Practice Quiz 12 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, SPRING 2024

NAME: Solutions A#: _____

Problem 1. Exercise 28.4a (10 points) Solve the following initial value problem using the Laplace transform.

$$y' + 9\,y = 0,$$

with $y(0) = 4$.

Solution:

$\mathcal{L}[y' + 9y] = \mathcal{L}[0]$

$\hookrightarrow sY(s) - y(0) + 9Y(s) = 0$

$\hookrightarrow (s+9)Y(s) - 4 = 0$

$\hookrightarrow Y(s) = \frac{4}{s+9}$

$\Rightarrow \boxed{y(t) = 4e^{-9t}}$

Problem 2. Exercise 29.4c (10 points) Using convolution compute the inverse Laplace transform of the following.

$$\frac{1}{s\,(s^2+4)}$$

Solution:

If $Y(s) = \frac{1}{s} \cdot \frac{1}{s^2+4}$ with $F(s) = \frac{1}{s}$, $G(s) = \frac{1}{s^2+4}$

$\Downarrow$ $\Downarrow$

$f(t) = 1$ $g(t) = \frac{1}{2} \cdot \sin(2t)$

$$\Rightarrow y(t) = \int_0^t (1) \cdot \left(\tfrac{1}{2}\sin(2t-2x)\right) dx$$

$$= \int_0^t \tfrac{1}{2}\sin(2x) \cdot (1)\, dx$$

$$= \frac{1}{2}\int_0^t \sin(2x)\, dx$$

$$= \frac{1}{2}\left(-\frac{1}{2}\cos(2x)\right)\Big|_0^t$$

$$= -\frac{1}{4}\left(\cos(2t) - \cos(0)\right)$$

$$= \frac{1}{4}\left(1 - \cos(2t)\right)$$