Practice Quiz 11 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, FALL 2023

NAME: Solutions A#: _____

Problem 1. 27.1.g Find the Laplace transform, $Y(s)$, of the solution of the following initial value problem. Just find $Y(s)$ using ideas illustrated in Examples 27.1 and 27.2. Do NOT solve the problem using methods developed before we started discussing Laplace transforms! Also, do not attempt to recover $y(t)$ from $Y(s)$.

$$y'' + 4\,y = 3\ step_2(t),$$

with $y(0) = 0$, and $y'(0) = 5$.

Solution:

$$\mathcal{L}[y'' + 4y] = \mathcal{L}[3\,step_2(t)]$$

$$\Rightarrow s^2 Y(s) - s\,y(0) - y'(0) + 4Y(s) = 3e^{-2s}$$

$$\Rightarrow (s^2 + 4)Y(s) - s(0) - 5 = 3e^{-2s}$$

$$\Rightarrow Y(s) = \frac{5}{s^2+4} + \frac{3e^{-2s}}{s^2+4}$$

Problem 2. Ex. 28.4.a (10 points) Solve the following initial value problem using the Laplace Transform.

$$y' + 9\,y = 0$$

with $y(0) = 4$.

Solution:

$$\mathcal{L}[y' + 9y] = \mathcal{L}[0]$$

$$\Rightarrow \quad s\,Y(s) - y(0) + 9\,Y(s) = 0$$

$$\Rightarrow \quad s\,Y(s) - 4 + 9\,Y(s)$$

$$= (s+9)\,Y(s) - 4 = 0$$

$$\Rightarrow \quad Y(s) = \frac{4}{s+9}$$

$$\Rightarrow \quad y(t) = 4 \cdot \mathcal{L}^{-1}\left[\frac{1}{s+9}\right]$$

$$= 4 \cdot e^{-9t}$$