Practice Quiz 8 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, SPRING 2024

NAME: Solutions A#: _____

Problem 1. Exercise 17.3b (10 points) Find the general solution of the following differential equation.

$$y'' + 2y' + y = 0$$

Solution:

$y'' + 2y' + y = 0$

↳ $r^2 + 2r + 1 = 0$

$\Rightarrow (r+1)^2 = 0$

$\Rightarrow r_1 = -1, \; r_2 = -1$ repeated

$\Rightarrow y_1 = e^{-x}, \; y_2 = xe^{-x}$

So,

$$y = c_1 e^{-x} + c_2 x e^{-x}$$

Problem 2. Exercise 19.1d (10 points) Using clever factoring of the characteristic polynomial find the general solution to the following differential equation.

$$y^{(4)} - 81\,y - 0$$

Solution:

$y^{(4)} - 81y = 0$

↳ $r^4 - 81 = 0$

$\Rightarrow (r^2-9)(r^2+9) = 0$

$\Rightarrow (r-3)(r+3)(r^2+9) = 0$ $\qquad r^2+9 \Rightarrow r^2 = -9 \Rightarrow r = \pm i3$

$\Rightarrow (r-3)(r+3)(r+3i)(r-3i)$

$r_1 = 3, r_2 = -3, r_3 = -3i, r_4 = 3i$

$\Rightarrow y_1 = e^{3x}, y_2 = e^{-3x}, y_3 = \cos(3x), y_4 = \sin(3x)$

$\longrightarrow y = C_1 e^{3x} + C_2 e^{-3x} + C_3 \cos(3x) + C_4 \sin(3x)$