Quiz 7 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, SPRING 2024

NAME: Solutions A#: _____

Problem 1. Exercise 17.2c (10 points) Solve the following initial value problem.

$$y'' - 8\,y' + 15\,y = 0, \qquad y(0) = 5, \quad y'(0) = 19$$

Solution:

The characteristic equation is

$r^2 - 8r + 15 = 0$

$\Rightarrow (r-3)(r-5) = 0$

$\Rightarrow r_1 = 3, r_2 = 5$ roots are distinct

$\Rightarrow \{y_1, y_2\} = \{e^{3x}, e^{5x}\}$

$y = c_1 y_1 + c_2 y_2$

$= c_1 e^{3x} + c_2 e^{5x}$

$y' = 3c_1 e^{3x} + 5c_2 e^{5x}$

$y(0) = c_1 e^0 + c_2 e^0 = c_1 + c_2 = 5$

$y'(0) = 3c_1 e^0 + 5c_2 e^0 = 3c_1 + 5c_2 = 19$

So, solve

$c_1 + c_2 = 5 \Rightarrow c_1 = 5 - c_2$

$3c_1 + 5c_2 = 19 \longrightarrow 3c_1 + 5(5 - c_2) = 3c_1 + 25 - 5c_2 = 25 - 2c_1 = 19$

$\Rightarrow -2c_1 = -6$

$\Rightarrow \boxed{c_1 = 3}$

$\Rightarrow c_2 = 5 - 3 = 2$

Thus

$y = 3e^{3x} + 2e^{5x}$

Problem 2. Exercise 17.3c (10 points) Find the general solution to the following differential equation.

$$4y'' - 4y + y = 0$$

Solution:

The characteristic equation..

$4r^2 - 4r + 1 = 0$

$\Rightarrow (2r-1)^2 = 0$

$\Rightarrow r_1 = \frac{1}{2}, r_1 = \frac{1}{2}$

$\Rightarrow y_1 = e^{\frac{1}{2}x}, y_2 = xe^{\frac{1}{2}x}$ ↑ use reduction of order

$\Rightarrow y = c_1 e^{\frac{1}{2}x} + c_2 x e^{\frac{1}{2}x}$