

Practice Quiz 9
2024

MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, SPRING

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A#: _____

Problem 1. Exercise 19.4a (10 points) Find the general solution for the following

$$y''' - 8y = 0$$

Solution:

$$y''' - 8y = 0$$

$$\hookrightarrow r^3 - 8 = (r-2)(r^2 + 2r + 4)$$

$$= (r-2)(r^2 + 2r + 4)$$

$$= (r-2)((r+1)^2 + 3) = 0$$

$$r_1 = 2, r_2 = -1 + \sqrt{3}i, r_3 = -1 - \sqrt{3}i$$

$$y_1 = e^{2x}, y_2 = e^{-x} \cos(\sqrt{3}x), y_3 = e^{-x} \sin(\sqrt{3}x)$$

$$\Rightarrow y = C_1 e^{2x} + C_2 e^{-x} \cos(\sqrt{3}x) + C_3 e^{-x} \sin(\sqrt{3}x)$$

$$\begin{array}{r} r-2 \overline{) r^3 - 8} \\ \underline{r^3 + 2r^2 + 4r} \\ -2r^2 - 8 \\ \underline{2r^2 - 4r} \\ -4r - 8 \\ \underline{4r + 8} \\ 0 \end{array}$$

$$(r+1)^2 + 3 = 0$$

$$\Rightarrow (r+1)^2 = -3$$

$$\Rightarrow r+1 = \pm \sqrt{3}i$$

$$\Rightarrow r = -1 \pm \sqrt{3}i$$

Problem 2. Exercise 20.1e (10 points) Find the general solution (in terms of real-valued functions) for the following Euler equation.

$$x^2 y'' - 5x y' + 9y = 0$$

Solution:

$$x^2 y'' - 5x y' + 9y = 0$$

$$\hookrightarrow r(r-1) - 5r + 9 = 0$$

$$\Rightarrow r^2 - 6r + 9 = 0$$

$$\Rightarrow (r-3)' = 0$$

$$\Rightarrow r_1 = 3, r_2 = 3$$

$$\Rightarrow y_1 = x^3, y_2 = x^3 \ln|x|$$

$$\Rightarrow y = C_1 x^3 + C_2 x^3 \ln|x|$$