Quiz 1 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, SPRING 2024

NAME: Solutions A#: _____

Problem 1. Exercise 1.4a (10 points) For the initial-value problems give below, three choices for a possible solution $y = y(x)$ are given. Determine whether each choice is or is not a solution of the given initial-value problem.

$$\frac{dy}{dx} = 4\,y$$

with $y(0) = 5$.

i.) $y(x) = e^{4x}$, ii.) $y(x) = 5\,e^{4x}$, iii.) $y(x) = e^{4x} + 1$

Solution:

First write

$$\frac{dy}{dx} - 4y = 0$$

Then

i.) $y = e^{4x} \Rightarrow \frac{dy}{dx} = 4e^{4x}$

$\Rightarrow \frac{dy}{dx} - 4y = 4e^{4x} - 4(e^{4x}) = 0$ ✓

and $y(0) = e^{4\cdot 0} = e^0 = 1 \neq 5$ ✗

So $y = e^{4x}$ does not satisfy the IVP.

ii.) $y = 5e^{4x} \Rightarrow y' = 20e^{4x}$

$\Rightarrow \frac{dy}{dx} - 4y = 20e^{4x} + 4(5e^{4x}) = 0$ ✓

and $y(0) = 5e^0 = 5\cdot(1) = 5$ ✓

y satisfies the IVP

iii.) $y = e^{4x} + 1 \Rightarrow y' = 4e^{4x}$

$\Rightarrow 4e^{4x} - 4(e^{4x} + 1) = -4 \neq 0$ ✗

So y cannot be a solution to the IVP. We do not need to check the point $y(0) = 5$

Problem 2. Exercise 2.4d (10 points) Solve the following initial problem (using the indefinite integral). Also, state the largest interval over which the solution is valid (i.e., the maximal possible interval of interest).

$$x\,\frac{dy}{dx} + 2 = \sqrt{x}$$

with $y(1) = 6$.

Solution:

Solve for $\frac{dy}{dx}$

$\Rightarrow x\,\frac{dy}{dx} = \sqrt{x} - 2$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{x}} - \frac{2}{x}$

$\Rightarrow \int \frac{dy}{dx}\,dx = \int \left(x^{-1/2} - \frac{2}{x}\right) dx$

$\Rightarrow y(x) + C_1 = 2x^{1/2} - 2\ln|x| + C_2$

$\Rightarrow y(x) = 2\sqrt{x} - \ln|x^2| + C_3$

$C_3 = C_2 - C_1$

The domain for y is

$\sqrt{x}$ $x \in [0, +\infty)$

$\ln(x^2)$ $x \neq 0$

intersect $\Rightarrow$ $x \in (0, +\infty)$