Assignment 4

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Download all python codes and latex-tikz codes from

https://github.com/jvinaykumar12/EE5609/tree/master/Assignment4

1 Problem

AD is the altitude of a isosceles $\triangle ABC$ in which $AB = AC$.Show that

1) AD bisects BC
2) AD bisects $\angle A$

2 Explanation

Given AD is altitude of the $\triangle ABC$. Therefore

$$(\mathbf{B}-\mathbf{C})^T(\mathbf{A}-\mathbf{D}) = 0 \tag{2.0.1}$$

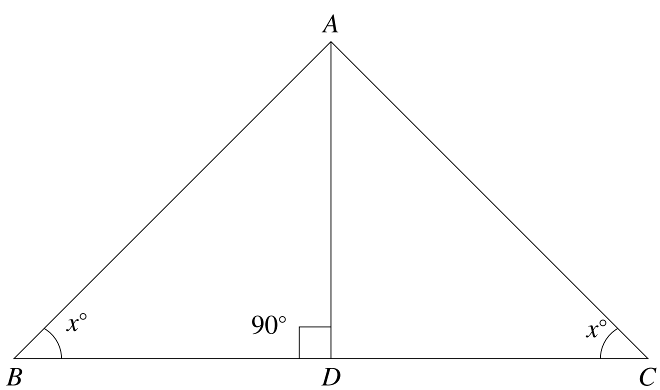

Fig. 0: $\triangle ABC$ with AD as altitude

$$\begin{aligned}\|\mathbf{A}-\mathbf{B}\|^2 &= (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{B})\\ &= (\mathbf{A}-\mathbf{B})^T((\mathbf{A}-\mathbf{D})-(\mathbf{B}-\mathbf{D}))\\ &= (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{D})-(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{D})\\ &= \mathbf{A}^T(\mathbf{A}-\mathbf{D})-\mathbf{B}^T(\mathbf{A}-\mathbf{D})-(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{D})\end{aligned} \tag{2.0.2}$$

Similarly for AC,

$$\begin{aligned}\|\mathbf{A}-\mathbf{C}\|^2 &= (\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{C})\\ &= (\mathbf{A}-\mathbf{C})^T((\mathbf{A}-\mathbf{D})-(\mathbf{C}-\mathbf{D}))\\ &= (\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{D})-(\mathbf{A}-\mathbf{C})^T(\mathbf{C}-\mathbf{D})\\ &= \mathbf{A}^T(\mathbf{A}-\mathbf{D})-\mathbf{C}^T(\mathbf{A}-\mathbf{D})-(\mathbf{A}-\mathbf{C})^T(\mathbf{C}-\mathbf{D})\end{aligned} \tag{2.0.3}$$

Given AB = AC. Therefore

$$\|\mathbf{A}-\mathbf{B}\| = \|\mathbf{A}-\mathbf{C}\| \tag{2.0.4}$$

By equating (2.0.2) and (2.0.3)

$$\begin{aligned}&\mathbf{B}^T(\mathbf{A}-\mathbf{D})+(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{D}) =\\ &\qquad \mathbf{C}^T(\mathbf{A}-\mathbf{D})+(\mathbf{A}-\mathbf{C})^T(\mathbf{C}-\mathbf{D})\\ &\implies (\mathbf{B}-\mathbf{C})^T(\mathbf{A}-\mathbf{D})+(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{D})\\ &\qquad = (\mathbf{A}-\mathbf{C})^T(\mathbf{C}-\mathbf{D}) \quad \text{From (2.0.1)}\\ &(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{D}) = (\mathbf{A}-\mathbf{C})^T(\mathbf{C}-\mathbf{D})\end{aligned} \tag{2.0.5}$$

Since $\triangle ABC$ is isosceles angle ABD is equal to angle ACD

$$\frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{D})}{\|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{B}-\mathbf{D}\|} = \frac{(\mathbf{A}-\mathbf{C})^T(\mathbf{C}-\mathbf{D})}{\|\mathbf{A}-\mathbf{C}\|\,\|\mathbf{C}-\mathbf{D}\|} \tag{2.0.6}$$

By refering the values from 2.0.4 and 2.0.5

$$\|\mathbf{B}-\mathbf{D}\| = \|\mathbf{C}-\mathbf{D}\| \tag{2.0.7}$$

Therefore BD = DC. In $\triangle ABD$

$$\cos\angle DAB = \frac{(\mathbf{D}-\mathbf{A})^T(\mathbf{A}-\mathbf{B})}{\|\mathbf{D}-\mathbf{A}\|\,\|\mathbf{A}-\mathbf{B}\|} \tag{2.0.8}$$

$$= \frac{(\mathbf{D}-\mathbf{A})^T(\mathbf{A}-\mathbf{C}+\mathbf{C}-\mathbf{B})}{\|\mathbf{D}-\mathbf{A}\|\,\|\mathbf{A}-\mathbf{B}\|} \tag{2.0.9}$$

$$= \frac{(\mathbf{D}-\mathbf{A})^T(\mathbf{A}-\mathbf{C})+(\mathbf{D}-\mathbf{A})^T(\mathbf{C}-\mathbf{B})}{\|\mathbf{D}-\mathbf{A}\|\,\|\mathbf{A}-\mathbf{B}\|} \tag{2.0.10}$$

By refering the values from (2.0.1) (2.0.4)

$$\frac{(\mathbf{D}-\mathbf{A})^T(\mathbf{A}-\mathbf{C})}{\|\mathbf{D}-\mathbf{A}\|\,\|\mathbf{A}-\mathbf{C}\|} = \cos\angle DAC \tag{2.0.11}$$

Therefore angle DAC is equal to angle DAB. Thus AD is the angular bisector of angle A.