Practice Quiz 1 Math 2280, Ordinary Differential Equations, Fall 2023

Name: Solution   A#: ⎯⎯⎯⎯⎯⎯

Problem 1. Section 1.3c (10 points) For each differential equation given three choices for a possible solution $y = y(x)$ are given. Determine whether each choice is or is not a solution to the given differential equation. (In each case, assume the interval of interest is the entire real line $(-\infty, \infty)$

$$\frac{d^2y}{dx^2} = 9\,y$$

i.) $y(x) = e^{3x}$ **ii.)** $y(x) = x^3$ **iii.)** $y(x) = sin(3x)$

Solution:

Check each possible solution for $\frac{d^2y}{dx^2} - 9y = 0$

i) $y(x) = e^{3x} \Rightarrow \frac{dy}{dx} = 3e^{3x}, \quad \frac{d^2y}{dx^2} = 9e^{3x}$

$\Rightarrow \frac{d^2y}{dx^2} - 9y = 9e^{3x} - 9(e^{3x}) = 0 \quad$ This is a solution

ii) $y(x) = x^3 \Rightarrow \frac{dy}{dx} = 3x^2, \quad \frac{d^2y}{dx^2} = 6x$

$\Rightarrow \frac{d^2y}{dx^2} - 9y = 6x - x^3 = x(6 - x^2) \neq 0 \Rightarrow$ Not a solution.

iii) $y(x) = sin(3x) \Rightarrow \frac{dy}{dx} = 3cos(3x), \quad \frac{d^2y}{dx^2} = -9sin(3x)$

$\Rightarrow \frac{d^2y}{dx^2} - 9y = -9sin(3x) - 9(sin(3x)) = -18 sin(3x)$

Problem 2. Section 1.4c (10 points) For each initial value problem given below, three choices for a possible solution, $y = y(x)$ are given. Determine whether each choice is or is not a solution to the given initial-value problem.

$$\frac{d^2y}{dx^2} - 9\,y = 0$$

with $y(0) = 1$ and $y'(0) = 9$.

i.) $y(x) = 2e^{3x} - e^{-3x}$ **ii.)** $y(x) = e^{3x}$ **iii.)** $y(x) = e^{3x} + 1$

Solution:

i) $y(x) = 2e^{3x} - e^{-3x} \Rightarrow \frac{dy}{dx} = 6e^{3x} + 3e^{-3x}, \quad \frac{d^2y}{dx^2} = 18e^{3x} - 6e^{-3x}$

$\Rightarrow \frac{d^2y}{dx^2} - 9y = 18e^{3x} - 6e^{-3x} - 9(2e^{3x} - e^{-3x})$

$= 0 + 0 = 0 \Rightarrow$ This solves the ODE

$y(0) = 2e^{(0)} - e^{0} = 2 - 1 = 1 \checkmark$

$y'(0) = 6e^{0} + 3e^{0} = 6 + 3 = 9 \checkmark$

$\Rightarrow$ The functions satisfies the initial conditions

ii) $y = e^{3x} \Rightarrow y' = 3e^{3x}, \quad y'' = 9e^{3x}$

$\Rightarrow y'' - 9y = 9e^{3x} - 9(e^{3x}) = 0 \Rightarrow$ this we solution

$y(0) = e^{0} = 1, \quad y'(0) = 3e^{0} = 3 \neq 9$

$\Rightarrow$ This means y does not satisfy the initial conditions.

iii) $y = e^{3x} + 1 \Rightarrow y' = 3e^{3x} + 0, \quad y'' = 9e^{3x}$

$\Rightarrow y'' - 9y = 9e^{3x} - 9(e^{3x} + 1) = 9e^{3x} - 9e^{3x} + 1 \neq 0$

This does not satisfy the DE