Trigonometry (1/5): Introduction and Overview Introduction to Engineering Mathematics

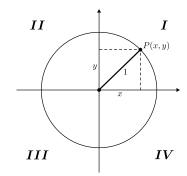
Prof. Joris Vankerschaver

Contents

- 1 Angles and points on the unit circle
- 2 Trigonometric functions as coordinates
- 3 Basic trigonometric identities

The unit circle

- The circle of radius 1 in the xy-plane, centered on the origin.
- Equation: $x^2 + y^2 = 1$
- Four quadrants: *I*, *II*, *III*, *IV*

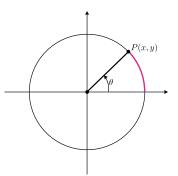


Example

If $P(\sqrt{3}/2,y)$ is a point on the unit circle, find the value of y.

Angles and points on the unit circle

- Each point P(x,y) defines an angle θ measured from the positive x-axis in counterclockwise direction.
- Angles measured in degrees or radians.
 - Value of θ in radians: length of arc subtended by θ (length of the red segment)

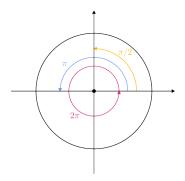


Converting between angles and radians

General formula to convert between degrees and radians:

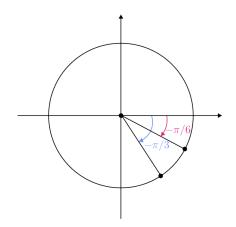
degrees
$$\times \frac{\frac{\pi}{180}}{\times \frac{180}{\pi}}$$
 radians

	Degrees	Radians
Full circle	360°	2π
Half circle	180°	π
Quarter circle	90°	$\pi/2$



Negative angles

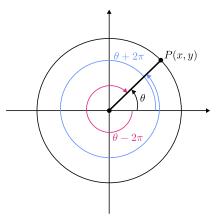
Measured from the positive x-axis, in clockwise direction.



Adding 2π to an angle

- Point P is determined by the angle θ .
- P stays same when adding $\pm 2\pi$ to θ .
- \Rightarrow All angles $\theta + 2k\pi$ with $k \in \mathbb{Z}$ give the same point P.

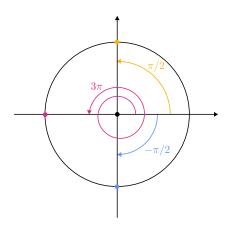
Principal angle: θ such that $-\pi < \theta \le \pi$.



Finding the coordinates of a point

Given an angle θ , find the coordinates of P(x,y).

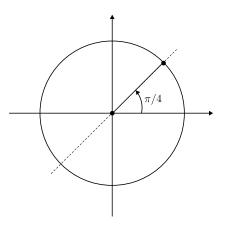
- **1** $\theta = \pi/2$
- $\theta = 3\pi$
- $\theta = -\pi/2$



Finding the coordinates of a point

Slightly more involved case:

4
$$\theta = \pi/4$$



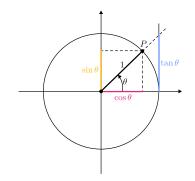
Important angles

Angle	x-coordinate	y-coordinate
0	1	0
$\pi/6$	$\sqrt{3}/2$	1/2
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/3$	1/2	$\sqrt{3}/2$
$\pi/2$	0	1
π	-1	0
2π	1	0

Trigonometric functions as coordinates

Let θ be an angle with point P(x,y).

Name	Notation	Definition
Cosine	$\cos \theta$	\overline{x}
Sine	$\sin \theta$	y
Tangent	$\tan \theta$	$\frac{\sin \theta}{\cos \theta}$
Cotangent	$\cot \theta$	$\frac{\cos \theta}{\sin \theta}$
Cosecant	$\csc \theta$	$\frac{1}{\sin \theta}$
Secant	$\sec \theta$	$\frac{1}{\cos \theta}$

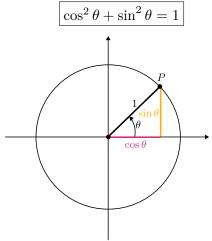


Example

Given that $\theta=\frac{\pi}{6}$, find the values of all 6 trigonometric functions.

Fundamental identity

- P(x,y) is on the unit circle: $x^2 + y^2 = 1$
- Put $x=\cos\theta$ and $y=\sin\theta$ to obtain the fundamental identity:



Aside: notation

Be very careful when you see $\sin^k \theta$.

• Positive exponent (power):

$$\sin^k \theta = (\sin \theta)^k.$$

• Negative exponent -1 (inverse function):

$$\sin^{-1} y = \arcsin y.$$

Fundamental identity: consequences

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Example

Suppose $\cos\theta=-\frac{4}{5}$ and θ is in quadrant III. Find $\sin\theta$ and $\tan\theta$.

Periodicity of sine and cosine

• Sine and cosine are 2π -periodic:

$$\sin(\theta \pm 2\pi) = \sin \theta$$
$$\cos(\theta \pm 2\pi) = \cos \theta$$

• The tangent is π -periodic:

$$\tan(\theta \pm \pi) = \tan\theta$$

Example: Compute $\tan\left(\frac{8093\pi}{4}\right)$

