Quiz 2

MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, SPRING 2024

NAME: Solutions A#: _____

Problem 1. Exercise 2.3i (10 points) Find a general solution for the following directly integrable equation. (Use indefinite instegrals).

$$1 = x^2 - 9\frac{dy}{dx}$$

Solution:

$$1 = x^2 - 9\frac{dy}{dx}$$

$$\Rightarrow 9\frac{dy}{dx} + 1 = x^2$$

$$\Rightarrow 9\frac{dy}{dx} = x^2 - 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2-1}{9}$$

$$\hookrightarrow \int \frac{dy}{dx}\,dx = \int \frac{x^2-1}{9}\,dx$$

$$\hookrightarrow y(x) + C_1 = \frac{1}{9}\int (x^2 - 1)\,dx$$

$$= \frac{1}{9}\left(\frac{1}{3}x^3 - x\right) + C_2$$

$$\hookrightarrow y(x) = \frac{1}{27}x^3 - \frac{1}{9}x + C_2 - C_1$$

$$= \frac{1}{27}x^3 - \frac{1}{9}x + C_3$$

Problem 2. Exercise 2.7f (10 points) Using definite integrals (as in Example 2.5 on page 25) find the solution of the following iitial-value problem. (In some cases, you may want to use the error function or the sine-integral function.)

$$x\,\frac{dy}{dx} = \sin(x^2)$$

with $y(0) = 0$.

Solution:

For $x\frac{dy}{dx} = \sin(x^2) \Rightarrow \frac{dy}{dx} = \frac{1}{x}\sin(x^2)$

$$\Rightarrow \int_0^x \frac{dy}{ds}\,ds = \int_0^x \frac{1}{s}\sin(s^2)\,ds$$

$$\hookrightarrow y(x) - y(0) = \int_0^x \frac{\sin(s^2)}{s}\,ds$$

$u = s^2$, $du = 2s\,ds$; $s=0,\ u=0$; $s=x,\ u=x^2$

$$= \int_0^x \frac{\sin(s^2)}{s^2}\cdot s\,ds$$

$$= \int_0^{x^2} \frac{\sin(u)}{u}\cdot\frac{1}{2}\,du$$

$$= \frac{1}{2}\int_0^{x^2} \frac{\sin(u)}{u}\,du$$

$$= \frac{1}{2}\,Si(x^2)$$