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1. Solve Exercise 4.9, parts (b)–(e) on page 247 in the book "Robust Adaptive Control" by Ioannou and Sun. (Assume part (a) is true.)

2. Implement the indirect MRAC control design example given in class via computer simulation. Plot the behavior of the plant state as well as control gains over time. Does loss of stabilizability ($\hat{b} \to 0$) actually happen? Investigate this in the following cases:

a) $b > 0$ and $\hat{b}$ is initialized with the correct sign ($\hat{b}(0) > 0$).

b) $b > 0$ but $\hat{b}$ is initialized with the wrong sign ($\hat{b}(0) < 0$).

Modify the update law for $\hat{b}$ to guarantee that loss of stabilizability will not happen. (Here you may use the initialization from part a).) Submit all plots (with and without modification) and compare them.

3. Consider the scalar system $\dot{x} = f(x) + g(x)u$ where $f(x) = -x\sin^2(x^2)$, $g(x) = \cos(x^2)$.

a) Construct a feedback law $u = k(x)$ which makes the closed-loop system GAS. (Justify this.)

b) Now suppose that the state measurements available to the feedback law are affected by an additive disturbance, resulting in the system $\dot{x} = f(x) + g(x)k(x + d)$ where f, g are the same as before and k is the feedback you found in part a). Is this system ISS with respect to d? Prove or disprove.

4. Consider again the system from the previous homework:

$$\begin{aligned} \dot{x} &= \theta x + \xi_1 \\ \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= u \end{aligned}$$

Use the adaptive ISS backstepping procedure given in Section 7.4.1 in the notes to design a feedback law that makes the closed-loop system ISS with respect to $(\tilde{\theta}, \dot{\hat{\theta}})^T$ (for an arbitrary tuning law). You need to do the initialization step as well as the two recursion steps.