Math 2280 HW 3 Solutions

1.4c $\frac{d^2y}{dx^2} - 9y = 0,\ y(0) = 1,\ y'(0) = 9$

i) $y(x) = 2e^{3x} - e^{-3x} \Rightarrow y' = 6e^{3x} + 3e^{-3x}$

$\Rightarrow y'' = 18e^{3x} - 9e^{-3x}$

$\Rightarrow (18e^{3x} - 9e^{-3x}) - 9(2e^{3x} - e^{-3x}) = 18e^{3x} - 9e^{-3x} - 18e^{3x} + 9e^{3x} = 0$ ✓ Satisfies the DE

$\Rightarrow y(0) = 2e^0 - e^0 = 2 - 1 = 1$ ✓

$\Rightarrow$ Solution for the IVP.

$\Rightarrow y'(0) = 6e^0 + 3e^0 = 6 + 3 = 9$ ✓

ii) $y(x) = e^{3x} \Rightarrow y' = 3e^{3x},\ y'' = 9e^{3x}$

$\Rightarrow y'' - 9y = -9e^{3x} + 9(e^{3x}) = 0$ ✓

$\Rightarrow y(0) = e^0 = 1,\ y'(0) = 3e^0 = 3$ X this does not satisfy the initial condition $y'(0)$. So this is not a solution of the IVP.

iii) $y(x) = e^{3x} + 1 \Rightarrow y' = 3e^{3x},\ y'' = 9e^{3x}$

$\Rightarrow y'' - 9y = 9e^{3x} - 9(e^{3x}+1) = -9 \neq 0.$

Since $y(x) = e^{3x} + 1$ does not satisfy the ODE y cannot be a solution of the IVP

2.7c $x\frac{dy}{dx} = \sin(x)\quad y(0) = 4\qquad (x_0, y_0) = (0, 4)$

$\Rightarrow \frac{dy}{dx} = \frac{\sin(x)}{x}$

$\Rightarrow \int_0^x \frac{dy}{ds}\,ds = \int_0^x \frac{\sin(s)}{s}\,ds$

$\Rightarrow \quad y(s)\Big|_0^x = Si(s)\Big|_0^x$

$\Rightarrow \quad y(x) - y(0) = Si(x) - \cancel{Si(0)}^{\nearrow 0}$

$\Rightarrow \quad y(x) - 4 = Si(x)$

$\Rightarrow \quad y(x) = 4 + Si(x)$

2.7f $\quad x\dfrac{dy}{dx} = \sin(x^2) \qquad y(0) = 0 \qquad (x_0, y_0) = (0,0)$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{\sin(x^2)}{x}$

$\Rightarrow \quad \displaystyle\int_0^x \frac{dy}{ds}\,ds = \int_0^x \frac{\sin(s^2)}{s}\,ds$

$\Rightarrow \quad y(x) - \cancel{y(0)}^{\nearrow 0} = \displaystyle\int_0^x \frac{\sin(s^2)}{s^2}\cdot \underline{s\,ds}$

$u = s^2 \Rightarrow du = 2s\,ds$

$\Rightarrow \tfrac{1}{2}du = \underline{s\,ds}$

$\Rightarrow s=0 \Rightarrow u=0$

$\Rightarrow s=x \Rightarrow u=x^2$

$= \displaystyle\int_0^{x^2} \frac{\sin(u)}{u}\cdot \frac{1}{2}\,du$

$= \displaystyle\frac{1}{2}\int_0^{x^2} \frac{\sin(u)}{u}\,du$

$= \tfrac{1}{2}\, Si(x^2)$

$\Rightarrow \quad y(x) = \tfrac{1}{2}\, Si(x^2)$

3.4h $\quad \sin(x+y) - y\dfrac{dy}{dx} = 0$

$\Rightarrow \quad y\dfrac{dy}{dx} = \sin(x+y)$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{\sin(x+y)}{y}$

the only way $\frac{dy}{dx} = 0$ is if $\sin(x+y)$

$\Rightarrow \quad y = n\pi x$ which depends on $x \Rightarrow$ __no__ constant solutions

3.4c $\frac{dy}{dx} - y^3 = 8$

$\Rightarrow \frac{dy}{dx} = y^3 + 8 = (y+2)(y^2 - 2y + 4) = 0$

$\hookrightarrow y = \frac{2 \pm \sqrt{(-2)^2 - 16}}{2} \notin$ real number

$\Rightarrow$ real root of $y = -2$

So, $y = -2$ is a constant solution

3.4.d $x^2 \frac{dy}{dx} + xy^2 = x$

$\Rightarrow \frac{dy}{dx} + \frac{1}{x} y^2 = \frac{1}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x}(1 - y^2)$

$\Rightarrow \frac{dy}{dx} = \frac{(1-y^2)}{x} = \frac{(1-y)(1+y)}{x}$

So, $\frac{dy}{dx} = 0$ when $y = 1, y = -1$. Then $y_1 = 1$ and $y_2 = -1$ are constant solutions.

3.4h $(y-2) \frac{dy}{dx} = (x-3)$

$\Rightarrow \frac{dy}{dx} = \frac{x-3}{y-2}$

There are not roots such that $\frac{dy}{dx} = 0$. $\Rightarrow$ no constant solutions

3.5 All

a.) $\frac{dy}{dx} = F(x,y) = 6x - 3xy$ depends on $x \Rightarrow$ not autonomous

b.) $\frac{dy}{dx} = \frac{\sin(x+y)}{y} = F(x,y)$ depends on $x \Rightarrow$ not autonomous

c.) $\frac{dy}{dx} = y^3 + 8 = F(x,y)$ only depends on $y \Rightarrow$ autonomous

d.) $\frac{dy}{dx} = \frac{1-y^2}{x} = F(x,y)$ depends on $x \Rightarrow$ not autonomous

e.) $\frac{dy}{dx} = y^2 + x = F(x,y)$ depends on $x \Rightarrow$ not autonomous

f.) $\frac{dy}{dx} = 25y - y^3 = F(x,y)$ depends only on $y \Rightarrow$ autonomous

g.) $\frac{dy}{dx} = \frac{y+3}{x-1} = F(x,y)$ depends on $x \Rightarrow$ not autonomous

h.) $\frac{dy}{dx} = \frac{x-3}{y-2} = F(x,y)$ depends on $x \Rightarrow$ not autonomous

i) $\frac{dy}{dx} = y^2 - 2y - 2 = F(x,y)$ depends only on $y \Rightarrow$ autonomous

j) $\frac{dy}{dx} = y^2 - (8-x)y - 8x = F(x,y)$ depends on $x \Rightarrow$ not autonomous

3.6 Set

$$\frac{dy}{dx} = 2\sqrt{y} \qquad y(1) = 0 \qquad \text{So } \frac{dy}{dx} - 2\sqrt{y} = 0$$

a. i) $y \equiv 0 \Rightarrow \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} - 2\sqrt{y} = 0 - 2\sqrt{0} = 0$ ✓

$y(1) = 0$ since for any x, $y = 0$

ii) $$y(x) = \begin{cases} 0 & x < 1 \\ (x-1)^2 & 1 \le x \end{cases}$$

$$y' = \begin{cases} 0 & x < 1 \\ 2(x-1) & 1 \le x \end{cases}$$

$$\Rightarrow \frac{dy}{dx} - 2\sqrt{y} = \begin{cases} 0 & x < 1 \\ 2(x-1) & 1 \le x \end{cases} - 2 \begin{cases} 2\sqrt{0} & x < 1 \\ 2\sqrt{(x-1)^2} & 1 \le x \end{cases}$$

$$= \begin{cases} 0 - 2(0) & x < 1 \\ 2(x-1) - 2\sqrt{(x-1)^2} & 1 \le x \end{cases}$$

$$= \begin{cases} 0 & x < 1 \\ 0 & 1 \le x \end{cases} = 0 \checkmark$$

Note $y(1) = 0$ also

(iii) $$y(x) = \begin{cases} 0 & x < 3 \\ (x-3)^2 & 3 \le x \end{cases}$$

$$y' = \begin{cases} 0 & x < 3 \\ 2(x-3) & 3 \le x \end{cases}$$

$$\Rightarrow \frac{dy}{dx} - 2\sqrt{y} = \begin{cases} 0 & x < 3 \\ 2(x-3) & 3 \le x \end{cases} - 2 \begin{cases} 0 \\ 2\sqrt{(x-3)^2} \end{cases} =$$

$$= \begin{cases} 0 & x < 3 \\ 0 & 3 \le x \end{cases} = 0.$$

Since $x_0 < 3 \Rightarrow 1 < 3$
$y(1) = 0 \checkmark$

We have found 3 solutions that work the problem in the theorem is $\frac{\partial F}{\partial y}$.

$$\frac{\partial F}{\partial y} = 2 \frac{\partial}{\partial y} \sqrt{y}$$

$$= \frac{1}{\sqrt{y}}$$

This is <u>not</u> continuous when $y = 0$. This is why the theorem is <u>not</u> applicable.