Math 2280 Lecture Notes: Day 1

- Cover the syllabus for the course on github.

A little review:

Algebraic Equations: Any equation that involves one or more unknown values.

Ex: $\frac{3x+2}{5} = 11 \Rightarrow 3x+2 = 55$

$\Rightarrow 3x = 55-2 = 53$

$\Rightarrow x = 53/3$ $\qquad$ $x = 53/3$ is a solution of the algebraic equation

Note: $\frac{3x+2}{5} = 11$ is equivalent to $3x+2=55$ which is equivalent to $3x=53$ which is equivalent to $x = 53/3$ which is equivalent to $x+4 = \frac{53}{3}+4$ ...

Ex: $\cos(x) = 1 \Rightarrow x = n\pi,\ n = 0, 2, 4, \ldots$

This equation has ∞-many solutions.

Ex: $\cos(x) = 17 \Rightarrow$ no solutions.

Ex: $x^2+y^2=4 \Rightarrow$ circle of radius 2 at $(0,0)$.

Ex: $x^2+y^2=-14 \Rightarrow$ no solution

1.1 DEs

Def: An equation that involves one or more derivatives of an unknown function is called a differential equation. If all derivatives are ordinary derivatives, the equation is referred to as an ordinary differential equation or ODE. If there are any partial derivatives the equation is called a partial differential equation or PDE

Examples:

$\rightarrow \quad \frac{dy}{dx} = 4x^3$

$\rightarrow \quad \frac{dy}{dx} + \frac{1}{x} y = x^2$

$\rightarrow \quad \frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 65\cos(2x)$

$\rightarrow \quad 4x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + [4x^2 - 1]y = 0$

$\rightarrow \quad \frac{d^4y}{dx^4} = 81y$

$\vdots$

lots and lots

ODEs

$\rightarrow \quad \frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \rightarrow u = u(x,t)$

$\rightarrow \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \rightarrow u = u(x,y)$

PDEs - Math [illegible]

Order of a DE.

the highest order of derivative in any DE is the order of the ODE

So,

$\frac{dy}{dx} = 4x^2$ is 1^{st} order

$\frac{dy}{dx} + \frac{4}{x} y = x^2$ is 1^{st} order

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3y = 6x\cos(t)$ is 2^{nd} order

$4x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + [4x^2 - 1]\, y = 0$ is 2^{nd} order

$\frac{d^4y}{dx^4} = 8y$ is 4^{th} order

Ex. For the DE

$$\frac{dy}{dx} - 3y = 0$$

Let $y = e^{3x} \implies \frac{dy}{dx} = 3e^{3x}$

$\implies \frac{dy}{dx} - 3y = \cancel{3e^{3x}} - \cancel{3(e^{3x})} = 0$

$y = \cos(3x) \implies \frac{dy}{dx} = -3\sin(3x)$

$\implies \frac{dy}{dx} - 3y = -3\sin(3x) - 3\cos(3x)$

$= -3(\sin(3x) + \cos(3x)) \neq 0$

So, we see $y = e^{3x}$ "satisfies" the equation of interest and $y = \cos(3x)$ does not.

Suppose we set

$$\frac{dy}{dx} - 3y = 0, \quad y_1(x) = e^{3x}, \quad y_2(x) = 7e^{3x}$$

$$y_1' = 3e^{3x}, \quad y_2' = 21e^{3x}$$

$$\frac{dy_1}{dx} - 3y_1 = (3e^{3x}) - 3(e^{3x}) = 0 \checkmark$$

$$\frac{dy_2}{dx} - 3y_2 = (21e^{3x}) - 3(7e^{3x}) = 0 \checkmark$$

2 solutions!

Ex: $y' = 4x^3 \Rightarrow \frac{dy}{dx} = 4x^3$

$$\Rightarrow \int \frac{dy}{dx}\,dx = \int 4x^3\,dx$$

$$\Rightarrow \underbrace{y(x) + C_1 = x^4 + C_2}_{\text{Constants of integration}}$$

$$\Rightarrow y(x) = x^4 + (C_2 - C_1)$$

$$= x^4 + C_3$$

What we need is a constant of integration for each integration we do!

Ex: $\frac{d^4y}{dx^4} = 81x$

$$\Rightarrow \frac{d^3y}{dx^3} = \int 81x\,dx = \frac{81}{2}x^2 + C_1 \leftarrow \text{one constant here}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \int \left(\frac{81}{2}x^2 + C_1\right)dx = \frac{27}{2}x^3 + C_1x + C_2$$

2nd Constant!

$$\Rightarrow \frac{dy}{dx} = \int \left(\frac{27}{2} x^3 + c_1 x + c_2 \right) dx$$

$$= \frac{27}{8} x^4 + \frac{c_1}{2} x^2 + c_2 x + c_3$$

$$\Rightarrow y = \int \left(\frac{27}{8} x^4 + \frac{c_1}{2} x^2 + c_2 x + c_3 \right) dx$$

$$= \frac{27}{32} x^5 + \frac{c_1}{6} x^3 + \frac{c_2}{2} x^2 + c_3 x + c_4$$

$$= \underline{\frac{27}{32} x^5 + b_3 x^3 + b_2 x^2 + b_1 x + b_0}$$

Due to the form we can integrate 4 times an find something. So, we see that there are a lot of function that are NOT solution and an infinite number up to constants will be part of the process

Initial Value Problem

Ex $\frac{d^4 y}{dx^4} = 81x$

$$\Rightarrow y = \frac{27}{32} x^5 + b_3 x^3 + b_2 x^2 + b_1 x + b_0$$

Now, if

$y(x_0) = y_0$ = constant, then

$$y(x_0) = \frac{27}{32} x_0^5 + b_3 x_0^3 + b_2 x_0^2 + b_1 x_0 + b_0 = y_0$$ This gives ICs

$$y'(x) = \frac{27}{32} x^4 + 3b_3 x^2 + 2b_2 x + b_1$$

$$y'(x_0) = \frac{27}{32} x_0^4 + 3b_3 x_0^2 + 2b_2 x_0 + b_1 = y_0'$$