CISC 489
Homework 3
April 28, 2022
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Problem 1

Here is the table from the HW3-MAS-2022.pdf replicated below:

		Train 2 *(top)*	
Train 1 *(left)*		Slow track	Fast track
	Slow track	-5, -5	-6, 0
	Fast track	0, -6	-10, -10

Problem 1a

I'm using what we talked about in lecture to solve this problem.

Find all pure strategy Nash equilibria:

- (Slow track, Fast Track)
    - That is, Train 1 takes the slow track, and Train 2 takes the fast track
- (Fast track, Slow Track)
    - That is, Train 1 takes the fast track, and Train 2 takes the slow track

These are our possibilities because if both trains take the fast track, they both stand to lose a lot, whereas if one of them take the fast track, one of them will gain huge utility.

Problem 1b

I'm using what we talked about in lecture to solve this problem.

Find all mixed strategy Nash equilibria:

- Train 1
    - Plays the mixed strategy of (Slow track, Fast track)
    - That equals (0.444, 0.556)
    - Expected payout is -5.556
- Train 2
    - Plays the mixed strategy of (Slow track, Fast track)
    - That equals (0.444, 0.556)
    - Expected payout is -5.556

Problem 1c

I'm using Chapter 11.3.3 "Pareto efficiency" on page 233 to solve this.

Find all pure Pareto optimal / efficient strategy profiles:

"We will say that an outcome is Pareto efficient if there is no other outcome that improves one player's utility without making somebody else worse off."

In other words, it is the outcome that is benefits both agents

		Train 2 *(top)*	
Train 1 *(left)*		Slow track	Fast track
	Slow track	-5, -5	-6, 0
	Fast track	0, -6	-10, -10

In our example, it would be left choosing slow track and top choosing fast track. Or the reverse: Left chooses the fast track, and top chooses the slow track.

Problem 1d

According to Lecture11.pdf, minmax is where we "minimize the other players max payoff".

		Train 2 *(top)*	
Train 1 *(left)*		Slow track	Fast track
	Slow track	-5, -5	-6, 0
	Fast track	0, -6	-10, -10

In that case, left should always choose the fast track. And top should always choose the fast track.

When left chooses the fast track, top can now only at best get -6 utility, and worst -10 utility.

When top chooses the fast track, left can now only at best get -6 utility, and worst -10 utility.

Problem 2a

Iteratively eliminate all strictly dominated pure strategies.

	A	B	C
D	-5, 5	-2, 2	2, -2
E	-1, 3	-1, 2	2, -2
F	-2, -1	-2, 3	4, -4

First, let us look at the B and C columns:

- B,D > C,D == 2 > -2
- B,E > C,E == 2 > -2
- B,F > C,F == 3 > -4

So, we can eliminate the C column.

	A	B
D	-5, 5	-2, 2
E	-1, 3	-1, 2
F	-2, -1	-2, 3

Next, let us look at the E and F rows:

- E,A > F,A == -1 > -2
- E,B > F,B == -1 > -2

So, we can eliminate the F row.

	A	B
D	-5, 5	-2, 2
E	-1, 3	-1, 2

Next, let us look A and B columns:

- A,D > A,B == 5 > 2
- A,E > B,E == 3 > 2

So we can eliminate the B column.

	A
D	-5, 5
E	-1, 3

Since the top player can only choose A, the only good choice for the left player is E, as when we look at the values, E > D, -1 > -5.

Problem 2b

	A
D	-5, 5
E	-1, 3

Looking now at the reduced table, we will look from the perspective of the top player.

The top player will only ever choose A, because that is their only choice.

For the left layer, they would only ever choose E, because -1 is greater than -5.

The Nash equilibrium of this game would be [E, A].

Problem 3a

We need to show that [0.5: (D, B); 0.5: (E,C)] is a correlated equilibrium by coin flip.

	A	B	C
D	-2, 1	6, 2 (50%)	-2, 1
E	-2, 3	-1, 3	2, 6 (50%)
F	2, -1	3, 1	2, -2

"It's only at equilibrium if we think this is better than acting on our own"

Utility(Top) = (.5)2 + (.5)6 = 4
Utility(Left) = (.5)6 + (.5)2 = 4

It is an equilibrium because the expected utility for both agents is the same. From the top agent's perspective, we would choose B if for the left agent chose the coin landed as heads, making the left agent choose D. And if the coin flip were to end up as tails, then the left agent would be choosing E, and then we would have to choose C, which is our best option. And this contract makes sense because this gives us the highest expected utility out of all the other options if the other follows the same pattern.

Coincidentally, both (D, B) and (E, C) are Nash equilibria's, and "Nash equilibria are correlated equilibria, but not visa versa..." (Lecture 11, page 25).

Problem 3b

We need to show that [[0.5: D, 0.5: E] ; [0.5: B, 0.5: C]] is **not** a Nash equilibrium

	A	B	C
D	-2, 1	6, 2 (25%)	-2, 1 (25%)
E	-2, 3	-1, 3 (25%)	2, 6 (25%)
F	2, -1	3, 1	2, -2

Utility(Top) = (.25)(2) + (.25)(3) + (.25)(1) + (.25)(6) = 3
Utility(Left) = (.25)(6) + (.25)(-1) + (.25)(-2) + (.25)(2) = 1.25

The top agent would not want to work with the left agent, because the top agent would always choose C to get a higher expected utility ((1)(0.5) + (6)(0.5) = 3.5). The top agent would prefer to gain more than 3 utilities. Therefore, it is not Nash, because the agents would not want to work together.

Problem 4a

This problem has the same principle as 1b. I'm using the equation from Lecture11.pdf, page 19.

Assume Top plays [p LW, (1-p) WL]
Then Uleft(LW) = Uleft(WL)

		Pitcher	
Batter		P throws fastball	P throws curve
	B anticipates fastball	0.30, 0.70	0.20, 0.80
	B anticipates curve	0.15, 0.85	0.35, 0.65

Assume Top plays [p * P throws fastball, (1-p) * P throws curve]
Then Uleft(LW) = Uleft(WL)
Then (0.3)p + (0.2)(1 - p) = (0.15)p + (0.35)(1 - p)
Then 0.3p + 0.2 – 0.2p = 0.15p + 0.35 – 0.35p
Then 0.1p = -0.2p + 0.15
Then 0.3p = 0.15
Then p = 0.5

Assume Left plays [p * B anticipates fastball, (1-p) * B anticipates curve]
Then Utop(LW) = Utop(WL)
Then (0.7)p + (0.85)(1 - p) = (0.80)p + (0.65)(1-p)
Then 0.7p + 0.85 – 0.85p = 0.80p + 0.65 – 0.65p
Then -0.15p + 0.85 = 0.15p + 0.65 => 0.15 = 0
Then p = …

NOTE: Professor Decker told me to setup the equation above this way^, and it positively does not work, because it cancels out the P's. I'm going to do just a random way based off the slides.

Throws Fastball = 0.70p + 0.85 – 0.85p
Throws curve = 0.80p + 0.65 – 0.65p
Calculate: p = 0.667

The expected payout of top is (0.5, 0.5).
The expected payout of left is (0.667, 0.333)

I worked on this problem with Professor Decker in office hours.

Problem 4b

Listed above.

Problem 4c

I have now updated the table below (changes highlighted):

		Pitcher	
Batter		P throws fastball	P throws curve
	B anticipates fastball	0.25, 0.70	0.20, 0.80
	B anticipates curve	0.15, 0.85	0.35, 0.65

Problem 5a

We need to rework the equations.

$$\pi_a = (p - c)q_a$$
$$\pi_b = (p - c)q_b$$

$$\pi_a = \big(\alpha - \beta(q_a + q_b)\big)q_a - c \times q_a$$
$$\pi_a = (\alpha - \beta \times q_a - \beta \times q_b - c) \times q_a$$
$$\pi_b = (\alpha - \beta \times q_a - \beta \times q_b - c) \times q_b$$

$$\frac{\delta q_a}{\delta q_a} = 0$$

$$\alpha - \beta \times 2q_a - \beta q_b - c = 0$$

$$\frac{\delta q_b}{\delta q_b} = 0$$

$$\alpha - \mathrm{\beta} \times \mathrm{q_a} - 2\beta q_b - c = 0$$

$$q_a = \frac{a - \beta q_b - c}{2\beta}$$

$$q_b = \frac{a - \beta q_a - c}{2\beta}$$

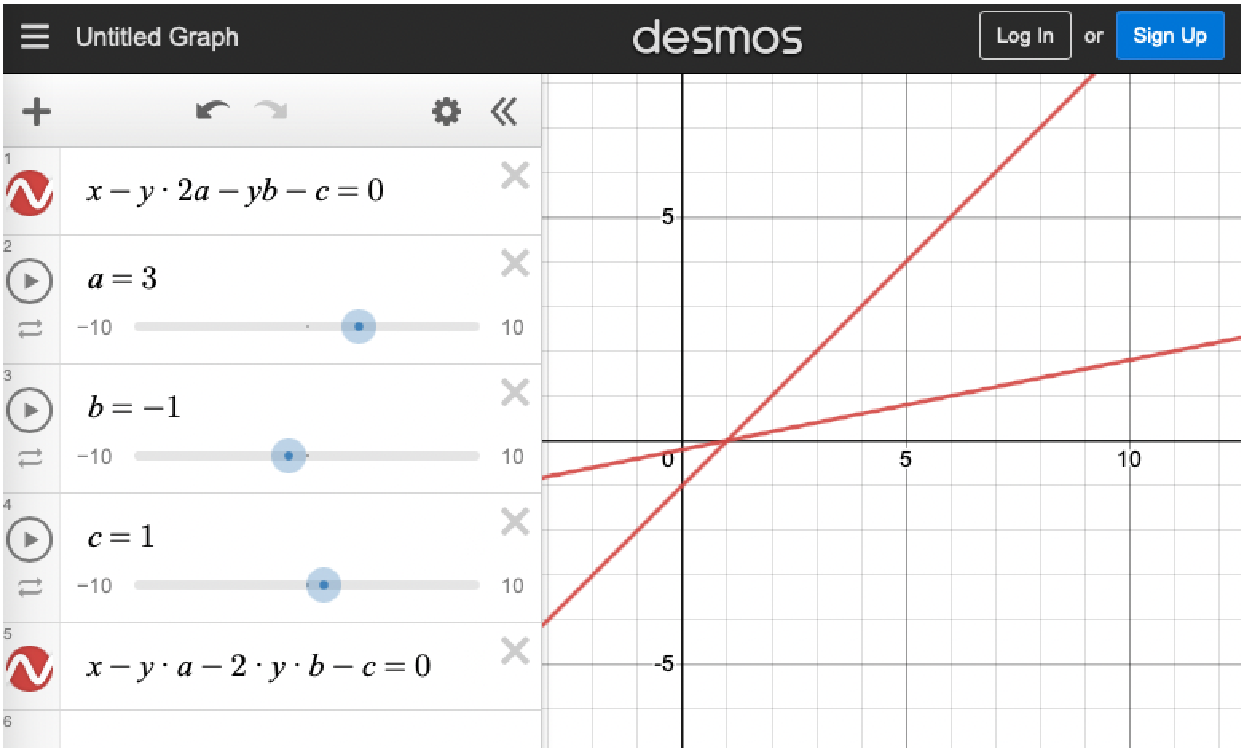
Problem 5b

If both q's must match, meaning the output of one agent is the same as the other agent. Then, both agents should have the same price so that they have similar profits. Cartels often set the same price for all of the members in the cartel.

Problem 5c

Agent A should choose a lower output because that will decrease the $-\beta(q_a + q_b)$ term and increase A's profits.

Problem 5d



The equilibrium choices for q_a and q_b would be $q_a = 3$ and $q_b = -1$

Problem 6a

	P1	P2	P3
	P2	P3	P1
	P3	P1	P2
Votes	*20*	*15*	*15*
Type	*1*	*2*	*3*

Total votes = 20 + 15 + 15 = 50

"Plurality voting is an electoral system in which a candidate, or candidates, who poll more than any other counterpart (that is, receive a plurality), are elected" (Wikipedia).

Since Proposal 1 polled higher in the first position than any of the other proposals, it would win in a plurality voting system.

And additionally, I think if we're looking more closely at the votes: Proposal 1 would still win. 20 voters put Proposal 1 first, and 15 voters put Proposal 1 first.

Problem 6b

Type 1		
Proposal	Votes	Points
P1	20	60
P2	20	40
P3	20	20

Type 2		
Proposal	Votes	Points
P2	15	45
P3	15	30
P1	15	15

Type 3		
Proposal	Votes	Points
P3	15	45
P1	15	30
P2	15	15

Total		
Proposal	Points	Total
P1	60 + 15 + 30	105
P2	40 + 45 + 15	100
P3	20 + 30 + 15	65

Proposal 1 would win in the Borda count system.

Problem 6c

Type 2 and Type 3 voters could agree to do some strategic voting and put Proposal 1 as both of their least preferred (third position) proposal. Proposal 1 only won by 5 points. If Type 3 voters put Proposal 1 to their least preferred position, then Proposal 2 would win.

Problem 7

I used information I learned in one-on-one office hours with Professor Decker and the following site to solve this problem:

https://courses.lumenlearning.com/waymakermath4libarts/chapter/instant-runoff-voting/

Five policy proposals being voted on: (v, w, x, y, z)

	Anderson	Brown	Clark	Davis	Evans	Foster	Garcia
1st	V	V	W	W	X	Y	Z
2nd	W	X	V	X	Y	X	Y
3rd	X	W	Y	V	Z	Z	X
4th	Y	Y	X	Y	V	W	W
5th	Z	Z	Z	Z	W	V	V

Assuming an IRV (Instant Runoff Voting) system where all policies/candidates that tie for the fewest votes are eliminated at the same time, under what conditions is an eventual majority winner guaranteed? [Put another way, under what conditions might there not be an ambiguous majority winner?] How will these conditions change if a new person, Harris, moves into town and votes?

Compute the IRV

Calculate the votes

Letter	Votes
V	2
W	2
X	1
Y	1
Z	1

There is no winner, so we eliminate the last place option. Since there is a tie for last place, and this is the first round, we randomly eliminate one. I'll eliminate Z. I'll also shift up all the votes.

	Anderson	Brown	Clark	Davis	Evans	Foster	Garcia
1st	V	V	W	W	X	Y	Y
2nd	W	X	V	X	Y	X	X
3rd	X	W	Y	V	V	W	W
4th	Y	Y	X	Y	W	V	V

Calculate the votes again

Letter	Votes
V	2
W	2
X	1
Y	2

Since X is in last place, I'll eliminate X.

	Anderson	Brown	Clark	Davis	Evans	Foster	Garcia
1st	V	V	W	W	Y	Y	Y
2nd	W	W	V	V	V	W	W
3rd	Y	Y	Y	Y	W	V	V

Let's count the votes, again.

Letter	Votes
V	2
W	2
Y	3

Now there is a tie for last place between V and W. We must look at the previous round to eliminate the candidate with the fewest first place votes. But still, V and W are tied with 2 votes. And so, we can randomly eliminate one of them. I chose W to eliminate.

	Anderson	Brown	Clark	Davis	Evans	Foster	Garcia
1st	V	V	V	V	Y	Y	Y
2nd	Y	Y	Y	Y	V	V	V

Let's count the votes, again.

Letter	Votes
V	4
Y	3

V now has a majority of votes, 4/7 = 0.57… and so V wins.

Professor Decker informed me that his version of IRV includes eliminating all ties at the same time. I did not do that. I am not going update the above tables because I do not have enough time, and I do not believe it would change the outcome.

To answer the question “under what conditions is an eventually majority winner guaranteed?”

It would be when if the population size is sufficiently large enough, and this process does eventually produce a majority winner.

To answer the question “How will these conditions change if a new person, Harris, moves into town and votes?”

Harris would make an even number of people, which would actually further complicate this process by making it more tie-tacular (tie likely).