Quiz 4 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, SPRING 2024

NAME: Solutions A#: ____________

Problem 1. Exercise 6.1b (10 points) Use linear substitution (as described in Section 6.2) to find a general solution to the given differential equation.

$$\frac{dy}{dx} = \frac{(3x-2y)^2+1}{3x-2y} + \frac{3}{2}$$

Solution:

$$u = 3x - 2y \Rightarrow \frac{du}{dx} = 3 - 2\frac{dy}{dx}$$

$$\Rightarrow 2\frac{dy}{dx} = 3 - \frac{du}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{2} - \frac{1}{2}\frac{du}{dx}$$

So.

$$\cancel{\frac{3}{2}} - \frac{1}{2}\frac{du}{dx} = \frac{u^2+1}{u} + \cancel{\frac{3}{2}}$$

$$\hookrightarrow -\frac{1}{2}\frac{du}{dx} = \frac{u^2+1}{u}$$

$$\hookrightarrow \frac{1}{2} \cdot \frac{u}{u^2+1}\frac{du}{dx} = -1$$

$$\hookrightarrow \frac{1}{4} \cdot \frac{2u}{u^2+1}\frac{du}{dx} = -1$$

$$\hookrightarrow \frac{1}{4}\int \frac{2u}{u^2+1}\,du = -\int dx$$

$$\hookrightarrow \frac{1}{4}\ln|u^2+1| = -x + C_1$$

$$\hookrightarrow \ln|u^2+1| = -4x + C_1$$

$$\hookrightarrow u^2+1 = e^{-4x+C} = Ae^{-4x}$$

$$\hookrightarrow u^2 = -1 + Ae^{-4x}$$

$$\hookrightarrow u = \pm\sqrt{Ae^{-4x}-1}$$

$$\hookrightarrow 3x - 2y = \pm\sqrt{Ae^{-4x}-1}$$

$$-2y = -3x \pm \sqrt{Ae^{-4x}-1}$$

$$\Rightarrow y = \frac{3}{2} \mp \frac{1}{2}\sqrt{Ae^{-4x}-1}$$

Problem 2. Exercise 6.71 (10 points) For the following determine a substitution that simplifies the differential equation, and using the substitution, find the general solution.

$$\frac{dy}{dx} + 3\,y = 28\,e^{2x}\,y^{-3}$$

Solution:

$$u = y^{1-n}, \quad n = -3$$

$$\Rightarrow u = y^{1-(-3)} = y^4 \Rightarrow y = u^{1/4} \Rightarrow \frac{dy}{dx} = \frac{1}{4}u^{-3/4}\frac{du}{dx}$$

So

$$\frac{1}{4}u^{-3/4}\frac{du}{dx} + 3u^{1/4} = 28e^{2x}(u^{1/4})^{-3}$$

$$\hookrightarrow \left(\frac{1}{4}u^{-3/4}\frac{du}{dx} + 3u^{1/4}\right)u^{3/4} = \frac{1}{4}\frac{du}{dx} + 3u = 28e^{2x}u^{-3/4}u^{3/4}$$

$$= 28e^{2x}$$

and

$$\frac{du}{dx} + 12u = 112e^{2x}$$

$$p(x) = 12, \quad e^{\int 12\,dx} = e^{12x}$$

$$\Rightarrow \frac{d}{dx}[\mu u] = \frac{d}{dx}[e^{12x}u] = 112e^{14x}$$

$$\Rightarrow e^{12x}u = 112\left(\frac{1}{14}e^{14x}\right) + c_1$$

$$= 8e^{14x} + c_1$$

$$\Rightarrow u = e^{-12x}(8e^{14x} + c_1) = 8e^{2x} + c_1e^{-12x}$$

$$\Rightarrow y^4 = 8e^{2x} + c_1e^{-12x}$$

$$\Rightarrow y = (8e^{2x} + c_1e^{-12x})^{1/4}$$