Math 2280 Homework # 11. Solutions

17.1b $y'' + 2y' - 24y = 0$

$\Rightarrow r^2 + 2r - 24 = (r+6)(r-4) = 0$

$r_1 = -6$, $r_2 = 4$

$y_1 = e^{-6x}$, $y_2 = e^{4x}$

$\Rightarrow y = c_1 e^{-6x} + c_2 e^{4x}$

17.1f $3y'' + 7y' - 6y = 0$

$\Rightarrow 3r^2 + 7r - 6 = 0$

$r = \frac{-7 \pm \sqrt{49 + 72}}{2(3)} = \frac{-7 \pm \sqrt{121}}{6} = \frac{-7 \pm 11}{6}$

$r_1 = \frac{4}{6} = \frac{2}{3}$, $r_2 = -3$

$y = c_1 e^{\frac{2}{3}x} + c_2 e^{-3x}$

17.2c $y'' - 9y = 0 \quad y(0) = 0, \; y'(0) = 1$

$r^2 - 9 = (r-3)(r+3) = 0$

$r_1 = 3$, $r_2 = -3$

$y_1 = e^{3x}$, $y_2 = e^{-3x}$

$\Rightarrow y = c_1 e^{3x} + c_2 e^{-3x} \Rightarrow y(0) = c_1 + c_2 = 0 \Rightarrow c_2 = -c_1$

$y' = 3c_1 e^{3x} - 3c_2 e^{-3x} \Rightarrow y'(0) = 3c_1 - 3c_2 = 1 \Rightarrow 3c_1 + 3c_1 = 1$

$\Rightarrow 6c_1 = 1 \Rightarrow c_1 = \frac{1}{6}$

$c_2 = -\frac{1}{6}$

$\Rightarrow y = \frac{1}{6} e^{3x} - \frac{1}{6} e^{-3x}$

17.3a $y'' - 10y' + 25 = 0$

$\Rightarrow r^2 - 10r + 25 = (r-5)^2 = 0$

$r_1 = 5, \quad r_2 = 5$

$y_1 = e^{5x}, \quad y_2 = xe^{5x}$

So $y = c_1 e^{5x} + c_1 x e^{5x}$

17.3e $16y'' - 24y' + 9y = 0$

$\Rightarrow 16r^2 - 24r + 9 = 0$

$r = \frac{24 \pm \sqrt{576 - 576}}{32} = \frac{24}{32} = \frac{3}{4}$

$\Rightarrow y_1 = e^{\frac{3}{4}x}, \quad y_2 = xe^{\frac{3}{4}x}$

$y = c_1 e^{\frac{3}{4}x} + c_1 x e^{\frac{3}{4}x}$

17.4. b $y'' - 8y' + 16y \qquad y(0) = 0, \quad y'(0) = 1$

$\Rightarrow r^2 - 8r + 16 = 0$

$\Rightarrow (r-4)^2 = 0$

$r_1 = 4, \quad r_2 = 4$

$y_1 = e^{4x}, \quad y_2 = xe^{4x}$

So $y = c_1 e^{4x} + c_2 x e^{4x} \Rightarrow 0 = y(0) = c_1 e^0 + c_2 (0) e^0 = c_1 = 0$

$y' = 4c_1 e^{4x} + c_2 (e^{4x} + 4x e^{4x}) \qquad y'(0) = 1 = 4c_1 + c_2 (1+0) = 4c_1 + c_2$

$\Rightarrow c_2 = 1 = 0 \cdot xe^{4x}$

17.5a $y'' + 25y = 0$

$\hookrightarrow r^2 + 25 = 0 \Rightarrow r = \pm\sqrt{-25} = \pm 5i$

$r_1 = 5i, \quad r_2 = -5i$

$y_1 = \cos(5x), \quad y_2 = \sin(5x)$

$\Rightarrow y = C_1\cos(5x) + C_2\sin(5x)$

17.5c $y'' - 2y' + 5y = 0$

$\hookrightarrow r^2 - 2r + 5 = 0$

$\hookrightarrow (r^2 - 2r + 1) + 4 = 0$

$\hookrightarrow (r-1)^2 = -4$

$\hookrightarrow r - 1 = \pm 2i \Rightarrow r = 1 \pm 2i$

$r_1 = 1 + 2i, \quad r_2 = 1 - 2i$

$y_1 = e^x\cos(2x), \quad y_2 = e^x\sin(2x)$

$\Rightarrow y = C_1 e^x\cos(2x) + C_2 e^x\sin(2x)$

17.6d $y'' - 4y' + 13y = 0 \qquad y(0) = 1, \quad y'(0) = 0$

$\hookrightarrow r^2 - 4r + 13 = 0$

$\hookrightarrow r^2 - 4r + 4 + 9 = 0$

$\hookrightarrow (r-2)^2 + 9 = 0$

$\hookrightarrow r = 2 \pm 3i$

$y_1 = e^{2x}\cos(3x), \quad y_2 = e^{2x}\sin(3x)$

$y = C_1 e^{2x}\cos(3x) + C_2 e^{2x}\sin(3x)$

$\rightarrow y(0) = 1 = C_1(1)(1) + 0 = 1$

$\Rightarrow C_1 = 1$

$y'(x) = C_1\left(2e^{2x}\cos(3x) - 3e^{2x}\sin(3x)\right) + C_2\left(2e^{2x}\sin(3x) + 3e^{2x}\cos(3x)\right)$

$y'(0) = C_1(2 - 0) + C_2(0 + 3) = 0$

$2C_1 + 3C_2 = 0 \Rightarrow C_2 = -\frac{2}{3}$

$y = e^{2x}\cos(3x) - \frac{2}{3}e^{2x}\sin(3x)$

17.8a

$y'' - 9y = 0$

$r^2 - 9 = 0 \Rightarrow (r-3)(r+3) = 0$

$\Rightarrow r_1 = 3,\ r_2 = -3$

$\Rightarrow y_1 = e^{3x},\ y_2 = e^{-3x}$

$\Rightarrow y = c_1 e^{3x} + c_2 e^{-3x}$

17.8b $y'' + 9y = 0$

$r^2 + 9 = 0 \Rightarrow (r+3i)(r-3i) = 0$

$r_1 = -3i,\ r_2 = 3i$

$y_1 = \cos(3x),\ y_2 = \sin(3x)$

$\Rightarrow y = c_1 \cos(3x) + c_2 \sin(3x)$

18.1 → Come su me!

(Grade on work done)

19.1.c

$y^{(4)} - 34y'' + 225y = 0$

↳ $r^4 - 34r^2 + 225 = 0$ $\quad s = r^2$

$\Rightarrow s^2 - 34s + 225 = 0$

$\Rightarrow (s-9)(s-25) = 0$

$\Rightarrow s = 9$ OR $s = 25$

$\Rightarrow r^2 = 9$ OR $r^2 = 25$

$\Rightarrow r = \pm 3$ OR $r = \pm 5$

$\Rightarrow r_1 = 3,\ r_2 = -3,\ r_3 = 5,\ r_4 = -5$

$\Rightarrow y_1 = e^{3x},\ y_2 = e^{-3x},\ y_3 = e^{5x},\ y_4 = e^{-5x}$

$\rightarrow y = C_1 e^{3x} + C_2 e^{-3x} + C_3 e^{5x} + C_4 e^{-5x}$

19.1.d

$y^{(4)} - 81y = 0$

↳ $r^4 - 81 = 0$

↳ $r^2 = \pm\sqrt{81} = \pm 9 \Rightarrow (r^2 - 9)(r^2 + 9)$

$\Rightarrow (r-3)(r+3)\cdot(r^2+9)$ $\longrightarrow r = \pm 3i$

$\Rightarrow y_1 = e^{3x},\ y_2 = e^{-3x},\ y_3 = \cos(3x),\ y_4 = \sin(3x)$

$\rightarrow y = C_1 e^{3x} + C_2 e^{-3x} + C_3 \cos(3x) + C_4 \sin(3x)$

19.2.c $y''' - 8y'' + 37y' - 50y = 0$

$\downarrow\ r^3 - 8r^2 + 37r - 50 = 0$

Try $x=1$, $(1)^3 - 8(1)^2 + 37(1) - 50 \neq 0$

Try $x=2$, $(2)^3 - 8(2)^2 + 37(2) - 50$

$= 8 - 32 + 74 - 50 = 0$ ✓

$\Rightarrow (r-2)$ is a factor!

$$r-2 \;\overline{)\; r^3 - 8r^2 + 37r - 50 \;}\; r^2 - 6r + 25$$

$r^3 - 2r^2$

$-6r^2 + 37r - 50$

$-6r^2 + 12r$

$25r - 50$

$25r - 50$

0 ✓

$y = c_1 e^{2x} + c_2 e^{3x}\cos(4x) + c_3 e^{3x}\sin(4x)$

$r^3 - 8r + 37r - 50$

$= (r-2)(r^2 - 6r + 25)$

$= (r-2)(r^2 - 6r + 9 + 16)$

$= (r-2)((r-3)^2 + 16)$

$\Rightarrow r_1 = 2,\ r_2 = 3+4i,\ r_3 = 3-4i$

19.3.a $y''' - 8y = 0$

$\downarrow\ r^3 - 8 = 0$

$\downarrow\ (r-2)(r^2 + 2r + 4)$

$\downarrow\ (r-2)((r^2 + 2r + 1) + 3)$

$\downarrow\ (r-2)((r+1)^2 + 3)$

$r_1 = 2,\ r_2 = -1 + \sqrt{3}i,\ r_3 = -1 - \sqrt{3}i$

$y = c_1 e^{2x} + c_2 e^{-x}\cos(\sqrt{3}x) + c_3 e^{-x}\sin(\sqrt{3}x)$

$$r-2 \;\overline{)\; r^3 - 8 \;}\; r^2 + 2r + 4$$

$r^3 - 2r^2$

$2r^2 - 8$

$2r^2 - 4r$

$4r - 8$