Practice Quiz 9 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, FALL 2023

NAME: Solutions A#: _____

Problem 1. Chapter 15 Ex. 17.8.b Find the general solution to the following. Write the solution in terms of real valued functions.

$y'' + 9\,y = 0$

Solution:

The characteristic equation is

$r^2 + 9 = 0 \Rightarrow r = \pm i3$

and

$y_1 = \cos(3x)$, $y_2 = \sin(3x)$

$\Rightarrow y = c_1 \cos(3x) + c_2 \sin(3x)$

Problem 2. Ex. 19.2.c (10 points) For the following differential equation find a root of the characteristic polynomial using factors of the constant term in the polynomial. Find the rest of the roots using long division and then write out the general solution of the differential equation.

$$y''' - 8\,y'' + 37\,y' - 50\,y = 0$$

Solution:

The characteristic equation is

$r^3 - 8r^2 + 37r - 50 = 0$ Try $1, 2, 5, 10, 25$ and $\pm$

$r=1 \Rightarrow 1 - 8 + 37 - 50 \neq 0$ ✗

$r=2 \Rightarrow 8 - 32 + 74 - 50 = -24 + 24 = 0$ ✓ $r=2$ is a root

Then,

$$(r-2) \overline{)\, r^3 - 8r^2 + 37r - 50 \;\;\; r^2 - 6r + 25}$$

$r^3 - 2r^2$

$-6r^2 + 37r - 50$

$-6r^2 + 12r$

$25r - 50$

$25r - 50$

0

$\Rightarrow r^3 - 8r^2 + 37r - 50$

$= (r-2)(r^2 - 6r + 25)$

$= (r-2)(r^2 - 6r + 9 + 16)$

$= (r-2)\left((r-3)^2 + 16\right)$

$\Rightarrow (r-3)^2 = -16$

$\Rightarrow (r-3) = \pm\sqrt{-16} = i(4)$

$\Rightarrow r = 3 \pm 4i$

So, $r_1 = 2,\ r_2 = 3 + 4i,\ r_3 = 3 - 4i$

$\Rightarrow y_1 = e^{2x},\ y_2 = e^{3x}\cos(4x),\ y_3 = e^{3x}\sin(4x)$

$\Rightarrow y = c_1 e^{2x} + c_2 e^{3x}\cos(4x) + c_3 e^{3x}\sin(4x)$