Practice Quiz 7 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, SPRING 2024

NAME: Solutions A#: _____

Problem 1. Exercise 15.2b (10 points) The following is a second order linear homogeneous differential equation and a pair of functions $y_1(x)$ and $y_2(x)$. Verify that the pair of functions forms a fundamental set of solutions to the given differential equation. That is, verify both $y_1(x)$ and $y_2(x)$ satisfy the equation on there own, then verify the solutions are independent of each other, and then write down the general solution for the differential equation. Finally apply the initial conditions to define the exact solution for the initial-value problem.

$$y'' - 4\,y = 0$$

with $y(0) = 0$ and $y'(0) = 12$. The functions are:

$$y_1(x) = e^{2x} \qquad y_2(x) = e^{-2x}$$

Solution:

$y_1 = e^{2x} \Rightarrow y_1' = 2e^{2x}$, $y_1'' = 4e^{2x}$

$\Rightarrow y_1'' - 4y_1 = (4e^{2x}) - 4(e^{2x}) = (4-4)e^{2x} = 0$ ✓

$y_2 = e^{-2x} \Rightarrow y_2' = -2e^{-2x}$, $y_2'' = 4e^{-2x}$

$\Rightarrow y_2'' - 4y_2 = (4e^{-2x}) - 4(e^{-2x}) = e^{-2x}(4-4) = 0$ ✓

Both satisfy the equation

$$W(e^{2x}, e^{-2x}) = \begin{vmatrix} e^{2x} & e^{-2x} \\ 2e^{2x} & -2e^{-2x} \end{vmatrix} = -2e^{0} - 2e^{0} = -4 \neq 0$$

$\Rightarrow y_1$ and y_2 are linearly independent ✓

So $\{y_1, y_2\}$ is a fundamental set of functions ✓

$y = c_1 e^{2x} + c_2 e^{-2x} \Rightarrow y' = 2c_1 e^{2x} - 2c_2 e^{-2x}$

$y(0) = c_1 e^{0} + c_2 e^{0} = c_1 + c_2 = 0 \Rightarrow c_2 = -c_1$ ✓

$y'(0) = 2c_1 e^{0} - 2c_2 e^{0} = 2c_1 - 2c_2 = 12 \rightarrow 2c_1 + 2c_1 = 12 \Rightarrow 4c_1 = 12 \Rightarrow c_1 = 3$

$c_1 = -3$ ✓

$y = -3e^{2x} + 3e^{-2x}$

Problem 2. Exercise 17.1d (10 points) Find the general solution to the following:

$$y'' + 3\,y' = 0$$

Solution:

$\Rightarrow y = e^{rx}$

$\Rightarrow r^2 + 3r = 0$

$\Rightarrow r(r+3) = 0$

$r_1 = 0, r_2 = -3$

$y_1 = 1, y_2 = e^{-3x}$

$\Rightarrow y = c_1 y_1 + c_2 y_2$

$= c_1 + c_2 e^{-3x}$