Quiz 10

MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, SPRING 2024

NAME: Solutions A#: _____

Problem 1. Exercise 24.1.g (10 points) Find the general solution of the following ordinary differential equation. Use variation of parameters even if another method might seem easier. For you convenience the equation is accompanied by a general solution to the corresponding homogeneous equation.

$$x^2\,y'' + x\,y' - y = \sqrt{x}, \qquad y_h = c_1\,x + c_2\,x^{-1}$$

Solution:

For this problem

$a = x^2$ $\quad g(x) = \sqrt{x} = x^{1/2}$ $\quad \Rightarrow \quad W = \begin{vmatrix} x & x^{-1} \\ 1 & -x^{-2} \end{vmatrix} = -x^{-1} - x^{-1} = -2x^{-1}$

$y_1 = x$ $\quad y_2 = x^{-1}$

So $y = y_1 u + y_2 v$ implies

$$u = -\int \frac{x^{-1}\cdot x^{1/2}}{(-2x^{-1})\,x^2} = +\int \frac{x^{1/2}}{2x^2}\,dx = \frac{1}{2}\int x^{-3/2}\,dx = \frac{1}{2}\cdot\left(-2x^{-1/2}\right) + C$$

$$= -x^{-1/2} + C$$

$$v = \int \frac{x\cdot x^{1/2}}{-2x}\,dx = -\frac{1}{2}\int x^{1/2}\,dx = -\frac{1}{2}\left(\frac{2}{3}x^{3/2}\right) + C_1 = -\frac{1}{3}x^{3/2} + C_2$$

$$\Rightarrow y = x\left(x^{-1/2} + C_1\right) + x^{-1}\left(-\frac{1}{3}x^{3/2} + C_2\right)$$

$$= x^{1/2} + C_1 x - \frac{1}{3}x^{1/2} + C_2 x^{-1}$$

$$= \underbrace{\frac{2}{3}x^{1/2}}_{y_p} + \underbrace{C_1 x + C_2 x^{-1}}_{y_h}$$