Quiz 5 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, FALL 2023

NAME: Solution A#: ____________

Problem 1. Chapter 6.6 (10 points) Use a substitution appropriate to a Bernoulli equation to solve the following initial value problem.

$$\frac{dy}{dx} - \frac{1}{x}\,y = \frac{1}{y}, \qquad y(1) = 3$$

Solution:

$\frac{dy}{dx} - \frac{1}{x} y = y^{-1}$

So, $n = -1$ and $1-n = 1-(-1) = 2$

$u = y^{v} = y^{1-n} = y^2$

$\Rightarrow y = u^{1/2} \Rightarrow \frac{dy}{dx} = \frac{1}{2} u^{-1/2} \frac{du}{dx}$

$\Downarrow$

$\frac{1}{2} u^{-1/2} \frac{du}{dx} - \frac{1}{x} u^{1/2} = (u^{1/2})^{-1}$

$\Rightarrow \frac{1}{2} u^{-1/2} \frac{du}{dx} - \frac{1}{x} u^{1/2} = u^{-1/2}$. mult. by $u^{1/2}$

$\Rightarrow \frac{1}{2} \frac{du}{dx} - \frac{1}{x} u = 1$

$\Rightarrow \frac{du}{dx} - \frac{2}{x} u = 2$

$\mu = e^{\int -2/x \, dx} = e^{-2 \ln|x|} = e^{\ln x^{-2}} = x^{-2}$

$\Rightarrow x^{-2} \frac{du}{dx} - \frac{2}{x} \cdot x^{-2} u = 2x^{-2}$

$\Rightarrow \frac{d}{dx}\left[x^{-2} u\right] = 2x^{-2}$

$\Rightarrow x^{-2} u = \int 2x^{-2} \, dx$

$\Rightarrow x^{-2} u = -2x^{-1} + C$

$u = -2x + Cx^2$

$y^2 = -2x + Cx^2$

$y(1) = 3 \Rightarrow$

$9 = -2(1) + C(1)^2$

$\Rightarrow C = 11$

$y^2 = -2x + 11x^2$

$\Rightarrow y = \pm\sqrt{-2x + 11x^2}$

7.4e

Problem 2. Section ~~6.1b~~ (10 points) The following differential equation is in exact form. Find the corresponding potential function and then find a general solution to the differential equation using that potential function (even if it can be solved by simpler means).

$$4\,x^3\,y + [x^4 - y^4]\,\frac{dy}{dx} = 0$$

Solution:

$$M(x,y) = 4x^3y \Rightarrow \frac{\partial M}{\partial y} = 4x^3$$
$$N(x,y) = x^4 - y^4 \Rightarrow \frac{\partial N}{\partial x} = 4x^3$$
✓

then

$$\frac{\partial \phi}{\partial x} = 4x^3y \Rightarrow \phi(x,y) = x^4y + p(y)$$

$$\frac{\partial \phi}{\partial y} = x^4 - y^4$$

$$\frac{\partial \phi}{\partial y} = \cancel{x^4} + p'(y) = \cancel{x^4} - y^4 \Rightarrow p'(y) = -y^4 \Rightarrow p(y) = -\frac{1}{5}y^5$$

So

$$\phi(x,y) = x^4y + \left(-\tfrac{1}{5}y^5\right) = x^4y - \tfrac{1}{5}y^5 = C$$

Define the solution

$$x^4y - \tfrac{1}{5}y^5 = C$$

$$\Rightarrow -\tfrac{1}{5}y^5 = C - x^4y$$

$$\Rightarrow y^5 = -5C + 5x^4y$$

$\vdots$