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Step 1

Fund Name	Ongoing Charge	Initial Charge
Jyske Portefølje Vækst Akk KL	1.84%	0.24%
Jyske Portefølje Balanceret Akk KL	1.34%	0.14%
Jyske Portefølje Stabil Akk KL	1.04%	0.29%
Jyske Portefølje Dæmpet Akk KL	0.47%	0.28%

Table 1: Charges for Jyske Portefølje funds

The table above shows that the riskier portfolios (listed from top to bottom) display the highest fees. Note that this information is provided by Financial Times, and we use the 'Ongoing Charge' as a measure for the total annual expense ratio, given as the percentage of the asset under management (AUM).

Portfolio	**3Y**			**5Y**			**12Y**		
	Ret. (%)	**Std.** (%)	**CVaR** (%)	**Ret.** (%)	**Std.** (%)	**CVaR** (%)	**Ret.** (%)	**Std.** (%)	**CVaR** (%)
Jyske Portefølje Balanceret Akk KL	5.37	7.31	19.62	4.19	7.15	18.52	4.56	7.87	20.76
Jyske Portefølje Dæmpet Akk KL	3.56	2.77	6.85	0.69	2.68	7.19	0.94	2.41	6.39
Jyske Portefølje Stabil Akk KL	4.11	5.13	3.88	2.28	4.99	13.41	2.55	4.87	12.98
Jyske Portefølje Vækst Akk KL	8.41	11.98	31.89	8.86	11.67	29.01	7.54	12.80	33.29

Table 2: Summary Statistics

Note: Returns are arithmetic averages of annualized returns. Standard deviation and CVaR(95%) are based on weekly data annualized to yearly frequency.

The arithmetic average of annualized return is a measure of how an investment of money earns each year on average. We see that the more risky portfolios Jyske Portøfjle Vækst and Jyske Portføjle Balanceret generate the highest arithmetic average annualized return for the three periods, respectively, with the 12Y horizon providing an average annualized return of 7.54% for Jyske Portføjle Vækst and 4.56% for Jyske Portføjle Balanceret.

Although the higher return comes at a cost for the riskier portfolios, reaching an annualized standard deviation of 7.87% and 12.80% on the same horizon. Recall that the annualized standard deviation is a measure of risk that indicates the deviation from the average annualized return.

We see that the Conditional Value at Risk (CVaR) is also substantially greater for these two portfolios in comparison to the Dæmpet and Stabil portfolios. Recall that the CVaR is a measure of risk, measuring the expected loss in the worst of 5% cases during extreme market situations. So, for the Jyske Porteføjle Vækst, we see the annualized CVaR is 33.29%, which indicates that on average, the annualized loss of portfolio value is 33.29% at the 5% worst-case tail of annual returns. We may remind ourselves that for a loss r.v. X then the CVaR is $\mathrm{CVaR}_{0.95}(X) = \mathbb{E}\left[X \mid X \geq \mathrm{VaR}_{0.95}(X)\right]$.

Step 2

Year	Coupon Payment (DKK)	Principal Returned (DKK)
1	22,500	0
2	22,500	0
3	22,500	0
4	22,500	0
5	22,500	0
6	22,500	0
7	22,500	0
8	22,500	0
9	22,500	0
10	22,500	1,000,000
Total	**225,000**	**1,000,000**

Table 3: Annual payments for bond DK0009924961

Note: Face value is 1,000,000 DKK with a fixed coupon rate of 2.25%. Coupon paid annually for 10 years; principal repaid at maturity. This example does not include the potential cost for this investment.

For the investment with a guarantee, we suggest investing in the Danish government bond 'Danske Stat 2035', which we view as a relatively safe investment in comparison to the Jyske portfolios. The strengths of such an investment lie in knowing the annual payment in advance and knowing with almost certainty that the coupon payment is paid out, unless the government defaults, which is an extremely rare case. Although a return of 2.25% annually might only be above or at the inflation level, and thus in real terms the investment barely has any gains. In comparison to the stock market, which on average has a return of 7% yearly, the money might be better placed elsewhere considering the inflation level, but at the cost of a more riskier investments in comparison to the government bond.

We acknowledge that the grandparents might not have a high risk profile considering their age and investment time-horizon, thus their utility might be maximized by considering safer assets due to the potential loss of riskier investments.

Step 3

For the feature selection process, we utilize the AI Feature Selection tool in the Investment Funnel project. The tool has historical data for about 1,866 ETFs, with at least 5 years of price data, trading volume of above $1,000,000, and expense ratios below 0.7%. The tool has two methods to reduce the universe, either by clustering or the minimum spanning tree method.

Thoroughly testing multiple clusters and trees, we found the highest Sharpe ratios (excess return to the risk-free rate, risk-adjusted, is well above zero in most cases) and the most appealing number of assets using the clustering method with 5 clusters, compared to the other trials. This method groups the ETFs into clusters based on return and risk characteristics, thereby avoiding redundancy among ETFs with similar profiles (i.e., strategies and risk). The method preserves diversity by selecting clusters of different types of assets. Additionally, working with fewer ETFs makes optimisation faster, rather than working with the entire universe. Our goal here is not to be too strict with reducing the universe, considering the limit of 30 to 50 ETFs, as we want the optimisation to be able to pick out assets in a diversified universe. We end up with a total of 45 assets.

Name	ISIN	TER(%)	Ann. Ret(%)	Net Ann. Ret(%)	Ann. STD(%)	CVaR 95%	TuW	Sharpe	Max Drawdown(%)	Skew	Ex. Kurtosis
Fundamental Invest Stock Pick II Akk	DK0060521854	1.84	24.73	22.89	17.07	16.2	1	0.52	-16.2	0.45	-0.53
Nordea Invest Danske Aktier Fokus KL 1	DK0060012466	0.14	19.61	19.47	14.93	12.62	1	0.61	-12.62	0.08	-0.84
Nordea Invest Danske Aktier Fokus KL 2	DK0061542719	0.26	19.61	19.35	14.93	12.62	1	0.61	-12.62	0.08	-0.84
Danske Invest Danm In x OMXC20 DKK Wd	DK0060786564	0.25	18.58	18.33	13.47	2.91	1	0.68	-2.91	1.14	0.89
Danske Invest Danm In ex OMXC20 DKK d	DK0060244242	0.35	18.55	18.20	13.47	3.00	1	0.68	-3.00	1.14	0.90
BGF Swiss Small MidCap Opps D2	LU0376447149	1.06	17.96	16.90	13.03	11.29	1	0.80	-11.29	-1.47	2.31
BGF Swiss Small MidCap Opps A2	LU0376446257	1.81	17.08	15.27	13.03	11.95	1	0.75	-11.95	-1.46	2.30
Nykredit Alpha Mira	DK0060158160	1.44	10.44	9.00	5.86	1.27	0	0.72	0.00	0.71	-2.32
Xtrackers MSCI World Utilities ETF 1C	IE00BM67HQ30	0.25	9.22	8.97	12.54	0.05	0	0.69	0.00	1.74	3.47
iShares Global Infras ETF USD Dist	IE00B1FZS467	0.65	8.04	7.39	12.62	2.99	1	0.44	-2.99	1.25	0.55
Xtrackers SP Global Infra Swap ETF 1C	LU0322253229	0.60	7.32	6.72	13.06	6.60	1	0.36	-6.60	0.78	-0.70
iShares STOXX Europe 600 Utilities (DE)	DE000A0Q4R02	0.46	6.26	5.80	15.20	5.99	1	0.36	-5.99	0.43	-0.30
Nordea Invest Euro High Yield Bonds KL 2	DK0061543600	0.54	5.14	4.60	2.94	3.19	1	0.40	-3.19	-1.57	2.44
Nordea Invest Euro High Yield Bnd KL 1	DK0016306798	1.08	5.14	4.06	2.94	3.19	1	0.40	-3.19	-1.57	2.44
Gudme Raaschou European High Yield	DK0016205255	1.24	4.35	3.11	3.35	3.45	1	0.31	-3.45	-1.41	1.76
Nordea Invest Globale Obligationer KL 2	DK0061544418	0.43	3.08	2.65	5.90	5.93	2	0.27	-5.93	0.62	1.39
Maj Invest Danske Obligationer	DK0060005098	0.21	2.61	2.40	1.25	0.14	0	0.17	0.00	-0.08	-1.00
Nordea Invest Globale Obligationer KL 1	DK0010170398	0.85	3.08	2.23	5.90	5.93	2	0.27	-5.93	0.62	1.39
Maj Invest Globale Obligationer	DK0060004950	0.35	2.43	2.08	2.70	0.34	1	0.19	-0.34	0.30	-1.60
Danske Invest Kommuner 4 KL	DK0016205685	0.34	1.80	1.46	1.05	0.45	1	-0.27	-0.45	-0.79	-0.86
Nykredit Invest Korte Obligation Akk KL	DK0060033975	0.19	1.59	1.40	1.06	0.00	1	-0.66	0.00	-0.48	-2.48
Nordea Invest Korte Obli Lagerbe KL 2	DK0061544921	0.16	1.51	1.35	0.92	0.39	0	-0.74	0.00	0.71	-0.04
Nordea Invest Korte Obligationer KL 2	DK0061545068	0.16	1.50	1.34	0.99	0.38	0	-0.75	0.00	-0.07	-2.29
Nordea Invest Korte Obl Lagerbe KL 1	DK0060014678	0.25	1.51	1.26	0.92	0.39	0	-0.74	0.00	0.71	-0.04
Nordea Invest Korte Obligationer KL 1	DK0060268506	0.27	1.50	1.23	0.99	0.38	0	-0.75	0.00	-0.07	-2.29
Wealth Invest HP Engros Korte Danske Obl	DK0061553245	0.21	1.36	1.15	0.86	0.45	1	-1.12	-0.45	-1.33	1.04
Wealth Invest HP Invest Korte Danske Obl	DK0061150984	0.22	1.36	1.14	0.86	0.45	1	-1.12	-0.45	-1.33	1.04
BankInvest Korte Danske Obligationer A	DK0016109614	0.19	1.30	1.11	0.65	0.06	0	-1.14	0.00	-0.11	-0.80
BankInvest Korte Danske Obligationer W	DK0060822468	0.24	1.33	1.09	0.65	0.18	0	-1.14	0.00	-0.03	-1.12
Wealth Invest HP Inv. Lange Dk Obl Akk A	DK0060227239	0.35	1.24	0.89	0.82	0.50	1	-1.29	-0.50	-1.09	-0.30
Danske Invest Teknologi Indeks KL	DK0016023229	0.50	0.99	0.49	1.02	0.44	0	-2.99	0.00	0.56	-0.49
Wealth Invest HP Invest Grønne Obl KL A	DK0060118610	0.57	0.89	0.32	0.77	0.91	1	-1.59	-0.91	-1.31	1.22
Maj Invest Vækstaktier	DK0060005254	1.35	0.90	-0.45	5.15	4.63	2	0.06	-4.63	-0.22	0.88
Danske Invest Euro IG Corp Bd A SEKH	LU0178670161	1.07	0.17	-0.90	6.87	6.13	2	-0.67	-6.44	-1.41	2.37
Danske Invest Danish Mortgage Bd A SEK h	LU0332084994	0.80	-0.64	-1.44	6.27	4.18	3	-1.16	-4.18	-1.47	3.11
Danske Invest Euro IG Corp Bd A NOKH	LU0178670245	1.07	-0.40	-1.47	9.04	5.22	2	-0.23	-5.51	1.46	2.02
BGF World Gold X2	LU0320298689	0.06	-7.32	-7.38	31.17	13.61	1	0.19	-13.61	1.30	1.47
BGF World Gold I2	LU0368252358	1.08	-8.25	-9.33	31.16	14.47	1	0.15	-14.47	1.30	1.47
iShares Gold Producers ETF USD Acc	IE00B6R52036	0.55	-8.87	-9.42	34.61	12.89	1	0.19	-12.89	2.01	4.28
BGF World Gold D4	LU0827889139	1.34	-8.49	-9.83	31.33	14.58	1	0.14	-14.58	1.31	1.52
BGF World Gold A2	LU0055631609	2.06	-9.17	-11.23	31.17	15.30	2	0.11	-15.76	1.30	1.48
BGF World Gold A4	LU0724618789	2.09	-9.17	-11.26	31.17	15.34	2	0.11	-15.80	1.30	1.47
BGF World Gold A2 HKD Hedged	LU0788108826	2.09	-9.83	-11.92	31.39	16.46	2	0.07	-18.35	1.29	1.38
BGF World Gold D2 SGD Hedged	LU0827889303	1.34	-10.96	-12.30	33.11	18.86	1	0.03	-18.86	0.74	0.03
BGF World Gold D2	LU0252968424	5.26	-8.48	-13.74	31.17	14.69	1	0.14	-14.69	1.30	1.48

Table 4: Summary Statistics of the Investment Universe in the date range 2013-01-09 to 2019-01-09

Table 4 is sorted by the net annualized return, subtracting the TER from the annualized returns. In addition to the already mentioned performance measures, we include the Time under Water (TuW), which is the maximum distance in time from a previous peak to a new peak in performance in years. The Sharpe ratio measures the annualized excess return given the annualized risk of the asset. Here, the annualized excess return is calculated by subtracting the risk-free rate of the government bond presented in step 2 (2.25%) from the annualized return of the asset. The maximum drawdown shows the largest decline in portfolio value from its peak to the subsequent lowest point. The skewness and kurtosis is a measure for the probability distribution of returns for the different assets. A positive skewness shows that there is a higher probability of positive returns and vice-versa for a negative skewness. A positive excess kurtosis displays that the returns of the asset have a heavier tail relative to a normal distribution and vice-versa for negative excess kurtosis.

The summary statistics shows a clear trade-off between returns and risk. The greater the annualized return the more risky the asset appears from the statistics. Fundamental Invest reaches the greatest net annualized return of 22.89% with an annualized standard deviation of 17.09%. Fundemental Invest appears only to be Under the Water for a single year (by the annualized returns), whilst reaching a substantial maximum drawdown of 16.2%. Looking into the distribution of this asset, the annualized returns show a greater probability of positive returns and a smaller tail in comparison to the normal distribution. We see a clear ranking of more stock focused asset performs best in comparison to the obligation and commodity/gold products seen in the middle and bottom of the table. **Note** that the covariance matrix is included in the code but not shown in the report, as the assignment does not explicitly say to include it, only to calculate it.

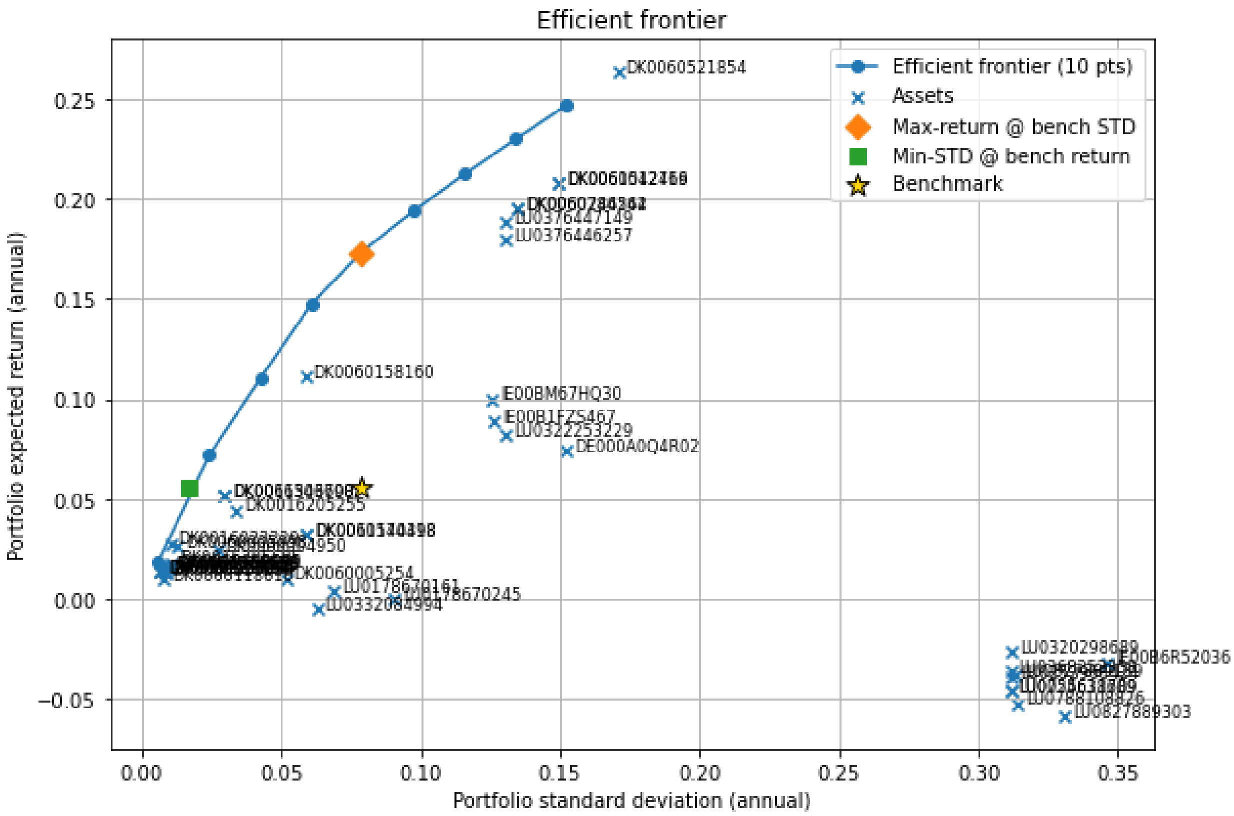

Figure 1: Efficient Frontier

Step 4

First, we formulate the Markowitz model.

$$\max(1-\lambda)\sum_{i=1}^{n}\mu_i x_i^p - \lambda\sum_{i=1}^{n}\sum_{j=1}^{n} x_i^p Q_{ij} x_j^p \quad \textbf{s.t} \tag{1}$$

$$\sigma^p \leq \sigma^b \quad \text{for pf. max ret std leq to bm.} \tag{2}$$

$$\mu^p \geq \mu^b \quad \text{for pf. min std with exp ret geq to bm.} \tag{3}$$

$$\sum_{i=1}^{n} x_i^p = \sum_{i=1}^{n} x_i^b = 1 \tag{4}$$

$$x_i^p, x_i^b \geq 0 \quad \forall i \tag{5}$$

The Markowitz optimization problem used in step 4.1 is formulated above. μ_i denotes the expected return for asset i, x_i^p denotes the asset weight in the portfolio, x_i^b denotes the asset weight in the benchmark. Q_{ij} is the covariance between asset i and j. The annualized standard deviation is denoted by σ^p and σ^b for the portfolio and benchmark respectively, and similarly the expected annualized return μ^p and μ^b.

Relevant for the first strategy, we maximize the expected return of the portfolio by setting $\lambda = 0$ in equation 1. Next we consider the constraint 2 for this strategy. Relevant for the second strategy, we set $\lambda = 1$ and minimize the standard deviation using the constraint 3. Please note that we use the annualized covariance matrix in the optimization, as we deem this to be consistent with the assignment mentioning the expected annual return.

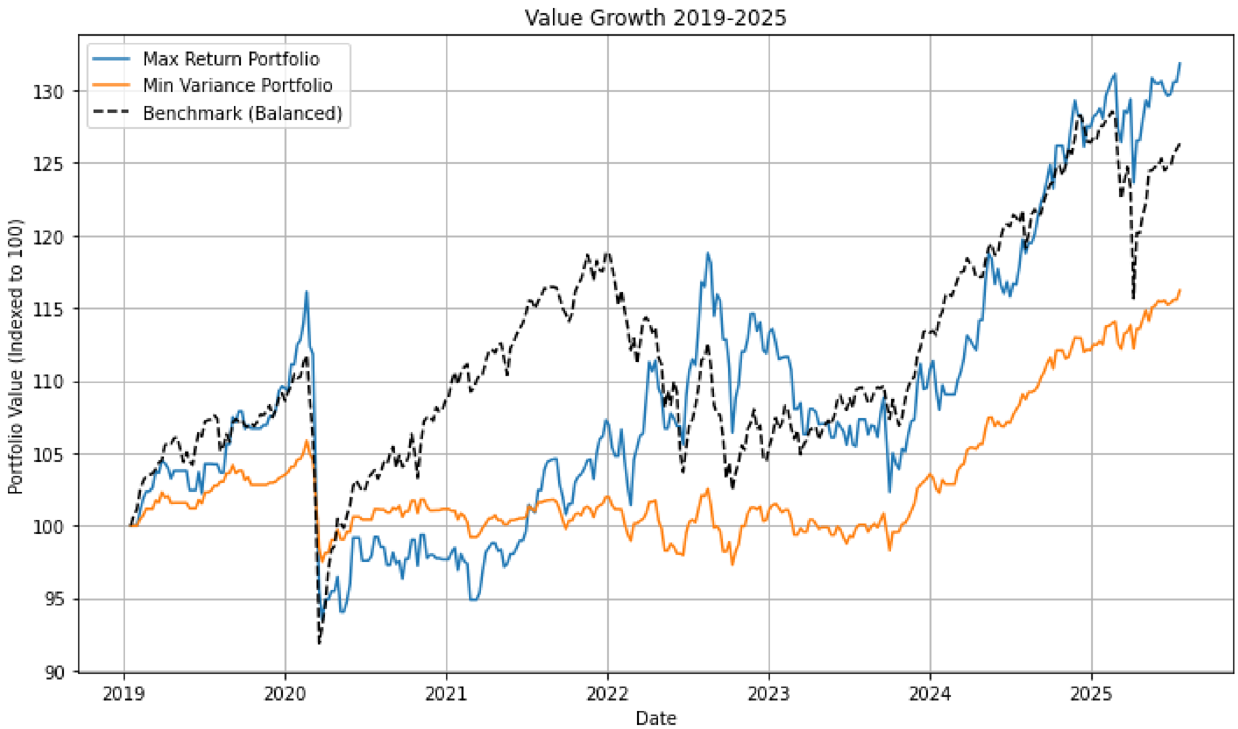

Figure 2: Performance using the date range 2019-01-16 to 2025-07-23

Figure 1 depicts the efficient frontier considering the selected investment universe. Note that it appears as ISIN code DK0060521842 lies beyond the efficient frontier. However, this is not the case, if we use more efficient frontier points than 10, the asset would appear on the frontier. Recall that the efficient frontier display the optimal portfolios providing the highest expected return given the level of risk. From this figure, it shows that there exists a portfolio performing better than the benchmark Jyske Portefølje Balanceret Akk KL with ISIN code DK0060259786 given the risk level. From the figure the strategy maximizing returns whilst having a risk level below or at the benchmark has a greater expected annual return. We pick this benchmark as it is a balanced strategy with both safe and risky assets, which we deem to be most appropriate considering the selected investment universe that consists of a variety of assets.

Figure 2 depicts the performance of the strategies, where the value is indexed to 100. From the graph, we see that the max return (maximizing expected return) strategy outperforms the benchmark in the long run, whilst the minimum variance portfolio (minimizing the standard deviation) underperforms. Intuitively, it makes sense that there is a trade-off between returns and risk, hence the minimum variance portfolio is lacking in return by selecting a composition of assets less volatile. Looking at the annualized measures in table 5, the Max Return strategy delivers the highest expected annual return, but this comes at the cost of greater volatility relative to the other strategies. The VaR(95%) indicates that, with 95% confidence, annual losses are not expected to exceed 8.25%. However, in the worst 5% of cases, the CVaR(95%) suggests an average loss of 10.88%. The strategy Max Return delivers the highest return, although it should coincide with the investor's risk profile and time horizon. The strategies considered here are buy-and-hold strategies.

Table 5: Portfolio performance comparison

Strategy	**Mean Ann. (%)**	**STD Ann. (%)**	**Sharpe**	**VaR(95%)**	**CVaR(95%)**
Max Return	4.39	9.23	0.24	8.25	10.88
Min Variance	2.35	3.93	0.00	1.98	2.15
Benchmark	3.79	7.93	0.20	8.56	12.03

Step 5

We will formulate the model we run in step 5. Consider scenarios $s = 1, \dots, n$ with associated probability p_s^*, target return g_s^*, value at end of period $V_s(x)^*$ and downside regret $y_l^* = \max\{0, g_s^* - V_s^*\}$, where $* \in \{b, p\}$ means it relates to portfolio and benchmark, respectively. For each asset $i = 1, \dots, m$ introduce initial value $P_{0,s}$, x_i the amount allocated in asset i and expected price at end of period $\bar{P}_i$. Let V_0 be the initial budget and $x^p = (x_1^p, \dots, x_m^p) \in \mathbb{R}_+^m$. The optimization problem then becomes

$$
\begin{aligned}
&\min \sum_{s=1}^{n} p_s^p y_s^p \\
\text{s.t} \quad &\sum_{i=1}^{m} \bar{P}_i x_i^p \geq V_0 \sum_{s=1}^{n} p_s^b \mu_s^b \\
&\sum_{i=1}^{m} P_{0,i} x_i^p = V_0 \\
&x_i \geq 0, \forall i,
\end{aligned}
$$

such that the target is to find the portfolio that minimizes expected downside regret constrained by having expected returns above the expected benchmark returns in addition to the budget constraint, i.e., no gearing, and non-negative portfolio-allocation, i.e. no shorting.

In step 5.4, the model was changed, and

$$
\begin{aligned}
&\max \sum_{s=1}^{n} \bar{P}_i x_i \\
\text{s.t} \quad &\sum_{s=1}^{n} p_s^p y_s^p \leq \sum_{s=1}^{n} p_s^b y_s^b \\
&\sum_{i=1^n} P_{0,i} x_i = V_0,
\end{aligned}
$$

This means that we maximized the expected portfolio return with respect to portfolio allocation constraining the downside regret from above by benchmark downside regret in addition to the budget constraint, i.e., no gearing, and non-negative portfolio-allocation, i.e. no shorting

Table 6: Downside Regret strategy performance comparison

Strategy	**Mean Ann. Ret (%)**	**STD Ann. (%)**	**Sharpe**	**VaR(95%)**	**CVaR(95%)**	**Exp. Downside Regret**
Min Downside Regret (5.3)	5.12	9.94	0.27	3.73	3.90	0.51
Max Return (5.4)	2.80	4.88	0.09	3.55	4.13	0.30
Benchmark	3.79	7.93	0.20	8.56	12.03	0.39

Table 6 summarizes the performance of the three strategies over the testing period from 2013-01-09 to 2019-01-09. Step 5.3, which minimizes downside regret while constraining expected return to exceed the benchmark, achieved higher mean returns than Step 5.4, which maximizes expected return while constraining downside regret to be below that of the benchmark. This outcome may seem surprising, but it can be explained by the differing constraints and thus portfolio compositions. Step 5.4's strict regret constraint, making its portfolio smaller and more concentrated, likely limited diversification benefits, which became particularly costly during periods of higher volatility in the testing phase. In contrast, Step 5.3 maintained a much larger and better diversified portfolio which allowed it to capture upside returns more consistently. This illustrates that maximizing expected returns in-sample does not necessarily translate into higher realized returns, particularly under tight constraints.

We note that VaR and CVaR were highest for benchmark, whereas expected downside regret was highest for Step 5.3 and lowest for 5.4. The expected downside regret is the mean of the raw regrets for each time period over the testing period. This means that both of the methods are perform well for more risk averse investors. So consider returns μ_i for time indices $i = 1, \dots, n$ with target $g = 1.02^{4/52}$, then expected

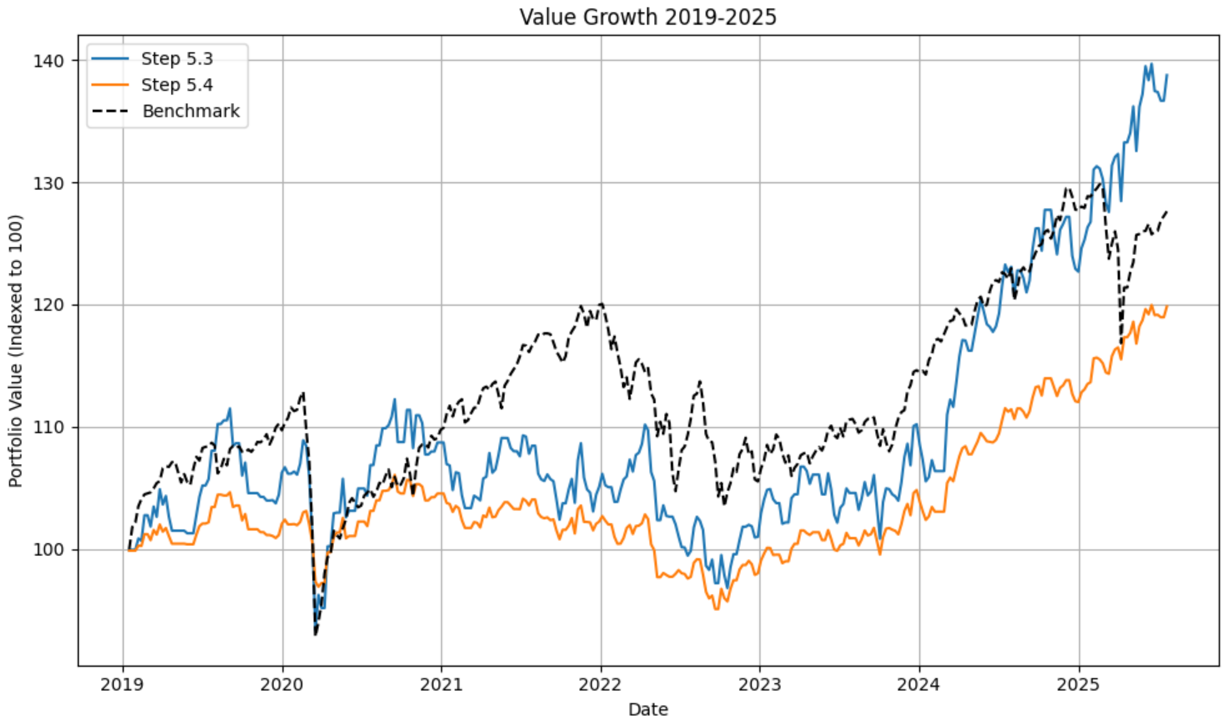

Figure 3: Portfolio Value

downside regret is EDR $= n^{-1}\sum_{i=1}^{n}(g-\mu_i)^+$. This is similar to what is done in the optimization, where the expectation was over the scenarios whereas it is now over the time series. Note here that $(\,\cdot\,)^+ = \max\{0,\,\cdot\,\}$.

Step 6

We now proceed to formulate the CVaR optimization problem in step 6.3.

$$\min CVaR \quad \textbf{s.t.} \tag{6}$$

$$\sum_{i=1}^{n} x_i = V_0 \tag{7}$$

$$Losses_s = V_0 - \sum_{i=1}^{n} R_{is}\cdot x_i^p \quad \forall s \tag{8}$$

$$VaRDev_s \geq Losses_s - VaR \quad \forall s \tag{9}$$

$$CVaR = VaR + \frac{\sum_{s=1}^{S} p_s \cdot VaRDev_s}{1-\alpha} \tag{10}$$

$$\mu_s^p = \frac{1}{S}\sum_{s=1}^{S} r_{is}\cdot x_i^p \geq \mu_s^b = \frac{1}{S}\sum_{s=1}^{S} r_{is}^b x_i^b \tag{11}$$

Here the goal is to minimize the Conditional Value-at-Risk (CVaR) whilst the expected return μ_s^p, using the scenarios, should be greater or equal to the benchmark expected return μ_s^b using the scenarios. Note that we use the same definitions for the scenarios s, weights of assets x_i, probabilities p_s as seen in the prior steps of this project.

For step 6.4, we formulate the optimization problem maximizing the expected return of the scenarios whilst the CVaR is constrained to be less or equal to the benchmark.

$$\max \mu_l^p \quad \textbf{s.t.} \tag{12}$$

$$\sum_{i=1}^{n} x_i = V_0 \tag{13}$$

$$Losses_s = V_0 - \sum_{i=1}^{n} R_{is} \cdot x_i^p, \quad \forall s \tag{14}$$

$$VaRDev_s \geq Losses_s - VaR, \quad \forall s \tag{15}$$

$$CVaR = VaR + \frac{\sum_{s=1}^{S} p_s \cdot VaRDev_s}{1-\alpha} \tag{16}$$

$$CVaR \leq CVaR^b \tag{17}$$

Where we calculate the CVaR of the benchmark using the same formula as the one described above, by replacing the returns by the benchmark returns.

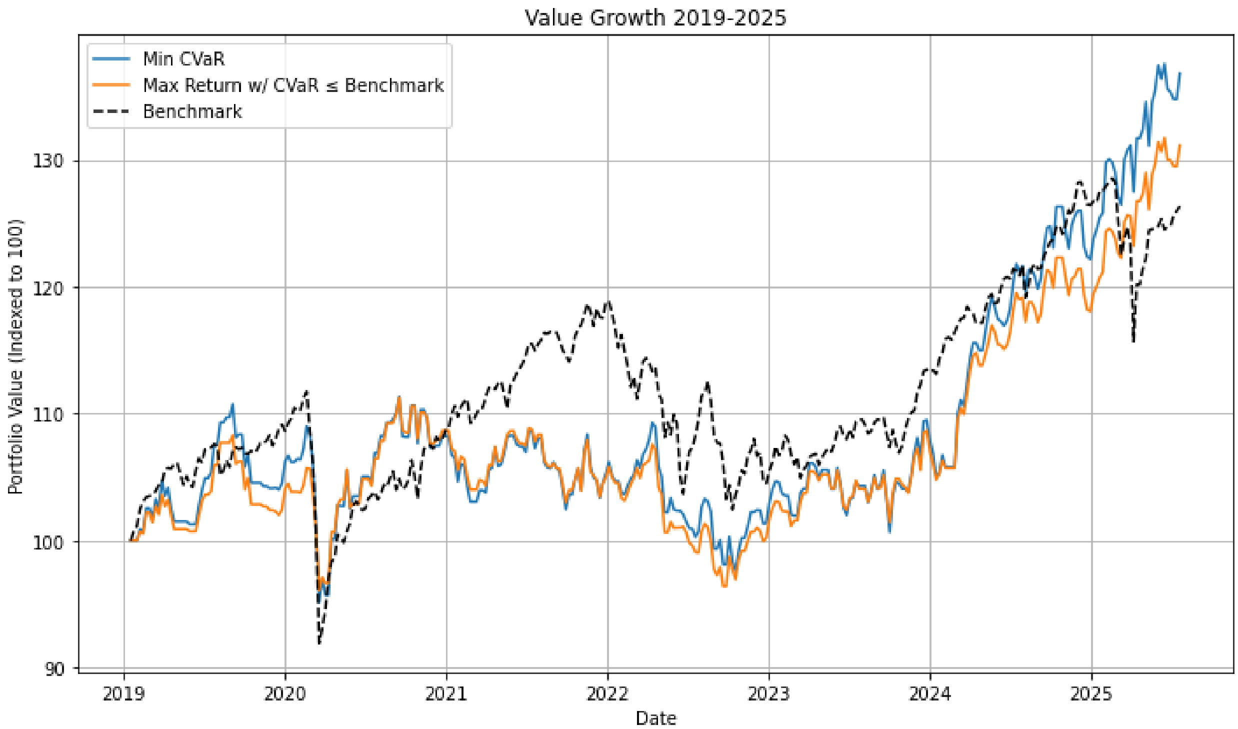

Figure 4: Performance using the date range 2019-01-16 to 2025-07-23

Interestingly, we see from figure 4 that the Min CVaR strategy outperforms the Max Return with CVaR less or equal to the benchmark CVaR in the evaluation period. This is a perfect example of the dispersion of training to reality. The optimization solves the problem at hand, but it does not necessarily mean that it works as intended in the evaluation period. The world changes, and so does the assets included in the portfolio, and thus we might not get the result that we were expecting, i.e., that the CVaR of the Min CVaR strategy would be lower than the alternative strategies. Looking into table 7, we see that the mean annualized return outperforms the two alternatives, whilst achieving higher risk measures, which may appear counter intuitive.

To verify that our optimisations solve the problems as intended, we checked that the CVaR of the Min CVaR strategy was substantially lower than the benchmark in the training period. Also, we checked that the Max Return strategy CVaR was just slightly below the benchmark CVaR in the training period as intended. So, the difference appears in the evaluation period.

Table 7: Portfolio performance comparison with CVaR constraint

Strategy	**Mean Ann.(%)**	**STD Ann.(%)**	**Sharpe**	**VaR(95%)**	**CVaR(95%)**
Min CVaR	4.89	9.07	0.27	3.42	3.71
Max Return, CVaR $\leq$ BM	4.18	7.83	0.23	4.20	4.54
Benchmark	3.79	7.93	0.20	8.56	12.03

Step 7

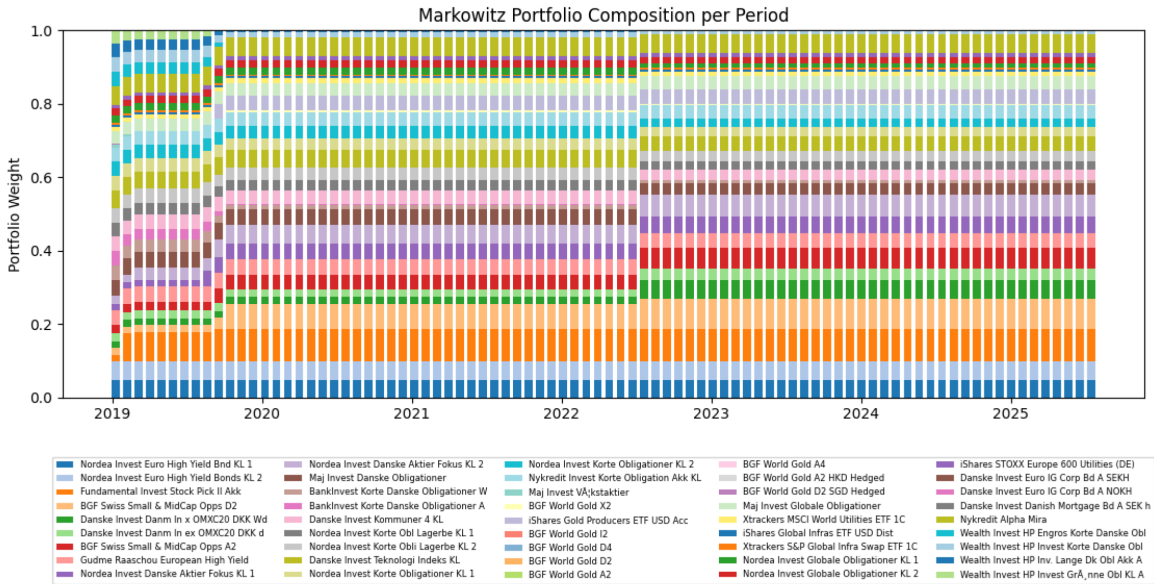

Figure 5: Portfolio composition Markowitz

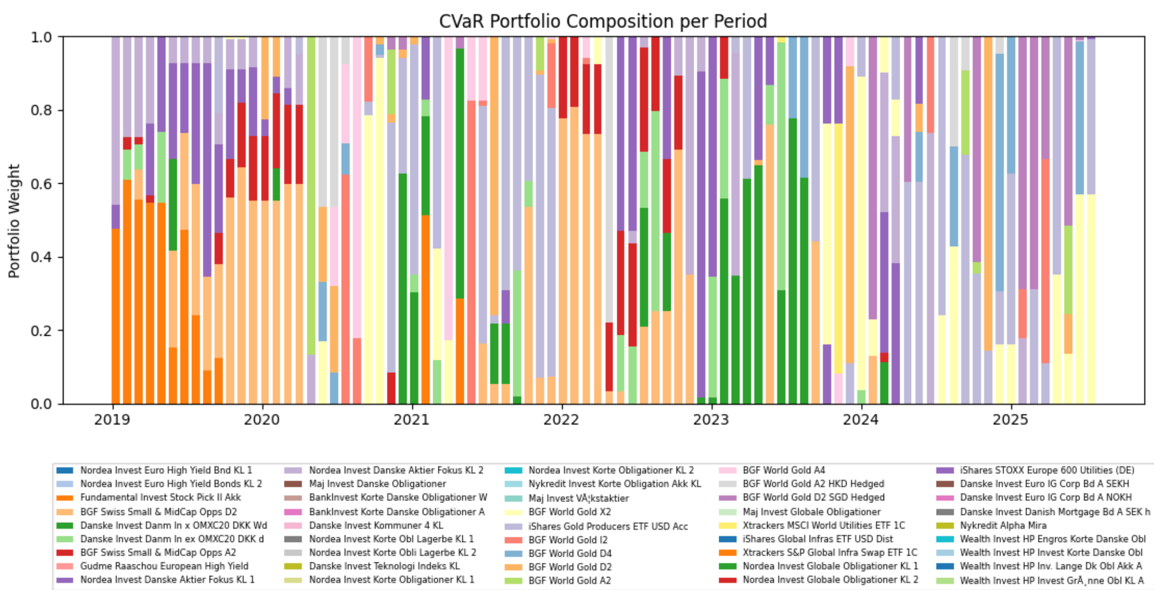

Figure 6: Portfolio composition CVaR

We selected the following two strategies:

1. Maximize return subject to CVaR $\leq$ benchmark CVaR.
2. Minimize standard deviation subject to expected return $\geq$ benchmark expected return.

These two strategies are our preferred choices for the grandparents. Both approaches explicitly address risk while still aiming to outperform the balanced benchmark.

From the weight composition seen in figures 5 and 6 we notice two things:

1. The Markowitz model has very few portfolio revisions; most often, it will start the next period with the portfolio from the previous period. From mid-2022 till the end of the testing period, virtually no updates are happening. Contrarily, the CVaR model frequently liquidates all its positions at the end of a period and reinvests it for the next period. This means we will see fewer transaction costs in the Markowitz model.

2. The Markowitz model has a very diversified portfolio, which frequently consists of around 25-30 assets. A few do dominate and have a pretty consistent portion of the portfolio, however this seems to be capped at around 10%. In constrast, the CVaR portfolio consists of fewer assets, sometimes only two, and with a single asset being up to over 90% of the portfolio, but most often with 3-4 assets.

Notice how these two points complement each other. The highly diversified Markowitz portfolio is relatively stable, such that small changes in individual asset returns or covariances have a limited impact on the overall allocation, so the optimizer often finds that the previous period's weights remain close to optimal, which results in fewer revisions. In contrast, the concentrated CVaR portfolio is much more sensitive to shifts in the risk profile of its few assets. Minor changes in one key asset can significantly affect the portfolio's total risk, resulting in more frequent and larger reallocations. In other words, portfolios with few large positions naturally require more adjustments to maintain optimality while highly diversified portfolios remain largely stable over time.

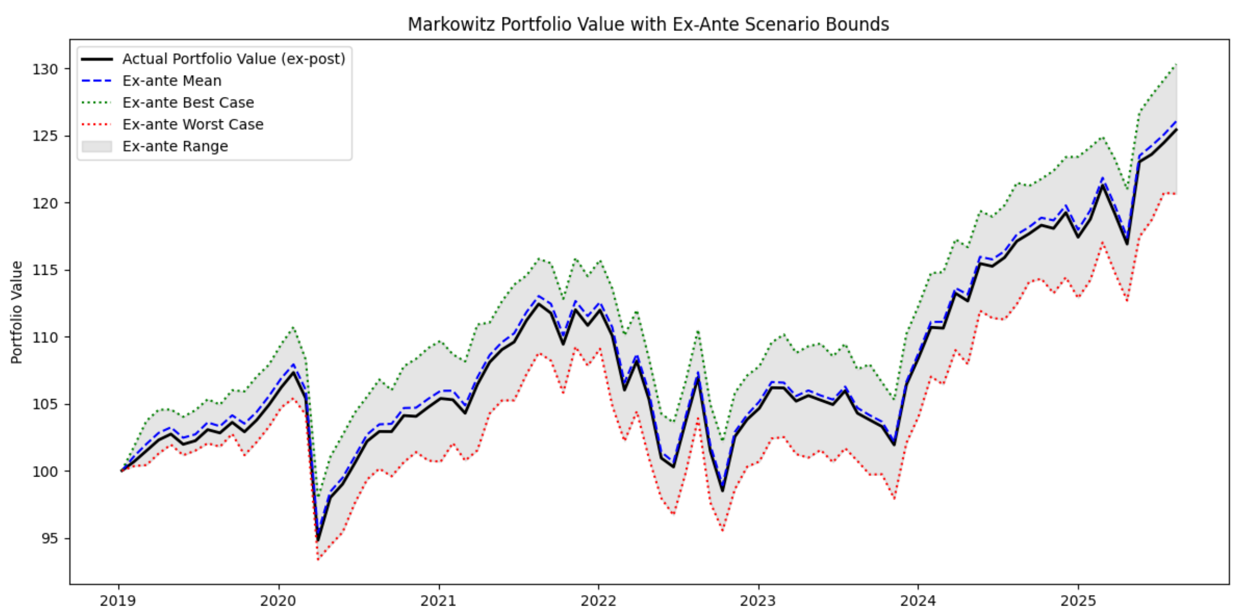

Figure 7: Markowitz portfolio value over time

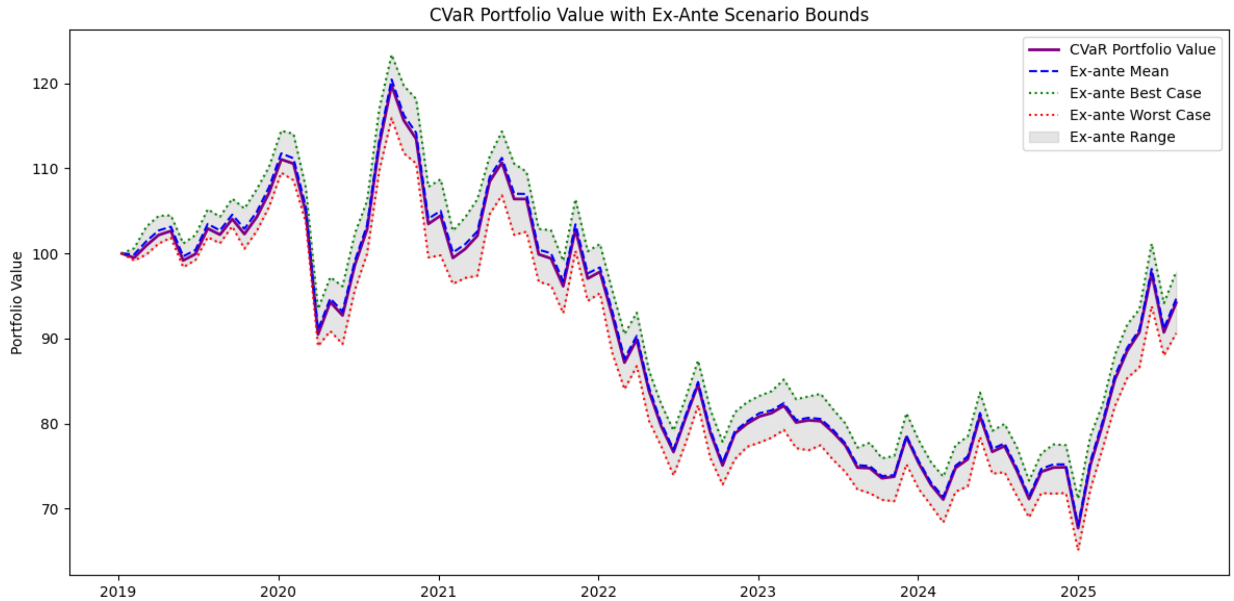

Figure 8: CVaR portfolio value over time

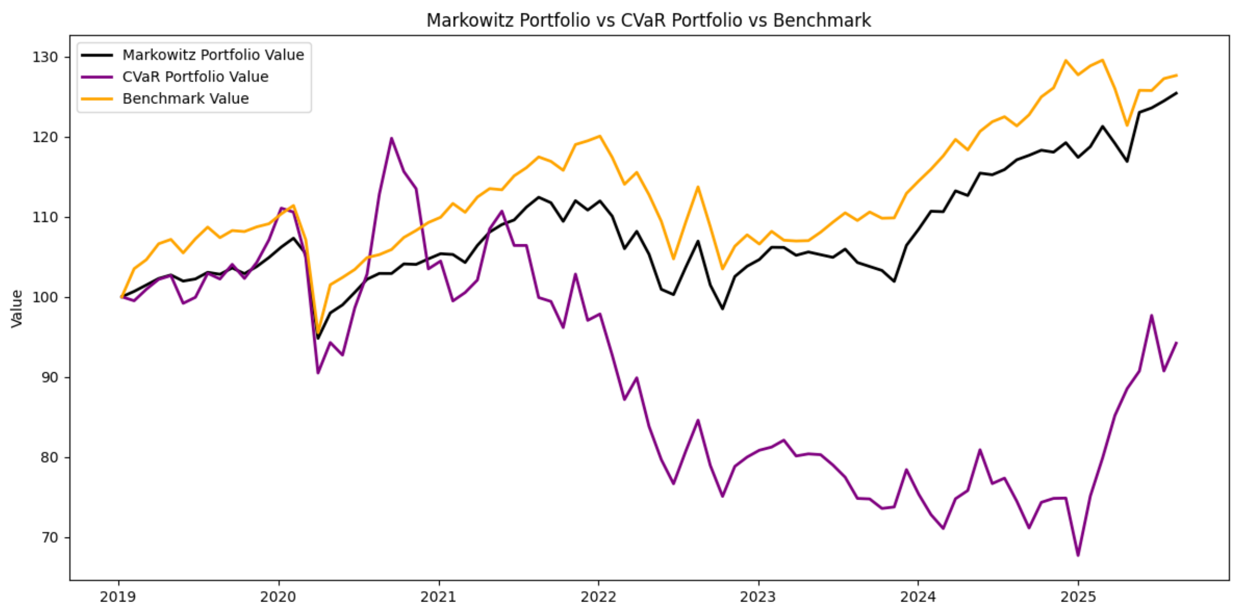

Figure 9: Result

We observe that the CVaR model performs poorly over the test period, ultimately delivering negative annualized returns. A recurring feature of this strategy is a heavy allocation to assets which may be seen as ”safe”, such as BFG World Gold 2 and Nordic Invest Danske Aktier Fokus KL 2. In theory, concentrating in a few assets with historically low tail risk and strong returns seems attractive. In practive, however, this concentration makes the portfolio highly vulnerable, that is, if one of these key assets turn negative, performance suffers a lot. This shows an important point also noted earlier: optimizing for low risk ex ante does not guarantee low realized risk ex post. When returns contain a large random component, holding very few assets can mean capturing more noise than signal.

In contrast, the Markowitz model ends the test period with only slightly below the benchmark in total return. Although it underperforms its own ex ante mean scenario consistently, it tracks the benchmark's trajectory closely. The Markowitz portfolio also exhibits lower volatility and tail risk, as measured by the CVaR, than the benchmark, as seen in Table 8.

From the scenario plots, we see that both models' realized performance generally has a closer range between their worst and best case scenario paths in times of lower volatility.

Table 8: Portfolio performance comparison with CVaR constraint

Strategy	**Mean Ann.(%)**	**STD Ann.(%)**	**Sharpe**	**VaR(95%)**	**CVaR(95%)**
Markwitz	3.48	7.48	0.16	4.48	6.33
CVaR	-0.90	16.55	-0.14	14.17	17.58
Benchmark	3.79	7.93	0.20	8.56	12.03

Step 8

Table 9: Portfolio performance comparison across strategies

Strategy	**Mean Ann. (%)**	**STD Ann. (%)**	**Sharpe**	**VaR(95%)**	**CVaR(95%)**	**Exp. Downside Regret**
Max Return (4.1)	4.39	9.23	0.24	8.25	10.88	–
Min Variance (4.1)	2.35	3.93	0.00	1.98	2.15	–
Min Downside Regret (5.3)	5.12	9.94	0.27	3.73	3.90	0.51
Max Return (5.4)	2.80	4.88	0.09	3.55	4.13	0.30
Min CVaR (6.3)	4.89	9.07	0.27	3.42	3.71	–
Max Return, CVaR $\leq$ BM (6.4)	4.18	7.83	0.23	4.20	4.54	–
Markowitz (7.)	3.48	7.48	0.16	4.48	6.33	–
CVaR (7.)	-0.90	16.55	-0.14	14.17	17.58	–
Benchmark	3.79	7.93	0.20	8.56	12.03	0.39

Table 9 shows the overall summary of all the strategies tested in this assignment. Two buy-and-hold strategies, 'Min Downside Regret (Step 5.3)' and 'Min CVaR (Step 6.3)' stand out on risk-adjusted metrics, such as Sharpe ratio. Over the evaluation period, Min Downside Regret delivered an average annual return of 5.12% with annualized standard deviation 9.94% and Sharpe ration 0.27, while Min CVaR returned 4.89% with STD 9.07% and Sharpe 0.27. By contrast, the benchmark (Jyske Portefølje Balanceret) shows mean 3.79%, STD 7.93% and Sharpe 0.20, and the high-growth Jyske Portefølje Vækst achieved 7.54% over 12 years but with much higher observed tail risk.

Min CVaR has the lowest CVaR among the top performers (3.71%) while Min Downside Regret has slightly higher CVaR (3.90%) but the highest mean return. The rebalanced strategies in step 7 performed worse, especially the CVaR-based one which had high realized volatility (Mean Ann. return 0.90%, Mean Ann. STD 16.55%) showing that frequent rebalancing and concentrated portfolios can amplify tail losses. The Markowitz model with rebalancing also underperformed compared to buy-and-hold Min CVaR and Min Downside Regret. This may be due to transaction costs and the fact that scenario-based simulations from the less volatile training period underestimated volatility in the testing period.

We may thus **conclude** on this with two practical lessons from the project: 1) Buy-and-hold strategies outperform the same models actively rebalanced and 2) optimizing over tail-aware measures, such as CVaR or regret, while constraining on returns, performs better than optimizing over returns while constraining on the risk measures.

In order to make a recommendation, we recall that the money should be used over 10 years. Consider a 20-year-event in Jyske Bank Vækst during the first year. A portfolio initially worth 100 drops to a value of 67.71 at the start of year 2. Recovering to the initial value would require annual returns of 4.6%. In contrast, under the Min Downside Regret model (step 5.3), the same event reduces the portfolio by only 3.9%, requiring just 0.4% annual returns over the next nine years. For soon-to-be retirees needing regular withdrawals, large drawdowns are to be avoided.

For the grandparents with a 10-year withdrawal horizon, our **recommandation** is conservative, and we will favour a strategy with lower returns, that is tail aware (in the sense it has low VaR/CVaR) and buy-and-hold. Concretely, consider a portfolio consisting of low-cost ETFs that targets performance similar to Min CVaR or Min Downside Regret, i.e. expected annual return 4.5% to 5.1% coupled with CVaR around 3.7-3.9%, This balances inflation protection with higher returns than the coupon bond while limiting tail exposure like the ones seen in the aggressive Jyske Bank Vækst. Furthermore, the Min CVaR/Min Downside Regret are both higher return but lower tail-risk than the benchmark, hence our recommendation.