Math 2280 Ordinary Differential Equation: Practice Exam #2 Name: ________________

Friday, March 1, 2024 A-Number: _______________

Directions: You must show all work to receive full credit for problems. Partial credit will only be given if the work is essentially correct with a minor error like a sign error. Please make sure that you write all work on the test in the space provided. There is a single problem on each page and you should have plenty of room to work the given problems on a single page. Also, make sure that

1. Your cell phone, ipod, calculator, tablet, PC, or any other electronic devices must be turned off,
2. you complete the work using a pencil and not a pen,
3. turn in your exam when it has been completed to your instructor, and

Students who do not follow these rules will be asked to leave the room. You will have 50 minutes to complete the exam.

For DRC Staff:
Please scan the test and email the pdf file to:
Joe Koebbe Joe.Koebbe@usu.edu
Please hold the original exam until next week so that all students will be able to complete the exam. There may be students who need to take the exam at a later date.

Thanks for your help with this course.

Problem 1. Use linear substitution to solve the following first order ODE.

$$\frac{dy}{dx} = \frac{1}{(2x - 5y + 7)^3}$$

Solution:

Problem 2. Compute the general solution of the following first order ODE. Use the substitution appropriate for a homogeneous first order ODE. Note that this is a nonlinear equation. So, the substitution involves $\frac{y}{x}$.

$$\frac{dy}{dx} = \frac{x^2\ y - x\ y^2}{x^3 + y^3}$$

Solution:

Problem 3. Solve the following Bernoulli equation using the substitution $u = y^{1-n}$.

$$\frac{dy}{dx} = \frac{3}{(x+1)}\, y - y^{5/2}$$

Solution:

Problem 4 Use the following potential function to define a first order ODE. From the differential equation, determine the two functions, $M(x, y)$ and $N(x, y)$ that arise when you apply the chain rule to get the differential equation. Even though you have defined the ODE from the given potential equation verify that the two functions, M and N, define an ODE in exact form.

$$\phi(x, y) = x^2 \ y + \sin(x \ y)$$

Solution:

Problem 5. Find the potential function for the following first order ODE.

$$sin(y) + [(1 + x)\ cos(y)]\ \frac{dy}{dx} = 0$$

Use the potential function to determine a general solution of the ODE. You can leave the solution in an implicit form.

Solution:

Problem 6. Compute the general solution for the differential equation

$$y'' = 4\ x\ \sqrt{y'}$$

using the substitution $v = y'$. Hint: There is no explicit dependence on y in the ODE.

Solution:

Problem 7. Compute the general solution for the differential equation

$$(y-3)\ y'' = (y')^2$$

using the substitution $v = y'$. Hint: There **is** explicit dependence on y in the ODE.

Solution:

Problem 8. Use reduction of order to determine the general solution of the following second order, linear, homogeneous ODE. Use $y_1 = e^{2x}$ as the seed for the general solution.

$$y'' - 3y' + 2y = 0$$

Find the unique solution for the initial conditions, $y(1) = 3$ and $y'(1) = 1$.

Solution: