

Ex: Suppose

$$\frac{dy}{dx} = (x+y)^2$$

This equation is a bit of a problem. Bernoulli.

$$u = x+y$$

$$\frac{du}{dx} = 1 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{du}{dx} - 1$$

$$\Rightarrow \frac{du}{dx} = u^2 + 1$$

$$\Rightarrow \frac{1}{u^2+1} du = dx$$

$$\Rightarrow \arctan(u) = x + C$$

$$\Rightarrow u = \tan(x+C)$$

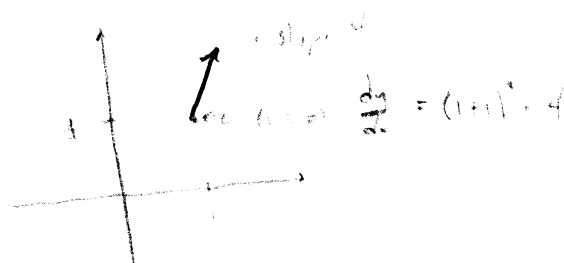
$$\Rightarrow x+y = \tan(x+C)$$

$$\Rightarrow y = \tan(x+C) - x$$

What if we want to know (qualitatively) how the function is changing?

$$\text{Ex: } \frac{dy}{dx} = (x+y)^2$$

$$x = 4, y = 1$$



Along with constant solutions we can get trajectories

$$\frac{du}{dx} = 0 \text{ at } u = \pm i$$

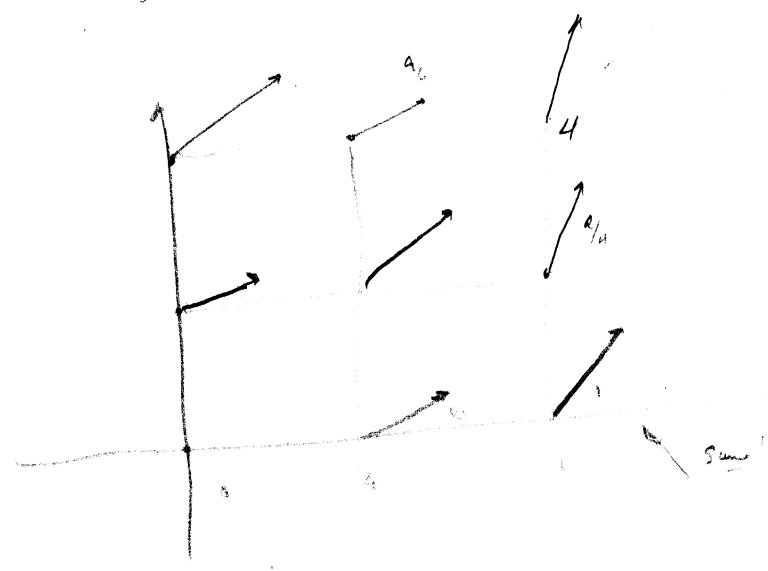
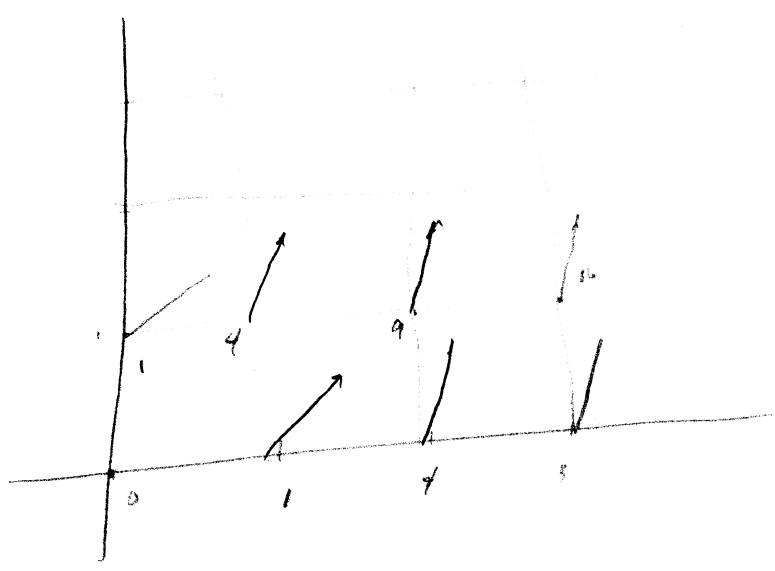
Then we already get

What happens if we perturb off (0,0)?

$$x=0, y=1 \Rightarrow \frac{dy}{dx} = y^2 = 1 \Rightarrow y = x + 1 \Rightarrow y = -\frac{1}{2}x + \frac{3}{2}$$

$$x=0, y=0 \Rightarrow \frac{dy}{dx} = 0 \Rightarrow y = \frac{1}{3}x^3$$

So, let's put a grid out for some of this



Show some results from back

Look at constant solutions:

1. Malthusian Model
2. Logistic Model
3. Sustainable Model