Practice Quiz 10 Math 2280, Ordinary Differential Equations, Spring 2024

+ Book hw 6.

$g(x) = e^{3x}$

Name: Solutions   A#: ____________________

Problem 1. Exercise 24.1d (10 points) Use variation of parameters to find the general solution for the following ODE, even if there is an easier method available. For your convenience the homogeneous solution is given.

$$y'' - 7\,y' + 10\,y = e^{3x}, \qquad y_h = c_1\,e^{2x} + c_2\,e^{5x}$$

Solution:

For this problem:

$a(x) = 1 \qquad y_1 = e^{2x} \Rightarrow y_1' = 2e^{2x} \qquad W(y_1, y_2) = \begin{vmatrix} e^{2x} & e^{5x} \\ 2e^{2x} & 5e^{5x} \end{vmatrix} = 3e^{7x}$

$g(x) = e^{3x} \qquad y_2 = e^{5x} \Rightarrow y_2' = 5e^{5x}$

For variation of parameters

$y = y_1 u + y_2 v$

$$\Rightarrow u = -\int \frac{y_2\, g}{W\, a}\,dx = -\int \frac{e^{5x} e^{3x}}{3e^{7x}\cdot(1)} = -\int \frac{e^{x}}{3}\,dx = -\int \frac{1}{3}e^{x}\,dx$$

$$= -\frac{1}{3}e^{x} + c_1$$

$$\Rightarrow v = \int \frac{y_1\, g}{W\, a}\,dx = \int \frac{e^{2x} e^{3x}}{3e^{7x}\cdot(1)}\,dx = \int \frac{e^{5x}}{3e^{7x}}\,dx = \int \frac{1}{3}e^{-2x}\,dx = -\frac{1}{6}e^{-2x} + c_2$$

$$= -\frac{1}{6}e^{-2x} + c_2$$

So

$$y = e^{2x}\left(-\frac{1}{3}e^{x} + c_1\right) + e^{5x}\left(-\frac{1}{6}e^{-2x} + c_2\right)$$

$$= -\frac{1}{3}e^{3x} + c_1 e^{2x} - \frac{1}{6}e^{3x} + c_2 e^{5x}$$

$$= -\frac{1}{2}e^{3x} + c_1 e^{2x} + c_2 e^{5x}$$

Problem 2. Exercise 26.8h (10 points) Find the Laplace transform of the following.

$3\, cos(2t) + 4\, sin(6t)$

Solution:

Due to linearity, we can write

$$\mathcal{L}\left[3\cos(2t) + 4\sin(4t)\right]$$

$$= 3\,\mathcal{L}\left[\cos(2t)\right] + 4\,\mathcal{L}\left[\sin(4t)\right]$$

$$= 3 \cdot \frac{s}{s^2+2^2} + 4 \cdot \frac{4}{s^2+4^2}$$

$$= 3 \cdot \frac{s}{s^2+4} + 4 \cdot \frac{4}{s^2+16}$$

← you can stop here in the ODE setting

$$= \frac{3s}{s^2+4} + \frac{16}{s^2+16}.$$