CISC 489

Homework 3

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**Problem 1**

Here is the table from the HW3-MAS-2022.pdf replicated below:

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | ***Train 2*** *(top)* | |
| ***Train 1*** *(left)* |  | Slow track | Fast track |
| Slow track | -5, -5 | -6, 0 |
| Fast track | 0, -6 | -10, -10 |

**Problem 1a**

I’m using what we talked about in lecture to solve this problem.

Find all pure strategy Nash equilibria:

* (Slow track, Fast Track)
  + That is, Train 1 takes the slow track, and Train 2 takes the fast track
* (Fast track, Slow Track)
  + That is, Train 1 takes the fast track, and Train 2 takes the slow track

These are our possibilities because if both trains take the fast track, they both stand to lose a lot, whereas if one of them take the fast track, one of them will gain huge utility.

**Problem 1b**

I’m using what we talked about in lecture to solve this problem.

Find all mixed strategy Nash equilibria:

* Train 1
  + Plays the mixed strategy of (Slow track, Fast track)
  + That equals (0.444, 0.556)
  + Expected payout is -5.556
* Train 2
  + Plays the mixed strategy of (Slow track, Fast track)
  + That equals (0.444, 0.556)
  + Expected payout is -5.556

**Problem 1c**

I’m using Chapter 11.3.3 “Pareto efficiency” on page 233 to solve this.

Find all pure Pareto optimal / efficient strategy profiles:

“We will say that an outcome is Pareto efficient if there is no other outcome that improves one player’s utility without making somebody else worse off.”

…

**Problem 1d**

…

**Problem 2a**

Iteratively eliminate all strictly dominated pure strategies.

|  |  |  |  |
| --- | --- | --- | --- |
|  | A | B | C |
| D | -5, 5 | -2, 2 | 2, -2 |
| E | -1, 3 | -1, 2 | 2, -2 |
| F | -2, -1 | -2, 3 | 4, -4 |

First, let us look at the B and C columns:

* B,D > C,D == 2 > -2
* B,E > C,E == 2 > -2
* B,F > C,F == 3 > -4

So, we can eliminate the C column.

|  |  |  |
| --- | --- | --- |
|  | A | B |
| D | -5, 5 | -2, 2 |
| E | -1, 3 | -1, 2 |
| F | -2, -1 | -2, 3 |

Next, let us look at the E and F rows:

* E,A > F,A == -1 > -2
* E,B > F,B == -1 > -2

So, we can eliminate the F row.

|  |  |  |
| --- | --- | --- |
|  | A | B |
| D | -5, 5 | -2, 2 |
| E | -1, 3 | -1, 2 |

Next, let us look A and B columns:

* A,D > A,B == 5 > 2
* A,E > B,E == 3 > 2

So we can eliminate the B column.

|  |  |
| --- | --- |
|  | A |
| D | -5, 5 |
| E | -1, 3 |

Since the top player can only choose A, the only good choice for the left player is E, as when we look at the values, E > D, -1 > -5.

**Problem 2b**

|  |  |
| --- | --- |
|  | A |
| D | -5, 5 |
| E | -1, 3 |

Looking now at the reduced table, we will look from the perspective of the top player.

The top player will only ever choose A, because that is their only choice.

For the left layer, they would only ever choose E, because -1 is greater than -5.

The Nash equilibrium of this game would be [E, A].

**Problem 3a**

We need to show that [0.5: (D, B); 0.5: (E,C)] is a correlated equilibrium by coin flip.

|  |  |  |  |
| --- | --- | --- | --- |
|  | A | B | C |
| D | -2, 1 | 6, 2  (50%) | -2, 1 |
| E | -2, 3 | -1, 3 | 2, 6  (50%) |
| F | 2, -1 | 3, 1 | 2, -2 |

“It’s only at equilibrium if we think this is better than acting on our own”

Utility(Top) = (.5)2 + (.5)6 = 4

Utility(Left) = (.5)6 + (.5)2 = 4

It is an equilibrium because the expected utility for both agents is the same. From the top agent’s perspective, we would choose B if for the left agent chose the coin landed as heads, making the left agent choose D. And if the coin flip were to end up as tails, then the left agent would be choosing E, and then we would have to choose C, which is our best option. And this contract makes sense because this gives us the highest expected utility out of all the other options if the other follows the same pattern.

Coincidentally, both (D, B) and (E, C) are Nash equilibria’s, and “Nash equilibria are correlated equilibria, but not visa versa...” (Lecture 11, page 25).

**Problem 3b**

We need to show that [ [0.5: D, 0.5: E] ; [0.5: B, 0.5: C] ] is **not** a Nash equilibrium

|  |  |  |  |
| --- | --- | --- | --- |
|  | A | B | C |
| D | -2, 1 | 6, 2  (25%) | -2, 1  (25%) |
| E | -2, 3 | -1, 3  (25%) | 2, 6  (25%) |
| F | 2, -1 | 3, 1 | 2, -2 |

Utility(Top) = (.25)(2) + (.25)(3) + (.25)(1) + (.25)(6) = 3

Utility(Left) = (.25)(6) + (.25)(-1) + (.25)(-2) + (.25)(2) = 1.25

The top agent would not want to work with the left agent, because the top agent would always choose C to get a higher expected utility ( (1)(0.5) + (6)(0.5) = 3.5 ). The top agent would prefer to gain more than 3 utilities. Therefore, it is not Nash, because the agents would not want to work together.

**Problem 4a**

This problem has the principle as 1b. I’m using the equation from Lecture11.pdf, page 19.

Assume Top plays [p LW, (1-p) WL]

Then Uleft(LW) = Uleft(WL)

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Pitcher | |
| Batter |  | P throws fastball | P throws curve |
| B anticipates fastball | 0.30, 0.70 | 0.20, 0.80 |
| B anticipates curve | 0.15, 0.85 | 0.35, 0.65 |

Assume Top plays [p \* P throws fastball, (1-p) \* P throws curve]

Then Uleft(LW) = Uleft(WL)

Then (0.3)p + (0.2)(1 - p) = (0.15)p + (0.35)(1 - p)

Then 0.3p + 0.2 – 0.2p = 0.15p + 0.35 – 0.35p

Then 0.1p = -0.2p + 0.15

Then 0.3p = 0.15

Then p = 0.5

Assume Left plays [p \* B anticipates fastball, (1-p) \* B anticipates curve ]

Then Utop(LW) = Utop(WL)

Then (0.7)p + (0.85)(1 - p) = (0.80)p + (0.65)(1-p)

Then 0.7p + 0.85 – 0.85p = 0.80p + 0.65 – 0.65p

**Problem 4b**

…

**Problem 4c**

…

**Problem 5a**

…

**Problem 5b**

…

**Problem 5c**

…

**Problem 5d**

…

**Problem 6a**

|  |  |  |  |
| --- | --- | --- | --- |
|  | P1 | P2 | P3 |
| P2 | P3 | P1 |
| P3 | P1 | P2 |
| **Votes** | *20* | *15* | *15* |
| **Type** | *1* | *2* | *3* |

Total votes = 20 + 15 + 15 = 50

“Plurality voting is an electoral system in which a candidate, or candidates, who poll more than any other counterpart (that is, receive a plurality), are elected” ([Wikipedia](https://en.wikipedia.org/wiki/Plurality_voting)).

Since Proposal 1 polled higher in the first position than any of the other proposals, it would win in a plurality voting system.

And additionally, I think if we’re looking more closely at the votes: Proposal 1 would still win. 20 voters put Proposal 1 first, and 15 voters put Proposal 1 first.

**Problem 6b**

|  |  |  |
| --- | --- | --- |
| Type 1 | | |
| Proposal | Votes | Points |
| P1 | 20 | 60 |
| P2 | 20 | 40 |
| P3 | 20 | 20 |

|  |  |  |
| --- | --- | --- |
| Type 2 | | |
| Proposal | Votes | Points |
| P2 | 15 | 45 |
| P3 | 15 | 30 |
| P1 | 15 | 15 |

|  |  |  |
| --- | --- | --- |
| Type 3 | | |
| Proposal | Votes | Points |
| P3 | 15 | 45 |
| P1 | 15 | 30 |
| P2 | 15 | 15 |

|  |  |  |
| --- | --- | --- |
| Total | | |
| Proposal | Points | Total |
| P1 | 60 + 15 + 30 | 105 |
| P2 | 40 + 45 + 15 | 100 |
| P3 | 20 + 30 + 15 | 65 |

Proposal 1 would win in the Borda count system.

**Problem 6c**

Type 2 and Type 3 voters could agree to do some strategic voting and put Proposal 1 as both of their least preferred (third position) proposal. Proposal 1 only won by 5 points. If Type 3 voters put Proposal 1 to their least preferred position, then Proposal 2 would win.

**Problem 7**

I used information I learned in one-on-one office hours with Professor Decker and the following site to solve this problem:

<https://courses.lumenlearning.com/waymakermath4libarts/chapter/instant-runoff-voting/>

Five policy proposals being voted on: (v, w, x, y, z)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Anderson | Brown | Clark | Davis | Evans | Foster | Garcia |
| 1st | V | V | W | W | X | Y | Z |
| 2nd | W | X | V | X | Y | X | Y |
| 3rd | X | W | Y | V | Z | Z | X |
| 4th | Y | Y | X | Y | V | W | W |
| 5th | Z | Z | Z | Z | W | V | V |

*Assuming an IRV (Instant Runoff Voting) system where all policies/candidates that tie for the fewest votes are eliminated at the same time, under what conditions is an eventual majority winner guaranteed? [Put another way, under what conditions might there not be an ambiguous majority winner?] How will these conditions change if a new person, Harris, moves into town and votes?*

Compute the IRV

Calculate the votes

|  |  |
| --- | --- |
| Letter | Votes |
| V | 2 |
| W | 2 |
| X | 1 |
| Y | 1 |
| Z | 1 |

There is no winner, so we eliminate the last place option. Since there is a tie for last place, and this is the first round, we randomly eliminate one. I’ll eliminate Z. I’ll also shift up all the votes.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Anderson | Brown | Clark | Davis | Evans | Foster | Garcia |
| 1st | V | V | W | W | X | Y | Y |
| 2nd | W | X | V | X | Y | X | X |
| 3rd | X | W | Y | V | V | W | W |
| 4th | Y | Y | X | Y | W | V | V |

Calculate the votes again

|  |  |
| --- | --- |
| Letter | Votes |
| V | 2 |
| W | 2 |
| X | 1 |
| Y | 2 |

Since X is in last place, I’ll eliminate X.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Anderson | Brown | Clark | Davis | Evans | Foster | Garcia |
| 1st | V | V | W | W | Y | Y | Y |
| 2nd | W | W | V | V | V | W | W |
| 3rd | Y | Y | Y | Y | W | V | V |

Let’s count the votes, again.

|  |  |
| --- | --- |
| Letter | Votes |
| V | 2 |
| W | 2 |
| Y | 3 |

Now there is a tie for last place between V and W. We must look at the previous round to eliminate the candidate with the fewest first place votes. But still, V and W are tied with 2 votes. And so, we can randomly eliminate one of them. I chose W to eliminate.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Anderson | Brown | Clark | Davis | Evans | Foster | Garcia |
| 1st | V | V | V | V | Y | Y | Y |
| 2nd | Y | Y | Y | Y | V | V | V |

Let’s count the votes, again.

|  |  |
| --- | --- |
| Letter | Votes |
| V | 4 |
| Y | 3 |

V now has a majority of votes, 4/7 = 0.57… and so V wins.