Practice Quiz 2 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, SPRING 2024

NAME: Solution A#: ____________

Problem 1. Exercise 2.4c (10 points) Solve the initial-value problem (using the indefinite inegral). Also, state the largest interval over which the the solution is valid (i.e., the maximal possible interval of interest).

$$\frac{dy}{dx} = \frac{x-1}{x+1}$$

with $y(0) = 8$.

Solution:

$$\frac{dy}{dx} = \frac{x-1}{x+1}$$

$$\hookrightarrow \int \frac{dy}{dx}\,dx = \int \frac{x-1}{x+1}\,dx$$

$$\hookrightarrow y(x) + C_1 = \int \frac{x+1-1-1}{x+1}\,dx$$

$$= \int \frac{(x+1)-2}{x+1}\,dx$$

$$= \int \left(1 - \frac{2}{x+1}\right) dx$$

$$= x - 2\ln|x+1| + C_2$$

$$\hookrightarrow y(x) = x - 2\ln|x+1| + C$$

$$\Rightarrow y(0) = 0 - \cancel{2\ln(1)}^{\,0} + C = 8$$

$$\Rightarrow y(x) = x - \ln(x+1)^2 + 8$$

Problem 2. Exercise 2.7d (10 points) Using definite integrals (as in Example 2.5 on page 25) find the solution of the following iitial-value problem. (In some cases, you may want to use the error function or the sine-integral function.)

$$\frac{dy}{dx} = e^{-9x^2}$$

with $y(0) = 1$.

Solution:

$$\frac{dy}{dx} = e^{-9x^2}, \quad x_0 = 0, \quad y_0 = y(x_0) = 1$$

$$\Rightarrow \int_{x_0}^{x} \frac{dy}{ds}\, ds = y(s)\Big|_{x_0}^{x} = y(x) - y(x_0) = y(x) - y(0) = y(x) - 1$$

$$\Rightarrow \int_{x_0}^{x} e^{-9s^2}\, ds = \int_0^x e^{-9s^2}\, ds$$

Note: $y^2 = (3s)^2 = 9s^2$

$\Rightarrow y = 3s$

$$\begin{cases} s = 0 \Rightarrow y = 0 \\ s = x \Rightarrow y = 3x \end{cases}$$

$ds = \frac{1}{3}\, dy$

$$= \int_0^{3x} e^{-y^2} \left(\tfrac{1}{3}\right) dy$$

$$= \frac{1}{3}\int_0^{3x} e^{-y^2}\, dy$$

$$= \frac{1}{3}\frac{\sqrt{\pi}}{2}\underbrace{\int_0^{3x} \frac{2}{\sqrt{\pi}} e^{-y^2}\, dy}_{erf(3x)}$$

$$= \frac{1}{6}\sqrt{\pi}\; erf(3x)$$

$$\Rightarrow y(x) - 1 = \frac{1}{6}\sqrt{\pi}\; erf(3x)$$

$$\Rightarrow y(x) = 1 + \frac{1}{6}\sqrt{\pi}\; erf(3x)$$