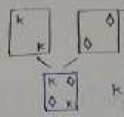
## Probability :-

Event
1) Exahustive
2) Equally likely : $P(A) = P(B)$
3) mutually Exclusivity .
$A \cap B = \phi$



$K \cap \lozenge = 1$

4) Independent .

⇒ The experiment whose result is unpredictable is called Random experiment

⇒ The Happening of each outcome is called as an event

(1) Exaustive : all possible outcomes are called exaustive eg. Head & T in Tossing coin.

(2) Equally Likely : $P(A) = P(B)$

(3) mutualy exclusivity : The Happening of an outcome privents the Happening of another outcomes eg: Head & Tail in tossing coin
$A \cap B = Null$
King and diamond are not mutually exclusive

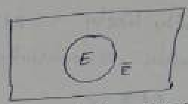
(4) Independent event : The ~~one~~ event does not influence the Happening of other outcome
eg: getting VOB A, B

$$\text{Probability } P = \frac{\text{no. of favarable outcomes}}{\text{no. of exahustive outcomes}} = \frac{m}{n}$$

## set definition :

Sample space :- Set of all exahustive events in a experiment is called sample space

$$P(E) = \frac{n(E)}{n(S)}$$



$n(E) + n(\bar{E}) = n(S)$

$\div$ by $n(S)$

$\frac{n(E)}{n(S)} + \frac{n(\bar{E})}{n(S)} = \frac{n(S)}{n(S)}$

$P(E) + P(\bar{E}) = P(S) = 1$

$\boxed{P(\bar{E}) = 1 - P(E)}$

### Axioms of Probability :-

(1) $P(S) = 1$
(2) $P(\phi) = 0$
(3) $0 \le P(E) \le 1$
(4) $P(E) + P(\bar{E}) = 1$
(5) If $A \subseteq B$ then $P(A) \le P(B)$ — $\subseteq$ ← subset of B
(6) If A, B are mutually excalusive $P(A \cap B) = P(\phi) = 0$
(7) If A, B are Independent $P(A \cap B) = P(A) \cdot P(B)$
(8) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
(9) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$
(10) odds in favour of an event 'E' $= P(E) : P(\bar{E})$
(11) odds ~~agg~~ against an event 'E' $= P(\bar{E}) : P(E)$

**Q.12** Exp: drawing '4' number from '14' numbers and x led

$\therefore n(S) = {}^{14}C_4 = 1001$

let E = Product to be +ve

$E = [4\text{+ve}, 4\text{-ve}, (2\text{+ve}, 2\text{-ve})]$

$= {}^6C_4 + {}^8C_4 + ({}^6C_2 \times {}^8C_2)$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{505}{1001}$

**Q.6** : box 1 = 3, 6, 9, 12, 15    box (2) : 6, 11, 16, 21, 26

$n(S) = {}^5C_1 \cdot {}^5C_1 = 25$

no. tobe even = {(even . even) + (odd . even) + (even.odd)}

$\bar{E} = \{odd\} = \{odd . odd\}$
$\{{}^3C_1 \cdot {}^2C_1\}$

$P(E) = 1 - P(\bar{E}) = 1 - \frac{n(\bar{E})}{n(S)} = 1 - \frac{6}{25} = 19/25$

**Q.7** :-

Box: (4) red (3) green    Box: (3) blue (4) green

$n(S) = {}^7C_1 \cdot {}^7C_1 = 49$

E = {one ball red & other blue}
E = {R, Blue)}

$n(E) = {}^4C_1 \cdot {}^3C_1 = 12$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{12}{49}$

**Q.8** :

4 - R
6 - B

~~n(S) = 10C3 n(S) = 10 × 9 × 8~~

let E = one red & 2 black drawn one after other without Replacement

E = { (R, B, B) (B, R, B) (B, B, R) }

$P(E) = P(R,B,B) + P(B,R,B) + P(B,B,R)$

$= P(R) \cdot P(B) \cdot P(B) + P(B) \cdot P(R) \cdot P(B) + P(B) \cdot P(B) \cdot P(R)$

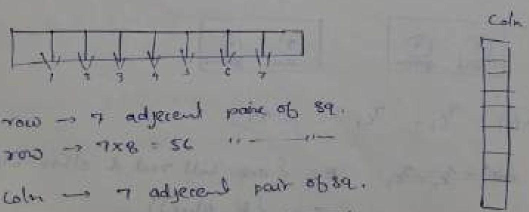
$= \frac{4}{10} \times \frac{6}{9} \times \frac{5}{8} + \frac{6}{10} \times \frac{4}{9} \times \frac{5}{8} + \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8}$

$= \frac{1}{2}$

**Q.9** Exp: drawing '2' squares from chess board.

$\therefore n(S) = {}^{64}C_2$

E = Selected '2' square to be adjecent (common side)



1 row → 7 adjecent paire of sq.
8 row → 7×8 = 56 " — "
1 colm → 7 adjecent pair of sq.
8 colm → 7×8 = 56 — " —

$\therefore n(E) = 56 + 56 = 112$

$\therefore P(E) = 112/{}^{64}C_2 = 1/18$

Q.10 Exp : arrangement of letters of word 'Probability'

Solve

$\therefore \boxed{n(S) = \frac{11!}{2!\,2!}}$ B → two times, I → two times.

E = arrangments in which, 2B's, 2I's together.

Consider 2B's one letter & 2I's one letter

$\therefore n(E) = \frac{9!}{1!}$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{9!}{11!} \times 2! \times 2! = \frac{2}{55}$

Q.11 Exp :- Arrangment of 'n' person at round table

$\therefore n(S) = (n-1)!$ ways.

let E = two specified Person don't sit together.

$\Rightarrow \bar{E}$ = two specified person sit together

$n(\bar{E}) = (n-2)! \times 2!$ → ($\because$ two specified consider as one person

$\therefore P(E) = 1 - P(\bar{E})$

$= 1 - \cancel{(n-2)}\ \frac{2}{n-1}$

$\boxed{P(E) = \frac{n-3}{n-1}}$

Q.50 $P(\omega) = 0.5$, $P(L) = 0.5$

$P(2^{nd}$ win $3^{rd}$ match$) = P(E) = ?$

$E = \{(L,\omega,\omega)\ (\omega, L, \omega)\}$

$P(E) = P(\omega, L, \omega) + P(L, \omega, \omega)$

$= P(\omega) \cdot P(L) \cdot P(\omega) + P(L)P(\omega) \cdot P(\omega)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{2}{2^3} = \frac{1}{4}$

Q.49

| |
|---|
| 200 nuts → def nuts $= \frac{3}{4} \times 200 = 150$ |
| 125 bolts → def bolts $= \frac{1}{5} \times 125 = 25$ |

Exp : drawing an item from bin

$\therefore n(S) = {}^{325}C_1 = 325$

let $E = n \cup b$ = nut or deffective

$\therefore P(E) = \frac{200}{325} + \frac{175}{325} - \frac{150}{325}$

$= \frac{225}{325} = \frac{9}{13}$

(or)

$P(E) = 1 - P(\text{non def bolt})$

$= 1 - \frac{100}{325} = \frac{225}{325} = \frac{9}{13}$

Q.45 x & y independent

$P(x \cap y) = P(x) \cdot P(y)$

$\Rightarrow \bar{x}, \bar{y}$ are also independent

$P(x) = .40$ $P(\bar{x}) = .60$

$P(x \cup y^c) = 0.7$

WKT :- $P(x \cup y^c) = P(x) + P(\bar{y}) - P(x \cap \bar{y})$

$0.7 = .40 + P(\bar{y}) - P(x) \cdot P(\bar{y})$

$.3 = P(\bar{y})[1 - 0.4]$

$P(\bar{y}) = .5$

$\therefore P(y) = 0.5$

$P(x \cup y) = \overset{P(x)}{.4} + \overset{P(y)}{.5} - \overset{P(x) \times P(y)}{.5 \times .4}$

$= .9 - .2$

$= .7$ Ans.

Q.18

| | |
|---|---|
| H | $\frac{1}{2}$ |
| T T H | $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ |
| T T T T H | $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ |

E = The required no. of Toses to get $1^{st}$ Head odd

$E = \{1, 3, 5, 7, \ldots\}$

E =

$\Rightarrow P(E) = \frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^5} \ldots \Rightarrow \frac{1}{2}\left[1 + \frac{1}{2^2} + \frac{1}{2^4} + \ldots \infty\right]$

$a = 1$

$r = \frac{1}{4}$

$\boxed{P(E) = \frac{1 - \frac{1}{4}}{1 - \frac{1}{2^2}} = \frac{3/4}{1/2} = \frac{3}{2}}$

$S_\infty = \frac{a}{1-r} = \frac{1}{1 - \frac{1}{2^2}} = \frac{4}{3}$

$P(E) = \frac{1}{2} S_\infty = \frac{2}{3}$

Q.19

$E = \{A, A_L B_L A_\omega, A_L B_L A_L B_L B_\omega \ldots\}$

$P(A_\omega) = \frac{1}{6}$, $P(A_L) = \frac{5}{6}$

$P(B_L) = \frac{5}{6}$

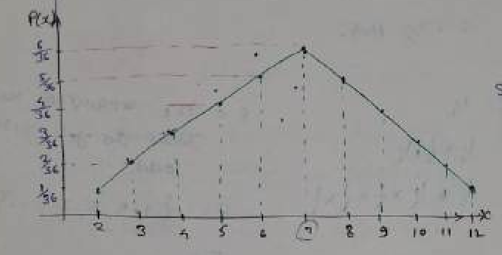
$P(E) = P(A_\omega) + P(A_L) \cdot P(B_L) \cdot P(A_\omega) + \ldots$

$= \frac{1}{6} + \left(\frac{5}{6}\right)^2 \frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{1}{6} + \ldots \infty$

$= \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \ldots \infty\right]$

$= \frac{1}{6}\left[\frac{1}{1 - \left(\frac{5}{6}\right)^2}\right] = \frac{6}{11}$ Ans.

Exp : Throwing a dice '2' times $\Rightarrow n(S) = 6 \times 6 = 36$




Symmetrical distribution at $x = 7$

⇓

Mean = mode = median = 7

mean $= \frac{\Sigma x_i}{n} = \frac{77}{11} = 7$

mode $\cdot x = 7$

median $= \left(\frac{n+1}{2}\right)^{th}$ term $= 7$ ⇒ from ⑦

$\boxed{\text{mode} = 3\,\text{median} - 2\,\text{mean}}$ → skew distribution

Note :- If the no of observations are even then after arranging The observation in increasing or decreasing order median = airthmatic mean of $\frac{n}{2}^{th}$ & $\left(\frac{n}{2}+1\right)^{th}$ term.

Q.1=1

$P(\text{sum} > 8) = P(9) + P(10) + P(11) + P(12)$

$= \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36}$

$= \frac{10}{36}$

Q.(13) ⇒

$$P(4) + P(6) + P(8) + P(12)$$

$$= \frac{3}{36} + \frac{5}{36} + \frac{5}{36} + \frac{1}{36} = \frac{14}{36}$$

Q.19

$$P(\overline{8 \cup 9}) = 1 - P(8 \cup 9)$$

$$= 1 - [P(8) + P(9)]$$

$$= 1 - \left[\frac{5}{36} + \frac{4}{36}\right] = \frac{27}{36} = \frac{3}{4}$$

Q.20

$$E = \{ S, (\overline{S \cup 7}, S), (\overline{S \cup 7}, \overline{S \cup 7}, S), \text{---} \}$$

$$P(S) = \frac{4}{36}, \quad P(\overline{S \cup 7}) = 1 - [P(S) + P(7)]$$

$$= 1 - \left[\frac{4}{36} + \frac{6}{36}\right] = \frac{26}{36}$$

$$P(E) = \frac{4}{36} + \frac{26}{36} \times \frac{4}{36} + \left(\frac{26}{36}\right)^2 \frac{4}{36} + \text{---}$$

$$= \frac{4}{36}\left[1 + \frac{26}{36} + \left(\frac{26}{36}\right)^2 + \text{--}\right]$$

$$\left\{ S_\infty = \frac{a}{1-r} \right.$$

$$= \frac{4}{36} \times S_\infty = \frac{2}{5}$$

Conditional Probability :

A, B are two independent events happening sequentially

If P(A) is given, prob. of 'B' is denoted by $P(B/A)$ called Conditional prob of 'B' when 'A' happened

If P(B) is given, Prob of 'A' is called condn prob of A when 'B' happened denoted by $P(A/B)$

Multiplication Theorem :

$$P(A \cap B) = P(A) \cdot P(B/A) = P(B) \cdot P(A/B)$$

$$\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)} \qquad P(B/A) = \frac{P(A \cap B)}{P(A)}}$$

Q.15

$$P\left(\frac{\text{sum}=7}{I_d > II_d}\right) : \frac{A}{B} = \frac{P(A \cap B)}{P(B)} = \frac{P(\text{Sum}=7 \ \& \ I_d > II_d)}{P(I_d > II_d)}$$

$$= \frac{\frac{3}{36}}{\frac{15}{36}} = \frac{3}{15} = \frac{1}{5}$$

Q.16

$$P\left(\frac{I^{st}}{II^{nd}}\right) = 0.6$$

~~P(A)~~ $P(I^{st}) = 0.3$

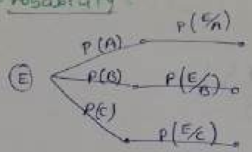
$P(II^{nd}) = 0.2$

$$P(B) \times P\left(\frac{A}{B}\right)$$

$$P(I^{st} \cap II^{nd}) = 0.2 \times 0.6$$

$$= 0.12$$

Total probability :




→ If E is an event happens in mutually exclusive and exhaustive events of exp.

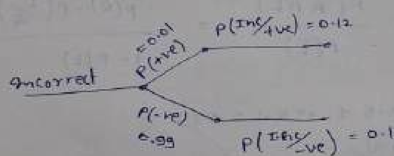
$$P(E) = P(A) \cdot P(E/A) + P(B) \cdot P(E/B) + P(C) \cdot P(E/C)$$

Condn Prob $P(A/E) = \frac{P(A \cap E)}{P(E)} = \frac{P(A) \cdot P(E/A)}{P(E)}$

$$P(B/E) = \frac{P(B \cap E)}{P(E)} = \frac{P(B) \cdot P(E/B)}{P(E)}$$

$$P(C/E) = \frac{P(C \cap E)}{P(E)} = \frac{P(C) \cdot P(E/B)}{P(E)}$$

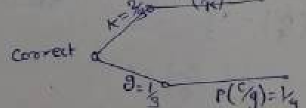
Q.23




$$P(\text{Incorrect}) = (0.01 \times 0.12) + (0.99 \times 0.15) = 0.1497$$

Q.24 :-

$$P\left(\frac{\text{Know}}{\text{Correct}}\right) = ? = P\left(\frac{K}{C}\right)$$



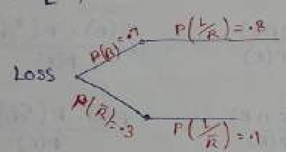

$$P(K/C) = \frac{P(K \cap C)}{P(C)}$$

$$= \frac{P(K) \cdot P(C/K)}{P(C)}$$

$$= \frac{2/3 \cdot \times 1}{\left(\frac{1}{3} \times \frac{1}{4}\right) + \left(\frac{2}{3} \times 1\right)} = \frac{2/3}{\frac{1}{12} + \frac{2}{3}} = \frac{8}{9}$$

Q.25

$$P\left(\frac{\bar{R}}{\bar{L}}\right) = ?$$




$$P\left(\frac{\bar{R}}{\bar{L}}\right) = \frac{P(\bar{R} \cap \bar{L})}{P(\bar{L})} = \frac{P(\bar{R}) \cdot P(\bar{L}/\bar{R})}{1 - P(L)}$$

$$P(L) = .7 \times .8 + .3 \times .1 = 0.59$$

$$P(\bar{L}) = 0.41$$

$$P(L/\bar{R}) = .1$$

$$P(\bar{L}/\bar{R}) = .9 \quad \rightarrow \quad P\left(\frac{\bar{R}}{\bar{L}}\right) = \frac{.3 \times (.9)}{.41}$$

$$= \frac{27}{41}$$

Q.47

$$P(P) = 1/4 \qquad P(P/Q) = \frac{1}{2}$$

$$P(Q/P) = 1/3 \qquad P(\bar{P}/\bar{Q}) = ?$$

$$P\left(\frac{\bar{P}}{Q}\right) = \frac{P(\bar{P} \cap \bar{Q})}{P(Q)}$$

Compliment law $\quad \overline{A \cup B} = \bar{A} \cap \bar{B}$

$$P\left(\frac{\bar{P}}{\bar{Q}}\right) = \frac{P(\bar{P} \cap \bar{Q})}{P(\bar{Q})} = \frac{P(\overline{P \cup Q})}{P(\bar{Q})}$$

$$= \frac{1 - [P(P \cup Q)]}{1 - P(Q)}$$

by $x^n$ theorm.

$$P(P \cap Q) = P(P) \cdot P(Q/P) = P(Q)\, P(P/Q)$$

$$\frac{1}{4} \cdot \frac{1}{3} = P(Q) \cdot \frac{1}{2}$$

$$\boxed{P(Q) = \frac{1}{6}}$$

$$\therefore P(P \cap Q) = P(P) \cdot P(Q/P) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

So

$$= \frac{1 - [P(P) + P(Q) - P(P \cap Q)]}{1 - P(Q)}$$

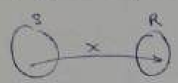
Q The probality that a given positive integer lying b/w 1 and 100 is not divisible by 2, 3 or 5

Solve $\quad$ 1 - divisible (2, 3, or 5)

Q:> 4 dice are Rolled the probability that sum of the result being 22 is $\frac{X}{1296}$ then the value of $X$ = ____

## * Random variable *

$X : S \longrightarrow R$



Ex: Tossing a coin '2' time

$S = \{(H,H)\ (H,T)\ (T,H)\ (T,T)\}$

Defining $X$ = no. of Heads

$X(H,H) = 2$ $\quad P(X=2) = \frac{1}{4}$

$X(H,T) = 1$ $\quad P(X=0) = \frac{1}{4}$

$X(T,H) = 1$ $\quad \Sigma P_i = 1$

$X(T,T) = 0$

Prob. distribution

D.R.V : $X \longrightarrow$ +ve int

1) Prob mass fn

$f(x_k) = P(X = x_k)$

② $\sum_{i=0}^{n} P(X = x_i) = 1$

③ cumulative dist. fn :

$F_X(x_k) = P(X \le x_k) = \sum_{i=0}^{k} P(X = x_i)$

④ Expectation 'x' = E(x) = mean of distribution

$$\boxed{= \sum_{i=0}^{n} x_i\, P(X = x_i)}$$

C.R.V : $X \longrightarrow \infty$ set of values

1) Prob. density fn

$$f(x) = \begin{cases} f_1(x) & x < a \\ f_2(x) & x \ge a \end{cases}$$

② $\int_{-\infty}^{\infty} f(x)\,dx = 1$

③ cdf : $F(x_k) = P(X \le x_k) = \int_{-\infty}^{x_k} f(x)\,dx$

$$\boxed{P(a \le x \le b) = \int_a^b f(x)\,dx}$$

Random variable X are of two types

① Discreate R.V. (X takes positive integer values)

② Continuous R.V. (X takes set of $\infty$ values)

Variance :-

$$Var(x) = E[x - \bar{x}]^2$$
$$= E[x^2] - E[x]^2$$

S.D $\quad \sigma_x = \sqrt{Var(x)} > 0$

Properties :

① $E[ax + b]$
$= aE(x) + E(b)$
$= aE(x) + b$

② $Var[Const] = 0$

③ $Var[ax + b] = a^2\, Var(x) + 0$

④ $\sigma(ax + b) = a\sqrt{Var} = a\,\sigma_x$

Q.27 : $f(x) = e^{-x}$, $x > 0$ find ① $P(x > 1)$ ② $P(|x-2| < 1)$

$P(x > 1) = 1 - P(x < 1)$

$$P(x > 1) = \int_1^{\infty} f(x)\,dx = \int_1^{\infty} e^{-x}\,dx = \left.\frac{e^{-x}}{-1}\right]_1^{\infty} = \frac{1}{e} \text{ Ans.}$$

② given $|x-2| < 1$

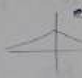
$\Rightarrow -1 < x - 2 < 1$

$1 < x < 3$

$$\therefore P(|x-2| < 1) = P(1 < x < 3) = \int_1^3 e^{-x}\,dx = \frac{1}{e} - \frac{1}{e^3} \text{ Ans.}$$

Q :) If $f(x) = k|x|e^{-|x|}$ $-\infty < x < \infty$ is a valid prob density function then find ① $k$. ② $E(x)$ ③ $E(x^2)$ ④ $Var(x)$ ⑤ $\sigma_x$ ⑥ $E[2x+3]$ ⑦ $\sigma(2x+3)$

Solve valid pdf $\Rightarrow \int_{-\infty}^{\infty} f(x)dx = 1$



even fn

$\cancel{x<0}$ $\cancel{x>0}$

$\cancel{-kxe^{x}}$ $kxe^{-x}$

$\because f(x)$ is valid

$1 = 2\int_0^{\infty} kxe^{-x} dx$

gama fn.

$= 2k\int x \cdot e^{-x} dx = 1 \Rightarrow 2k\Gamma 2 = 1$

$k = 1/2$

$\therefore$ Pdf : $f(x) = \frac{1}{2}|x|e^{-|x|}$ , $x \in (-\infty, \infty)$

② $E[x] = \int_{-\infty}^{\infty} x f(x)dx = \int_{-\infty}^{\infty} x\left[\frac{1}{2}|x|e^{-|x|}\right] dx = 0$

③ $E[x^2] = \int_{-\infty}^{\infty} x^2 f(x)dx = \int_{-\infty}^{\infty} x^2 \frac{1}{2}|x|e^{-|x|} dx$

$= 2 \times \frac{1}{2} \times \int_0^{\infty} e^{-x} x^{4-1} dx$

$= \Gamma 4 = 3! = 6$

④ $Var(x) = E(x^2) - (E(x))^2$

$= 6 - 0 = 6$

⑤ $\sigma_x = \sqrt{Var(x)} = \sqrt{6}$

⑥ $E(2x+3) = 2E(x) + 3$

$= 2(0) + 3 = 3$

⑦ $\sigma(2x+3)$

$= \sqrt{Var(2x+3)}$

$= \sqrt{2^2 \, Var(x) + 0} = 2 \times \sqrt{6}$

Q.29

| x | 1 | 2 | 3 |
|---|---|---|---|
| P(x) | $\frac{2+5P}{5}$ | $\frac{1+3P}{5}$ | $\frac{1.5+2P}{5}$ |

find ① P = ? ② $E(x)$ ③ $E(x^2)$ ④ $Var(x)$ ⑤ $\sigma_x$ ⑥ $E(2x+3)$ ⑦ $Var[2x+3]$ ⑧ mode ?

Solve

① ~~P(x)~~ WKT $\sum P_i = 1$

$\frac{2+5P}{5} + \frac{1+3P}{5} + \frac{1.5+2P}{5} = 1$

$10P + 4.5 = 5$

$P = 0.05$

$E(x) = \sum_{i=1}^{3} x_i P_i$

$= 0.45 \times 1 + 0.23 \times 2 + 3 \times .32 = 1.87$

$E(x^2) = \sum x^2 P_i = 1 \times .45 + (.23)^2 \times (4) + 9 \times (.32)$

$= 4.25$

④ $Var(x) = 4.25 - (1.87)^2$

$= 0.7531$

⑤ $\sigma_x = \sqrt{Var(x)} = .867$

⑥ $E(2x+3) = 2E(x) + 3$

$= 6.73$

⑦ $Var[2x+3] = 2^2 \times 0.7531 =$

⑧ Mode : The value of 'x' where $P(x)$ is max

$\therefore \boxed{x = 1 = \text{mode}}$

$\boxed{\text{Median} = \frac{\text{Mode} + 2\,\text{mean}}{3}}$

Q.54

$E(x) = E(L) = \sum x_i P_i$

x = length of the word. = 3, 4, 5

| x | 3 | 4 | 5 |
|---|---|---|---|
| P(x) | 4/9 | 2/9 | 3/9 |

$E(x) = 3 \times \frac{4}{9} + 4 \times \frac{2}{9} + 5 \times \frac{3}{9} = 3.88$

Q :) A ~~fair~~ coin tass till a first Head appears. then find the Expectation of the no. of tosses Required ? where P = the probability of Head

(a) P (b) $P \times (1-P)$

(c) $1/P$ (d) $1/P \times (P-1)$

$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots$

$\frac{d}{dx}(1-x)^{-1} = \frac{d}{dx}(1 + x + x^2 + x^3 + \cdots)$

$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots$

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| H | TH | TTH | TTTH --- |
| P | P(1-P) | $(1-P)^2 P$ | $(1-P)^3 P$ --- |

Refer Q.18

$E(x) = 1 \cdot P + 2 \cdot P(1-P) + 3 \cdot P(1-P)^2 + 4 \cdot P(1-P)^3 + \cdots$

$= P(1 + 2(1-P) + 3(1-P)^2 + 4(1-P)^3 + \cdots$

$= P(1 - (1-P))^{-2}$

$= 1/P$ // Ans.

Probability Distribution :

$\boxed{\begin{array}{l}\text{Mean} = np \\ \text{variance} = npq\end{array}}$

① Binomial Distribution :

$B(n, p)$

n = no. of times experiment repeat.

P = Prob of success

q = 1 - P = Prob of failure

Probability of 'r' success in 'n' trials is given by $\boxed{P(x=r) = {}^nC_r \, p^r q^{n-r}}$

## Poision's Distribution :

$P(n, \lambda)$ , $n \longrightarrow$ large

$\lambda$ : Avg = mean = varience = $np$

$P$ = prob of succes

Probability of 'r' success

$$\boxed{P(x = r) = \frac{e^{-\lambda} \lambda^r}{\lfloor r}}$$

**Q.31**

$E = n(H) > n(T)$ in Tossin '4' times, $n = 4$

$P(E) = P(4H) + P(3H)$

Define $x$ = no. of succes = no of Heads

$$P(E) = P(x=4) + P(x=3)$$

$$= {}^4C_4 \left(\tfrac{1}{2}\right)^4 \left(\tfrac{1}{2}\right)^0 + {}^4C_3 \times \left(\tfrac{1}{2}\right)^3 \times \left(\tfrac{1}{2}\right)$$

$$= 1 \times \left(\tfrac{1}{2}\right)^4 + 4 \times \left(\tfrac{1}{2}\right)^4$$

$$= 5 \times \frac{1}{16} = 5/16$$

(or) $P(HHHH) + 4(P(HHHT))$

$$= \frac{1}{16} + \frac{4}{16} = \frac{5}{16}$$

**Q.32**

$P(4^{th}$ H in $10^{th}$ toss$) = P(3H$ in 9 tosses$) \times P(H)$

$$= {}^9C_3 \left(\tfrac{1}{2}\right)^3 \left(\tfrac{1}{2}\right)^6 \times \frac{1}{2}$$

$$= \frac{9 \times 8 \times 7}{3 \times 2} \times \left(\tfrac{1}{2}\right)^9 \times \frac{1}{2}$$

**Q.83**

$E$ = getting sum = 12 in '5' throws

$E$ = 4 time '3' occurs in '5' throws

Define $x$ = no. of times '3' appear.

$P(E) = P(\text{sum} = 12) = P(x=4)$

$$= {}^5C_4 \left(\tfrac{2}{6}\right)^4 \left(\tfrac{4}{6}\right)^1$$

$$= 5 \times \left(\tfrac{1}{3}\right)^4 \times \frac{2}{3}$$

**Q.37 :**

Avg no. of events in a year = 3

Define $x$ = no. of events happen in '2' yr duration

The $\lambda = 2 \times 3 = 6$ per '2' year duration

$$P(x \le 2) = P(x=0) + P(x=1) + P(x=2)$$

$$= \frac{e^{-\lambda}\lambda^0}{\lfloor 0} + \frac{e^{-\lambda}\lambda^1}{\lfloor 1} + \frac{e^{-\lambda}\lambda^2}{\lfloor 2}$$

$$= e^{-6}\left[1 + 6 + \frac{36}{2}\right] = 25e^{-6} = 0.062$$

**Q.38**

$P(\text{def condensor}) = \frac{1}{100} = P$

Define $x$ = no. of faulty condensor per box

$\therefore \lambda = np = 100 \times \frac{1}{100} = 1$ per box

$$P(x \ge 3) = 1 - P(x < 3)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$

$$= 1 - \left[\frac{e^{-\lambda}\lambda^0}{\lfloor 0} + \frac{e^{-\lambda}\lambda^1}{\lfloor 1} + \frac{e^{-\lambda}\lambda^2}{\lfloor 2}\right]$$

$$= 1 - e^{-1}\left[1 + 1 + \tfrac{1}{2}\right]$$

$$= 1 - \tfrac{5}{2}e^{-1}$$

**Q.**

If 'x' follows poisson Dist. with 2nd ~~movement~~ moment '2' then find $var(x) = ?$

**Sol**

given $E(x^2) = 2$, $V(x) = ?$

& P.D $\Rightarrow E(x) = Var(x) = \lambda$

WKT $Var(x) = E[x^2] - [E(x)]^2$

$$\boxed{\begin{array}{l} E[X] \longrightarrow 1^{st} \text{ moment} \\ E[x^2] \longrightarrow \text{II}^{nd} \text{ moment} \end{array}}$$

$\Rightarrow \lambda = 2 - \lambda^2$

$\lambda^2 + \lambda - 2 = 0$

$\lambda = 1$ , $\cancel{-2}$ = var (x) always positive

## Normal Distribution : $N(\mu, \sigma^2)$

$n \rightarrow$ large $\rightarrow \infty$

$x \longrightarrow$ CRV , with given $\mu, \sigma$

**Prob density fn :**

$$f(x_k) = P(x = x_k) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\left(\frac{x_k - \mu}{\sigma}\right)^2 / 2}$$

**Cumulative pdf :**

$$F_x(x_k) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{x_k} e^{-\left(\frac{x-\mu}{\sigma}\right)^2/2}$$

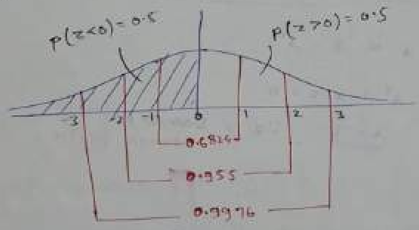
let $z = \frac{x - \mu}{\sigma}$ (standered normal variable)

p.d.f : $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$

and $\int_{-\infty}^{\infty} \phi(z)\,dz = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{(-z^2/2)}\,dz = 1$

c.d.f $F_z(z_k) = \int_{-\infty}^{z_k} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\,dz$

Standered




$$P(\mu - \sigma \le x \le \mu + \sigma) = P(-1 \le z \le 1) = 0.6826$$

$$P(\mu - 2\sigma \le x \le \mu + 2\sigma) = P(-2 \le z \le 2) = 0.955$$

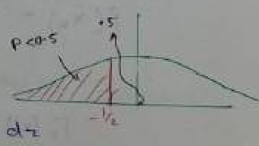
$$P(\mu - 3\sigma \le x \le \mu + 3\sigma) = P(-3 \le z \le 3) = 0.9976$$

**Q.48 :**

$z = \frac{x - \mu}{\sigma}$

$z = \frac{0 - 1}{2}$

$z = -\frac{1}{2}$




$$P(z < -\tfrac{1}{2}) = \int_{-\infty}^{-1/2} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\,dz$$

Ans (b)

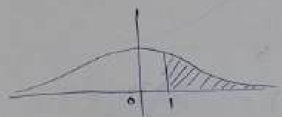
~~ref~~ $z < -\frac{1}{2}$

* whenever find the cumulative probability conversion of function is take place
eg: in above Normal distribution convert in z term game fun

Q.44 let $x$ = annual precipitation data of city with $\mu = 1000$, $\sigma = 200$

$P(x > 1200) = ?$

for $x = 1200 \Rightarrow z = \frac{1200 - 1000}{200} = 1$




$P(z > 1) = P(z > 0) - P(0 < z < 1)$

$= 0.5 - \frac{0.6824}{2}$

$< 50\%$

Q.42 $\mu = 100$

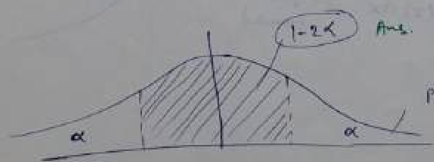
$P(x \geq 110) = \alpha$

$P(90 \leq x \leq 110) = ?$

$x = 90 \Rightarrow z = \frac{90 - 100}{\sigma} = \frac{-10}{\sigma}$

$x = 110 \Rightarrow z = \frac{110 - 100}{\sigma} = \frac{10}{\sigma}$

$\therefore P(90 \leq x \leq 110) = P\left(\frac{-10}{\sigma} \leq z \leq \frac{10}{\sigma}\right)$

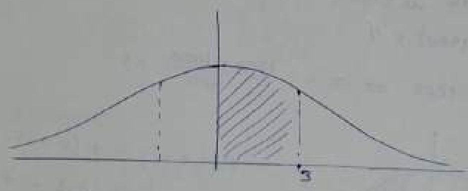



$P(x \geq 110) = P(z \geq \frac{10}{\sigma}) = \alpha$

Q ⇒ 'x' is N RV with $\mu = 30$, $\sigma = 5$ then find ① $P(30 \leq x \leq 45) = ?$ ② $P(x \geq 45) = ?$

$x = 30 \Rightarrow z = 0$

$x = 45 \Rightarrow z = 3$




① $P(30 \leq x \leq 45) = \frac{.9976}{2} =$

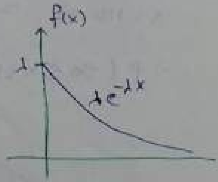
② $P(x \geq 45) = 0.5 - \frac{0.9976}{2} =$

## Exponential distribution :- $X \to$ CRV

p.d.f. $f(x) = \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$

mean $= \frac{1}{\lambda}$ $\quad$ $var(x) = \frac{1}{\lambda^2}$

$E(x^r) = \int_0^\infty x^r f(x)\, dx \to$ rth moment



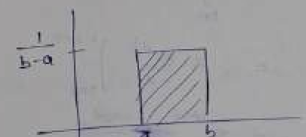

## Uniform Distribution : $x \to [a, b]$

p.d.f. $f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{else} \end{cases}$

mean $= E[X] = \frac{a+b}{2}$

varience $= V[x] = \frac{(b-a)^2}{12}$

rth moment $= E[x^r] = \int_a^b x^r f(x)\, dx$




Q.39 $x \to [2, 10] \Rightarrow f(x) = \frac{1}{(10-2)}$

mean $= E[x] = \frac{10+2}{2} = 6$

var $= V(x) = \frac{(b-a)^2}{12} = \frac{(10-2)^2}{12} = \frac{64}{12}$

$E[x^3] = \int_2^{10} x^3 f(x)\, dx$

$= \int_2^{10} x^3 \cdot \frac{1}{8} dx = \frac{1}{8} \frac{x^4}{4} \Big]_2^{10}$

Q.40 $f(x) = \frac{1}{5} e^{-x/5}$, $x \geq 0$

$x$ = duration in minutes of telephonic conversion

$\therefore P(x > 5) = \int_5^\infty f(x)\, dx$

$= \int_5^\infty \frac{1}{5} e^{-x/5} dx$

$= \frac{1}{5} \frac{e^{-x/5}}{-1/5} \Big]_5^\infty = \frac{1}{e}$

Q.41 $f(t) = \alpha e^{-\alpha t}$, $t \geq 0$

$t$ = life of bulb

$P(100 \leq t \leq 200) = \int_{100}^{200} \alpha e^{-\alpha t} dt$

$= \alpha \frac{e^{-\alpha t}}{-\alpha} \Big]_{100}^{200}$

$= e^{-100\alpha} - e^{-200\alpha}$

## LU - Decomposition :

$A_{n \times n} = L \cdot U$

$A_{22} = L_{2\times2} \cdot U_{2\times2}$

$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{bmatrix}$

where either $l_{11} = l_{22} = 1$

(or) $u_{11} = u_{22} = 1$