Practice Quiz 4 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, FALL 2023

NAME: Solution A#: ______________

Problem 1. Chapter 5. 5.c Using the methods developed in Chapter 5 for First Order Linear differential equations, find the general solution of the following ODE.

$$\frac{dy}{dx} = 4\,y + 16\,x$$

Solution:

First, we change the form to that of a first order linear ODE

$$\frac{dy}{dx} - 4y = 16x$$

Then

$$\mu(x) = e^{\int p(x)\,dx} = e^{\int -4\,dx} = e^{-4x}$$

So

$$\mu\left(\frac{dy}{dx} - 4y\right) = \mu\,(16x)$$

$$\Rightarrow e^{-4x}\frac{dy}{dx} - 4e^{-4x}y = 16xe^{-4x}$$

$$\Rightarrow \frac{d}{dx}\left(e^{-4x}y\right) = 16xe^{-4x}$$

$u = x \quad dv = e^{-4x}$

$du = dx \quad v = -\frac{1}{4}e^{-4x}$

$$\Rightarrow e^{-4x}y = 16\int xe^{-4x}$$

$$\Rightarrow e^{-4x}y = 16\left(-\frac{1}{4}xe^{-4x} - \int -\frac{1}{4}e^{-4x}\,dx\right)$$

$$\Rightarrow e^{-4x}y = -4xe^{-4x} + 16\int \frac{1}{4}e^{-4x}\,dx$$

$$= -4xe^{-4x} + 16\cdot\left(-\tfrac{1}{16}\right)e^{-4x} + c$$

$$\Rightarrow e^{-4x}y = -4xe^{-4x} - e^{-4x} + c$$

$$\Rightarrow y = -4x - 1 + Ce^{4x}$$

Problem 2. Chapter 6.2 (10 points) Using substitutions appropriate to homogeneous first order differential equations, as described in Section 6.3, find the general solution for the differential equation.

$$\frac{dy}{dx} = \frac{y}{x} + \frac{x}{y}$$

Solution:

We will let $u = y/x \Rightarrow y = xu \Rightarrow \frac{dy}{dx} = u + x\frac{du}{dx}$. So

$$\frac{dy}{dx} = y/x + 1/(y/x) \Rightarrow \cancel{u} + x\frac{du}{dx} = \cancel{u} + \frac{1}{u}$$

$$\Rightarrow x\frac{du}{dx} = \frac{1}{u}$$

$$\Rightarrow u\frac{du}{dx} = \frac{1}{x}$$

$$\Rightarrow u\,du = \frac{1}{x}\,dx$$

$$\Rightarrow \int u\,du = \int \frac{1}{x}\,dx$$

$$\Rightarrow \frac{1}{2}u^2 = \ln|x| + c$$

$$\Rightarrow u^2 = 2\ln|x| + 2c$$

$$\Rightarrow u^2 = \ln(x^2) + 2c$$

$$\Rightarrow u = \pm\sqrt{\ln(x^2) + 2c}$$

$$\Rightarrow \frac{y}{x} = \pm\sqrt{\ln(x^2) + 2c}$$

$$\Rightarrow y = x \pm \sqrt{\ln(x^2) + 2c}$$