Math 2280 Lecture Notes: Day 17 ①

Ex: $-\frac{y}{x^2+y^2} + \frac{x}{x^2+y^2}\frac{dy}{dx} = 0$

$$M(x,y) = -\frac{y}{x^2+y^2} \Rightarrow \frac{\partial M}{\partial y} = -\frac{(1)(x^2+y^2) - y(2y)}{(x^2+y^2)^2} = \frac{y^2-x^2}{x^2+y^2}$$

$$N(x,y) = \frac{x}{x^2+y^2} \Rightarrow \frac{\partial N}{\partial x} = \frac{(1)(x^2+y^2)-(2x)x}{(x^2+y^2)^2} = \frac{y^2-x^2}{x^2+y^2}$$

So we can try to compute a potential function

$$\frac{\partial \psi}{\partial x} = M(x,y) = \frac{-y}{x^2+y^2} = \frac{-y}{y^2}\cdot\frac{1}{(x/y)^2+1}$$

So,
$$\psi(x,y) = \int -\frac{1}{y}\frac{1}{(x/y)^2+1}\,dx$$

$$= -\frac{1}{y}\int \frac{1}{\left(\frac{x^2}{y^2}\right)+1}\,dx \qquad u = x/y \Rightarrow du = \frac{1}{y}dx$$

$$= -\frac{1}{y}\int \frac{1}{u^2+1}\,y\,du$$

$$= -(1)\int \frac{1}{u^2+1}\,du = -\arctan(u) + p(y)$$

$$= -\arctan\left(\frac{x}{y}\right) + p(y)$$

Then
$$\frac{\partial \psi}{\partial y} = -\frac{1}{1+(x/y)^2}\cdot\frac{\partial}{\partial y}\left(\frac{x}{y}\right) + p'(y)$$

$$= -\frac{1}{1+x^2/y^2}\cdot x(-y^{-2}) + p'(y)$$

$$= \frac{x}{1+x^2/y^2}\cdot\frac{1}{y^2} + p'(y)$$

$$= \frac{x}{\cancel{y^2+x^2}} + p'(y) = \frac{x}{\cancel{x^2+y^2}} \Rightarrow p'(y) = 0 \Rightarrow p(y) = \text{const.}$$

Alg. 1. Write the equation as

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

2. Test for exactness: Is

$$\frac{\partial M}{\partial y} \overset{?}{=} \frac{\partial N}{\partial x}$$

3. Set $\frac{\partial \phi}{\partial x} = M$ and integrate

$$\phi(x,y) = \int M(x,y)\,dx + p(y)$$

4. Diff. the result

$$\frac{\partial \phi}{\partial y} = \frac{\partial}{\partial y}\int M(x,y)\,dx + p'(y)$$

5. Compare to $N(x,y)$

6. Find $p(y)$

7. Set $\phi(x,y) = C$.

Slope field tools - online

page 150 - 151 Logistic Model

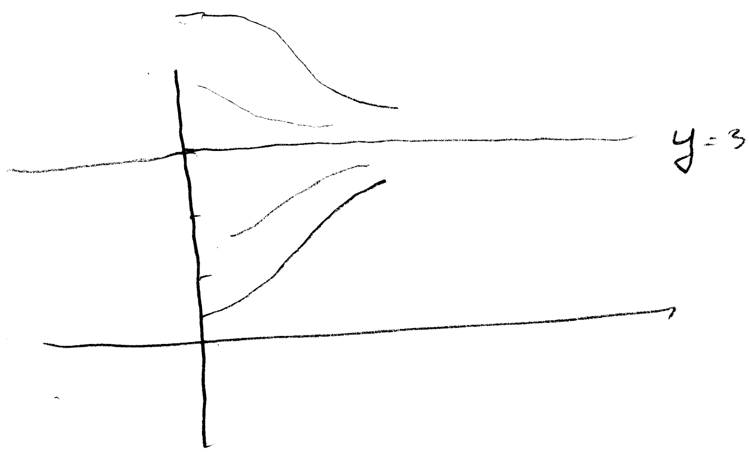
Make copies for logistic curve

Long Term Behavior

Constant Solution and Stability.

<u>Ex</u>: $\frac{dy}{dx} = \frac{x}{4}(3-y)$

$y = 3 \Rightarrow$ constant solution.



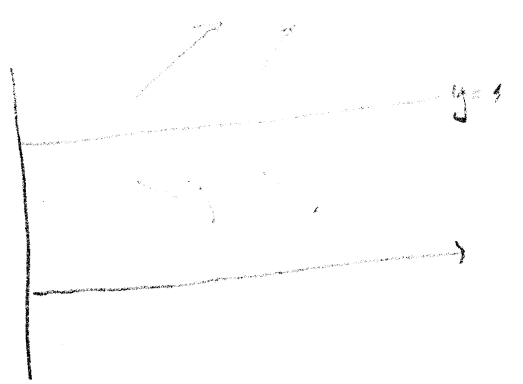
$\longrightarrow$ long term behavior in $+x$ direction.

If the solutions all approach the same constant, the solution is said to be stable

Ex $\frac{dy}{dx} = \frac{1}{3}(y-3)^{1/3}$

$y \geq 3 \Rightarrow \frac{dy}{dx} \geq 0$

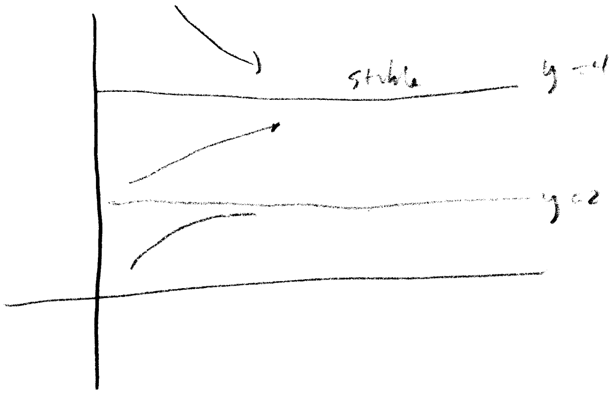
$y < 3 \Rightarrow \frac{dy}{dx} < 0$



Solutions move away from the constant solutions. If this is the case, the constant solution is unstable.

Ex: $\frac{dy}{dx}$ $\frac{y-2}{6e^{x/2}-2}$ $\Rightarrow$ not asymptotically stable.

Ex: $\frac{dy}{dx} = \frac{1}{2}(4-y)(y-2)^{4/3}$



Skip: Chapter 10 - 56201 ←

Chapter 11.

The problem is: We are given a breeding pair of rabbits with no predators. We release the pair on limitless acreage. How many rabbits will we have in 5 yrs.

Set up the models:

t = number of months since release

$R = R(t)$ = number of rabbits at time t.

$R(0) = R(2)$ ← implies initial value problem

$\frac{dR}{dt}$ = rate of change in number of rabbits.

$\frac{dR}{dt}$ = change in number of rabbits per month

= number of births per month − number of deaths per month.

number of births per month

= number of births per female rabbit per month

$\times$ number of female rabbits that month

= number of births per female rabbits per month

$\times \frac{1}{2} R(t)$

Define $\beta = \frac{1}{2} \times$ number of births per female rabbit per month

$\Rightarrow$ number of births per month $= \beta R$

Checking with the extension office on average each rabbit will have 6 litters per year. with 5 bunnies per litter. So

$$\beta = \left(\tfrac{1}{2}\right) \cdot \left(\tfrac{1}{12}\right) \cdot (6 \cdot 5) = 5/4$$

So, we are ready to write

$$\boxed{\frac{dR}{dt} = \frac{5}{4} R, \quad R(0) = 2}$$

$\Rightarrow R(t) = A e^{5/4 t} \quad \Rightarrow t = 0 \Rightarrow Ae^0 = 2 \Rightarrow A = 2$

$\Rightarrow R(t) = 2e^{5/4 t}$.

Note:

$$R(60) = 2 \cdot e^{\frac{5}{4} 60} = 2 \cdot e^{75} \approx 7.47 \times 10^{32}$$

weight

$$W = 2.2 \times 10^{33} \text{ kilos.}$$

mass of the earth $= 6 \times 10^{24}$ kg

2×10^{30} kg << rabbits!

This motivates other models.

$$\frac{dR}{dt} = \underbrace{\tfrac{5}{4} R}_{\text{groth}} - \gamma R^2$$

might work better

- Radio Active Decay
- Mixing Models
- Thermo dynamics Models! DWR example