Quiz 9 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, SPRING 2024

NAME: Solutions A#: ______

Problem 1. Exercise 20.4b (10 points) Compute the general solution of the following Euler equation.

$$x^3\, y''' + 2\, x^2\, y'' + x\, y' - y = 0$$

Solution:

$x^3 y''' + 2x^2 y'' + xy' - y = 0$

$\hookrightarrow\ r(r-1)(r-2) + 2r(r-1) + r - 1 = 0$

$\Rightarrow (r-1)(r(r-2) + 2r - 1) = 0$

$\Rightarrow (r-1)(r^2 - \cancel{2r} + \cancel{2r} - 1) = 0$

$\Rightarrow (r-1)(r^2-1) = 0$

$\Rightarrow (r-1)^2(r+1) = 0$

$r_1 = 1,\ r_2 = 1,\ r_3 = -1$

$y_1 = x,\ y_2 = x\ln|x|,\ y_3 = x^{-1}$

$\Rightarrow y = c_1 x + c_2 x\ln|x| + c_3 x^{-1}$

Problem 2. Exercise 21.12 (10 points) Comsoder the following nonhomogeneous linear differential equation.

$$y^{(4)} + y'' = 1$$

a. Verify that one particular solution to this equation is

$$y_p(x) = \frac{1}{2}x^2$$

b. Find the general solution of the differential equation.

Solution:

a. $y_p = \frac{1}{2}x^2$

$y_p' = x$ $\Rightarrow$ $y_p^{(4)} + y'' = 0 + 1 = 1$ ✓

$y_p'' = 1$

$y_p''' = 0$

$y_p^{(iv)} = 0$

b. Then solve the homogeneous ODE

$r^4 + r^2 = 0$

$\Rightarrow r^2(r^2+1) = 0$

$r_1 = 0, r_2 = 0, r_3 = +i, r_4 = -i$

$y_1 = 1, y_2 = x, y_3 = \cos(x), y_4 = \sin(x)$

So $y_h = C_1(1) + C_2 x + C_3 \cos(x) + C_4 \sin(x)$

and

$y = y_p + y_h = \frac{1}{2}x^2 + C_1 + C_2 x + C_3 \cos(x) + C_4 \sin(x)$