

Ex: $xy^2 \frac{dy}{dx} = x^3 + y^3$

$$\Rightarrow \frac{dy}{dx} = \frac{x^3 + y^3}{xy^2} = \frac{x^3(1 + (y/x)^3)}{x^3(y^2/x^2)}$$

$$= \frac{1 + [y/x]^3}{[y/x]^2}$$

hmm. suppose we change variables?

$$u = y/x$$

Let's replace y dependent with u dependent

$$\rightarrow u = y/x \rightarrow y = x \cdot u$$

$$\Rightarrow \frac{dy}{dx} = u + x \frac{du}{dx}$$

So

$$\frac{dy}{dx} = \frac{x^3 + y^3}{xy^2}$$

Replaces

$$u + x \frac{du}{dx} = \frac{1 + u^3}{u^2}$$

$$\Rightarrow x \frac{du}{dx} = \frac{1 + u^3}{u^2} - u$$

$$\Rightarrow x \frac{du}{dx} = \frac{1 + u^3 - u^3}{u^2} = \frac{1}{u^2}$$

$$\Rightarrow u^2 \frac{du}{dx} = \frac{1}{x}$$

$$\Rightarrow \int u^2 du = \int \frac{1}{x} dx$$

$$\frac{u^3}{3} = \ln|x| + C_1$$

$$\Rightarrow u^3 = 3 \ln|x| + 3C_1$$

$$\Rightarrow u = \sqrt[3]{\ln x^3 + C_2}$$

$$\Rightarrow \frac{y}{x} = \sqrt[3]{\ln x^3 + C_2}$$

$$\Rightarrow y = x \sqrt[3]{\ln x^3 + C_2}$$

$$y = x (\ln|x| + C_1)^{1/3}$$

Chapter 8

1. 1st order lin
2. separable
3. separable
4. sub $u = x + y$
5. directly integrable
6. do some algebra

$$xy \frac{dy}{dx} - y^2 = \sqrt{x^2 + y^2} \Rightarrow xy \frac{dy}{dx} = y^2 + x \cdot \sqrt{x^2 + y^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2}{xy} + \frac{1}{y} \sqrt{x^2 + y^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \sqrt{\frac{x^2 + y^2}{y^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \sqrt{\frac{x^2}{y^2} + 1}$$

$$\Rightarrow \frac{dy}{dx} = \left[\frac{y}{x} \right] + \sqrt{1 + \frac{x^2}{y^2}}$$

$$\Rightarrow \frac{dy}{dx} = \left[\frac{y}{x} \right] + \left(\frac{x^2}{y^2} + 1 \right)^{1/2}$$

$$u = y/x \Rightarrow \frac{dy}{dx} = u + x \frac{du}{dx}$$

$$= u + x \frac{du}{dx} = u + \frac{1}{u} (u^2 + 1)^{1/2}$$

$$\Rightarrow x \frac{du}{dx} = \frac{1}{u} (u^2 + 1)^{1/2}$$

$$\Rightarrow \frac{u}{(u^2 + 1)^{1/2}} \frac{du}{dx} = \frac{1}{x}$$

*7: factor

$$\frac{dy}{dx} = (y - x)^2$$

$$\Rightarrow u = y - x \Rightarrow y = u + x \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + 1$$

$$\Rightarrow \frac{du}{dx} + 1 = u^2$$