Math ~~1220~~ 2280 Lecture Notes Day 32.

Bray 109 6-8

Let's do examples:

Theorem: A general solution any given nonhomogeneous linear DE is given by

$$y = y_p + y_h$$

where y_p is the particular solution to the given DE and y_h is the associated general solution of the homogeneous ODE.

Ex: $y_1(x) = e^{3x}$ Satisfies

$$y_p'' - 4y_p = 9e^{3x} - 4e^{3x} = 5e^{3x} \checkmark \Rightarrow y_p = \tfrac{1}{5}e^{3x}$$

Suppose we find y_h from solving

$$y_h'' - 4y_h = 0 \Rightarrow y_1 = e^{2x},\ y_2 = e^{-2x}$$

So

$$y_h(x) = C_1e^{2x} + C_2e^{-2x}$$

$$\Rightarrow y = y_p + y_h = \tfrac{1}{5}e^{3x} + \underbrace{C_1e^{2x} + C_2e^{-2x}}_{L y_h = 0}$$

$y_p' = 3Ae^{3x}$

$y_p'' = 9Ae^{3x}$

$= (9-4)e^{3x}A$

$= 5e^{3x}Ae^{3x}$

The idea is that $y_p = Ae^{3x} \Rightarrow A = 1/5$

Ex: $y'' - 4y = 2x^2 - 8x + 3$

$y_h = c_1 e^{2x} + c_2 e^{-2x}$

$y_p = ? = Ax^2 + Bx + C$

$y_p' = 2Ax + B$

$y_p'' = 2A$

$2A - 4(Ax^2 + Bx + C) = 2x^2 - 8x + 3$

$\Rightarrow -4Ax^2 - 4Bx + (2A - 4C) = 2x^2 - 8x + 3$

$\Rightarrow$ $-4A = 2 \Rightarrow A = -\frac{1}{2}$

$-4B = -8 \Rightarrow B = 2$

$(2A - 4C) = 3 \Rightarrow C = -1$

$\boxed{y_p = -\frac{1}{2}x^2 + 2x - 1}$

Ex: $y'' - 2y' - 3y = 36e^{5x}$

y_h: $r^2 - 2r - 3 = 0$

$\Rightarrow (r-3)(r+1) = 0$

$r_1 = 3,\ r_2 = -1$

$y_1 = e^{3x};\ y_2 = e^{-x} \Rightarrow \boxed{y_h = c_1 e^{3x} + c_2 e^{-x}}$

$y_p = Ae^{5x}$

$y_p' = 5Ae^{5x} \Rightarrow 25Ae^{5x} - 2(5Ae^{5x}) - 3Ae^{5x}$

$y_p'' = 25Ae^{5x}$

$= 12Ae^{5x} = 36e^{5x} \Rightarrow \Rightarrow A = 3$

$y'' - 2y' - 3y =$ [illegible]

$y_p =$