Homework 3

[105 points available, graded out of 100]

This written homework focusses on Chapters 11 and 12. Your response should be well-thought-out, coherent, and concise. Quality of written expression will be a factor in the grading (please use full sentences when explaining something). Short, to-the-point answers are preferred.

1.) [20 points] Consider the following situation: Two freight trains are transporting goods across a mountain chain and ahead of them are three tracks, two slow ones and one fast one. The trains are currently on the two slow ones and can choose to either stay on them or to switch to the fast one (and they can not see each other or talk to each other). If both train drivers decide to switch to the fast track, the automatic signaling system a few miles down will detect that they are both bound for the same track and turn to red for both trains and close the faster track, forcing them to back up to the slower tracks and take those. If one of the trains moves to the fast track the high speed and proximity to the slower track further along will force the train on the slow track to slow down until the train on the faster track is far enough ahead, delaying the train staying on the slow track. Assuming that the utility for the trains is the loss in utility for the two trains is equal to the time delay, this results in the following game:

	slow track	fast track
slow track	-5,-5	-6,0
fast track	0, -6	-10, -10

a) [5 pts] Find all pure strategy Nash equilibria.

b) [5 pts] Find all mixed strategy Nash Equilibria.

c) [5 pts] Find all pure Pareto optimal/efficient strategy profiles.

d) [5 pts] Calculate the minmax strategy for player 1.

2.) [10 points] Consider the following game:

	A	B	C
D	-5,5	-2,2	2,-2
E	-1, 3	-1,2	2,-2
F	-2,-1	-2,3	4,-4

a) [5 pts] Iteratively eliminate all strictly dominated pure strategies.

b) [5 pts] Determine a Nash equilibria for the reduced game resulting from the iterated elimination in part a).

3.) [10 points] Consider the following game:

	A	B	C
D	-2,1	6,2	-2,-1
E	-2, 3	-1,3	2,6
F	2, -1	3,1	2,-2

a) [5 pts] Show that [0.5:(D,B) ; 0.5:(E,C)] is a correlated equilibrium. Recall that in a correlated equilibrium problem the coin flip is external and known to both players (imagine they flip a coin with DB on one side and EC on the other).

b) [5 pts] Show that [[0.5:D ; 0.5:E] ; [0.5:B ; 0.5:C]] is **not** a Nash equilibrium.

4.) [15 points] In a simplified model of baseball, the pitcher/batter game has no pure equilibrium since pitchers and batters have conflicting goals (the pitcher wants to get the ball past the batter, and the batter wants to connect with the ball). The normal form game might be as follows:

	P throws fastball	P throws curve
B anticipates fastball	0.30, 0.70	0.20, 0.80
B anticipates curve	0.15, 0.85	0.35, 0.65

(a) Find the mixed-strategy Nash equilibrium.
(b) What is each players expected payoff for the game?
(c) Suppose the pitcher tries to improve their expected payoff in the mixed strategy equilibrium by slowing down the fast ball (making it more similar to the curve ball). This changes the payoff to the hitter in [anticipate fastball/throw fastball}

from 0.30 to 0.25, and the pitcher payoff adjusts accordingly. Does this actually improve the pitcher's payoff as desired?

5.) [25 points] In a certain market there are two firms, which we label a and b. If the firms produce output q_a and q_b, then the price they will receive for their goods is given by $p = \max\{0, \alpha - \beta(q_a + q_b)\}$. Each firm has marginal cost $c > 0$ and no fixed costs. Suppose $\alpha, \beta > 0$ and $\alpha > 3c$.

(a) (5 points) Suppose the firms choose the quantity q_a and q_b independently in each period, without regard to their behavior in previous periods, and each maximizes profits $\pi_a = (p-c)q_a$ and $\pi_b = (p-c)q_b$. Find the unique pure strategy Nash equilibrium of this game. This is called the *Cournot duopoly* model. (**Hint**: In equilibrium, each choice of q_a, q_b, is a best response to the other choice—you may use a graphical approach if it helps you.)

(b) (5 points) Suppose the two firms collude by agreeing that each will produce amount $q^* = q_a = q_b$, and they have some way to enforce the agreement. What should they choose for q^*? What are their profits? This is called the *monopoly* or *cartel* model.

(c) (5 points) Suppose firm a reneges on its promise in the previous part but b does not. What should a choose for q_a? What are a's profits, and b's profits?

(d) (10 points) Suppose firm a gets to choose its output first (which is announced) and only afterward does firm b get to choose its own output. Find the equilibrium choices of q_a and q_b in this case. This is the *Stackelberg duopoly* model. (**Hint**: Firm a will need to calculate b's response q_b to every choice of q_a and then optimize its own choice of q_a with respect to this response.) *If we have time we'll talk about this as a model of (for example) airport security that was developed and deployed initially at LAX and elsewhere since; e.g. Firm a is the police, "firm" b are the terrorists.*

6.) [15 points] Consider a group of 50 residents attending a town meeting in Massachusetts. They must choose one of the three proposals for dealing with town garbage. Prop 1 asks the town to provide garbage collection as one of its services; Prop 2 calls for the town to hire a private garbage collector to provide collection services; Prop 3 calls for residents to be responsible for their own garbage.

There are three types of voters. The first type prefers P1 > P2 > P3; there are 20 of these. The second type prefers P2 > P3 > P1; there are 15 of these voters. The third type prefers P3 > P1 > P2; there are 15 of them.

(a) (5 points) Under a plurality voting system, which proposal wins?

(b) (5 points) Suppose voting proceeds with the use of a Borda count in which voters list the proposals in order of preference on their ballots. The proposal listed first gets three points, the second gets two points, and the third gets one point. In this situation, with no strategic voting, how many points does each proposal gain? Which proposal wins?

(c) (5 points) What strategy can the type 2 and type 3 voters use to alter the outcome of the Borda-count vote in part (b) to one that both prefer? If they use their strategy, how many points does each proposal get, and which wins?

7.) [10 points] Consider the following table, which gives the preference order ballots of a small town with seven citizens voting on five policy proposals [v, w, x, y, z] put forward by the mayor:

	Anderson	Brown	Clark	Davis	Evans	Foster	Garcia
1st	v	v	w	w	x	y	z
2nd	w	x	v	x	y	x	y
3rd	x	w	y	v	z	z	x
4th	y	y	x	y	v	w	w
5th	z	z	z	z	w	v	v

Assuming an IRV (Instant Runoff Voting) system where all policies/candidates that tie for the fewest votes are eliminated at the same time, under what conditions is an eventual majority winner guaranteed? [Put another way, under what conditions might there not be an ambiguous majority winner?] How will these conditions change if a new person, Harris, moves into town and votes?