# Quiz 1

MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, FALL 2023

NAME: Solutions A#:

**Problem 1. Section 1.3d** (10 points) For each differential equation given three choices for a possible solution $y = y(x)$ are given. Determine whether each choice is or is not a solution to the given differential equation. (In each case. assume the interval of interest is the entire real line $(-\infty, \infty)$

$$\frac{d^2y}{dx^2} = -9\,y$$

**i.)** $y(x) = e^{3x}$ **ii.)** $y(x) = x^3$ **iii.)** $y(x) = sin(3x)$

**Solution:**

$\Rightarrow \frac{d^2y}{dx^2} + 9y = 0$

i) $y = e^{3x} \Rightarrow \frac{dy}{dx} = 3e^{3x}, \frac{d^2y}{dx^2} = 9e^{3x}$

$\Rightarrow \frac{d^2y}{dx^2} + 9y = 9e^{3x} + 9e^{3x} = 18e^{3x} \neq 0$ Not a solution

ii) $y = x^3 \Rightarrow \frac{dy}{dx} = 3x^2, \frac{d^2y}{dx^2} = 6x$

$\Rightarrow \frac{d^2y}{dx^2} + 9y = 6x + 9(x^3) = 3x(2 + 3x^2) \neq 0$ Not a solution

iii) $y = \sin(3x) \Rightarrow \frac{dy}{dx} = 3\cos(3x), \frac{d^2y}{dx^2} = -9\sin(3x)$

$\Rightarrow \frac{d^2y}{dx^2} + 9y = -9\sin(3x) + 9\sin(3x) = 0$ ✓

This is a solution

**Problem 2. Section 1.4d** (10 points) For each initial value problem given below, three choices for a possible solution, $y = y(x)$ are given. Determine whether each choice is or is not a solution to the given initial-value problem.

$$x^2 \frac{d^2y}{dx^2} - 4\,x \frac{dy}{dx} + 6\,y = 36\,x^6$$

with $y(1) = 1$ and $y'(1) = 12$.

**i.)** $y(x) = 2e^{3x} - e^{-3x}$ **ii.)** $y(x) = e^{3x}$ **iii.)** $y(x) = e^{3x} + 1$

**Solution:**

$$\Rightarrow x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y - 36x^6 = 0$$

i) $y = 2e^{3x} - e^{-3x} \Rightarrow \frac{dy}{dx} = 6e^{3x} + 3e^{-3x}, \quad \frac{d^2y}{dx^2} = 18e^{3x} - 9e^{-3x}$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y - 36x^6$$

$$= x^2(18e^{3x} - 9xe^{-3x}) - 4x(6e^{3x} + 3e^{-3x}) + 6(2e^{3x} - e^{-3x}) - 36x^6$$

$$= 18x^2e^{3x} - 9x^2e^{-3x} - 24xe^{3x} - 12e^{-3x} + 12e^{3x} - 6e^{-3x} - 36x^6$$

$\neq 0$. Not a solution

ii) $y = e^{3x} \Rightarrow y' = 3e^{3x}, \quad y'' = 9e^{3x}$

$$\Rightarrow x^2(9e^{3x}) - 4x(3e^{3x}) + 6e^{3x} - 36x^2$$

$$= 9x^2e^{3x} - 12xe^{3x} + 6e^{3x} - 36x^6 \neq 0$$ Not a solution

iii) $y = e^{3x} + 1, \quad y' = 3e^{3x}, \quad y'' = 9e^{3x}$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y - 36x^6$$

$$= x^2(9e^{3x}) - 4x(3e^{3x}) + 6e^{3x} - 36x^6 \neq 0$$ Not a solution