Quiz 5

MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, SPRING 2024

NAME: A#:

Problem 1. Exercise 8.28 (10 points) Use any of the methods for first order ODEs to find a general solution for the following ODE.

$$ln|y| + \left[\frac{x}{y} + 3\right] \frac{dy}{dx} = 0$$

Solution:

Let's try "exact form"

$M(x, y) = \ln|y|$ $\quad \frac{\partial M}{\partial y} = 1/y$ ✓

$N(x, y) = \left[\frac{x}{y} + 3\right]$ $\quad \frac{\partial N}{\partial x} = 1/y + 0 = 1/y$

Then

$\frac{\partial \phi}{\partial x} = \ln|y| \Rightarrow \phi(x, y) = x \ln|y| + g(y)$

$\Rightarrow \frac{\partial \phi}{\partial y} = \frac{x}{y} + g'(y)$

$= \left[\frac{x}{y} + 3\right]$

$\Rightarrow \frac{\partial \phi}{\partial y} = g'(y) = 3$

$\Rightarrow g(y) = 3y + C$

So,

$\phi(x, y) = x \ln|y| + 3y + C$

the solution can be found from $\phi(x, y) = C_2$

$\Rightarrow \underline{x \ln|y| + 3y = C_3}$

Problem 2. Exercise 8.40 (10 points) Use any of the methods for first order ODEs to find a general solution for the following ODE.

$$y^2 - y^2 \cos(x) + \frac{dy}{dx} = 0$$

Solution:

For this problem, solve for $\frac{dy}{dx}$.

$$\Rightarrow \frac{dy}{dx} = y^2 \cos(x) - y^2$$

$$\Rightarrow \frac{dy}{dx} = (\cos(x) - 1)\, y^2$$

$\underline{y = 0}$ is a constant solution

$$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} = \cos(x) - 1$$

$$\Rightarrow \int \frac{1}{y^2}\, dy = \int (\cos(x) - 1)\, dx$$

$$\Rightarrow -\frac{1}{y} = \sin(x) - x + C_1$$

$$\Rightarrow \frac{1}{y} = x - \sin(x) - C_1$$

$$\Rightarrow y = \frac{1}{x - \sin(x) - C_1}$$