NAME: A#:

Problem 1. Section 5.3b (10 points) Find the solution the following initial-value problem using the techniques from Chapter 5.

$$\frac{dy}{dx} - 3\,y = 6 \qquad y(0) = -2$$

Solution:

$$\frac{dy}{dx} - 3y = 6$$

$$\uparrow$$

$$p(x) = -3 \implies \mu = e^{\int -3dx} = e^{-3x}$$

$$\implies \left(e^{-3x}\frac{dy}{dx} - 3e^{-3x}y\right) = 6e^{-3x}$$

$$\implies \frac{d}{dx}\left[e^{-3x}y\right] = 6e^{-3x}$$

$$\implies e^{-3x}y = -2e^{-3x} + C$$

$$\implies y = -2 + Ce^{3x}$$

I.C. $y(0) = -2$

$$\implies -2 = -2 + Ce^{0} = -2 + C \implies C = 0$$

So $\boxed{y = -2}$

Problem 2. Section 6.1b (10 points) Use lienar substitutiosn as in Section 6.2 to find the solution of the following.

$$\frac{dy}{dx} = \frac{(3x - 2y)^2 + 1}{3x - 2y} + \frac{3}{2}$$

Solution:

$u = 3x - 2y$

$$\frac{du}{dx} = 3 - 2\frac{dy}{dx} \implies -2\frac{dy}{dx} = \frac{du}{dx} - 3 \implies \frac{dy}{dx} = -\frac{1}{2}\frac{du}{dx} + \frac{3}{2}$$

So

$$-\frac{1}{2}\frac{du}{dx} + \frac{3}{2} = \frac{u^2 + 1}{u} + \frac{3}{2}$$

$$\implies -\frac{1}{2}\frac{u}{u^2+1}\frac{du}{dx} = 1$$

$$\implies \frac{1}{2}\frac{u}{u^2+1} du = -dx$$

$$\implies \int \frac{\frac{1}{2}u}{u^2+1} du = -\int dx = -x + C$$

$$\implies \ln|u^2+1| = -x + C$$

$$\implies u^2 + 1 = e^{-x+C} = Ae^{-x}$$

$$\implies u^2 = Ae^{-x} - 1$$

$$\implies u = \pm\sqrt{Ae^{-x} - 1}$$

$$\implies 3x - 2y = \pm\sqrt{Ae^{-x} - 1}$$

$$\implies -2y = -3x \pm \sqrt{Ae^{-x} - 1}$$

$$\implies y = \frac{3}{2}x \mp \frac{1}{2}\sqrt{Ae^{-x} - 1}$$