17.1a

$y'' - 7y' + 10 = 0$

$\hookrightarrow r^2 - 7r + 10 = 0$

$\Rightarrow (r-2)(r-5) = 0$

$\Rightarrow r_1 = 2,\ r_2 = 5$

$\Rightarrow y_1 = e^{2x},\ y_2 = e^{5x}$ $\Rightarrow$ $\underline{y = c_1 e^{2x} + c_2 e^{5x}}$

17.2a $y'' - 8y' + 15y = 0$

$\hookrightarrow r^2 - 8r + 15 = 0$

$\Rightarrow (r-5)(r-3) = 0$

$\Rightarrow y_1 = e^{5x},\ y_2 = e^{3x}$

$\Rightarrow y = c_1 e^{5x} + c_2 e^{3x} \rightarrow y(0) = c_1 + c_2 = 1 \Rightarrow c_2 = 1 - c_1 \rightarrow c_2 = 1 - \left(-\frac{3}{2}\right) = \frac{5}{2}$

$y' = 5c_1 e^{5x} + 3c_2 e^{3x} \rightarrow y'(0) = 5c_1 + 3c_2 = 0 \longrightarrow 5c_1 + 3(1 - c_1) = 0$

$\Rightarrow 5c_1 + 3 - 3c_1 = 0$

$\Rightarrow 2c_1 = -3 \Rightarrow c_1 = -\frac{3}{2}$

$\Rightarrow \underline{y(x) = -\frac{3}{2} e^{5x} + \frac{5}{2} e^{3x}}$

17.3b. $y'' + 2y' + y = 0$, $y(0) = 6$, $y'(0) = -5$

$\hookrightarrow r^2 + 2r + 1 = 0$

$\Rightarrow (r+1)^2 = 0$

$\Rightarrow r_1 = -1,\ r_2 = -1$

$\Rightarrow y_1 = e^{-x},\ y_2 = e^{-x} \cdot x$ $\Rightarrow$ $\underline{y = c_1 e^{-x} + c_2 x e^{-x}}$

17.4f $4y'' + 4y' + y = 0$, $y(0) = 4$, $y'(0) = 1$ $\qquad r_1 = -\frac{1}{2},\ r_2 = -\frac{1}{2}$

$\hookrightarrow 4r^2 + 4r + 1 = 0$ $\qquad \Rightarrow y_1 = e^{-\frac{1}{2}x},\ y_2 = x e^{-x/2}$

$\Rightarrow (2r+1)^2 = 0$

17.4f cont.

$y = C_1 e^{-\frac{1}{2}x} + C_2 x e^{-\frac{1}{2}x} \longrightarrow y(0) = C_1 + 0 = 6 \Rightarrow C_1 = 6$

$y' = -\frac{1}{2} C_1 e^{-x/2} + C_2 e^{-\frac{1}{2}x} - \frac{1}{2} C_2 x e^{-x/2} \longrightarrow y'(0) = -\frac{1}{2} C_1 + C_2 + 0 = -\frac{1}{2} C_1 + C_2 = -5$

$= -3 + C_2 = -5 \Rightarrow C_2 = -2$

So, $y = 6e^{-x/2} - 2xe^{-x/2}$

17.7

$4y'' + y = 0$

$\hookrightarrow 4r^2 + 1 = 0$

$\Rightarrow r^2 = -1/4 \Rightarrow y = C_1 \cos(\frac{1}{2}x) + C_2 \sin(\frac{1}{2}x)$

$\Rightarrow r = \pm i \frac{1}{2}$

17.8c

$y'' + 10y' + 25y = 0$

$\hookrightarrow r^2 + 10r + 25 = 0$

$\Rightarrow (r+5)^2 = 0 \Rightarrow y = C_1 e^{-5x} + C_2 x e^{-5x}$

$\Rightarrow r_1 = -5, r_2 = -5$

$\Rightarrow y_1 = e^{-5x}, y_2 = xe^{-5x}$

17.9

$9y'' - y = 0$

$\hookrightarrow 9r^2 - 1 = 0$

$\Rightarrow (3r-1)(3r+1) = 0$

$r = 1/3, \; r = -1/3$

$y_1 = e^{x/3}, y_2 = e^{-x/3} \Rightarrow y = C_1 e^{x/3} + C_2 e^{-x/3}$

17.8a

$y'' + 4y' + 4y = 0$

$\hookrightarrow r^2 + 4r + 4 = 0$

$(r+2)^2 = 0$

$\Rightarrow r = -2, \; r = -2 \Rightarrow y = C_1 e^{-2x} + C_2 e^{-2x} \cdot x$

17.8f $y'' - 2y' - 15y = 0$

$\hookrightarrow r^2 - 2r - 15 = 0$

$\Rightarrow (r-5)(r+3) = 0$

$\Rightarrow r_1 = 5,\ r_2 = -3$ $\Rightarrow y = c_1 e^{5x} + c_2 e^{-3x}$

$\Rightarrow y_1 = e^{5x},\ y_2 = e^{-3x}$

17.9a $\sin^2(t) + \cos^2(t) = 1$

$$\left(\frac{e^{it} - e^{-it}}{2i}\right)^2 + \left(\frac{e^{it} + e^{-it}}{2}\right)^2 = \left(\frac{e^{2it} - 2 + e^{-2it}}{-4}\right) + \left(\frac{e^{2it} + 2 + e^{-2it}}{4}\right)$$

$$= \cancel{-\tfrac{1}{4}e^{2it}} + \tfrac{1}{2} - \tfrac{1}{4}e^{-2it} + \cancel{\tfrac{1}{4}e^{2it}} + \tfrac{1}{2} + \tfrac{1}{4}e^{-2it}$$

$$= \tfrac{1}{2} + \tfrac{1}{2} = 1$$

17.9c $\cos(A+B) - \cos(A)\cos(B) + \sin(A)\sin(B) \stackrel{?}{=} 0$

$$\frac{e^{i(A+B)} + e^{-i(A+B)}}{2} - \frac{e^{iA} + e^{-iA}}{2} \cdot \frac{e^{iB} + e^{-iB}}{2} + \frac{e^{iA} - e^{-iA}}{2i}\,\frac{e^{iB} - e^{-iB}}{2i}$$

$$= \frac{e^{iA}e^{iB} + e^{-iA}e^{-iB}}{2} - \tfrac{1}{4}\left(e^{iA}e^{iB} + e^{iA}e^{-iB} + e^{-iA}e^{iB} + e^{-iA}e^{-iB}\right) - \tfrac{1}{4}\left(e^{iA}e^{iB} - e^{iA}e^{-iB} - e^{-iA}e^{iB} + e^{-iA}e^{-iB}\right)$$

$$= e^{iA}e^{iB}\left(\tfrac{1}{2} - \tfrac{1}{4} - \tfrac{1}{4}\right) + e^{-iA}e^{-iB}\left(+\tfrac{1}{2} - \tfrac{1}{4} - \tfrac{1}{4}\right) + e^{iA}e^{-iB}\left(-\tfrac{1}{4} + \tfrac{1}{4}\right) + e^{-iA}e^{iB}\left(-\tfrac{1}{4} + \tfrac{1}{4}\right)$$

$$= 0 + 0 + 0 + 0 = 0 \checkmark$$

17.9d NOTE: There is a typo in the book.

$2\sin(A)\sin(B) - \cos(A-B) + \cos(A+B)$

$$= 2\left(\frac{e^{iA} - e^{-iA}}{2i}\right)\left(\frac{e^{iB} - e^{-iB}}{2i}\right) - \frac{e^{i(A-B)} + e^{-i(A-B)}}{2} + \frac{e^{i(A+B)} + e^{-i(A+B)}}{2}$$

$$= -\frac{2}{4}\left(e^{iA}-e^{-iA}\right)\left(e^{iB}-e^{-iB}\right) - \frac{1}{2}\left(e^{iA}e^{-iB} + e^{-iA}e^{-iB} - e^{iA}e^{iB} - e^{-iA}e^{-iB}\right)$$

$$= -\frac{1}{2}\left(e^{iA}e^{iB} - e^{iA}e^{-iB} - e^{-iA}e^{iB} + e^{-iA}e^{-iB}\right)$$

$$-\frac{1}{2}\left(e^{-iA}e^{iB} + e^{iA}e^{-iB} \qquad\right) = 0$$

18.1 a. We are given

$$|F_0| = 2 \ \frac{\text{kg - meter}}{\text{second}}$$

for $y_0 = 1/2$. So,

$$K = \frac{|F_0|}{|y_0|} = \frac{2}{1/2} = 4$$

b. For the undamped system and $m = 16$

$$m\frac{d^2y}{dt^2} = -Ky = -4y$$

$$\Rightarrow \quad my'' + 4y = 0$$

$$\Rightarrow \quad mr^2 + K = 0 \quad \Rightarrow \quad 16r^2 + 4 = 0 \quad \Rightarrow \quad \omega_0 = \sqrt{K/m} = \sqrt{1/4} = 1/2$$

$$\Rightarrow \quad r = \pm i\,1/2$$

$$\Rightarrow \quad P_0 = 2\pi/\omega_0 = 2\pi/(1/2) = 4\pi \quad \Rightarrow \quad V_0 = 1/P_0 = \frac{1}{4\pi}.$$

c. i) The expression for y is

$$y = C_1\cos(x/2) + C_2\sin(x/2) \longrightarrow y(0) = C_1 + 0 = 2$$

$$\Rightarrow \quad y' = -\frac{C_1}{2}\sin(x/2) + \frac{C_2}{2}\cos(x/2) \longrightarrow y'(0) = 0 + \frac{C_2}{2} = 0 \Rightarrow C_2 = 0$$

$$\Rightarrow \quad y(x) = 2\cos\left(\tfrac{1}{2}x\right) = A\cdot\cos(\omega_0 x + \phi)$$

$$\Rightarrow A = 2, \ \phi = 0$$

ii) – (v) are the same

18.4a $K=37, \ m=4, \ \gamma=4$

Substitution gives

$$4y''+4y'+37y=0$$

$$\hookrightarrow 4r^2+4r+37=0$$

$$\Rightarrow r=\frac{-4\pm\sqrt{16-16(37)}}{8}=\frac{-4\pm4\sqrt{1-37}}{8}=-\frac{1}{2}\pm\frac{1}{2}\sqrt{-36} \Rightarrow 0<\gamma<2\sqrt{37\cdot4}$$

$\Rightarrow$ underdamped

b. The decay coeff is

$$\alpha=-1/2,$$

$$\omega=\frac{1}{2}\sqrt{36}=\frac{6}{2}=3$$

$$p=\frac{2\pi}{3}$$

$$\nu=\frac{1}{\pi}$$

$$\Rightarrow y=c_1e^{-x/2}\cos(3x)+c_2e^{-x/2}\sin(3x)$$

Use this to compute c_1 and c_2, then back out A, ϕ

$\vdots$

18.8 Solution for a.

$K=4, \ m=1, \ \gamma=4 \Rightarrow y''+\gamma y'+Ky=y''+4y'+4=0$

$$\Rightarrow r^2+4r+4=0 \Rightarrow (r+2)^2=0$$

So, $r_1 = -2,\ r_2 = -2$

$\Rightarrow\ y = C_1 e^{-2x} + C_2 e^{-2x} x$

non trig function - no oscillation!

19.1a $y^{(4)} - 4y''' = 0$

$\hookrightarrow\ r^4 - 4r^3 = 0$

$\Rightarrow\ r^3(r-1) = 0$

$r_1 = 0,\ r_2 = 0,\ r_3 = 0,\ r = 1 \quad \Rightarrow\ y = C_1 + C_2 x + C_3 x^2 + C_4 e^x$

19.1b $y^{(4)} + 4y'' = 0$

$\hookrightarrow\ r^4 + 4r^2 = 0 \ \Rightarrow\ r^2(r^2+4) = 0$

$r_1 = 0,\ r_2 = 0,\ r_3 = 2i,\ r_4 = -2i \quad \Rightarrow\ y = C_1 + C_2 x + C_3 \cos(2x) + C_4 \sin(2x)$

19.1c $y^{(4)} - 34y'' + 225y = 0$ $\qquad S = r^2$

$\Rightarrow\ r^4 - 34r^2 + 225 = 0$

$\Rightarrow\ S^2 - 34S + 225 = 0$

$\Rightarrow\ (S-15)(S-9) = 0$

$\Rightarrow\ (r^2-25)(r^2-9) = 0$

$\Rightarrow\ (r-5)(r+5)(r-3)(r+3) \quad \Rightarrow \quad y(x) = C_1 e^{5x} + C_2 e^{-5x} + C_3 e^{3x} + C_4 e^{-3x}$

19. 1d

$y^{(4)} - 81y = 0$

$r^4 - 81 = 0$

$\Rightarrow (r^2 - 9)(r^2 + 9) = 0$

$\Rightarrow (r-3)(r+3)(r+3i)(r-3i) = 0$

$\Rightarrow r_1 = 3,\ r_2 = -3,\ r_3 = -3i,\ r_4 = 3i$

$\Rightarrow y = C_1 e^{3x} + C_2 e^{-3x} + C_3 \cos(3x) + C_4 \sin(3x)$