Practice Quiz 3 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, FALL 2023

NAME: Solutions A#: [redacted]

Problem 1. Section 3.4j (10 points) Rewrite each of the following in derivative formula form and then find all constant solutions. (In some cases, you may have to use the quadratic formula formula to find any constant solutions.

$$\frac{dy}{dx} + (8 - x)\, y - y^2 = -8\, x$$

Solution:

Rewrite:

$$\frac{dy}{dx} = y^2 - (8-x)y - 8x = F(x,y)$$

Next, set the right hand to zero.

$$F(x,y) = 0 \Rightarrow y^2 - (8-x)y - 8x = 0$$

$$\Rightarrow y = \frac{(8-x) \pm \sqrt{(8-x)^2 - 4(1)(-8x)}}{2}$$

$$= \frac{(8-x) \pm \sqrt{64 - 16x + x^2 + 32x}}{2}$$

$$= \frac{(8-x) \pm \sqrt{x^2 + 16x + 64}}{2}$$

$$= \frac{(8-x) \pm \sqrt{(x+8)^2}}{2}$$

$$= \frac{(8-x) \pm (x+8)}{2}$$

"+" $\Rightarrow$ $y = \frac{(8-x)+(x+8)}{2} = \frac{16}{2} = 8 \Rightarrow$ $y = 8$ is a constant solution

"−" $\Rightarrow$ $y = \frac{(8-x)-(x+8)}{2} = \frac{-2x}{2} = -x$ * Not a constant

Problem 2. Section 3.5g (10 points) Is the given equation an autonomous differential equation?

$$(x-2)\,\frac{dy}{dx} = y+3$$

Solution:

Rewrite:

$$\frac{dy}{dx} = \frac{y+3}{x-2} = F(x,y)$$

Since $F(x,y)$ is dependent on the independent variable, x, the equation is not autonomous.