Practice Quiz 5 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, SPRING 2024

NAME: Solution A#: ________________

Problem 1. Exercise 8.27 (10 points) Use any of the methods for first order ODEs to find a general solution for the following ODE.

$$1-(x+2\,y)\,\frac{dy}{dx}=0$$

Solution:

$M(x,y)=1 \qquad \partial M/\partial y = 0$

$N(x,y)=-(x+2y) \qquad \frac{\partial N}{\partial x}=-1 \neq$

This equation is not in exact form

$\frac{dy}{dx}=\frac{1}{x+2y} \qquad u=x+2y$

$\frac{du}{dx}=1+2\frac{dy}{dx} \Rightarrow \frac{dy}{dx}=\frac{1}{2}\left(\frac{du}{dx}-1\right)$

$\Rightarrow \frac{1}{2}\left(\frac{du}{dx}-1\right)=\frac{1}{u}$

$\Rightarrow \frac{du}{dx}=\frac{2}{u}+1=\frac{2+u}{u}$

$\Rightarrow \int \frac{u}{2+u}\,du=\int dx$

$\Rightarrow \int \frac{2+u-2}{2+u}\,du = x+C_1$

$\Rightarrow \int\left(1-\frac{2}{u+2}\right)du = x+C_1$

$\Rightarrow u-2\ln|u+2| = x+C_1$

$\Rightarrow \cancel{x}+2y-2\ln|x+2y+2| = \cancel{x}+C_1$

$\Rightarrow 2y-2\ln(x+2y+2)=C_1$

$\Rightarrow y-\ln(x+2y+2)=C_2$

Problem 2. Exercise 8.39 (10 points) Use any of the methods for first order ODEs to find a general solution for the following ODE.

$$1 + y\, e^{xy} + x\, e^{xy} \frac{dy}{dx} = 0$$

Solution:

$M(x,y) = 1 + y e^{xy}$ $\quad \frac{\partial M}{\partial y} = e^{xy} + y e^{xy} \cdot x = e^{xy}(1+yx)$ ✓ equal

$N(x,y) = x e^{xy}$ $\quad \frac{\partial N}{\partial x} = e^{xy} + x e^{xy} \cdot y = e^{xy}(1+xy)$

So, the equation is in exact form.

$\frac{\partial \varphi}{\partial x} = 1 + y e^{xy} \Rightarrow \phi(x,y) = x + y \cdot \frac{1}{y} e^{xy} + g(y) = x + e^{xy} + g(y)$

$\Rightarrow \frac{\partial \phi}{\partial y}(x,y) = 0 + \cancel{e^{xy} \cdot x} + g'(y)$

$= \cancel{x e^{xy}}$

$\Rightarrow g'(y) = 0 \Rightarrow g(y) = C_1$

$\Rightarrow \phi(x,y) = x + e^{xy} = C_1$

The solution is $\phi(x,y) = C_2 \Rightarrow \boxed{x + e^{xy} = C}$

$\Rightarrow e^{xy} = C - x$

$\Rightarrow xy = \ln|C - x|$

$\Rightarrow y = \frac{1}{x} \ln|C - x|$