Please type your answers and submit them as a PDF on Canvas.

1) Exercise 1.1.2: Express each English statement using logical operations ∨, ∧, ¬ and the propositional variables t, n, and m defined below. The use of the word "or" means inclusive or.

    t: The patient took the medication.
    n: The patient had nausea.
    m: The patient had migraines.

    1. The patient took the medication, but still had migraines.
       t ^ m

    2. The patient had nausea or migraines.
       n v m

    3. Despite the fact that the patient took the medication, the patient had nausea.
       t ^ n

2) Exercise 1.2.7: Write a logical expression for the requirements under the following conditions:

    B: Applicant presents a birth certificate.
    D: Applicant presents a driver's license.
    M: Applicant presents a marriage license.

    1. The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.
       (B ^ D) v (B ^ M) v (D ^ M)

3) Exercise 1.3.6: Give an English sentence in the form "If...then...." that is equivalent to each sentence.

    1. Maintaining a B average is sufficient for Joe to be eligible for the honors program.
       If Joe maintains a B average, then he will be eligible for the honors program.

    2. Maintaining a B average is necessary for Joe to be eligible for the honors program.
       If Joe does not maintain a B average, then he will not be eligible for the honors program.

3. Rajiv can go on the roller coaster only if he is at least four feet tall.
If Rajiv is not at least four feet tall, then he cannot go on the roller coaster.

4. Rajiv can go on the roller coaster if he is at least four feet tall.
If Rajiv is at least four feet tall, then he can go on the roller coaster.

4) Exercise 1.4.6: Translate each English sentence into a logical expression using the propositional variables defined below. Then negate the entire logical expression using parentheses and the negation operation. Apply De Morgan's law to the resulting expression and translate the final logical expression back into English.

    p: the applicant has written permission from his parents
    e: the applicant is at least 18 years old
    s: the applicant is at least 16 years old

1. The applicant has written permission from his parents or is at least 18 years old.
First, equate the sentence to (p v e).
Then, ~(p v e), becomes (~p ^ ~e).
~p ^ ~e is the same as "The applicant does not have written permission from his parents and is not at least 18 years old".

5) Exercise 1.5.2: Use the laws of propositional logic to prove the following:

1. ~(p v (~p ^ q)) == ~p ^ ~q
Distribute p using Distributive Law: ~((p v ~p) ^ (p v q)) == ~p ^ ~q
Apply Complement Law to (p v ~p): ~((T) ^ (p v q)) == ~p ^ ~q
Apply Identity Law to T ^ (p v q): ~(p v q) == ~p ^ ~q
Distribute negation using DeMorgan's: *~p ^ ~q == ~p ^ ~q*

6) Exercise 1.6.1: Indicate whether the argument is valid or invalid. For valid arguments, prove that the argument is valid using a truth table. For invalid arguments, give truth values for the variables showing that the argument is not valid.

1. (p ∨ q) → r | ∴ (p ∧ q) → r
This statement is invalid. One example is when p = T, q = F, and r = F. The proposition (p v q) → r comes out to F, while (p ^ q) → r is T.

2. q → p , p | ∴ ¬(p → q)
The statement is invalid. When p = T and q = T, q → p is T, p is T, but ~(p → q) is F.

7) 1.6.3: Which of the following arguments are invalid and which are valid? Prove your answer by replacing each proposition with a variable to obtain the form of the argument. Then prove that the form is valid or invalid.
   1. The patient has high blood pressure or diabetes or both.
      The patient has diabetes or high cholesterol or both.
      ∴ The patient has high blood pressure or high cholesterol.
      P: High blood pressure
      Q: Diabetes
      R: High Cholesterol
      P v Q
      Q v R
      ∴ P v R
      Invalid. When P is F, Q is T, and R is F, P v R is F.

8) 1.7.2: Use the rules of inference and the laws of propositional logic to prove that each argument is valid. Number each line of your argument and label each line of your proof "Hypothesis" or with the name of the rule of inference used at that line. If a rule of inference is used, then include the numbers of the previous lines to which the rule is applied.
   1. (p ^ q) → r , ~r , q | ∴ ~p

1.	**(p ^ q) → r**	**Hypothesis**
2.	**~r**	**Hypothesis**
3.	**q**	**Hypothesis**
4.	**~(p ^ q)**	**Modus tollens**
5.	**~p v ~q**	**DeMorgan's law**
6.	**~p**	**Disjunctive syllogism**

9) 1.7.4: Prove that each argument is valid by replacing each proposition with a variable to obtain the form of the argument. Then use the rules of inference to prove that the form is valid.
   1. If it was not foggy or it didn't rain (or both), then the race was held and there was a trophy ceremony. The trophy ceremony was not held.
      ∴ It rained.
      p: it was foggy.
      q: It rained.
      r: The race was held.
      s: The trophy ceremony was held.

      ~(p v q) → (r ^ s)

~s
∴ q

1.	**~(p v q) → (r ^ s)**	**Hypothesis**
2.	**~s**	**Hypothesis**
3.	**~s v ~r**	**Addition**
4.	**~r v ~s**	**Commutative law**
5.	**~(r ^ s)**	**DeMorgan's law**
6.	**~(~p v ~q)**	**Modus tollens**
7.	**~~p ^ ~~q**	**DeMorgan's Law**
8.	**p ^ q**	**Double Negation law**
9.	**r**	**Simplification**

10) A woman's purse has been stolen inside an office building. There are 4 people on her office building's floor. One of these 4 people stole her purse. The police interview each of these people and they offer the following statements:

Adam: David is lying and Carl is telling the truth.
Bridget: Adam is telling the truth or Carl is lying.
Carl: If Adam is lying then Bridget is lying.
David: Either Bridget is lying or Carl is telling the truth.

Assuming that every innocent person ends up lying to protect their friends while the guilty person actually tells the truth, who took the woman's purse? Justify your answer.

Adam	Bridget	Carl	David	~D ^ C	A v ~C	~A → ~B	~B v C
T	**T**	**T**	**T**	**F**	**T**	**T**	**T**
T	**T**	**T**	**F**	**T**	**T**	**T**	**T**
T	**T**	**F**	**T**	**F**	**T**	**T**	**F**
T	**T**	**F**	**F**	**F**	**T**	**T**	**F**
T	**F**	**T**	**T**	**F**	**T**	**T**	**T**
T	**F**	**T**	**F**	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**F**	**T**	**T**	**T**
T	**F**	**F**	**F**	**F**	**T**	**T**	**T**
F	**T**	**T**	**T**	**F**	**F**	**F**	**T**
F	**T**	**T**	**F**	**T**	**F**	**F**	**T**
F	**T**	**F**	**T**	**F**	**T**	**F**	**F**
F	**T**	**F**	**F**	**F**	**T**	**F**	**F**
F	**F**	**T**	**T**	**F**	**F**	**T**	**T**
F	**F**	**T**	**F**	**T**	**F**	**T**	**T**
F	**F**	**F**	**T**	**F**	**T**	**T**	**T**
F	**F**	**F**	**F**	**F**	**T**	**T**	**T**

We can start his problem by eliminating potential possibilities. First, let us assume Adam is guilty, meaning he tells the truth. This means David is lying and Carl is telling the truth. However, if the innocent is assumed to be lying, then Carl cannot be telling the truth, therefore we certainly eliminate Adam as the suspect, meaning his statement MUST be false.

Next, we can assume Bridget is guilty, thus telling the truth. We already know Adam MUST be lying, so if Bridget is truthful, Carl must be false.

If we assume Carl is guilty, we must assume that both Bridget and Adam are lying.

Lastly, if David is guilty, we must Carl is lying, as only one person can tell the truth. Bridget must then be lying as well.

This leaves the truth table with two possibilities: 1) Adam and Bridget are lying, Carl and David are truthful. This cannot be. 2) Adam, Bridget, David are lying, Carl is truthful. If we assume the latter to be true, then Bridget and Adam MUST both be lying, which checks out with their statements. David lying also means Bridget is lying OR Carl is truthful. We can simplify Bridget lying or Carl being truthful to just Carl being truthful, which settles all the criteria necessary. Therefore, Carl MUST be guilty.