# Trigonometry (5/5): Inverse Trigonometric Functions

Introduction to Engineering Mathematics

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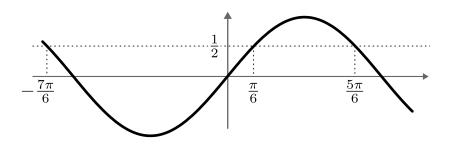
#### Overview

- 1 Definition of the inverse trigonometric functions
- 2 Examples

### Inverting the sine function

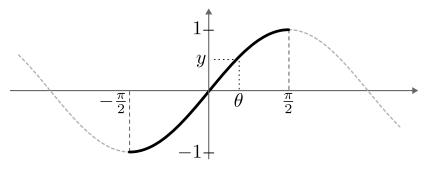
- The sine function turns angles into sine values.
- The **inverse sine** turns sine values back into angles.
- Notation:  $\sin^{-1}(x)$ ,  $\arcsin x$

**Problem:** Many values in the range correspond to the same angle!



# Restricting the domain

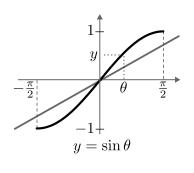
Solution: restrict the domain of the sine function so that there is exactly one angle corresponding to each value.

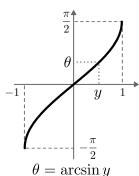


This gives us a meaningful way to define the inverse sine.

#### The inverse sine function

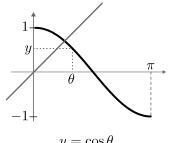
- Domain: [-1,1], range:  $[-\pi/2,\pi/2]$
- Cancellation properties:
  - $\sin(\arcsin(y)) = y$  for all  $y \in [-1, 1]$
  - $\arcsin(\sin(\theta)) = \theta$  for all  $\theta \in [-\pi/2, \pi/2]$ .



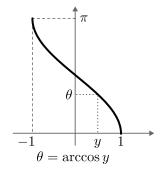


#### The inverse cosine function

- Domain: [-1, 1], range:  $[0, \pi]$
- Cancellation properties:
  - $\cos(\arccos(y)) = y$  for all  $y \in [-1, 1]$
  - $\arccos(\cos(\theta)) = \theta$  for all  $\theta \in [0, \pi]$

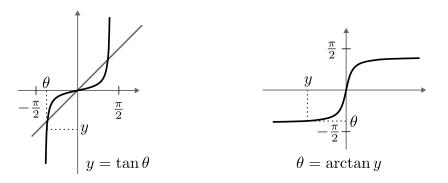






# The inverse tangent function

- Domain: all of  $\mathbb{R}$ , range:  $[-\pi/2, \pi/2]$
- Cancellation properties:
  - $\tan(\arctan(y)) = y$  for all  $y \in \mathbb{R}$
  - $\arctan(\tan\theta) = \theta$  for all  $\theta \in [-\pi/2, \pi/2]$



# Notation (warning)

Don't confuse

$$\sin^k \theta = (\sin \theta)^k$$
, for  $k \ge 0$ 

with

$$\sin^{-1} x = \arcsin x.$$

## Example

Simplify the following expression:  $\tan\left(\sin^{-1}\frac{2}{3}\right)$ .

# Example

Show that  $\arccos x = \frac{\pi}{2} - \arcsin x$ .