Math 2280 HW #4 Solutions

4.3a $\frac{dy}{dx} = 3y^2 \cdot y^2 \sin(x)$

$= y^2(3 - \sin(x)) = (3-x)\,y^2 = f(x)\cdot g(y)$

4.3c $x\frac{dy}{dx} = (x-y)^2 \Rightarrow \frac{dy}{dx} = \frac{1}{x}(x-y)^2$ not separable

4.3e $\frac{dy}{dx} + 4y = 8 \Rightarrow \frac{dy}{dx} = 8 - 4y = (1)\cdot(8-4y) = f(x)\cdot g(y)$

4.3f $\frac{dy}{dx} + xy = 4x \Rightarrow \frac{dy}{dx} = 4x - xy = x(4-y) = f(x)\cdot g(y)$

4.3g $\frac{dy}{dx} + 4y = x^2 \Rightarrow \frac{dy}{dx} = x^2 - 4y$ not separable

4.3i $\frac{dy}{dx} = \sin(x+y) \Rightarrow$ not separable

$= \sin(x)\cos(y) + \cos(x)\sin(y)$ ← can't factor!

4.4c $xy\frac{dy}{dx} = y^2 + 9$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x}\cdot\frac{y^2+9}{y}$

$\Rightarrow \frac{y}{y^2+9}\,dy = \frac{1}{x}\,dx$

$\Rightarrow \int \frac{y}{y^2+9}\,dy = \int \frac{1}{x}\,dx = \ln|x| + C$

$u = y^2+9,\ du = 2y\,dy$

$\frac{1}{2}\ln|u| = \ln|x| + C$

$\Rightarrow \int \frac{\frac{1}{2}}{u}\,du = \ln|x| + C$

$\Rightarrow \frac{1}{2}\ln|u| = \ln|x| + C$

$\Rightarrow \ln|u| = 2\ln|x| + C$

$u = x^2 \cdot e^C$

$= Ax^2$

$\Rightarrow y^2 + 9 = Ax^2$

$\Rightarrow y = \pm\sqrt{Ax^2 - 9}$

4.4f $\frac{dy}{dx} = e^{2x-3y} = e^{2x} \cdot e^{-3y}$

$\Rightarrow e^{3y} \frac{dy}{dx} = e^{2x}$

$\Rightarrow e^{3y} \frac{dy}{dx} dx = e^{2x} dx$

$\Rightarrow e^{3y} dy = e^{2x} dx$

$\Rightarrow \int e^{3y} dy = \int e^{2x} dx$

$\Rightarrow \frac{1}{3} e^{3y} = \frac{1}{2} e^{2x} + C$

$\Rightarrow e^{3y} = \frac{3}{2} e^{3x} + 3C$

$\Rightarrow 3y = \ln\left(\frac{3}{2} e^{3x} + 3C\right)$

$\Rightarrow y(x) = \frac{1}{3} \ln\left(\frac{3}{2} e^{3x} + 3C\right)$

4.5a $\frac{dy}{dx} = \frac{x}{y}$ $y(1) = 3$

$\Rightarrow \int y\, dy = \int x\, dx$

$\Rightarrow \frac{1}{2} y^2 = \frac{1}{2} x^2 + C$

$\Rightarrow \frac{1}{2} y^2 = \frac{1}{2} x^2 + 4$

$\Rightarrow y^2 = x^2 + 8$

$\Rightarrow y = \pm\sqrt{x^2 + 8}$

$y(1) = 3$

$\Rightarrow \frac{1}{2}(3)^2 = \frac{1}{2}(1)^2 + C$

$\Rightarrow \frac{9}{2} = \frac{1}{2} + C \Rightarrow C = \frac{8}{2} = 4$

4.5b $\frac{dy}{dx} = 2x - 1 + 2xy - y$ $y(0) = 2$

$= 2x - 1 + y(2x - 1)$

$= (2x - 1)(1 + y)$

$\Rightarrow (1+y)^{-1} \frac{dy}{dx} = (2x - 1)$

$\Rightarrow (1+y)^{-1} dy = (2x - 1)\, dx$

$\Rightarrow \ln|y-3| = \frac{1}{2}x^2 - 2x + C$

$\Rightarrow y-3 = e^{\frac{1}{2}x^2 - 2x} e^C$

$\Rightarrow y = 3 + Ae^{\frac{1}{2}x^2 - 2x}$

4.7e $\frac{dy}{dx} = \frac{y}{x}$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x}$

$\Rightarrow \frac{1}{y}\,dy = \frac{1}{x}\,dx$

$\Rightarrow \int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$

$\Rightarrow \ln|y| = \ln|x| + C$

$\Rightarrow y = e^{\ln|x| + C} = Ax$

$y=0$

4.7h $(y^2-1)\frac{dy}{dx} = 4xy^2$

$\Rightarrow \frac{y^2-1}{y^2}\frac{dy}{dx} = 4x$

$\Rightarrow \int \frac{y^2-1}{y^2}\,dy = 4\int x\,dx$

$\Rightarrow \int (1 - \frac{1}{y^2})\,dy = 4(\frac{x^2}{2}) + C$

$\Rightarrow y + \frac{1}{y} = 2x^2 + C$

$\Rightarrow y^2 + 1 = (2x^2 + C)y$

$\Rightarrow y^2 - (2x^2 + C)y + 1 = 0$

$y = \frac{(2x^2+C) \pm \sqrt{(2x^2+C)^2 - 4}}{2}$

$y=0$

4.7i

$\frac{dy}{dx} = e^{-y}$ — no const. solution

$\Rightarrow e^y \frac{dy}{dx} = 1$

$\Rightarrow \int e^y \, dy = \int dx$

$\Rightarrow e^y = x + c \Rightarrow \boxed{y = \ln|x+c|}$

4.8a $\frac{dy}{dx} - 2y = -10 \qquad y(0) = 8$

$\Rightarrow \frac{dy}{dx} = 2y - 10$

$= 2(y-5)$ ⟶ $y=5$ is const. solution

$\Rightarrow \frac{1}{y-5} \frac{dy}{dx} = 2$

$\Rightarrow \int \frac{1}{y-5} \, dy = \int 2 \, dx$

$\Rightarrow \ln|y-5| = 2x + c$ ⟶ $y(0)=1 \Rightarrow \ln$

$\Rightarrow y - 5 = e^{2x} \cdot A$

$\Rightarrow y = 5 + Ae^{2x}$ ⟶ $y(0) = 8 \Rightarrow 5 + A^0 = 8 \Rightarrow A = 3$

$\Rightarrow y = 5 + 3e^{2x}$

4.8e $x \frac{dy}{dx} = y^2 - y \qquad y(1) = 2$

$\frac{1}{y^2-y} \frac{dy}{dx} = \frac{1}{x}$

$\Rightarrow \int \frac{1}{y^2-y} \, dy = \int \frac{1}{x} \, dx$

$\frac{1}{y^2-y} = \frac{1}{y(y-1)} = \frac{A}{y} + \frac{B}{y-1}$

$1 = A(y-1) + By$

$y = 1 \Rightarrow B = 1$

$y = 0 \Rightarrow A = -1$

$\Rightarrow \int \left(\frac{-1}{y} + \frac{1}{y-1}\right) dy = \ln|x| + C$

$\Rightarrow -\ln|y| + \ln|y-1| = \ln|x| + C$

$\Rightarrow \ln\left|\frac{y-1}{y}\right| = \ln|x| + C$

$y(1) = 2$

$\Rightarrow \ln\left|\frac{2-1}{2}\right| = \ln|1| + C$

$\Rightarrow C = \ln(1/2)$

$\Rightarrow \ln\left|\frac{y-1}{y}\right| = \ln|x| + \ln|1/2|$

$\Rightarrow \frac{y-1}{y} = e^{\ln|x| + \ln(1/2)}$

$\Rightarrow \frac{y-1}{y} = x \cdot (1/2)$

$\Rightarrow y - 1 = \frac{1}{2} x y$

$\Rightarrow y - \frac{1}{2} x y = 1$

$\Rightarrow y(1 - \frac{1}{2}x) = 1 \Rightarrow y = \frac{1}{1 - \frac{1}{2}x} \cdot \frac{2}{2} = \frac{2}{2-x}$

4.10a $\frac{dy}{dx} - 2y = -10$, $y(0) = 8$

$\Rightarrow y = 5 + 3e^{2x}$

y is defined for any $x \in (-\infty, +\infty)$

4.11 $x \frac{dy}{dx} = y^2 - y$ $\quad y(1) = 0$

$\Rightarrow \ln\left|\frac{y-1}{y}\right| = \ln|x| + C$

$\Rightarrow \frac{y-1}{y} = A \cdot x$

$\Rightarrow y - 1 = A x y$

$\Rightarrow y = A x y - 1$

$y(1 - Ax) = 1$

$\Rightarrow y = \frac{1}{1 - Ax}$

$x \neq 1/A$