Math 2280 Lecture Notes Day 6.

Ex: Directly Integrable:

$$y'(x) = x\cos(x) \Rightarrow \int y'\,dx = y(x) = \int x\cos(x)\,dx$$

$u = x \quad dv = \cos(x)\,dx$

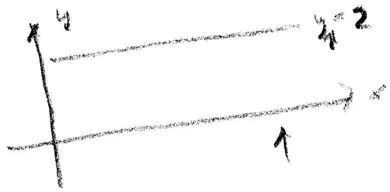
$du = dx \quad v = \sin(x)$

$$\Rightarrow y(x) = x\sin(x) - \int \sin(x)\,dx$$

$$= x\sin(x) + \cos(x) + C.$$

Ex: Constant Solutions:

$$y' = xy(y-2) \Rightarrow y_1 = 0, \quad y_2 = 2$$



$$y' = \sin(y) \longrightarrow y' = 0 \Rightarrow \sin(y) = 0$$

$y = 0, \; y = \pm\pi, \; y = \pm 2\pi$

$\Rightarrow y = n\pi$ for $n = \pm 1$

Ex: $\frac{dy}{dx} = x^2y^2 + x^2$

$= x^2(y^2+1)$

$$\Rightarrow \frac{1}{y^2+1}\frac{dy}{dx} = x^2$$

$$\Rightarrow \frac{1}{y^2+1}\underbrace{\frac{dy}{dx}\,dx}_{dy} = x^2\,dx$$

$$\Rightarrow \frac{1}{y^2+1}\,dy = x^2\,dx$$

$$\Rightarrow \int \frac{1}{y^2+1}\,dy = \int x^2\,dx$$

$$\Rightarrow \arctan(y) = \frac{1}{3}x^3 + c$$

$$\Rightarrow y(x) = \tan\left(\frac{1}{3}x^3 + c\right)$$

Ex: $\frac{dy}{dx} = -\frac{x}{y-3}$, $y(0)=1$

$$\Rightarrow (y-3)\frac{dy}{dx} = -x$$

$$\Rightarrow (y-3)\frac{dy}{dx}\,dx = -x\,dx$$

$$\Rightarrow (y-3)\,dy = -x\,dx$$

$$\Rightarrow \int (y-3)\,dy = -\int x\,dx$$

$$\Rightarrow \frac{1}{2}y^2 - 3y = -\frac{x^2}{2} + c$$ implicit form

$$y(0)=1 \Rightarrow \frac{1}{2}(1)^2 - 3(1) = 0 + c$$

$$\Rightarrow c = \frac{1}{2} - 3 = -5/2$$

$$\Rightarrow \frac{1}{2}y^2 - 3y = -\frac{x^2}{2} - 5/2$$

$$\Rightarrow y^2 - 6y = -x^2 - 5 \Rightarrow y^2 - 6y + (5 + x^2) = 0$$

$$\Rightarrow y = \frac{6 \pm \sqrt{36 - 4(5+x^2)}}{2}$$ explicit form

Ex: $y' = 2x(y-3)$

$y' = 0 \Rightarrow y - 3 = 0 \Rightarrow y = 3$

$\Rightarrow \frac{1}{y-3}\, dy = 2x\, dx$

$\Rightarrow \int \frac{1}{y-3}\, dy = \int 2x\, dx$

$\Rightarrow \ln|y-3| = x^2 + c$

$\Rightarrow y - 3 = e^{x^2+c} = e^c e^{x^2} = Ae^{x^2} \qquad A = e^c$

$\Rightarrow y = 3 + Ae^{x^2}$

general soln

Ex: Falling body problem

$$\frac{dv}{dt} = -9.8 - kv$$

$$0 = -9.8 - kv \Rightarrow v_0 = \frac{-9.8}{k} = -\frac{9.8m}{\gamma} = \text{term. vel.}$$

For this problem we can separate variables

$$\frac{1}{9.8 + kv}\frac{dv}{dt} = -1$$

$$\Rightarrow \frac{1}{9.8 + kv}\frac{dv}{dt} \cdot dt = -dt$$

$$\Rightarrow \int \frac{1}{9.8 + kv} \cdot dv = -\int dt$$

$$\Rightarrow \frac{1}{k}\ln|9.8 + kv| = -t + c_1 = -t + c$$

$$\Rightarrow \ln|9.8 + kv| = -kt + c_2 = -kt + kc$$

So

$$|9.8 + kv| = e^{-k t + k t} \cdot e^{k(t - c)}$$

$$= e^{-kt} e^{kc} = A e^{kt}$$

$$kv = -9.8 \pm A e^{kt}$$

$$v(t) = \frac{1}{k}\left[-9.8 \pm A e^{kt}\right]$$

To find y, integrate

Separability requires : $\frac{dy}{dx} = f(x) \cdot \underline{g(y)}$

↑ look for constant solution.

First Order Linear ODEs

→ ODEs of the form :

$$\boxed{\frac{dy}{dx} + p(x)\, y = g(x)}$$

This comes from.

$$\frac{dy}{dx} = \underbrace{f(x)}_{y^0} - \underbrace{p(x)\, y}_{y^1}$$