdots$	$\vdots$

(b) [5 points] Compute the Shapley value, $\phi_i(N,v)$, for each student $i \in N$.

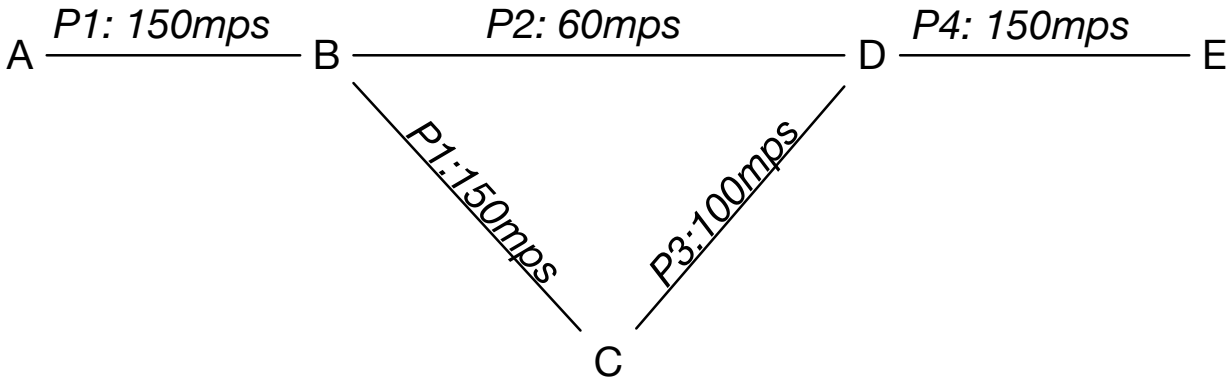
(c) [2 point] True or false: this game is additive.

(d) [2 point] True or false: this game is superadditive.

(e) [2 point] True or false: this game is constant-sum.

(f) [2 point] True or false: the core of this game is empty.

(2) [30 points] Assume the following network links with the capacities and providers [P1, P2, P3, P4] shown:



Service requests:

- Customer 1: A->E, 70mps, will pay $6
- Customer 2: C->E, 80mps, will pay $4
- Customer 3: B->E, 50mps, will pay $1
- Customer 4: B->D, 20mps, will pay $2

The providers can work together (collude) to provide virtual circuits. The entire capacity of a circuit has to follow the same route, I.e., no individual request can be divided among multiple routes. Requests have to be handled completely or not at all: the customers will not pay anything for partial handling. If the request is not handled the customer will not pay of course.

(a) [5 pts] Show the value (collected revenue) of each possible provider coalition. You may assume that the coalition gets all the customers that it wants to service. Take into account the lack of appropriate links as well as bandwidth limitations that may preclude the coalition from handling all customers.
(b) [5 pts] What is the social welfare maximizing solution?
(c) [5 pts] What is the corresponding coalition structure?
(d) [5 pts] Is that coalition structure stable according to the core solution concept (justify your answer)? Note that deviating coalitions may service customers the were serviced in the original coalition structure as well as those who were not.
(e) [10 pts] How would the answers to a,b,c,d change if customer 4 would only be willing to pay $1 instead of $2?

(3) [20 points] You are in the market for a used car and see an ad for the model that you like. The owner has not set a price but invites potential buyers to make offers. Your pre-purchase inspection gives you only a very rough idea of the value of the car; you think it is equally likely to be anywhere in the range of $1,000 to $5,000 (so your calculation of the average of this value is $3,000). The current owner knows the exact value and will accept your offer if it exceeds that value. If your offer is accepted and you get the car, you will find out the truth. But you have some special repair skills and know that when you own the car, you will be able to work on it and increase its value by a third (33.3%) of whatever it is worth.

(a) [10 pts] What is your expected profit if you offer $3,000? Should you make such an offer?

(b) [10 pts] What is the highest offer that you can make without losing money on the deal?

(4) [25 pts] Resource Allocation. Consider a combinatorial auction with three items {a,b,c} and three players {I, II, III}. The valuations each player has for each subset of

	a	b	c	ab	bc	ac	abc
I	1	2	1	6	4	5	11
II	3	5	3	5	5	3	5
III	4	5	0	7	5	7	9

the items is shown below.
What is the allocation produced under the VCG (Vickrey-Clarke-Groves) mechanism and what are the VCG payments of the players?