Chapter 15

FUZZY REASONING

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15.1 INTRODUCTION

The derivation of mathematical models that can efficiently describe real-world problems is most of the time an overwhelming or even impossible task due to the complexity and the inherent ambiguity of characteristics that these problems may possess. As Zadeh (1973), the founder of the theory of fuzzy sets, puts it,

> ... as the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics.

Fuzzy Reasoning is based on the theory of fuzzy sets and it encompasses Artificial Intelligence, information processing and theories from logic to pure and applied mathematics, like graph theory, topology and optimization. The theory of fuzzy sets was introduced in 1965. In his introductory paper, Zadeh, while stating his intention ("to explore in a preliminary way some of the basic properties and implications" of fuzzy sets) he noted that

> ... the notion of a fuzzy set provides a convenient point of departure for the construction of a conceptual framework which parallels in many respects the framework used in the case of ordinary sets, but is more general than the latter and, potentially, may prove to have a much wider scope of applicability, particularly in the fields of pattern classification and information processing.

Table 15.1. A chronology of critical points in the development of fuzzy reasoning.

First paper on fuzzy systems	Zadeh, 1965
Linguistic approach	Zadeh, 1973
Fuzzy logic controller	Assilian and Mamdani, 1974
Heat exchanger control based on fuzzy logic	Ostergaard, 1977
First industrial application of fuzzy logic: cement kiln control	Homblad and Ostergaard, 1982
Self-organizing fuzzy controller	Procyk and Mamdani, 1979;
Fuzzy pattern recognition	Bezdek, 1981
Fuzzy controllers on Tokyo subway shuttles	Hitachi, 1984
Fuzzy chip	Togai and Watanabe, 1986
Takagi–Sugeno fuzzy modeling	Takagi and Sugeno, 1985
Hybrid neural–fuzzy systems	Kosko, 1992

Indeed, in subsequent years, the theory of fuzzy sets was more decisively established as a new approach to complex systems theory and decision processes. The application of fuzzy logic has dramatically increased since 1990, ranging from production, finance, marketing and other decision-making problems to micro-controller-based systems in home appliances and large-scale process control systems (Sugeno and Yasukawa, 1993; Karr and Gentry, 1993; Lee, 1990). For systems involving nonlinearities and lack of a reliable analytical model, fuzzy logic control has emerged as one of the most promising approaches. Without doubt, fuzzy inference is a step towards the simulation of human thinking.

The main advantage of fuzzy logic techniques, i.e. techniques based on the theory of fuzzy sets, over more conventional approaches in solving complex, nonlinear and/or ill-defined problems lies in their capability of incorporating a priori qualitative knowledge and expertise about system behavior and dynamics. This renders fuzzy logic systems almost indispensable for obtaining a more transparent and tactile qualitative insight for systems whose representation with exact mathematical models is poor and inadequate. Besides, fuzzy schemes can be used either as enabling to other approaches or as self-reliant methodologies providing thereby a plethora of alternative structures and schemes.

In fact, fuzzy control theory generates nonlinear functions according to a representation theorem by Wang (1992), who stated that any continuous nonlinear function can be approximated as exactly as needed with a finite set of fuzzy variables, values and rules. Therefore, by applying appropriate design procedures, it is always possible to design a fuzzy controller that is suitable for the nonlinear system under control. Table 15.1 depicts some benchmarks in the history of fuzzy logic, particularly in the domain of fuzzy control.

This chapter is intended to present an overview of the basic notions of the theory of fuzzy sets and fuzzy logic. The chapter is organized as follows: in the next section, an introduction to the theory of fuzzy sets is presented, covering topics of the most commonly used types of membership functions, logical and transformation operators, fuzzy relations, implication and inference rules, and fuzzy similarity measures. Section 15.3 introduces the basic structure of a fuzzy inference system and its elements are described. Section 15.4 presents the topic of fuzzy control system and an example is demonstrated. In particular a fuzzy controller is proposed for the control of a plug flow tubular reactor, which is a typical nonlinear distributed parameter system. The proposed fuzzy controller is compared with a conventional proportional–integral (PI) controller. In the same section an introduction to the field of fuzzy adaptive control systems is given and the self-organizing scheme is presented. In Section 15.5 reviews are given on the topics of model identification and stability of fuzzy systems, respectively. Conclusions and perspectives of fuzzy reasoning are given in Section 15.6.

15.2 BASIC DEFINITIONS OF FUZZY SET THEORY

15.2.1 Fuzzy Sets and the Notion of Membership

A classical set A is defined as a collection of elements or objects. Any element or object x either belongs or does not belong to A. The membership $\mu_A(x)$ of x in A is a mapping:

$$\mu_A : X \to \{0, 1\}$$

that is, it may take the value 1 or 0, which represent the truth value of x in A. It follows that, if is the complement set of A and $\cap$ represents intersection of sets, then

$$A \cap \bar{A} = \emptyset$$

Fuzzy logic is a logic based on fuzzy sets, i.e. sets of elements or objects characterized by truth-values in the [0,1] interval rather than crisp 0 and 1, as in the conventional set theory. The function that assigns a number in [0,1] to each element of the universe of discourse of a fuzzy set is called the membership function.

15.2.2 Membership Functions

Let X denote the universe of discourse of a fuzzy set A. A is completely characterized by its membership function μ_A:

$$\mu_A : X \to [0, 1]$$

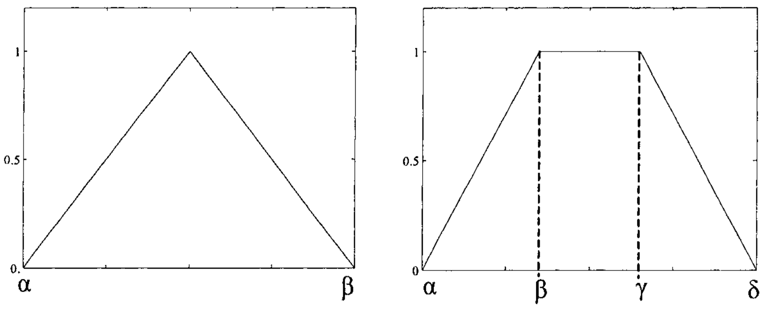


Figure 15.1. (a) Triangular, (b) trapezoid membership function.

and is defined as a set of pairs:

$$A = \{(x, \mu_A(x))\}$$

The most commonly used membership functions are the following (Dubois and Prade, 1980; Zimmermann, 1996):

- triangular membership function
- trapezoid membership function
- linear membership function
- sigmoidal membership function
- Π-type membership function
- Gaussian membership function.

The *triangular* membership function, see Figure 15.1(a), is defined as

$$\mathrm{Tri}(x; \alpha, \beta, \gamma) = \begin{cases} 0 & x < \alpha \\ \dfrac{x - a}{\beta - \alpha} & \alpha \leq x \leq \beta \\ -\dfrac{x - \gamma}{\gamma - \beta} & \beta \leq x < \gamma \\ 0 & x \geq \gamma \end{cases}$$

The *trapezoid* membership function (Figure 15.1(b)) is defined as

$$\mathrm{Tra}(x; \alpha, \beta, \gamma, \delta) = \left[\begin{array}{ll} 0 & x < \alpha \\ \dfrac{x - \alpha}{\beta - \alpha} & \alpha \leq x < \beta \\ 1 & \beta \leq x < \gamma \\ -\dfrac{x - \delta}{\delta - \gamma} & \gamma \leq x < \delta \\ 0 & x \geq \delta \end{array} \right]$$

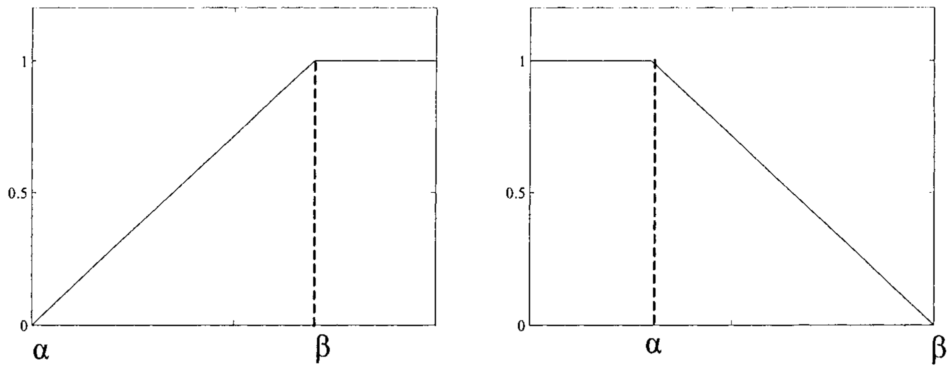


Figure 15.2. (a) Monotonically increasing linear, (b) monotonically decreasing linear membership function.

The *monotonically increasing linear* membership function (Figure 15.2(a)) is given by

$$L(x;\alpha,\beta)=\begin{cases}0 & x<\alpha\\ \dfrac{x-\alpha}{\beta-\alpha} & \alpha\le x\le\beta\\ 1 & x>\beta\end{cases}$$

The *monotonically decreasing linear* membership function (Figure 15.2(b)) is given by

$$L(x;\alpha,\beta)=\begin{cases}1 & x<\alpha\\ -\dfrac{x-\alpha}{\beta-\alpha} & \alpha\le x\le\beta\\ 0 & x>\beta\end{cases}$$

The *monotonically increasing sigmoidal* membership function (Figure 15.3(a)) is given by

$$S(x;\alpha,\beta,\gamma)=\begin{cases}0 & x<\alpha\\ 2\left(\dfrac{x-\alpha}{\gamma-\alpha}\right)^2 & \alpha\le x\le\beta\\ 1-2\left(\dfrac{x-\alpha}{\gamma-\alpha}\right)^2 & \beta\le x\le\gamma\\ 1 & x>\gamma\end{cases}$$

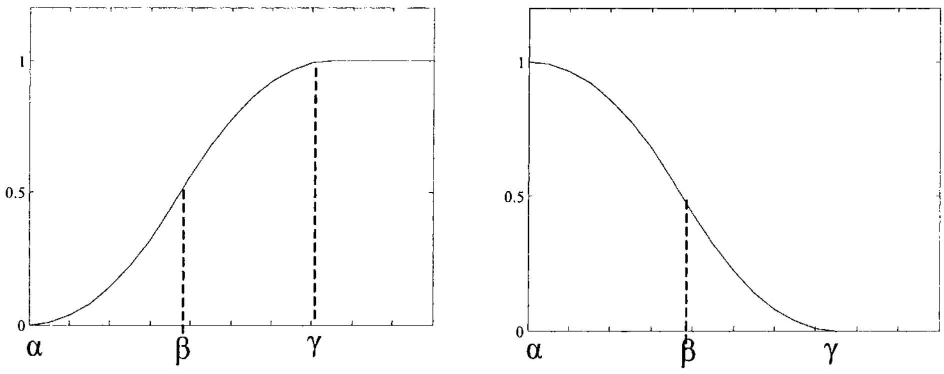


Figure 15.3. (a) Monotonically increasing sigmoidal, (b) monotonically decreasing sigmoidal membership function.

The *monotonically decreasing sigmoidal* membership function (Figure 15.3(b)) reads as

$$S(x;\alpha,\beta,\gamma)=\begin{cases}1 & x<\alpha\\ 1-2\left(\dfrac{x-\alpha}{\gamma-\alpha}\right)^2 & \alpha\le x\le\beta\\ 2\left(\dfrac{x-\alpha}{\gamma-\alpha}\right)^2 & \beta\le x\le\gamma\\ 0 & x>\gamma\end{cases}$$

The Π-membership function (Figure 15.4(a)) is defined as

$$\Pi(x;\beta,\gamma)=\begin{cases}S\left(x;\gamma-\beta,\dfrac{\gamma-\beta}{2},\gamma\right) & x\le\gamma\\ 1-S\left(x;\gamma,\dfrac{\gamma+\beta}{2},\gamma+\beta\right) & x>\gamma\end{cases}$$

The *Gaussian* membership function (Figure 15.4(b)) is given by

$$G(x;k,\sigma)=\exp\left(-\frac{(\gamma-x)^2}{2\sigma^2}\right)$$

where σ is the standard deviation.

Examples on Fuzzy Sets Maintaining a "comfortable" room temperature is of great importance for the work productivity. Fuzzy logic climate control is one of the many successive commercial applications of the theory. For example, the room temperature for low-level activities could be described by the following five fuzzy sets, where a temperature around 18° C is a comfortable

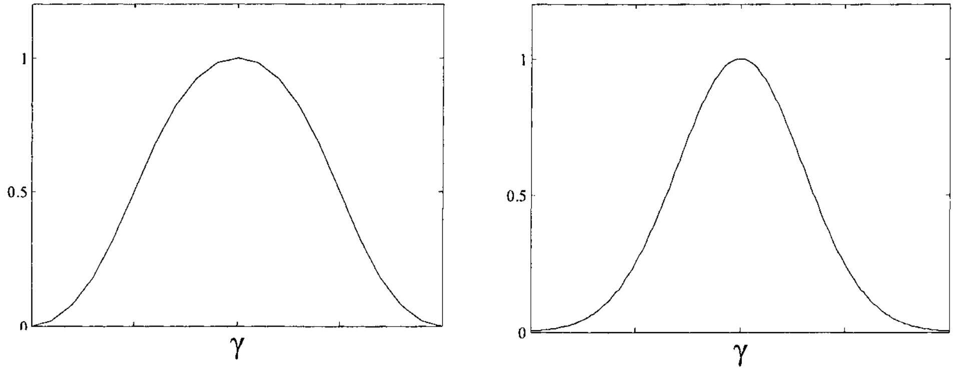
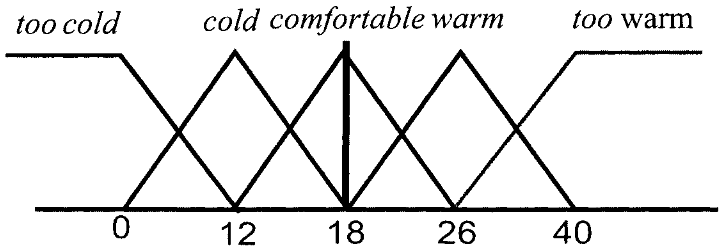


Figure 15.4. (a) Π, (b) Gaussian membership functions.

one, around 26° C a warm one (though not during summer!), while above 40° C is definitely too warm, and around 12° C can be characterized as cold and below that too cold.



"Fast" cars can be described by the horsepower (HP) using the following membership function:

$$\mu(x) = \left\{ \begin{array}{ll} 0 & 0 \leq x \leq 75 \\ \dfrac{x - 120}{25} & 75 \leq x \leq 120 \\ 1 & 120 \leq x \leq 150 \\ 0 & x \geq 150 \end{array} \right\}$$

Notice that in the above characterization, for a horsepower above 150 HP "fast" cars have a zero membership. A nonzero membership could have been assigned in another fuzzy set (e.g. the fuzzy set of "very fast" cars).

15.2.3 Fuzzy Set Operations

Knowledge and understanding of the operations of the theory of fuzzy sets is important for the design of fuzzy systems. The fuzzy set operations are defined with respect to the sets' membership functions.

Two fuzzy sets A and B on the universe of discourse X are ***equal*** if their membership functions are equal for each $x \in X$:

$$\forall x \in X : \mu_A(x) = \mu_B(x)$$

A fuzzy set A is a *subset* of B $(A \subseteq B)$ if

$$\forall x \in X : \mu_A(x) \leq \quad \mu_B(x)$$

For the operation of *intersection* $\cap$ of two fuzzy sets A and B, there is a plethora of definitions in the bibliography. The choice is application dependent:

$$\forall x \in X : \mu_{A \cap B} = \left\{ \begin{array}{c} \min(\mu_A(x), \mu_B(x)) \\ \dfrac{\mu_A(x) + \mu_B(y)}{2} \\ \mu_A(x)\mu_B(y) \\ \dots \end{array} \right\}$$

The *union* $\cup$ of two fuzzy sets A and B is also defined in several ways:

$$\forall x \in X : \mu_{A \cup B} = \left\{ \begin{array}{c} \max(\mu_A(x), \mu_B(y)) \\ \dfrac{2\min(\mu_A(x), \mu_B(y)) + 4\max(\mu_A(x), \mu_B(y))}{6} \\ \mu_A(x) + \mu_B(y) - \mu_A(x)\mu_B(y) \\ \dots \end{array} \right.$$

The *complement* A' of a fuzzy set A is defined as

$$\forall x \in X : \mu_{A'}(x) = 1 - \mu_A(x)$$

Examples on Fuzzy Set Operations Let us consider the fuzzy sets A and B:

$$\begin{aligned} A &= \{0/1 + 0.2/2 + 0.8/3 + 1/4 + 1/5) \\ B &= \{0.1/1 + 0.4/2 + 0.5/3 + 0.7/4 + 0.3/5) \end{aligned}$$

Table 15.2. Examples of transformation operators.

Very	$\mu_{\tilde{A}}(x) = (\mu_A(x))^n \quad n > 1$
More/less	$\mu_{\tilde{A}}(x) = (\mu_A(x))^n \quad 0 < n < 1$
More than (lt)	
$= 1 - \mu_A(x)$ for $x < x_0$	$M_{lt(A)}(x) = 0$ for $x \geq x_0, x_0 : \mu_A(x_0) = \max \mu_A(x)$
More/less (mt)	
$= 1 - \mu_A(x)$ for $x > x_0$	$\mu_{mt(A)}(x) = 0$ for $x \leq x_0, x_0 : \mu_A(x_0) = \max \mu_A(x)$

Then

$$
\begin{aligned}
A \cap B &= \{0/1 + 0.2/2 + 0.5/3 + 0.7/4 + 0.3/5) \\
&\quad \text{using the min operator} \\
&= \{0/1 + 0.08/2 + 0.4/3 + 0.7/4 + 0.3/5) \\
&\quad \text{using the product operator} \\
A \cup B &= \{0.1/1 + 0.4/2 + 0.8/3 + 1/4 + 1/5) \\
&\quad \text{using the max operator} \\
A' &\text{ (complement of A)} = \{1/1 + 0.8/2 + 0.2/3 + 0/4 + 0/5\} \\
B' &= \{0.9/1 + 0.6/2 + 0.5/3 + 0.3/4 + 0.7/5) \\
(A \cap B)' &= A' \cup B' = \{1/1 + 0.8/2 + 0.5/3 + 0.3/4 + 0.7/5) \\
&\quad \text{using the max operator}
\end{aligned}
$$

15.2.4 Transformation Operators

The transformation operator (or hedge or modifier) acts on a membership function to modify the concept of the linguistic term that describes the fuzzy set. For example, in the clause "number *very* close to 10", the transformation operator *very* acts on the linguistic term "close to 10" which corresponds to a fuzzy set. Examples of such operators are given in Table 15.2 (Ross, 1995; Zimmermann, 1996; Pappis and Mamdani, 1977).

Example on Transformation Operators Let us consider the fuzzy set "young" on the discrete set $U = \{0, 20, 40, 60, 80\}$:

$$F_{\text{young}} = \{(0, 1), (20, 0.75), (40, 0.52), (60, 0.23), (80, 0)\}$$

Then we can derive the fuzzy set $F' =$ "very young" by using the relevant transformation operator. Choosing $\nu = 1.5$ we obtain

$$F' = \{(0, 1), (20, 0.6495), (40, 0.0.375), (60, 0.1103), (80, 0)\}$$

15.2.5 Cartesian Inner Product of Fuzzy Sets

If $A_1, A_2, \ldots, A_\nu$ are fuzzy sets defined in $U_1, U_2, \ldots, U_\nu$, their Cartesian inner product is a fuzzy set $F = A_1 \times A_2 \times \cdots \times A_\nu$ in $U_1 \times U_2 \times \cdots \times U_\nu$ with membership function $\mu_F(u_1, u_2, \ldots, u_\nu) = \cap_{i=1,\nu}\mu_{A\iota}(u_i)$, e.g.

$$\mu_F(u_1, u_2, \ldots, u_\nu) = \min\{\mu_{A1}(u_1), \mu_A 2(u_2), \ldots, \mu_A \nu(u_\nu)\}$$

or

$$\mu_F(u_1, u_2, \ldots, u_\nu) = \mu_{A1}(u_1), \mu_A 2(u_2), \ldots, \mu_A \nu(u_\nu)$$

Example The objective in climate control is to find the optimum conditions in terms of both temperature T and humidity H. Suppose that the discrete sets of temperature and humidity are given by $T = \{T_1, T_2, T_3, T_4\}$ and $H = \{H_1, H_2, H_3\}$ respectively, and that of the desired temperature by the discrete fuzzy set

$$A = \frac{0.12}{T_1} + \frac{0.65}{T_2} + \frac{1}{T_3} + \frac{0.25}{T_4}$$

while that for the desired level of humidity by

$$B = \frac{0.5}{H_1} + \frac{0.9}{H_2} + \frac{0.1}{H_3}$$

Then the Cartesian product $A \times B$ reads as

$$\begin{aligned} A \times B \quad = \quad & \frac{0.12}{T_1, H_1} + \frac{0.12}{T_1, H_2} + \frac{0.1}{T_1, H_3} + \frac{0.5}{T_2, H_1} + \frac{0.65}{T_2, H_2} + \frac{0.1}{T_2, H_3} \\ & + \frac{0.5}{T_3, H_1} + \frac{0.9}{T_3, H_2} + \frac{0.1}{T_3, H_3} + \frac{0.25}{T_4, H_1} + \frac{0.25}{T_4, H_2} + \frac{0.1}{T_4, H_3} \end{aligned}$$

Then the optimum conditions are those for $T = T_3$ and $H = H_2$.

15.2.6 Fuzzy Relations

Let U_1 and U_2 be two universes of discourse and the membership function $\mu_R : U_1 \times U_2 \rightarrow [0, 1]$. Then a fuzzy relation R on $U_1 \times U_2$ is defined as (Zimmermann, 1996)

$$R = \int_{U \times B} \mu_R \frac{(u_1, u_2)}{(u_1, u_2)} \text{ if } U_1, U_2 \quad \text{are continuous}$$

or

$$R_d = \sum_{U \times V} \mu_R \frac{(u_1, u_2)}{(u_1, u_2)} \text{ if } U_1, U_2 \quad \text{are discrete}$$

Example Consider the coordinates of three atoms, denoted by i, j, k in a cubic crystal with a lattice constant of 3 A and their corresponding x, y, z coordinates $U = \{(0, 0, 0), (0.5, 0.5, 0.5), (1.2, 1.2, 1.2)\}$ (in A). Then the fuzzy relation "near neighbors" can be described by the following fuzzy relation:

$$R = \frac{1.0}{i, i} + \frac{1}{j, j} + \frac{1}{k, k} + \frac{0.9}{i, j} + \frac{0.1}{i, k} + \frac{0.9}{j, i} + \frac{0.6}{j, k} + \frac{0.1}{k, i} + \frac{0.6}{k, j}$$

(note that for the particular problem we should have excluded the pairs (i, i), (j, j) and (k, k), but we have kept them for the completeness of the example).

15.2.7 Fuzzy Set Composition

Let R_1 and R_2 be two fuzzy relations on $U_1 \times U_2$ and $U_2 \times U_3$ respectively, then the composition C of R_1 and R_2 is a fuzzy relation defined as follows:

$$\begin{aligned} C &= R_1 \cdot R_2 = \{(u_1, u_3) \cup (\mu_{R_1}(u_1, u_2) \cap \mu_{R_2}(u_1, u_2))\} \\ & \quad u_1 \in U_1, u_2 \in U_2, u_3 \in U_3 \end{aligned}$$

Example Consider the following fuzzy relations (in matrix form):

$$R = \begin{bmatrix} 0.2 & 0.6 \\ 0.9 & 0.4 \end{bmatrix} \quad \text{and} \quad S = \begin{bmatrix} 1 & 0.4 & 0.3 \\ 0.8 & 0.5 & 0.1 \end{bmatrix}$$

Then using a min operator for $\cap$ and a max operator for $\cup$ their composition gives

$$T = R \cdot S = \begin{bmatrix} 0.2 & 0.6 \\ 0.9 & 0.4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0.4 & 0.3 \\ 0.8 & 0.5 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.6 & 0.5 & 0.2 \\ 0.9 & 0.4 & 0.3 \end{bmatrix}$$

15.2.8 Fuzzy Implication

Let A and B be two fuzzy sets in U_1, U_2 respectively. The implication $I : A \to B \in U_1 \times U_2$ is defined as (Ross, 1994; Zimmermann, 1996):

$$I = A \times B = \int_{U_1 \times U_2} \mu_A(u_1) \cap \mu_B(u_2)/(u_1, u_2)$$

The rule "*If* the error is negative big *then* control output is positive big" is an implication: error x implies control action y.

Let there be two discrete fuzzy sets $A = \{(u_i, \mu_A(u_i)), i = 1, \ldots, n\}$ defined on U and $B = \{(v_j, \mu_B(v_j)), j = 1, \ldots, m\}$ defined on V. Then the implication $A \to B$ is a fuzzy relation R:

$$R = \{((u_i, v_j), \mu_R(u_i, v_j)), i = 1, \ldots, n, j = 1, \ldots, m\}$$

defined on $U \times V$, whose membership function $\mu_R(u_i, v_j)$ is given by

$$\begin{bmatrix} \mu_A(u_1) \\ \mu_A(u_2) \\ \cdots \\ \mu_A(u_n) \end{bmatrix} \times [\mu_B(v_1)\mu_B(v_2)\ldots\mu_B(v_m)]$$

$$= \begin{bmatrix} \mu_A(u_1) \wedge \mu_B(v_1) & \mu_A(u_1) \wedge \mu_B(v_2) & \cdots & \mu_A(u_1) \wedge \mu_B(v_m) \\ \mu_A(u_2) \wedge \mu_B(v_1) & \mu_A(u_2) \wedge \mu_B(v_2) & \cdots & \mu_A(u_2) \wedge \mu_B(v_m) \\ \cdots & \cdots & \cdots & \cdots \\ \mu_A(u_n) \wedge \mu_B(v_1) & \mu_A(u_n) \wedge \mu_B(v_2) & \cdots & \mu_A(u_n) \wedge \mu_B(v_m) \end{bmatrix}$$

15.2.9 Inference Rules

Let R be a fuzzy relation on $U_1 \times U_2$ and A be a fuzzy set in U_1. The composition $A \cdot R = B$ is a fuzzy set in U_2, which represents the conclusion made from the fuzzy set A (fact) based on the implication R (rule). Let there be a multiple-input–single-output (MISO) rule base with N rules. The ith rule is given by

If A_{i1} and A_{i2} and ... and A_{in} *then* B_i

where n is the number of input variables x_i, A_{ij} is the fuzzy set of input variable x_j in the ith rule, and B_i is the fuzzy set of output variable y_j in the ith rule. The ith rule is the implication

$$I_i = A_i \rightarrow B_i, A_i = A_{i1} \cap A_{i2} \cap \ldots \cap A_{in} = \cap_{i=1}^{n} A_{ij}$$

Then the implication I_{tot} of N rules is given by

$$I_{tot} = R_1 \cup R_2 \cup \ldots \cup R_N = \cup_{i=1}^{N} R_i = \cup_{i=1}^{N} A_i \rightarrow B_i$$

15.2.10 The Inverse Problem

The inverse problem is defined as follows: given two fuzzy relations S and T find R such that $R \cdot S = T$. In application terms, the problem may be defined as follows: let S be the input–output relation describing a system and T a desired output of the system. Find input R, which produces T.

Sanchez (1976) showed an existence condition of the solutions associated with their least upper bound and presented a method for obtaining it analytically. Pappis (1976) and Pappis and Sugeno (1985) presented a method to obtain the whole set of solutions.

15.2.11 Fuzzy Similarity Measures

The fuzzy similarity measures introduce the notion of approximate equality (or similarity) between fuzzy sets. The most commonly used fuzzy similarity measures are the following (Pappis, 1991; Pappis and Karacapilidis, 1993, 1995; Wang, 1997; Wang et al., 1995; Cross and Sudkamp, 2002):

L-fuzzy Similarity Measure The $L(A, B)$ similarity measure of two fuzzy sets A, B is defined as

$$L(A, B) = 1 - \max_{x \in X} |A(x) - B(x)|$$

M-fuzzy Similarity Measure The $M(A, B)$ similarity measure of two fuzzy sets $A, B \in X$ is defined as

$$M(A, B) = \begin{cases} 1 & \text{if } A = B = \emptyset \\ \dfrac{\sum_{x \in X} \min(A(x), B(x))}{\sum_{x \in X} \max(A(x), B(x))} & \text{in every other case} \end{cases}$$

Two fuzzy sets are ε-"almost" equal ($A \sim B$) if and only if $M(A, B) \leq \varepsilon$, where $\varepsilon \in [0, 1]$.

S-fuzzy Similarity Measure The $S(A, B)$ similarity measure of two fuzzy sets $A, B \in X$ is defined as

$$S(A, B) = \begin{cases} 1 & \text{if } A = B = \emptyset \\ 1 - \dfrac{\sum_{x \in X} |A(x) - B(x)|}{\sum_{x \in X} (A(x) + B(x))} & \text{in every other case} \end{cases}$$

W-fuzzy Similarity Measure The $W(A, B)$ similarity measure of two fuzzy sets $A, B \in X$ is defined as

$$W(A, B) = 1 - \sum_{x \in X} |A(x) - B(x)|$$

P-fuzzy Similarity Measure The $P(A, B)$ similarity measure of two fuzzy sets $A, B \in X$ is defined as

$$P(A, B) = \frac{\sum_{x \in X} A(x) B(x)}{\max\left(\sum_{x \in X} A(x) A(x), \sum_{x \in X} B(x) B(x) \right)}$$

15.3 BASIC STRUCTURE OF A FUZZY INFERENCE SYSTEM

The basic structure of a fuzzy inference system consists of a fuzzification unit, a fuzzy logic reasoning unit (process logic), a knowledge base, and a

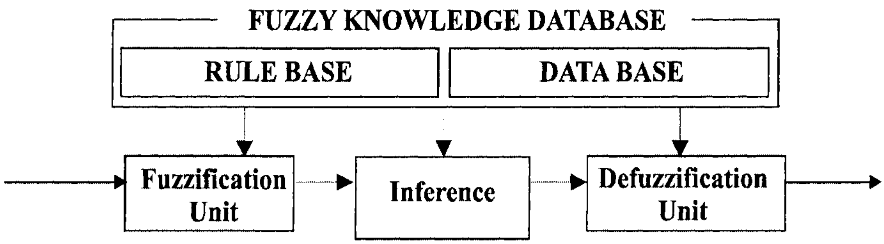


Figure 15.5. Basic structure of a fuzzy inference system.

defuzzification unit (Figure 15.5). The key element of the system is the fuzzy logic-reasoning unit that contains two main types of information:

1 A data base defining the number, labels and types of the membership functions the fuzzy sets used as values for each system variable. These are of two types: the input and the output variables. For each one of them the designer has to define the corresponding fuzzy sets. The proper selection of these is one of the most critical steps in the design process and can dramatically affect the performance of the system. The fuzzy sets of each variable form the universe of discourse of the variable.

2 A rule base, which essentially maps fuzzy values of the inputs to fuzzy values of the outputs. This actually reflects the decision-making policy. The control strategy is stored in the rule base, which in fact is a collection of fuzzy control rules and typically involves weighting and combining a number of fuzzy sets resulting from the fuzzy inference process in a calculation, which gives a single crisp value for each output. The fuzzy rules incorporated in the rule base express the control relationships usually in an IF-THEN format. For instance, for a two-input-one-output fuzzy logic controller, that is the case in this work, a control rule has the general form

$$\text{Rule } i\text{: IF } x \text{ is } A_i \text{ and } y \text{ is } B_i \text{ THEN } z \text{ is } C_i$$

where x and y are input variables, z is the output variable; A_i, B_i and C_i are linguistic terms (fuzzy sets) such as "negative", "positive" or "zero". The *if-part* of the rule is called condition or premise or antecedent, and the *then-part* is called the consequence or action.

Usually the actual values acquired from or sent to the system of concern are crisp, and therefore fuzzification and defuzzification operations are needed to map them to and from the fuzzy values used internally by the fuzzy inference system.

The fuzzy reasoning unit performs various fuzzy logic operations to infer the output (decision) from the given fuzzy inputs. During fuzzy inference, the following operations are involved for each fuzzy rule:

1 Determination of the degree of match between the fuzzy input data and the defined fuzzy sets for each system input variable.

2 Calculation of the fire strength (degree of relevance or applicability) for each rule based on the degree of match and the connectives (e.g. AND, OR) used with input variables in the antecedent part of the rule.

3 Derivation of the control outputs based on the calculated fire strength and the defined fuzzy sets for each output variable in the consequent part of each rule.

Several techniques have been proposed for the inference of the fuzzy output based on the rule base. The most common used are the following:

- the Max–Min fuzzy inference method
- the Max-product fuzzy inference method.

Assume that there are two input variables, e (error) and ce (change of error), one output variable, cu (change of output), and two rules:

Rule 1 *If* e is A_1 AND ce is B_1 THEN cu is C_1

Rule 2 *If* e is A_2 AND ce is B_2 THEN cu is C_2

In the Max–Min inference method, the fuzzy operator AND (intersection) means that the minimum value of the antecedents is taken:

$$\mu_A \text{ AND } \mu_B = \min\{\mu_A, \mu_B\}$$

while for the Max-product one the product of the antecedents is taken:

$$\mu_A \text{ AND } \mu_B = \mu_A \mu_B$$

for any two membership values μ_A and μ_B of the fuzzy subsets A, B, respectively. All the contributions of the rules are aggregated using the union operator, thus generating the output fuzzy space C.

15.3.1 Defuzzification Unit

Defuzzification typically involves weighting and combining a number of fuzzy sets resulting from the fuzzy inference process in a calculation, which gives a single crisp value for each output.

The most commonly used defuzzification methods are those of mean of maximum, centroid, and center of sum of areas (Lee, 1990; Ross, 1995; Driankov et al., 1993).

Mean of Maximum Defuzzification Technique The technique of the mean value of maximum is given by the following equation (Yan et al., 1994):

$$x = \frac{\sum_{i=1}^{n} \alpha_i H_i x_i}{\alpha_i H_i}$$

where x is the control (output) value to be applied, n is the number of rules in a MISO system, H_i is the maximum value of the membership function of the output fuzzy set, which corresponds to rule I_i, x_i is the corresponding control (output) value, and α_i is the degree that the rule i is fired.

Centroid Defuzzification Technique This is the most prevalent and intuitively appealing among the defuzzification methods (Lee, 1990; Ross, 1995). This method takes the center of gravity of the final fuzzy space in order to produce a result (the value u of the control variable) sensitive to all rules; it is described by the following equation (Ross, 1995):

$$u = \frac{\sum_{i=1}^{n} \alpha_i M_i}{\sum_{i=1}^{n} \alpha_i A_i}$$

where M_i is the value of the membership function of the output fuzzy set of rule i, A_i is the corresponding area and α_i is the degree that the rule i is fired. Note that the overlapping areas are merged (Figure 15.6(a)).

In the case of a continuous space (universe of discourse), the output value is given by (Ross, 1994; Taprantzis et al., 1997)

$$u = \frac{\int_U u \mu_U(u) \mathrm{d}u}{\int_U \mu_U(u) \mathrm{d}u}$$

Center of Sums Defuzzification Technique A similar technique to the centroid technique, but computationally more efficient, in terms of speed, is that of the center of sums. The difference is that the overlapping (between the output fuzzy sets) areas are not merged (Figure 15.6(b)). The discrete value of the output is given by (Lee, 1990; Driankov et al., 1993)

$$u = \frac{\sum_{i=1}^{l} u_i \cdot \sum_{k=1}^{n} \mu_k (u_i)}{\sum_{i=1}^{l} \sum_{k=1}^{n} \mu_k (u_i)}$$

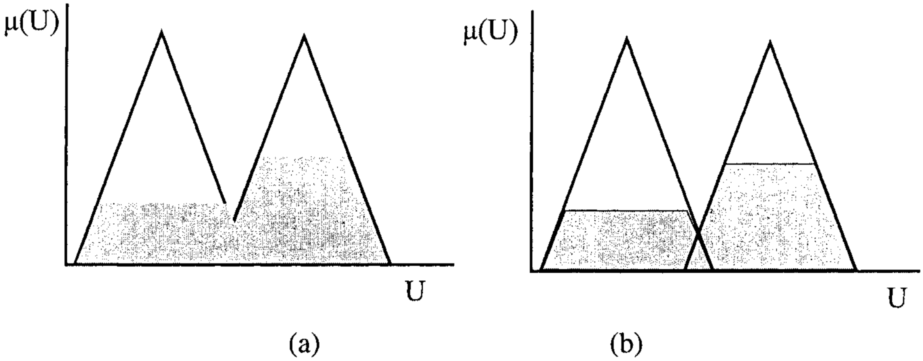


Figure 15.6. Defuzzification techniques: (a) centroid, (b) center of sums.

15.3.2 Design of the Rule Base

Two are the main approaches in the design of rule bases (Yan et al., 1994):

- the heuristic approach,
- the systematic approach.

Heuristic approaches (Yan et al., 1994; King and Mamdani, 1977; Pappis and Mamdani, 1977) provide a convenient way to build fuzzy control rules in order to achieve the desired output response, requiring only qualitative knowledge for the behavior of the system under study. For a two-input (e and ce) one-output (cu) system these rules are of the form

IF e is P (positive) AND ce is N (negative) THEN cu is P (positive)
IF e is N (negative) AND ce is P (positive) THEN cu is N (negative).

The reasoning for the construction of the fuzzy control rules can be summarized as follows:

- If the system output has the desired value and the change of the error (ce) is zero then keep the control action constant.
- If the system output diverges from the desired value then the control action changes with respect to the sign and the magnitude of the error e and the change of error ce. Table 15.3 compresses the design of a rule base for the linguistic term sets NB (negative big), NM (negative medium), NS (negative small), ZE (zero), PS (positive small), PM (positive medium) and PB (positive big) of the fuzzy variables e, ce, cu. The input variables are laid out along the axes, and each matrix element represents the output variable.

Table 15.3. A fuzzy rule base with two inputs and one output.

		ce						
		NB	NM	NS	Z	PS	PM	PB
e	PB	ZE	PS	NM	NB	NB	NB	NB
	PM	PS	ZE	NS	NM	NM	NB	NB
	PS	PM	PS	ZE	NS	NS	NM	NB
	Z	PB	PM	PS	ZE	NS	NM	NB
	NS	PB	PM	PS	PS	ZE	NS	NM
	NM	PB	PB	PM	PM	PS	ZE	ZE
	NB	PB	PB	PB	PB	PM	ZE	ZE

Systematic approaches provide the decision-making strategy (rule base) with the aid of system identification and pattern recognition techniques from input–output data.

15.4 A CASE STUDY: A FUZZY CONTROL SYSTEM

15.4.1 The Fuzzy Logic Control Closed Loop

Over the last 20 years, a large number of conventional modeling and control methods have been proposed to cope with nonlinear and/or time-varying systems including input-state linearization (Isidori, 1995), input–output linearization (Cravaris and Chung, 1987; Henson and Seborg, 1990), model predictive schemes (Patwardhan et al., 1992) and various direct and indirect adaptive control schemes (Isermann, 1989; Batur and Kasparian, 1991).

However, the poor modeling of system uncertainties and the inherent difficulty of incorporating a priori qualitative information about the system dynamics limit the efficiency and the applicability of the classical approaches. The fuzzy logic approach to process control provides a convenient way to build the control strategy, by requiring only qualitative knowledge for the behaviour of the control system. The heuristics employed offer a very attractive way of handling imprecision in the data and/or complex systems, where the derivation of an accurate model is difficult or even impossible. On the other hand, modeling and control techniques based on fuzzy logic comprise very powerful approaches of handling imprecision and nonlinearity in complex systems. The basic structure of a fuzzy logic controller is given in Figure 15.7. Usually the input and output variables are normalized through scaling factors G_{in} and G_{out} in the interval $[-1, 1]$.

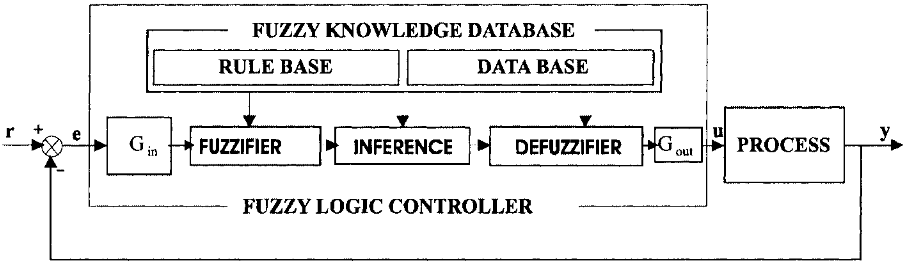


Figure 15.7. The fuzzy logic control closed loop.

15.4.2 Fuzzy Logic Controllers in Proportional–Integral (PI) and Proportional–Differential (PD) Forms

In what follows, in order to enable a comparison basis, the fuzzy logic controller (FLC) with two input variables, the error and the change of error, is represented in PI- and PD-like forms.

PI-like Fuzzy Controller The PI controller in the z-domain has the following form (Stephanopoulos, 1984):

$$C(z) = \frac{u(z)}{e(z)} = K_c\left(1+K\frac{1}{1-z^{-1}}\right)$$

In the time domain the above equation can be rewritten as

$$cu = K_c ce + (K_c K)e$$

where e is the error between a predefined set point and the process output, ce is the change in error, and u is the control output signal. In order to generate an equivalent fuzzy controller, the same inputs e, ce and the same output, cu, will be used in its design.

Based on the above, a two-input–single-output FLC is derived with the following variables:

- input variables: $e(t) = r(t) - y(t)$
  $ce(t) = e(t) - e(t-1)$
- output variable: $cu(t) = u(t) - u(t-1)$

where $r(t)$ is the set point at time t (set point moisture), $y(t)$ is the process output at time t (output moisture), $e(t)$, $ce(t)$ are the error and the change of error at time t, respectively, and $cu(t)$ is the change in the control variable at time t.

In a general form the control action cu can be represented as a nonlinear function of the input variables $e(t)$, $ce(t)$:

$$cu = f(e', ce', t) = f(GEe, GCEce, t)$$

For small perturbations δe, δce around equilibrium, the above equation is approximated by the linearized equation

$$cu = \left[\frac{\partial f}{\partial e}\right]_{ce} \delta e + \left[\frac{\partial f}{\partial ce}\right]_{e} \delta ce$$

By substitution one finally obtains the simplified discretized equation (Mizumoto, 1995)

$$cu(t) = GEe(t) + GCEce(t)$$

which gives the incremental control output at time t. GE and GCE are the scaling factors for the error and change of error, respectively.

PD-like Fuzzy Controller In an analogous manner the PD-like fuzzy controller is of the form

$$u(t) = GEe(t) + GCEce(t)$$

Note that the above expressions are derived using the max-product inference technique.

15.4.3 An Illustrative Example

The design procedure of a fuzzy controller (FLC) is demonstrated through an illustrative example: the system under study is a plug flow tubular reactor, which is a nonlinear distributed parameter with time lag system. The design of the FLC is based on a heuristic approach. The proposed controller is compared with a conventional PI controller, which is tuned with two methods: the process reaction curve tuning method and by using time integral performance criteria such as integral of absolute error (IAE). Based on dynamic performance criteria, such as IAE, ISE, ITAE, it is shown that the proposed fuzzy controller exhibits a better performance compared to the PI controller tuned by the process reaction curve tuning method and an equivalent, if not better, dynamic behavior, compared to the optimal tuned via the time performance criteria PI controller, for a wide range of disturbances.

Case Study: Fuzzy Control of a Plug Flow Tubular Reactor The process of concern is shown in Figure 15.8. It is the problem of the control of a jacketed tubular reactor in which a simple exothermic reaction $A \rightarrow B$ with first-order kinetics takes place. Assuming plug flow conditions, constant temperature for

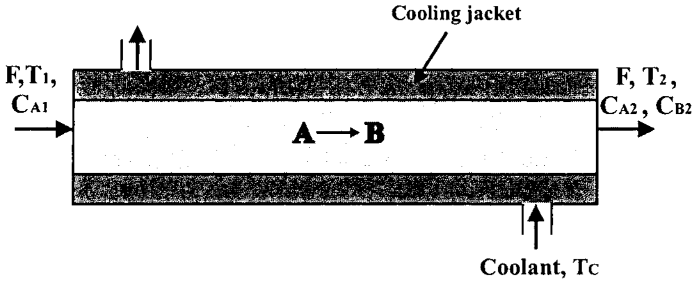


Figure 15.8. The process under study: control of a plug flow tubular reactor.

the coolant, which flows around the tube of the reactor, the governing equations consists of a set of nonlinear time-dependent partial differential equations listed below. The system is a nonlinear distributed parameter with time delay system:

$$\frac{\partial C_A}{\partial t} + u\frac{\partial C_A}{\partial z} = -kC_A$$

$$c_p\rho A\frac{\partial T}{\partial t} + c_p\rho uA\frac{\partial T}{\partial t} = hA_t\left(T_C - T\right) + \left(-DH_R\right)kAC_A$$

$$k = k_O \exp\left(-\frac{E}{RT}\right)$$

The nominal values of the tubular reactor parameters are as follows:

$C_{A1} = 1.6$ kmol m^{-3}	$C_{A2} = 0.11$ kmol m^{-3}
$T_1 = 440$ K	$T_2 = 423$ K
$k_o = 3.34 \times 10^8$ min^{-1}	$E/R = 8600$ K
$DH_R = -44\,000$ kcal kmol^{-1}	$U = 25$ kcal m^2 min^{-1} grad^{-1}
$c_p = 25$ kcal kmol^{-1} K^{-1}	$\rho = 47$ kmol m^{-3}
$T_{co} = 293$ K	$A_t = 0.01$ m^2
$A = 0.002$ m^2	$U = 2$ m min^{-1}

The solution of nonlinear, time-dependent, partial differential equations is possible only by means of modern computer-aided methods. The choice here is the combination of Galerkin's method of weighted residuals and finite-element basis functions (Zienkiewicz and Morgan, 1983).

The control objective is to maintain the control variable, which is the composition of the reacting mixture at the output of the reactor, within the desired operational settings and, particularly, to keep the A reactant concentration at the output below its nominal steady-state value, eliminating mostly input concentration disturbances. The manipulated variable is taken to be the coolant temperature. The incremental fuzzy controller, a two-input–single-output FLC, is derived with the following variables: $e(t) = r(t) - y(t)$,

$ce(t) = e(t) - e(t-1)$, $cu(t) = u(t) - u(t-1)$, where $r(t)$ is the set point at time t (set point moisture), $y(t)$ is the process output at time t (output moisture), and $e(t)$, $ce(t)$ are the error and the change of error at time t.

For the fuzzification of the input–output variables, seven fuzzy sets are defined for each variable, $e(t)$, $ce(t)$ and $cu(t)$ with fixed triangular shaped membership functions normalized in the same universe of discourse, as it is shown in Figure 15.9. For the development of the rule base a heuristic approach was employed.

Given the fact that a reduction in the coolant temperature decreases the output concentration, and inversely, the reasoning for the construction of the fuzzy control rules is outlined as follows:

- Keep the output of the FLC constant if the output has the desired value and the change of error is zero.
- Change the control action of the FLC according to the values and signs of the error, e, and the change of error, ce:

  1 *If* the error is negative (the process output is above the set point) *and* the change of error is negative (at the previous step the controller was driving the system output upwards), *then* the controller should turn its output downwards. Hence, considering negative feedback, the change in control action should be positive, i.e. $cu > 0$, since $u(t) = u(t-1) + cu$.

  2 *If* the error is positive (the process output is below the set point) *and* the change of error is positive (at the previous step the controller was driving the system output downwards), *then* the controller should turn its output upwards. Hence, considering negative feedback, the change in control action should be negative, i.e. $cu < 0$, since $u(t) = u(t-1) + cu$.

  3 *If* the error is positive (the process output is below the set point) *and* the change of error is negative, implying that at the previous step the controller was driving the system output upwards, trying to correct the control deviation, *then* the controller need not take any further action.

  4 *If* the error is negative (the process output is above the set point) *and* the change of error is positive, implying that at the previous step the controller was driving the system output downwards, *then* the controller need not take any further action.

Table 15.4 compresses the design of the control rules for the term sets (nb: negative big, nm: negative medium, ns: negative small, ze: zero, ps: positive small, pm: positive medium, pb: positive big) of the fuzzy variables e, ce, cu.

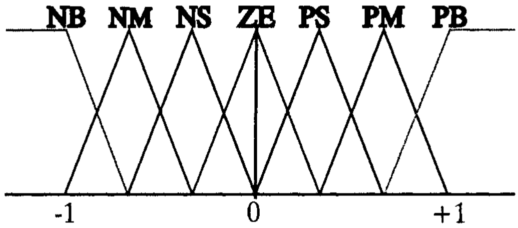


Figure 15.9. Input–output fuzzy sets.

Table 15.4. Fuzzy control rules.

		ce						
		nb	**nm**	**ns**	**ze**	**ps**	**pm**	**pb**
e	**pb**	ze	ze	nm	nb	nb	nb	nb
	pm	ps	ze	ns	nm	nm	nb	nb
	ps	pm	ps	ze	ns	ns	nm	nb
	ze	pb	pm	ps	ze	ns	nm	nb
	ns	pb	pm	ps	ps	ze	ns	nm
	nm	pb	pb	pm	pm	ps	ze	ze
	nb	pb	pb	pb	pb	pm	ze	ze

The input variables are laid out along the axes, and each matrix element represents the output variable. This structure of the rule base provides negative feedback control in order to maintain stability under any condition. For the evaluation of the rules, the fuzzy reasoning unit of the FLC has been developed using the Max–Min fuzzy inference method (Lee, 1990; Driankov et al., 1993). In the particular FLC, the centroid defuzzification method (Zimmermann, 1996; Driankov et al., 1993) is used. Finally, for the projection of the input and output variable values to the normalized universe of discourse, the following values of scaling factors have been chosen: $G_{e(t)} = 5$, $G_{ce(t)} = 45$, $G_{cu(t)} = 2.5$.

Performance Analysis: Results and Discussion To study the performance of the FLC controller, a comparison with a conventional PI controller is made. The parameters of the PI controller are adjusted using two methods of tuning. First it is assumed that the dynamics of the process are poorly known and the tuning of the PI controller is based on the process reaction curve, an empirical tuning method, which provides an experimental model of the process near the operating point. The results of this analysis are: $Gain_{\mathrm{I}} = 350$, $Integral\ time\ constant_{\mathrm{I}} = 1.5$ min.

In the second approach, the optimal values of the PI controller are determined by minimizing the integral of absolute value of error (IAE) of the control variable for a predetermined disturbance in input concentration. Here the optimal parameters of the PI controller are adjusted by minimizing the IAE at the +20% step disturbance in input reactant concentration. The resulting tuning parameters are $Gain_{\mathrm{II}} = 155$, *integral time constant*$_{\mathrm{II}} = 1.0$ min. The relatively large deviation between the parameters obtained by minimizing the IAE and those obtained by the process reaction curve method is results from the fact that the process reaction curve method is based on the approximation of the open loop process response by a first-order system plus dead time.

In the case under study, this approximation seems to be rather poor. In order to objectively compare the FLC controller with the conventional PI controller, in addition to the IAE criterion, the integral of time multiplied by the absolute value of error (ITAE) and the integral of the square of the error (ISE) performance criteria are used for both control and manipulated variables.

Simulation results are presented for step change disturbances ranging from -5% up to -20% in input reactant concentration. Figure 15.11 depicts in histograms the calculated three dynamic performance criteria IAE, ISE, ITAE for the fuzzy and the PI controller tuned with the two different methods.

The performance criteria are determined for both control and manipulated variables. Based on Figure 15.10, it is apparent that the overall performance of the FLC seems better than the conventional PI controller tuned by the empirical process reaction curve method (controller PI-1) and equivalent, if not better, than the optimal PI controller tuned by minimizing the IAE (controller PI-2). The PI-1 controller has the highest values of IAE (Figure 15.10(a, b)), ISE (Figure 15.10(c, d)) and ITAE (Figure 15.10(e, f)) criteria. As is shown, the fuzzy controller exhibits up to 60% lower IAE (Figure 15.10(b)), up to 30% lower ISE (Figure 15.10(c)) and up to 200% lower ITAE (Figure 15.10(f)) compared to the PI controller tuned by the process reaction curve method. In comparison to the PI controller, whose parameters are optimally adjusted by minimizing the IAE criterion, the fuzzy controller shows an equivalent, if not better, performance, based on IAE, ISE and ITAE criteria for all the range of step disturbances (from 5% up to 20%).

However, the approach of optimally adjusting the parameters of the PI controller to some dynamic performance criterion, such as IAE, requires an exact mathematical model of the process, which in real-world processes is very difficult, if not impossible, to derive. In contrast, the design of the fuzzy logic controller is based on a heuristic approach and a mathematical model of the process is not vital.

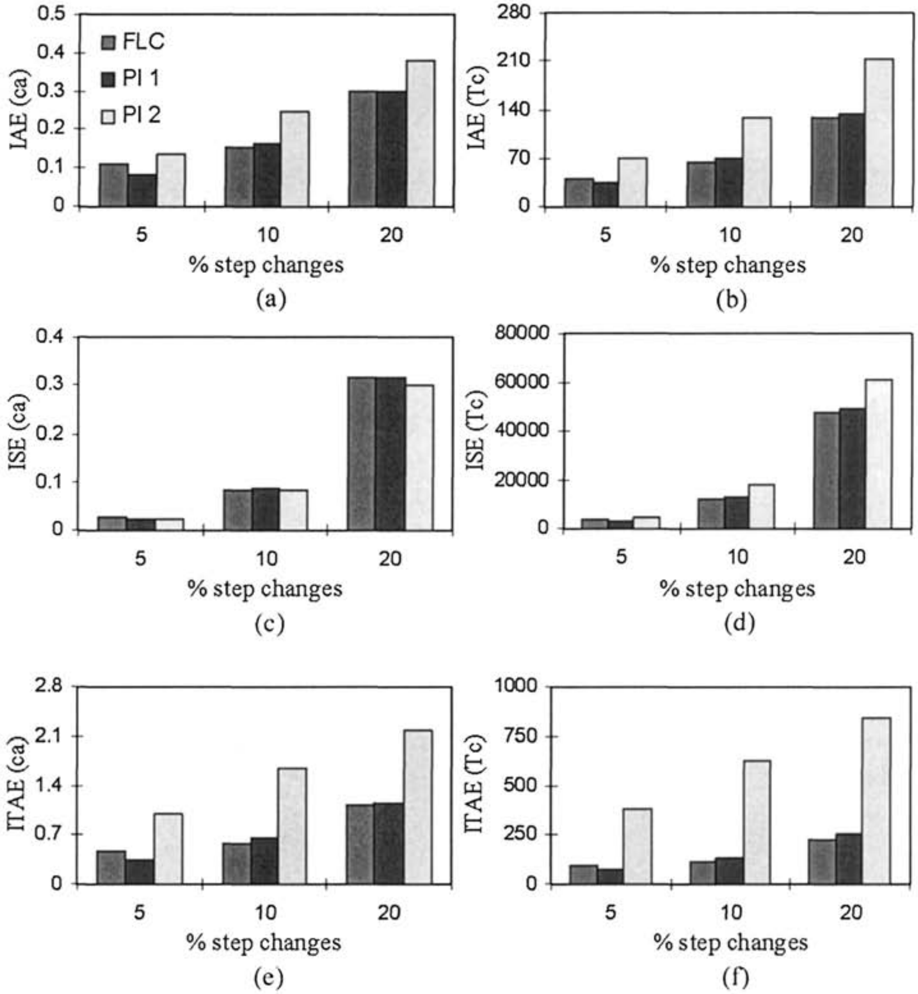


Figure 15.10. Performance comparison of the fuzzy and the PI controller tuned by the process reaction method (PI-1), by minimizing the IAE criterion (PI-2): (a) IAE of the control variable, (b) IAE of the manipulated variable, (c) ISE of the control variable, (d) ISE of the manipulated variable, (e) ITAE of the control variable, (f) ITAE of the manipulated variable.

15.4.4 Fuzzy Adaptive Control Schemes

A major problem encountered in nonlinear and/or time-dependent systems is the degradation of the closed-loop performance as the system shifts away from the initial operational settings. This drawback imposes the need of using adaptive controllers, i.e. controllers, which adjust their parameters optimally, according to some objective criteria (Astrom, 1983).

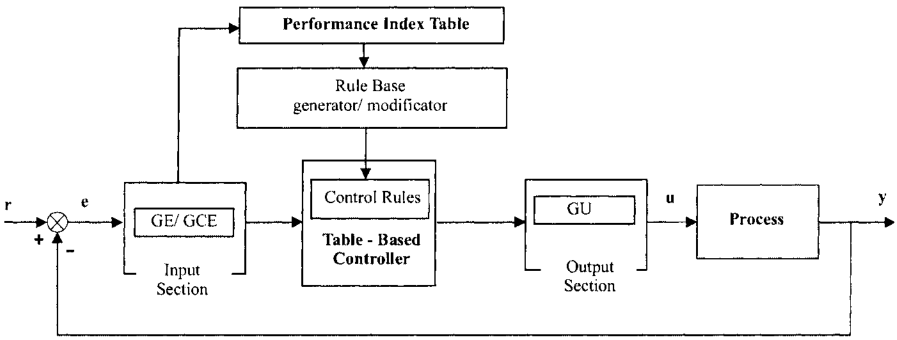


Figure 15.11. The self-organizing fuzzy logic controller.

To date, many schemes have been proposed for fuzzy adaptive control, including self-organizing control (Procyk and Mamdani, 1979; Siettos et al., 1999), membership functions adjustment (Batur and Kasparian, 1991; Zheng, 1992) and scaling factors adjustment (Maeda and Murakami, 1992; Daugherity et al., 1992; Palm, 1993; Jung et al., 1995; Chou and Lu, 1994; Chou, 1998; Sagias et al., 2001). Maeda and Murakami and Daugherity et al. proposed adjustment mechanisms for the tuning of scaling factors by evaluating the control result based on system performance indices such as overshoot, rising time, amplitude and settling time. Palm (1993) addressed the method of adjusting optimally the scaling factors by measuring on-line the linear dependence between each input and output signal of the fuzzy controller. According to the above method the scaling factors are expressed in terms of input–output cross-correlation functions. Jung et al. proposed a real-time tuning of the scaling factors, based on a variable reference tuning index and an instantaneous system fuzzy performance according to system response characteristics. Chou and Lu presented an algorithm for the adjustment of the scaling factors using tuning rules, which are based on heuristics. Sagias et al. (2001) presented a model-identification fuzzy adaptive controller for real-time scaling factor adjustment.

Among the first attempts to apply a fuzzy adaptive system for the control of dynamic systems was that of Procyk and Mamdani (1979), who introduced the self-organizing controller (SOC). The configuration of the proposed controller is shown in Figure 15.11.

It has a two-level structure, in which the lower level consists of a table-based controller with two inputs and one output. The upper level consists of a performance index table, which relates the state of the process to the deviation from the desired overall response, and defines the corrections required in the table-based controller to bring the system to the desired state. From this point of view, the SOC performs two tasks simultaneously: namely, (a) performance evaluation of the system and (b) system performance improvement by creation and/or modification of the control actions based on experience gained from past system states. Hence, the controller accomplishes its learning through repetition over a sequence of operations. The elements of the table are the control actions as they are calculated from a conventional or a fuzzy controller for a fixed operational range of input variables. Here, the (i, j) element of the table contains the changes in control action inferred for the ith value of error and jth value of change of error.

15.5 MODEL IDENTIFICATION AND STABILITY OF FUZZY SYSTEMS

15.5.1 Fuzzy Systems Modeling

Mathematical models, which can describe efficiently the dynamics of the system under study, play an essential role in process analysis and control. However, most of the real-world processes are complicated and nonlinear in nature, making the derivation of mathematical models and/or subsequent analysis rather formidable tasks. In practice, such models are not available. For these cases, models need to be developed based solely on input–output data. Many approaches based on nonlinear time series (Hernadez and Arkun, 1993; Ljung, 1987), several nonlinear approaches (Henon, 1982; Wolf et al., 1985) and normal form theory (Read and Ray, 1998a,b,c) have been applied in nonlinear system modeling and analysis. During the last decade, a considerable amount of work has been published on the dynamic modeling of nonlinear systems using neural networks (Narendra and Parthasarathy, 1990; Chen and Billings, 1992; Shaw et al., 1997; Haykin, 1999) and/or fuzzy logic methodologies (Sugeno and Yasukawa, 1993; Laukoven and Pasino, 1995; Babuska and Verbruggen, 1996). Most of them, excluding normal forms and fuzzy-based approaches, are numerical in nature providing therefore only black-box representations. On the other hand, fuzzy logic methodologies (Laukoven and Pasino, 1995; Park et al., 1999; Sugeno and Kang, 1988; Sugeno and Yasukawa, 1993; Takagi and Sugeno, 1985) can incorporate a priori qualitative knowledge of the system dynamics. In Siettos et al. (2001) and Alexandridis et al. (2002) fuzzy logic and Kohonen's neural networks are combined for the derivation of truncated time series models.

Fuzzy logic can incorporate expertise and a priori qualitative knowledge of the system. In the last 20 years, strikingly results have been obtained by using various fuzzy design methods. In many cases the fuzzy control systems outperform other more traditional approaches. However, the extensive applicability of the former is limited due to the deficiency of formal and systematic design techniques, which can fulfill the two essential requirements of a control system: the requirement for robust stability and that of satisfactory performance. As a consequence, due to the complexity of nonlinear processes, it is rather difficult to construct a proper fuzzy rule base based only on observation. Moreover, the lack of a mathematical model, which characterizes fuzzy systems, often limits their applicability, since various vital tasks, such as stability analysis, are difficult to accomplish.

15.5.2 Stability of Fuzzy Systems

The problem of designing reliable fuzzy control systems in terms of stability and performance has found a remarkable resonance among engineers and scientists. So far, various approaches to this problem have been presented. One of the first contributions to this topic was that of Braae and Ratherford (1979), where they utilized the phase plane method for analyzing the stability of a fuzzy system. Kickert and Mamdani (1978) proposed the use of describing functions for the stability analysis of unforced fuzzy control systems. In Kiszka et al. (1985) the notion of the energy of fuzzy relations to investigate the local stability of a free fuzzy dynamic system is introduced. Motivated by the work of Tanaka and Sugeno (1992), many schemes have been proposed for analyzing the stability of fuzzy systems (Feng, et al., 1997; Kiriakidis et al., 1998; Leung et al., 1998; Kim et al., 1995; Thathachar and Viswanath, 1997; Wang et al., 1996). The main idea behind this approach lies in the decomposition of a global fuzzy model into simpler linear fuzzy models, which locally represent the dynamics of the whole system. In Kiendl and Ruger (1995) and Michels (1997) the authors proposed numerical methods for the stability analysis for fuzzy controllers in the sense of Lyapunov's direct method. In Fuh and Tung (1997) and Kandel et al. (1999) the stability analysis of fuzzy systems using Popov–Lyapunov techniques is proposed. In recent years, the problem of designing stable robust and adaptive fuzzy controllers with satisfactory performance based on the sliding mode approach has attracted much attention (Chen and Chang, 1998; Chen and Chen, 1998; Chen and Fukuda, 1998; Palm, 1992; Tang, et al., 1999; Wang, 1994; Yi and Chung, 1995; Yu et al., 1998). The design of such schemes is based on the Lyapunov direct method. The proposed schemes take advantage of both sliding and fuzzy features. A systematic practical way of deriving analytical expressions for fuzzy systems for use in control, system identification and stability using well-established classical theory methods is presented in Siettos et al. (2001). Finally, in Siettos and Bafas (2001) singular perturbation methods (Kokotovic et al., 1976) based on a Lyapunov approach are implemented for the derivation of sufficient conditions for the semiglobal stabilization with output tracking of nonlinear systems having internal dynamics, incorporating fuzzy controllers.

15.6 TRICKS OF THE TRADE

Newcomers to the field of fuzzy reasoning often ask themselves (and/or other more experienced fuzzy researchers) questions such as "What is the best way to get started with fuzzy reasoning?", or "Which papers should I read?"

A very helpful tutorial, a step by step introduction to the basic ideas and definitions of fuzzy set theory, with simple and well designed illustrative examples, is available at http://www.mathworks.com/access/helpdesk/help/toolbox/fuzzy/fuzzytu2.html

A variety of other sources of information are available, including the first publication on fuzzy reasoning by L. A. Zadeh, the founder of fuzzy logic, which appeared in 1965, as well as his subsequent publications (notably "Outline of a new approach to the analysis of complex systems and decision processes" *IEEE Trans. Syst., Man. Cybernet.* **3**:28–44, 1973), which inspired so many researchers in this new and fascinating field of research.

Another question often asked is "How should I be acquainted with the world of fuzzy systems and fuzzy reasoning?". This question is best answered by consulting information available on the Web. For example, in http://www.cse.dmu.ac.uk/~rij/tools.html, information useful to practitioners is given about fuzzy logic tools and companies. Information may also be found about books and journals as well as research groups and national and international associations and networks, whose members are researchers and practitioners working on fuzzy sets and systems. For this and other relevant information see the next section.

15.7 CONCLUSIONS AND PERSPECTIVES

In this chapter an overview of the basics of fuzzy reasoning has been presented. The theory of fuzzy sets has been introduced and definitions concerning the membership function, logical and transformation operators, fuzzy relations, implication and inference rules, and fuzzy similarity measures have been stated. The basic structure of a fuzzy inference system and its elements have been described. Fuzzy control is introduced and an example of a fuzzy logic controller has been demonstrated, which applies to the control of a plug flow tubular reactor. The issue of fuzzy adaptive control systems has been discussed and the self-organizing scheme has been presented. Subsequently, the topics of stability and model identification of fuzzy systems have been outlined and the presentation has concluded with an introduction of fuzzy classification and clustering systems in pattern recognition.

The above are only an elementary attempt to outline only a small part of the introductory concepts and areas of interest of fuzzy reasoning, whose theory and applications are fast developing. Indeed, during recent years, the literature on fuzzy logic theory and applications has exploded. Areas of current research include an enormous set of topics, from basic fuzzy set-theoretic concepts and fuzzy mathematics to fuzzy methodology and fuzzy logic in practice. The statement by H.-J. Zimmermann that "theoretical publications are already so specialized and assume such a background in fuzzy set theory that they are hard to understand" (Zimmermann, 1985) holds much more today than 20 years ago, when it was first stated.

In particular, research is continuing on the various basic fuzzy set-theoretic concepts, including possibility theory (Ben Amor et al., 2002), fuzzy operators (Pradera et al., 2002; Yager, 2002; Ying, 2002; Wang et al., 2003), fuzzy relations (Wang et al., 1995; Naessens et al., 2002; Pedrycz and Vasilakos, 2002), measures of information and comparison (Hung, 2002; Yager, 2002), etc.

In the area of fuzzy mathematics, research focuses on various issues of non-classical logics (Biacino and Gerla, 2002; Novak, 2002), algebra (Di Nola et al., 2002), topology (Albrecht, 2003), etc.

The research on fuzzy methodology is very extensive. It encompasses issues related to inference systems (del Amo et al, 2002; Marin-Blazquez and Shen; 2002), computational linguistics and knowledge representation (Intan and Mukaidono, 2002), production scheduling (Adamopoulos et al., 2000, Karacapilidis et al., 2000), neural networks (Alpaydin et al., 2002; Oh et al., 2002), genetic algorithms (Spiegel and Sudkamp, 2002), information processing (Liu et al., 2002; Hong et al., 2002; Nikravesh et al., 2002), pattern analysis and classification (Gabrys and Bargiela, 2002; de Moraes et al., 2002; Pedrycz and Gacek, 2002), fuzzy systems modeling and control (Mastrocostas and Theocharis, 2002; Pomares et al., 2002; Tong et al., 2002; Yi and Heng, 2002), decision making (Yager, 2002c; Wang, 2000; Zimmermann et al., 2000; Wang, 2003), etc.

Finally, extensive research is also reported on various applications of fuzzy logic, including process control (Tamhane et al., 2002), robotics (Lin and Wang, 1998; Ruan et al., 2003), scheduling (Muthusamy et al., 2003), transportation (Chou and Teng, 2002), nuclear engineering (Kunsch and Fortemps, 2002), medicine (Blanco et al., 2002; Kilic et al., 2002), economics (Kahraman et al., 2002), etc.

It is this last area and the reported applications of fuzzy reasoning, which proves the relevance and vigor of this new approach to understanding, modeling and solving many problems of modern society.

SOURCES OF ADDITIONAL INFORMATION

A most valuable source of additional information about fuzzy reasoning is the site http://www.abo.fi/~rfuller/fuzs.html. It includes information on almost everything one might like to know about the world of fuzzy systems and fuzzy reasoning, from L. A. Zadeh, the founder of fuzzy logic, fuzzy national and international associations and networks, personal home pages of fuzzy researchers, and fuzzy-mail archives, to fuzzy logic tools and companies, Conferences and Workshops on fuzzy systems, fuzzy logic journals and books and research groups. An excellent Internet course on fuzzy logic control and fuzzy clustering can be found in the site http://fuzzy.iau.dtu.dk/ from Jan Jantzen, Professor at the Technical University of Denmark, Oersted-DTU.

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