Math 2280 Ordinary Differential Equation: Exam #2   Name: _____

Friday, March 1, 2024   A-Number: _____

Directions: You must show all work to receive full credit for problems. Partial credit will only be given if the work is essentially correct with a minor error like a sign error. Please make sure that you write all work on the test in the space provided. There is a single problem on each page and you should have plenty of room to work the given problems on a single page. Also, make sure that

1. Your cell phone, ipod, calculator, tablet, PC, or any other electronic devices must be turned off,

2. you complete the work using a pencil and not a pen,

3. turn in your exam when it has been completed to your instructor, and

Students who do not follow these rules will be asked to leave the room. You will have 50 minutes to complete the exam.

Problem 1. Use linear substitution to solve the following first order ODE.

$$\frac{dy}{dx} = \frac{1}{(x - 2y + 1)}$$

Solution:

Problem 2. Compute the general solution of the following first order ODE. Use the substitution appropriate for a homogeneous first order ODE. Note that this is a nonlinear equation. So, the substitution involves $\frac{y}{x}$.

$$\frac{dy}{dx} = \frac{x}{y} + \frac{y}{x}$$

Solution:

Problem 3. Solve the following Bernoulli equation using the substitution $u = y^{1-n}$.

$$\frac{dy}{dx} + 2y = y^{1/2}$$

Solution:

Problem 4. Test the following ODE for being in exact form. If the equation is in exact form, find the associated potential function and define a solution for the ODE in terms of the potential function.

$$1 + 2\ x\ y^2 + \left[2\ x^2\ y + 2\ y\right] \frac{dy}{dx} = 0$$

Solution:

Problem 5. Compute the general solution for the third order ordinary differential equation

$$y''' = 4\,x\,\sqrt{y''}$$

using the substitution $v = y''$. Hint: There is no explicit dependence on y or y' in the ODE.

Solution:

Problem 6. Compute the general solution for the differential equation

$$y \, y'' = -(y')^2$$

using the substitution $v = y'$. Hint: There **is** explicit dependence on y in the ODE.

Solution:

Problem 7. Use reduction of order to determine the general solution of the following second order, linear, homogeneous ODE. Use $y_1 = x$ as the seed for the general solution.

$$2\,x^2\,y'' - x\,y' + y = 0$$

Find the unique solution for the initial conditions, $y(1) = 3$ and $y'(1) = 1$.

Solution: