## 1. Linear algebra:

linear algebra deals with linear systems linear transformation and their applications

every LT & LS can be expressed in terms of madiress

me no ob rows n = no ob colors

If m +n > Arxn is ractangular matrix BE m= N => Anxn is so madrie

Colomn Vector A = [aij]

raw vector: A= [ai] -xn

if 
$$\alpha_{ij} = 0$$
 for  $i \neq j$   
 $\alpha_{ij} = K$   $\forall i$   
 $S = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 

Froduct of two matrix:

D AB & BA (Hot commutative) 2) A(BC) = (AB) C (Associative)

1) the no ob x13 occasived to compute (A-B)mag = mpq

2) the no. of +" " " (A-B) = m(P-1)2

let Para, Boxy, Revy be 3 matrices find the Minimum no of X" Required to compute POR?

Soln --4×2×4 + 4×4×1

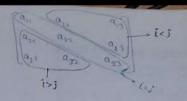
we know that p(g, R) = (PP)R

consider: P(9R)

P4x2 (QR) = No of x = 4x2x1 = 8

Consider: (PQ) R

P422 P4x4 -7 NO 00 X" = 4x2x4=31 No of X = 4x4x1= (PR) KAKI



lower triangular metrix :-

if dos for (4)

## upper triangular matrix :-

$$V = \begin{bmatrix} u y \end{bmatrix}_{m \times n} \quad D \quad U.T. M$$

$$if \quad V_{i,j} = 0 \quad \text{for} \quad (>j) \quad V = \begin{bmatrix} -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

Diagonal matrix :

Scaller matrix :-

the minimum no ob multiplication require find POR = 16

maximum of no of multiplication required - 40 . The minimum one of addition required in finding paper

Transpose of matrix :

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 & 3 \\ 4 & 2 & 3 \end{bmatrix} \qquad 5 \quad A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

Peoperties : (AT) = 1

Symmetric matrics -

$$A = [a_{1j}]_{n \times n}$$
 is sym. matrix iff  $[A^T = A]$  (or)  $[(a_{1j}) = a_{\overline{j}\overline{i}}]$ 

Properties: let AB are sym. matrics (:e AT = A; B = B

Then 
$$\bigcirc$$
 A<sup>n</sup> is sym matrix

(a) & A, A + A, B (4 sym)

(b) AB + BA (3 sym)

(c) AB + BA (3 sym)

(d) AB + BA (3 sym)

(d) AB + BA (3 sym)

(e) AB + BA (3 sym)

(f) BA - AB

(f) BA - AB

(f) AB + BA (3 sym)

(f) BA - AB

(f) AB + BA (3 sym)

(f) BA - AB

(f) BA -

$$(AB + BA)^{T} = (AB)^{T} + (BA)^{T}$$

$$= BA + AB$$

$$= BA + AB$$

$$\Rightarrow (AB)^{T} = B^{T} B^{T} = B B$$

$$A = [a_{ij}]_{n \times n}$$
 is skew sm matric iff  $A^{T} = -A$  (or)  $[a_{ij} = -a_{j}i]$  and  $[a_{i\ell} = 0]$ 

Properties: let 
$$A$$
,  $B$  are skew sym matrix  
 $C = A^T = -A$ ,  $B^T = -B$ 

Brew Hermition :-

$$A = \left[ \overline{a_{ij}} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$\left[ \overline{A} = -A^T \right]$$

$$(a_{ij} = a_{ij} = a_{ji} \text{ in againary}$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{ is $k$ thm. If }$$

$$A = \left[ \overline{A}_{ij} \right]_{n \times m} \text{$$

The Properties are some at steam symptoni

Note: every square matrix can be expressed as sum of symmetric and skyw symmetric matrixes

$$A_{n\times n} = \frac{1}{2} \left[ A + A^{T} \right] + \frac{1}{2} \left[ A - A^{T} \right]$$

$$\downarrow S_{VM} \qquad \qquad \downarrow S_{KM} - S_{NM} + \cdots$$

$$A = \frac{1}{2} (2A)$$

$$= \frac{1}{2} [A + A]$$

$$= \frac{1}{2} [(A + A^T) + (A - A^T)]$$

$$= \frac{1}{2} [A + A^T] + \frac{1}{2} [A - A^T]$$

AB' is show symmetric of AB+BA = null madric 
$$(A^{m})^{T} = (A^{T})^{m} = (-1A)^{m} = (-1)^{m}A^{T}$$

$$(A^{m})^{T} = B^{T}A^{T} = BA = AB = a$$

$$A = a = a$$

domplex matries or

Conjugate of matrix:

$$\mathcal{E}_{X}$$
: A-  $\begin{bmatrix} \hat{t} & \hat{t}+\hat{t} \\ 2 & 2\hat{t} \end{bmatrix}$ 

$$\overline{A} = \begin{bmatrix} -1 & 1-1 \\ 2 & -2^{\circ} \end{bmatrix}$$

Hermitian matrix ?-

(A) = A = AT | air = yeal

$$(a)^T = A$$
  $a_{ii} = veal$   $(a)^T = A$   $b = a_{ij} = a_{ji}$ 

It is them as then "In" is show the oud

orthogonal matrix :

Then 
$$A = I = A^T \cdot A$$

(oR)  $A^T = A^T$ 

$$M^{-1} = M^{T}$$
  $\Rightarrow$   $M - M^{T} = I$ 

$$\begin{bmatrix} 3_5 & 4_5 \\ x & 8_5 \end{bmatrix} \begin{bmatrix} 4_5 & x \\ 4_5 & 3_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\frac{3}{5}x + 4_5 + \frac{3}{5} = 0$$

$$\Rightarrow x = -4_5$$

properties: - 36 AP A.S are orthogonal/ unitary then ( 10.89 , 1' B' n' B' are orthogon uniday valires.

Unitary matrix : A·ft = I = A.A Them 'A' is unitrary (or) A-1 = A. Idempotent matrix :-96 A2=A. Properties: let A,B are idempotent matin ie:  $A^2 = A$   $B^2 = B$  then @ A+B is idenpotent if and only if AB+BA (A+B) = A2+B2 + AB+BA = (A+B) (2) AB is idempotent if and only if ABAB=1 3 A" = A , 3" = B Note: O of AB=P. ABA=A then A=A, B=B (1) 98 AB : A & BA = B , then A2 = A , B2 = P  $\{A^2 - A \cdot A = A \cdot (BA) = (AB)A - BA \cdot = A \}$ Q=0 xy = y & yx = x then x2+y2 = x+y x5+45= x+4 27+4n = x+4

x8 = x5. x3

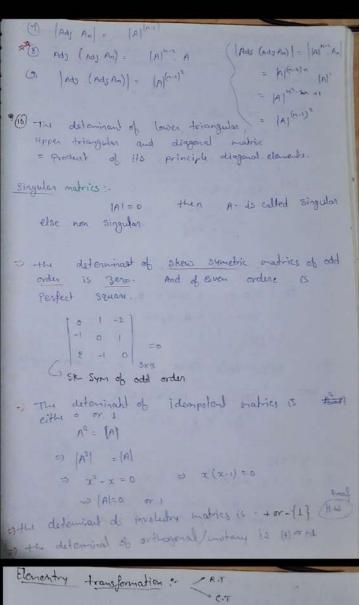
: | X8 = X3 |

Go A, B are of Same Sizer and AB= B & DA=A then (AM) == ? a) 25 c) 26(A+B) d) 25 (A+B) given AB=B & BA=A = A=H  $(\overline{n}+\overline{p})^2 = \overline{n^2} + \overline{p^2} + \overline{n}\overline{p} + \overline{p}\overline{n}$ = A+B + A+B = e[A+A] - 0 [A+B] = 2[A+B] = 2[2(A+B)] = 2[A+B] [A+B] = 2 [A+B]2 = 2º [0[n+B]] = 20 [A+B] X14 by (A+P) on bis (A+B) = 23 (A+B)2 (A+B)5 = 29 (8 (A+B)) Involutory matrix: 96 A2 = I then A is involutely making Aant = A where is = cube the roc 9-19 :of with i.e. w=169 Determinant of matrix .

Detainistion:

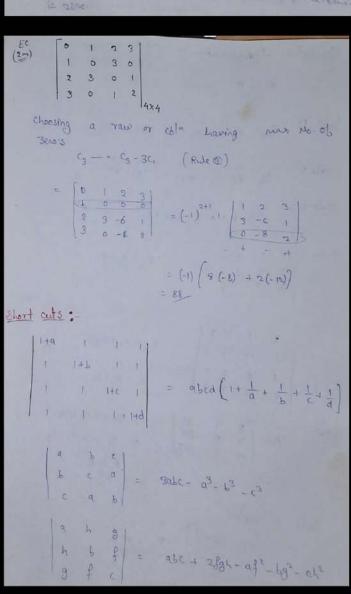
Detainition:  $\begin{cases}
A_{n\times n} = \sum_{j=1}^{n} \{-1\}^{j+1} & a_{ij} \in S_{ij} \\
A_{n\times n} = \sum_{j=1}^{n} \{-1\}^{j+1} & a_{ij} \in S_{ij} \\
A_{n\times n} = \sum_{j=1}^{n} \{-1\}^{j+1} & a_{ij} \in S_{ij} \\
A_{n\times n} = \sum_{j=1}^{n} \{-1\}^{j+1} & a_{ij} \in S_{ij} \\
A_{n\times n} = \sum_{j=1}^{n} \{-1\}^{j+1} & a_{ij} \in S_{ij} \\
A_{n\times n} = \sum_{j=1}^{n} \{-1\}^{j+1} & a_{ij} \in S_{ij} \\
A_{n\times n} = \sum_{j=1}^{n} A_{n\times n}$ 

E | R Amon = K" | Amon



```
Rule 1: 36 R; ( => (A) = -(B)
   Rule 2:- 31 RI - KRI - |B| = KIN]
 Rule 3: 96 R_i \longrightarrow R_i + KR_j \Rightarrow |B| = |A|
                                                                                       2 3 = 0
                                                                                                                       3 4 R3 = R1+R
                                                A = \begin{bmatrix} 9 & 4 & 45 \\ 7 & 8 & 105 \end{bmatrix}
                                                                                                                                                          45
                                                                                                                                                                                                               1 R2 -> R2+R3
                                                                                       13 2 195
                                                                                                                                                                                                                           2. C1 -> C, -C3
                                             (B)=?
               Rule (8) Applied
                                                                             |\beta| = |A| = \begin{cases} 9 & 45 \\ 13 & 2 & 95 \\ 13 & 2 & 195 \end{cases}
                                                                                                       (A)= 15 \( \frac{3}{7} & \frac{4}{8} & \frac{3}{1} \\ \frac{1}{1} & \fra
```

```
A set of vectors (Rows and colons) or . In an are said to be linearly dependent if the linear
     \alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_n x_n = 0
   for all not all ar = 0
        9, x, + a, x, + a, x, = 0
     (ep) \alpha_{1}x_{1} + d_{1}x_{2} = 0
               X= K, OX
 ex: Sin x, Gsx, tanx
      16 9, sinx, + 9, 00x + b2 tanx =0
            Then a = 01=03=0
        : LI
  Ex sin2x, cos2x, cos2x
    6082x = 1652x-15143
             fg = f1-f2
  O Ri = KR;
 @ Ri = A, Rj + Az Rx
Note: It any two rows are allows in a making
     are linearly dependent them and the determ
```



$$\frac{2m-2c}{1 + b^{2}} = (a-b)(b-c)(c-a)$$

$$\frac{2m-2c}{1 + c}$$

$$\frac{2m-2$$

(3) 
$$\begin{bmatrix} 2 & 1 & 1 & 2 \\ -1 & -1 & 1 & 1 \\ 1 & 0 & 2 & 3 \\ 0 & -1 & 3 & 4 \end{bmatrix}$$

\*\*  $R_1 + R_2 = R_3$ 
 $R_3 + R_3 = R_4$ 

\*\*  $R_1 + R_2 = R_3$ 

Results are Ind

\*\*  $R(A) = ra$  of Ind row ( $ran = 2$ )

So where  $ran = ran = ran$ 

Note (either find the Relation of either row or colongest both are colonn and row antyses & time wasting) because

A numer | 2 3-1 | = 2(-1) - 3(3)-1(3) | | 3 | 3 | 3 | 3 | 4 0 |

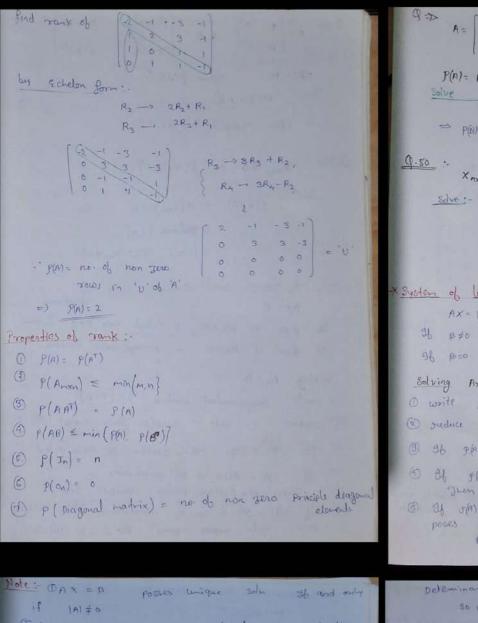
\* R. R. R. R. R. are ind (or) highest orders of non row = 2

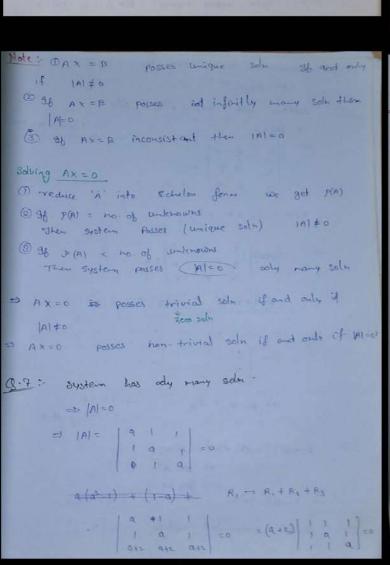
Rank of matrix: The no of linearly independent stone and colum in a matrix is called mande The highest order of none sero-inor is called Rank. let A3x3 96 | A3x3) # 0 Thengal = order 96 | A3x3 = 0 => P(n) < order They if I attend one non zero minor of order 2×2 Then 5(A) = 2 Else 9(a) = 1/ find rank & 1 2 3 1 -1 0 2 1 3 " R.+R2 = R9 & minor from 8 1 2 = -1-2 \$0 => R., R2 are ind rows => P(A) = no ob Ind orows/coln (independent) = 2

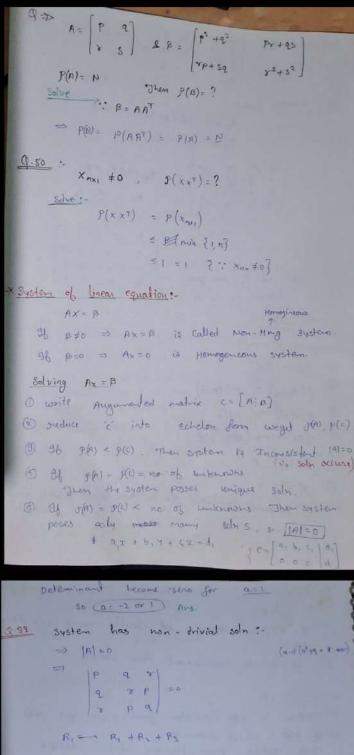
Then 
$$x = 9$$

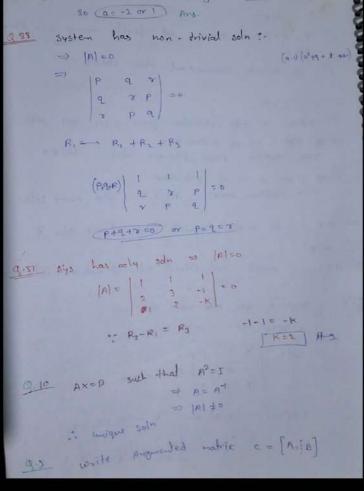
Soln: given  $p(A_{3x3}) = 2 < \text{orden}$ 

$$|A_{3x3}| = 0$$









reduce a into Echelon form R2-R, , A3-R1 0 for no soln => P(n) < F(c)he of hen sens rows in A < no of no zero row in c for 1-6=0 1 m= 20 10 3(A)= 2 < p(c)=3 .. for 1 = 6, when a system is incommistant (B) why many soln P(A) = P(c) < No ch subraw-P(A) = P(C) < 3 Ib x = 6, -4=20 => P(N) = P(O) = 2 < no of when @ unique sola: p(A) = p(c) = no of thes for 146 so no of man you rows in n = 3 = no of non-seno moras in y = no of who Hence for 2 + 6, system posses

(b) {(1,-1,0)}, (1,0,-1)} is a linear 2rd selected but it is not a basis (c) 'x' is not a subspace of R3 Theor chiftenent region  $x = \left\{ (x_1, x_2, x_3) \in (R^3) / x_1 + x_2 + x_3 = 0 \right\}$ let x, = k1 . x3 = k2 x1 = -x2 - x3 x,= -K,-K2 -- for different values of k, , kz , X is is generated  $x_1 + x_2 + \alpha_3 = 0$ , therefore set of { (-1,1,0) , (-1,0,1) } spains x -> Since (-1, 1,0) and (-1,0,1) are linearly independent minor is non zero? - Hence set of {(-1,+1,0)}, (-1,0,1)} forms a basis of dimension a there for hullity = 2

migue 30h

 $P = \begin{bmatrix} -10 \\ -1 \end{bmatrix}$   $Q = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$   $R = \begin{bmatrix} 2 \\ -7 \\ 12 \end{bmatrix}$ Of an orthogonal set of vectors having spain spain spain spain.

= no ob zunkn - p(A) = 3-1 = 2

rullity = din of space of sol ob Ax=B

2.88 write c= [A:B] 1 2-3 9 austen is consistant =) P(A) = P(c) > row / dependency in it & h' must be same : 3R1+R2 = R3 on 1 = 3R1+R2 = R3 onc : 3a+b = c e(er) put . By action form \* nullity : millity = the dimension of the space of the in the Bosis of the space of the soln to Ax = B = (no ob coloms - Rank ob A) = (no ob Unknowns - Rank ohA) the Mullity = no of Coloms - Rank (Ph)) Dimension = mullity = n-x go consider set ob colomb vectors defined by x= {x, x, x3 e R3 } such that x,+x+= estich of the following is true

(1, -1, 0) (1, 0, -1) } is a casis of x

(a)  $\begin{bmatrix} -6 \\ -8 \\ 3 \end{bmatrix}$ (b)  $\begin{bmatrix} -4 \\ 2 \\ 7 \end{bmatrix}$ (c)  $\begin{bmatrix} -3 \\ 2 \\ 2 \end{bmatrix}$ (d) none.

(e)  $\begin{bmatrix} -3 \\ 2 \\ 2 \end{bmatrix}$ (f) The following vector is linearly dependent up on soln to Privious prob.

(a)  $\begin{bmatrix} -3 \\ 2 \\ 2 \end{bmatrix}$ (b)  $\begin{bmatrix} -17 \\ -17 \\ 30 \end{bmatrix}$ (c)  $\begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}$ (d)  $\begin{bmatrix} 13 \\ 2 \\ -3 \end{bmatrix}$ Solve (D)  $\begin{bmatrix} -17 \\ 30 \end{bmatrix}$ (c)  $\begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}$ orthogonal =  $x_1 \cdot x_2 = 0$ option

(a)  $\begin{bmatrix} -17 \\ 2 \end{bmatrix}$ orthogonal =  $x_1 \cdot x_2 = 0$ option

(b)  $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$ (c)  $\begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}$ orthogonal =  $x_1 \cdot x_2 = 0$ option

(a)  $\begin{bmatrix} -17 \\ 2 \end{bmatrix}$ orthogonal =  $x_1 \cdot x_2 = 0$ option

(a)  $\begin{bmatrix} -17 \\ 2 \end{bmatrix}$ and  $\begin{bmatrix} -17 \\ 3 \end{bmatrix}$ orthogonal =  $\begin{bmatrix} -17 \\ 3 \end{bmatrix}$ orthogonal =  $\begin{bmatrix} -17 \\ 3 \end{bmatrix}$ orthogonal self of vectors drawing a spain that contain (semental) of vectors drawing a spain that contain (semental)  $\begin{bmatrix} -17 \\ -30 \end{bmatrix}$ orthogonal =  $\begin{bmatrix} -17 \\ -30 \end{bmatrix}$ orthogonal =  $\begin{bmatrix} -17 \\ -30 \end{bmatrix}$ orthogonal self of vectors drawing a spain that contain (semental) of vectors drawing a spain that contain (semental)  $\begin{bmatrix} -17 \\ -30 \end{bmatrix}$ if  $a_1 = 3$   $a_1 = 3$   $a_2 = 4$ 

=2 3x, + 4x2=

S= { (2,1,-2) (-2,-1,2) (4,2,-4)} ob R3 ts a basis or net? find Eigen Vector A + 0 . " Spani (5) = R3  $a_1x_1 + a_2x_2 + a_3x_3 = R^2$ 1 -2 .. R3 = 2R, :. S' to not laving and and set of vector of the at can't be besid Eigen values & Figen value vactor. let Anxa for any scalar 1. 7 x 40 such that Ax=Ax Then it is Eigen value of Anxn & x to is called Eigen valor corresponding to an Eigen value 's find Eigen values. Ax = Ax AX - AIX = D (A-AI) X=0 |- Honog Soln Should posses non sew soln => [ A-XI | =0 - chandenite enumber of A Adj (adj Az) = /A/2 A = /A/3 A = /A/1 xly (1) to the given characteristic cer => -13-13+211 +45=0 .. coust term = [A] = 45 Properties of eigen values ? (1) A, + A2 + A2 + - - + An = a11 + a22 + - - + ann A. A2 - A3 - - An = [A] (3) IAI = 0 if and only if atten adjust one of the eigen 1A1 to iff non of the eigenvalue is zero value is zero & The no. of non zero eigen values = P(A) of matrice A and AT have the same to eigen value @ the eigen values of any triangular matrix and diagonal matrix are its principle diagonal elements 1 the eigenvalues of Real symmetrics / Hemition matrice are real no. (8) the eigen values of stew synetric / skew tremition are or purly imaginary

1 the eigen values of orthogonal / thirtary are with

(16) let the 1 be the eiger value of Anna them

. do 1, -1, -1, i

[4]=1

(m) J+K

(i) in a is the eigen value of A"

KI is the eigen value of

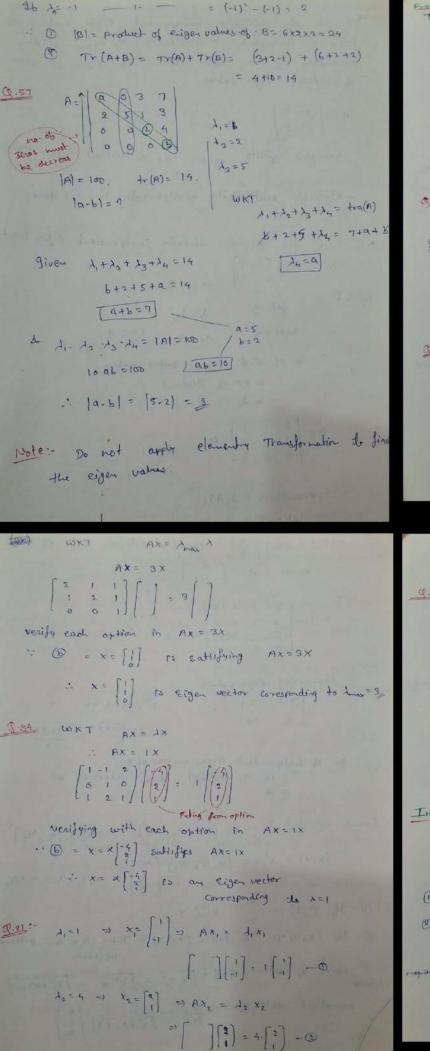
15----

Sub in (A-11) X=0 Solving equations - we get x = 0 organization of America | An AI | = (-1)" A" + (-1)" + acc(A) A" + --Constant term in characteristic plo polynomial of Anni [A] It A. As are Eigen values of As XAS Then [ADD-AI] = (-1)3(A)3+(-1)3 (A,+A2+A3) A2+(-1)(A,A+A,A+A + 1, 1, 1, = 0 4.55 Constant term of chan polyl A = IN = 0 1: R3 = 2R1 f(t) = t" + cont that + --- + c, t + co Convert into Standard form X14 by (-1)" \$(t) = (-1)" t"+ (-1)" cn, t" + - - + (-1)" c .: const term of flet = (Unco = |A| 84 16 23+22-212-45 =0 is a characteristic equation of coefficient natrice Asks in homogeneous system Ax =0. Then adj (adj As)= 9 (a) 42a (b) -45q2 (d.) 45 I # a. I + a, A + a, A2+ - - + a, A" Note: The eigen vector x corresponding to an eigen value of is same for the matrices the A", KA, A+KI and polynomial in A Q. 96 let i' be eigen value of i' E.C. (Ax = Ax) - O xly by 'A' on b.s.  $A^2 x = \lambda(Ax)$ A3 x = A3 X 1 N' x = 2" x 1 x The eigen vector'x for the matrixes A and At is Same therefore the eigen vector'x for A3 is same (11) let I be the eight value of non-singular matrix A. | M/O then (i) is the eigen value of A" (11) [A] is the eigen value of (Adj A) Note. The eigen vector & ob A, A, Adj A is same [AX = XX |- 0 xir by A-1 on bs. adjax - 1x x = x(A i A) (adia) x = x I A-1 x = +x - 0

weget composed the comments

```
And the eigen values of M (Heravition realms) are
9.17 The secon matrix is the inverse of first
                                                                  Real o hence a and R statement one true.
   matrix there for eigen value are 1/2, to Pros &
(E) The eigen values of involutory matrix are (+ or - 1)
           " be sign value "
                [ 1 · ± 1 ]
1 the eigenvalues of idempoded matric are 0,1
                                                             S.II WKT
           A^2 = A
        let ibe liger value of 'A'
        \lambda_n^2 = \lambda_n
        22- 1 =0 1 (1-1)=0 (1=0,1)
 9-39 · P3 P
        A3 = A
        13-1=0 A(1-1)=0 A(1+1)(1-1)=0
                                                                 Cal
        (d=0,1,-1)
     The determinent of skew syndric matrices of odd order is seno there for 100
 9-20: - Ans 0
                                                                     1, 12 13 = 1A1=0
aus Pasi
    => p-1 = 03
 => (P) #0 (8) #0 => Ap #0 , Aq #0
 1 17: Since Ar ari = real & aij aje
          there fore M is nemitian matrix
         and 'im' is skew Hermitton
                                                                    let d d de are the
   R. = R. + R2 + R3
                              1 1-2 1
      A= 3,0
   and 130
     1,+12+13 = +r(A)
      3+0+23 = 3
          [13-0]
0.13
       WKT : A_1 + A_2 + A_3 - - A_n = a_{11} + a_{12} + - - + a_{nn}
            = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + - - + \frac{1}{\ln(\ln a)}
                                                             $4.
               1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+--\frac{1}{h}-\frac{1}{h+1}
                                                                     A 100+I = ?
            = 1- 1 / /
                                                                   let B= 100 +I
       WKT lower triangular matrix
             1, 1 1 1 1 (A=L)
                       = |L|
              = a,, a2, a3, --- ann
                              = 1 \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} \cdot - \frac{1}{m} = \frac{1}{m}
        Given | A-I) = 0 = | A-11 |
           > | A, = 1
    WKT 1 + 12 + 2 = 13
             ( de+d3 = 12) - 0
          A. da da = 32
 Brown 1 = 1 ( Telly = 31) (3)
                                     : 12+12+12 = 12+8+42
             13=4
```

9.82: eigen balue one real the matrix 13 a Menition matrice then when AT ais = The  $a_{21} = \overline{a}_{11}$   $\left[ x = 5 + -j \right]$  $\lambda_1 + \lambda_2 + \lambda_3 = \alpha_0 + \alpha_{22} + \alpha_{33} =$ & 1. A2 23 = [A] = | 9 1 0 = A(a2-1)-1(a) , => a3-29/ (1) A, A2, A3 = d3 + (A) (B) A1. A2. A3 = 0.4.29 = 0 #(A) A, A2 A3 = ax -29 x 29 = -403 # 1A1 1. 12:13 = ax (a+52) x(a-12) = a3-2a=191 1 fats A= | 1 1 1 | 1, +1, +13 = tra (A) = 3 (A) Since the rank of this matrice = 1 and track= 3 there and be only one non zero eigen value = = trace(A) = 3 there for 1 = 0,0,3 WKT- A, + A2 = T8 (A) = 2+ K >0 =) K>-2 => KE (-2, 0) A1-A2 = 101 = 2K-1 >0 = ドン塩 K € (4,00) Brace both condition to be satisfied KE (-2,0) A (1/2,00) Ke (12,00) to K>12 An= 1,-1,0 Eigen value of B = 100+1 96 20=1 => - -111 - = 100 +1 = 2 th inc-1 => -= (-1)00+1=2 Hence |B| = |A+I| = Productor ob eigen values of = QX 2 X 1 = 4 918 96 B= A2-A . A=+3,2A find 10 101 (A+B)=? The eigen values of 'B' = 12 - 14 36 A3 A -11-



The no ob linearly independent Anxn = geometric multiplicity of Anxn = the dimension of the eigen space of Anxn = No of district eign values. A= (2) A= 2,2 Geometric xlicity of Az no ob distinct eigen values - Algebraic multiplicity of eigenvalue 2' = 2' times. the no ob linearly independent eight vectors A 5 -1 - 1 -3 1= 8,3 Solve :- |A| = 9 - 1, 1 = 3 | 1 = 3.3 : no ob Ind Eigen value vectors = no. ob distinct 'a'  $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \Leftrightarrow \lambda_1 = 1$ (trace(A)= 4 = 1,+12 |A| = 3 = 1, 12 remainin eigen values of given by 1 Lang = 3 : (c) = A = [3 2] Exactly satisfies both () R @

Hence  $A = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$  exactly satisfies both (1) less that  $A = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$ .  $A = \begin{bmatrix} 2 & 2 & 0 & | & 0 \\ 2 & 1 & 0 & | & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix} \lambda_1 \quad \lambda_2 = 3$ Sternaining eign values  $\begin{vmatrix} 2 - \lambda & 2 \\ 2 & 1 - \lambda \end{vmatrix} = 0 \quad \lambda_2^2 - 3\lambda + 2 = 0$   $\lambda_1 = 3 + \sqrt{9 + 6} = 3 + \sqrt{17}$ The of Independent Eigen vectors of A.  $A = N_0 \quad \text{ob distinct eigen values of } A$ 

= No ob distinct sign values of A+
= '4'

Inner Product:  $x = [x_1, x_2, x_3]^T$ ,  $y = [y_1, y_2, y_3]^T$   $(x,y) = x^T y = [x_1, x_2, x_3] \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = x_1y_1 + x_2y_2 + x_3y_3$   $(x,y) = 0 \implies x \perp^2 y$ 

(1) 34 (x,4) =0 X I 4 (1) 34 (x,4) =0 X I 4 ||x|| = ||Y|| = 1 Then 34 are orthonormore vectors.

Normalised vector of  $x = \frac{1}{|x|!} = \int (x, x) = \int x^{\frac{1}{2}} + x^{\frac{$ 

Property ( ) - The eigen vector of real symmetric matrix are orthogonal to each other [inner product of each paire is seno]  $X = \{x_1 \ x_2 \ x_3\}^T$   $y = \{y_1 \ y_2 \ y_3\}^T$  are the - eigen vectors of A real symmetric matrix of order of 3x3 corresponding to two distinct eigen values then x, y, + x242 + x343 =0 " X, y are the eigen vectors of Real symmetric matrix : the inner product of X y = XT y = x,4,+x,4+x,4=0  $A = \begin{bmatrix} 3/2 & 0 & -1/2 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{(i)} \quad$ Then another Ergen vectoroff= (a)  $\begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix}$  (b)  $\begin{bmatrix} 1 \\ -1 \\ 6 \end{bmatrix}$  (c)  $\begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$  d)  $\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$ c Real (-3)

A - is yequetric matrix was eigen to vectors are orthogonal to each other. choosing an orthogonal vector to the given vector from the options we get  $x_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ Cayley hemilton theorn: - every square matrix satisfy its characteristic equation -> we can replace I by A in the characteristic equation of A. (A - AI = 0 | A - AI | = 0 EC-18 A 5-9 18-9 5) 19A + 30I 116 + NT (p 171 + NT (n) Solar Pland the characteristic equation ob Azzo |Aex = AI = (-1) x2 + (-1) (-5) x + C = 0 +ra(n) 1A) = A + S A + 6 = 0 => A = -2, ~3 by c- 4 theorem (OR) 1=-0,-3 suplace 'd' by 'h' (a) A3 = 15A + 121 A2+5A+6I = 0 13 = 151 + 12 A2 = -5A -6 I 1=-2 = -8 +-30+12 xly by "A" on bs. (B) A3 = 19A + 30I A3 = -5A2-6A 13 = 191 + 30 A3 = -5 (-5A-61) - 6A A=-2 => -8=-8L A3 = 19A + 361 1:-3 = -27 = -27 (3) It Eigen values of Ages are 1,1,3 Then characteristic equation of matrix - A ? 3 A-1 Gadja Characteristic equation of Assa [Asxs - IA] = (-1)3, 13 + (-1)2 [1+1+3] 12 + (-1) [1+1+3] 1 + (1.1-3)=0 = -13+52-71+3=0 characteristic equation of (-A): replace it by -1 in above equation we got = -(1)3 + 5(-1)2 -7(-1)+3=0

Property (1) :- The eigen vector of Real Symmetric matrix are orthogonal to each other [inner product of each poire is zero] X = [x, x, x3] y= [4, 4, 43] are the reigen vectors of A real symmetric matrix of order 3x 3x3 corresponding to two distinctions where the two distinct eigen values them x, y, + x2 y2 + x3 y3 =0 " X, y are the eigen vectors of Real symmetric matrix : the inner product of x, y = xT y = x,y,+x,y+x,y=0 A - is symphic matrix whos eigen to vectors an orthogonal do each other. Choosing an ontegonal vector to the given vector from the options we get  $x_e = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ Cayley hemilton theorn: - every square matrix satisfy its characteristic equation -> we can replace I by A in the characteristic equation of A. d is replaced by n | |A-AI| = 0 A- AI =0 + A-AI) = 23+ 522 + 72+3=0 8tand form: (G) adj A = JAI => replace i by IAI = = m0 replace 'i' by 'si in I we get (2A- AI) = To replace it by 'th' in (1) ue get | A-1 - 11 | = (P.3) Inverse of matrix:  $A^{-1} = \frac{AdJ A}{|A|} , \qquad |A| \neq 0$ Af V= (x q)  $Q \Rightarrow find inverse ob <math>A = \begin{bmatrix} 1+qi & 2 \\ 1 & 1-2i \end{bmatrix}$  $A^{-1} = \frac{1}{(1^2 + 2^2) - 2} \begin{bmatrix} 1 - 2 & -2 \\ -1 & 1 + 2i \end{bmatrix}$  $A'' = \frac{1}{3} \left[ \begin{array}{c} 0 \\ 0 \end{array} \right]$ 

adjoint 
$$A = (cofector matrix)^T$$

$$= [(-1)^{i+j} \delta_{ij}]^T$$

$$A = [a_{ij}]_{3x3} , a_{ij} = i^2 - j^2 + ij$$

$$A^1 = 0$$

$$A = [a_{ij}]_{3x3} = \begin{bmatrix} 0 & -3 & -2 \\ 3 & 0 & -5 \\ 8 & 5 & 0 \end{bmatrix}$$

$$a_{11} = i^2 - i^2 = 0$$

$$a_{12} = i^2 - 2^2 = 43$$

$$a_{13} = i^2 - 3^2 = -8$$

$$a_{13} = 4 - 3 = -8$$
Given that  $A = B, c, D, E$  are non singular
$$A^{-1}, B^{-1}, c^{-1}, D^{-1}, E^{-1} \text{ exists}$$
Given  $DABEC = I$ 

$$B^{-1} = 0$$

$$A^{-1}, B^{-1}, c^{-1}, D^{-1}, E^{-1} \text{ exists}$$

$$A^{-1}, B^{-1}, C^{-1}, D^{-1}, E^{-1}, D^{-1}, D^$$

Pre X by E and post x4 by A on b.s.

BT = ECDA Ans.