Practice Quiz 3 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, SPRING 2024

NAME: A#:

Problem 1. Exercise 3.4b (10 points) Rewrite the following in derivative form, and then find all constant solutions. (In some cases, you may have to use the quadratic formula to find any constant solutions.)

$$sin(x+y) - y\,\frac{dy}{dx} = 0$$

Solution:

$$\sin(x+y) - y\,\frac{dy}{dx} = 0$$

$$\hookrightarrow\ y\,\frac{dy}{dx} = \sin(x+y)$$

$$\hookrightarrow\ \boxed{\frac{dy}{dx} = \frac{\sin(x+y)}{y}}$$

For $\frac{dy}{dx} = 0 \Rightarrow \frac{\sin(x+y)}{y} = 0$ we assume $y \neq 0$

$$\Rightarrow \sin(x+y) = 0$$

$$\Rightarrow x+y = n\pi$$

$$\Rightarrow y = n\pi - x$$

depends on x $\Rightarrow$ No constant solutions.

Problem 2. Exercise 4.7e (10 points) Find the general solution of the following. Where possiible, write your answer as a an explicit solution.

$$\frac{dy}{dx} = \frac{y}{x}$$

Solution:

$$\frac{dy}{dx} = \frac{y}{x} \Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x}$$

$$\hookrightarrow \frac{1}{y}\frac{dy}{dx}\cdot dx = \frac{1}{x}\,dx$$

$$\hookrightarrow \frac{1}{y}\,dy = \frac{1}{x}\,dx$$

$$\hookrightarrow \int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$$

$$\hookrightarrow \ln|y| = \ln|x| + c$$

$$\hookrightarrow e^{\ln|y|} = \pm e^{\ln|x| + c}$$

$$\hookrightarrow y = \pm e^{\ln|x|}\cdot e^{c}$$

$$= \pm A x$$

Since A is arbitrary, we can leave off the "$\pm$"

$$\Rightarrow y(x) = Ax$$