NAME: Solutions   A#: ———————

Problem 1. Exercise 27.5a (10 points) Determine the Laplace using the tables provided. ou will have to use two identities.

$$t\, e^{4t}\, \sin(3t)$$

Solution:

$$\mathcal{L}\left[\sin(3t)\right] = \frac{3}{s^2 + 9}$$

$$\Rightarrow \quad \mathcal{L}\left[e^{4t} \cdot \sin(3t)\right] = \frac{3}{(s-4)^2 + 9}$$

$$\Rightarrow \quad \mathcal{L}\left[t\left(e^{4t}\sin(3t)\right)\right] = -\frac{d}{ds}\left(\frac{3}{(s-4)^2 + 9}\right)$$

$$= -3\frac{d}{ds}\left((s-4)^2 + 9\right)^{-1}$$

$$= -3\left((s-4)^2 + 9\right)^{-2}(-2)(s-4)$$

$$= 6\frac{s-4}{\left((s-4)^2 + 9\right)^2}$$

Problem 2. Exercise 28.8a (10 points) Using the Laplace transformation and translation identity, solve the following IVP.

$$y'' - 8\,y' + 17\,y = 0$$

with $y(0) = 3$ and $y'(0) = 12$.

Solution:

$$\mathcal{L}\left[y'' - 8y' + 17y\right] = \mathcal{L}[0]$$

$$\hookrightarrow \left(s^2 Y(s) - s\,y(0) - y'(0)\right) - 8\left(s\,Y(s) - y(0)\right) + 17\,Y(s) = 0$$

$$\hookrightarrow \left(s^2 - 8s + 17\right) Y(s) - 3(s) - 12 + 24 = 0$$

$$\hookrightarrow \quad s^2 - 8s + \quad = 3s - 12$$

$$\hookrightarrow \quad Y(s) = \frac{3(s-4)}{(s-4)^2 + 1}$$

$$= 3\,\frac{s-4}{}$$

$$y = 3\,e^{4t}\cos t$$