Quiz 3 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, SPRING 2024

NAME: Solutions A#: ______

Problem 1. Exercise 4.5b (10 points) Using the basic procedure, find the solution to the following initial-value problems.

$$\frac{dy}{dx} = 2\,x - 1 + 2\,x\,y\ - y$$

with $y(0) = 2$.

Solution:

$$\frac{dy}{dx} = 2x - 1 + 2xy - y$$

$$\hookrightarrow \frac{dy}{dx} = (2x-1) + y(2x-1)$$

$$= (2x-1)(1+y)$$

There is a constant solution $\frac{dy}{dx} = 0 \Rightarrow (2x-1)(y+1) = 0$

$$\Rightarrow y+1 = 0 \Rightarrow y = -1$$

Next, separate variables.

$$\frac{dy}{dx} = (2x-1)(1+y)$$

$$\hookrightarrow \frac{1}{1+y} \cdot \frac{dy}{dx} = 2x+1$$

$$\hookrightarrow \frac{1}{1+y} \cdot \frac{dy}{dx}\, dx = (2x+1)\, dx$$

$$\hookrightarrow \frac{1}{1+y}\, dy = (2x+1)\, dx$$

$$\hookrightarrow \int \frac{1}{1+y}\, dy = \int (2x+1)\, dx$$

$$\hookrightarrow \ln|1+y| = x^2 + x + c$$

$y(0) = 2.$

$$\Rightarrow \ln|1+2| = 0^2 + 0 + c$$

$$\Rightarrow \underline{c = \ln|3|}$$

$$\hookrightarrow \ln|1+y| = x^2 + x + \ln|3|$$

$$\Rightarrow 1+y = e^{x^2+x+\ln|3|} = e^{x^2+x} \cdot e^{\ln|3|} = 3e^{x^2+x} \Rightarrow y = -1 + 3e^{x^2+x}$$

Problem 2. Exercise 4.8b (10 points) Solve the following initial-value problem. If possible, express each solution as an explicit expression.

$$y\,\frac{dy}{dx} = \sin(x)$$

with $y(0) = -4$.

Solution:

$$y\frac{dy}{dx} = \sin(x)$$

$$\hookrightarrow \quad y\frac{dy}{dx}\,dx = \sin(x)\,dx$$

$$\hookrightarrow \quad y\,dy = \sin(x)\,dx$$

$$\hookrightarrow \quad \int y\,dy = \int \sin(x)\,dx$$

$$\hookrightarrow \quad \frac{1}{2}y^2 = -\cos(x) + C$$

$$\hookrightarrow \quad \frac{1}{2}(-4)^2 = -\cos(0) + C$$

$$\hookrightarrow \quad 8 = -1 + C \Rightarrow C = 9$$

So

$$\Rightarrow \frac{1}{2}y^2 = 9 - \cos(x)$$

$$\hookrightarrow \quad y^2 = 18 - 2\cos(x)$$

$$\hookrightarrow \quad y = \pm\sqrt{18 - 2\cos(x)}$$

Since $x = 0 \Rightarrow y = -4$

$$\Rightarrow \quad y = -\sqrt{18 - 2\cos(x)}$$

from I.C.