

Practice Quiz 1 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, SPRING 2024

NAME: Solutions

A#: _____

Problem 1. Exercise 1.3e (10 points) For each differential equation given three choices for a possible solution $y = y(x)$ are given. Determine whether each choice is or is not a solution to the given differential equation. (In each case, assume the interval of interest is the entire real line $(-\infty, \infty)$)

$$x \frac{dy}{dx} - 2y = 6x^4$$

i.) $y(x) = x^4$ ii.) $y(x) = 3x^4$ iii.) $y(x) = 3x^4 + 5x^2$

Solution:

First, let's write

$$x \frac{dy}{dx} - 2y - 6x^4 = 0$$

i) $y = x^4 \Rightarrow \frac{dy}{dx} = 4x^3$

$$\Rightarrow x(4x^3) - 2x^4 - 6x^4 = 4x^4 - 2x^4 - 6x^4 = (4 - 2 - 6)x^4 = -4x^4 \neq 0$$

not a solution

ii) $y = 3x^4 \Rightarrow \frac{dy}{dx} = 12x^3$

$$\Rightarrow x(12x^3) - 2(3x^4) - 6x^4 = 12x^4 - 6x^4 - 6x^4 = (12 - 6 - 6)x^4 = 0 \checkmark$$

this is a solution

iii) $y = 3x^4 + 5x^2 \Rightarrow \frac{dy}{dx} = 12x^3 + 10x$

$$\Rightarrow x(12x^3 + 10x) - 2(3x^4 + 5x^2) - 6x^4$$

$$= 12x^4 + 10x^2 - 6x^4 - 10x^2 - 6x^4 = 0$$

Problem 2. Exercise 2.3g (10 points) Find a general solution for the following directly integrable equations. Use an indefinite integral to compute the solution.

$$x = (x^2 - 9) \frac{dy}{dx}$$

Solution:

$$\frac{dy}{dx} = \frac{x}{x^2 - 9}$$

$$\Rightarrow \int \frac{dy}{dx} dx = \int \frac{x}{x^2 - 9} dx$$

$$u = x^2 - 9$$

$$du = 2x dx \Rightarrow x dx = \frac{1}{2} du$$

$$\Rightarrow y(x) + C_1 = \int \frac{1}{2u} du$$

$$= \frac{1}{2} \ln |u| + C_2$$

$$= \frac{1}{2} \ln |x^2 - 9| + C_2$$

$$\Rightarrow y(x) = \frac{1}{2} \ln |x^2 - 9| + C_3$$

$$C_3 = C_2 - C_1$$