Quiz 2 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, FALL 2023

NAME: Solutions A#: ________________

Problem 1. Section 2.3g (10 points) Find a general solution for the following directly integrable differential equation. (Use an indefinite integral in this case.

$$x = (x^2 - 9)\,\frac{dy}{dx}$$

Solution:

First, solve for $\frac{dy}{dx}$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{x^2-9}$$

Now integrate

$$\int \frac{dy}{dx}\,dx = \int \frac{x}{x^2-9}\,dx$$

$$y(x) = \int \frac{x}{x^2-9}\,dx$$

$$u = x^2 - 9$$

$$du = 2x\,dx \Rightarrow x\,dx = \tfrac{1}{2}\,du$$

$$= \int \frac{1/2}{u}\,du$$

$$= \tfrac{1}{2}\ln|u| + C$$

So, $$y(x) = \tfrac{1}{2}\ln|x^2-9| + C$$

Problem 2. Section 2.4a (10 points) Solve the following initial problem (using the indefinite integral). Also state the largest interval over which the solution is valid (i.e, the maximum possible intergal of interest.

$$\frac{dy}{dx} = 4\,x + 10e^{2x}$$

with $y(0) = 4$.

Solution:

In this case we can integrate

$$\int \frac{dy}{dx}\,dx = \int (4x + 10e^{2x})\,dx$$

$$\Rightarrow y(x) = 4\int x\,dx + 10\int e^{2x}\,dx$$

$$= 4\left(\tfrac{1}{2}x^2\right) + 10\left(\tfrac{1}{2}e^{2x}\right) + C$$

combines all 3 integration constants

$$= 2x^2 + 5e^{2x} + C$$

Then $y(0) = 4$ implies

$$y(0) = 2(0)^2 + 5e^0 + C = 5 + C = 4$$

$$\Rightarrow C = -1$$

So

$$\boxed{y(x) = 2x^2 + 5e^{2x} - 1}$$