# Quiz 3

MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, FALL 2023

NAME: A#:

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**Problem 1. Section 2.7f** (10 points) Using definite integrals (as in Example 2.5 on page 25 of the textbook) find the solution of the initial value problem. (In some cases, you may want to use the error function or the sine-integral function.)

$$x\,\frac{dy}{dx} = \sin\left(x^2\right)$$

with $y(0) = 0$

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**Solution:**

First, rewrite in derivative formula form

$$\frac{dy}{dx} = \frac{\sin(x^2)}{x}$$

Now, integrate

$$\int_0^x \frac{dy}{ds}\,ds = \int_0^x \frac{\sin(s^2)}{s}\,ds$$

$u = s^2$

$du = 2s\,ds \Rightarrow s\,ds = \frac{1}{2}du$

$s = 0 \Rightarrow u = 0$

$s = x \Rightarrow u = x^2$

$$\Rightarrow y(s)\Big|_0^x = \int_0^x \frac{\sin(s^2)}{s^2}\cdot s\,ds$$

$$\Rightarrow y(x) - y(0) = \frac{1}{2}\int_0^{x^2} \frac{\sin(u)}{u}\,du$$

$y = x^2$

$$= \frac{1}{2}\int_0^{y} \frac{\sin(u)}{u}\,du$$

$$= \frac{1}{2}\,Si(y) = \frac{1}{2}\,Si(x^2)$$

$$\Rightarrow y(x) = y(0) + \frac{1}{2}\,Si(x^2) = 0 + \frac{1}{2}\,Si(x^2)$$

$$= Si(x^2)$$

**Problem 2. Section 3.4b** (10 points) Rewrite the following in derivative formula form and then find all constant solutions. (In some cases, you may have to use the quadratic formula to find any constant solutions.)

$$\sin(x+y) - y\,\frac{dy}{dx} = 0$$

**Solution:**

We can rewrite this as

$$y\frac{dy}{dx} = \sin(x+y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin(x+y)}{y}$$

the only way to have $\frac{dy}{dx} = 0$ is for $\sin(x+y) = 0 \Rightarrow x+y = n\pi$ and then $y = n\pi - x$. This means $y$ is dependent on $x$ and therefore not a constant. No constant solutions