ECE 517, Fall 19 **PROBLEM SET 3** **Due Wednesday, Oct 9**

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1. The purpose of this exercise is to extend the backstepping procedure beyond pure integrator backstepping. Consider the system

$$\begin{aligned}\dot{x} &= f(x) + g(x)\xi \\ \dot{\xi} &= f_1(x,\xi) + g_1(x,\xi)u\end{aligned}$$

where x, ξ, u are all scalar variables and $g_1(x,\xi) \neq 0\ \forall\, x,\xi$. Assume that a Lyapunov function $V_0(x)$ and a feedback law $k_0(x)$ (with $k_0(0) = 0$) are known such that

$$\frac{\partial V_0}{\partial x}\big(f(x) + g(x)k_0(x)\big) \leq -W(x) < 0 \qquad \forall\, x \neq 0$$

Construct a control Lyapunov function $V_1(x,\xi)$ and a stabilizing feedback law $k_1(x,\xi)$ for the 2-D system.

2. Consider the system

$$\begin{aligned}\dot{x} &= \theta x + \xi_1 \\ \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= u\end{aligned}$$

where x, ξ_1, ξ_2, u are all scalar variables and θ is an unknown parameter. Continue the adaptive backstepping procedure given in Section 5.2 in the lecture notes and design an adaptive control law that asymptotically stabilizes this system.

3. Prove Lemma 4 from Section 6.2 of the lecture notes. Be sure to give explicit expressions for λ, c in terms of α_0, T_0 and vice versa.

4. As defined in the lecture notes, a vector-valued signal ϕ is *persistently exciting* (PE) if for some $\alpha_0, T_0 > 0$ we have $\int_t^{t+T_0} \phi(s)\phi^T(s)ds \geq \alpha_0 T_0 I$ for all t, where the inequality means that the difference is a positive semidefinite matrix. Prove that the vector signal

$$\phi(t) := \begin{pmatrix} A\sin(\omega t + \alpha) \\ \sin \omega t \end{pmatrix}$$

is PE under suitable constraints on the numbers A, ω, α. Be sure to specify these constraints.

5. Solve Exercise 4.2 on page 246 in the book "Robust Adaptive Control" by Ioannou and Sun. (Use the link provided earlier by email to access the electronic version of the book.)