Math 2280 Lecture Notes Day 29.

Ex: $\left(\frac{d}{dx}+2\right)\left(\frac{dx}{dx}+3\right)$

Ex: $\left(\frac{d}{dx}+5x\right)^3$

Now, $ay''+by'+cy=0$

Ex: $2\frac{dy}{dx}+6y=0$ now $ay'+by=0$

$\Rightarrow \frac{dy}{dx}=-3y$ $\Rightarrow y'=-\frac{b}{a}y$

$\Rightarrow \frac{1}{y}\frac{dy}{dx}=-3$ $\Rightarrow \underline{y(x)=Ae^{-b/a\,x}}$

$\Rightarrow \ln|y|=-3x+c_1$

$\Rightarrow y=e^{-3x+c}=Ae^{3x}$

Ex: $y''+7y'=0$

$e^{rx}\Rightarrow r^2e^{rx}+7re^{rx}$

$=e^{rx}(r^2+7r)=0$

$\Rightarrow r^2+7r=r(r+7)=0$

$\Rightarrow r_1=0,\ r_2=-7$

$\Rightarrow y_1=e^0,\ y_2=e^{-7x}$

$\Rightarrow y_1=1,$ ___

then

$y_h=c_1y_1+c_2y_2=c_1+c_2e^{-7x}$

Ex: $y'' + 2y' + y = 0$

$\Downarrow$

$(r^2 + 2r + 1) = (r+1)^2$

$r_1 = -1,\ r_2 = -1$

$y_1 = e^{-x},\ \cancel{y_2 = e^{-x}}$

same!

So we retreat to reduction of order

$y = e^{-x} u$

$y' = -e^{-x} u + e^{-x} u'$

$y'' = e^{-x} u - 2e^{-x} u' + e^{-x} u''$

$\Rightarrow (\cancel{e^{-x} u} - 2e^{-x} u' + e^{-x} u'') + 2(\cancel{-e^{-x} u} + e^{-x} u') + \cancel{e^{-x} u}$

$= \cancel{-2e^{-x} u'} + \cancel{2e^{-x} u'} + e^{-x} u''$

$= e^{-x} u'' = 0$

$\Rightarrow u'' = 0$ easy! $\Rightarrow u' = C_1,\ u = C_1 x + C_2$

$\Rightarrow y = e^{-x}(C_1 x + C_2)$

$= C_1 x e^{-x} + C_2 e^{-x}$ ✓

Ex: $y'' + 4y = 0$

$\Downarrow$

$r^2 + 4 = 0$

$r_1 = 2i,\ r_2 = -2i$

Why? $y_1 = e^{2i\theta}$, $y_2 = e^{-2i\theta}$

Euler's Formula!

$$\begin{cases} e^{i\theta} = \cos(\theta) + i\sin(\theta) \\ e^{-i\theta} = \cos(\theta) - i\sin(\theta) \end{cases}$$

$$\begin{cases} \cos(\theta) = \dfrac{e^{i\theta} + e^{-i\theta}}{2} \\ \sin(\theta) = \dfrac{e^{i\theta} - e^{-i\theta}}{2i} \end{cases}$$

So, we can use cosine and sine forms for complex roots.

Ex: $y'' + 25y = 0$

$\Rightarrow r^2 + 25 = 0 \Rightarrow r = \pm 5i$

$\Rightarrow y_1 = \cos(5x)$, $y_2 = \sin(5x)$

$\Rightarrow y = C_1\cos(5x) + C_2\sin(5x)$

Ex: $y'' - 6y' + 13y = 0$

$\Rightarrow r^2 - 6r + 13 = 0$

$r = \dfrac{6 \pm \sqrt{36-52}}{2} = 3 \pm \sqrt{16/4} = 3 \pm \sqrt{4} = 3 \pm 2i$

$r_1 = 3 - 2i$, $r_2 = 3 + 2i$

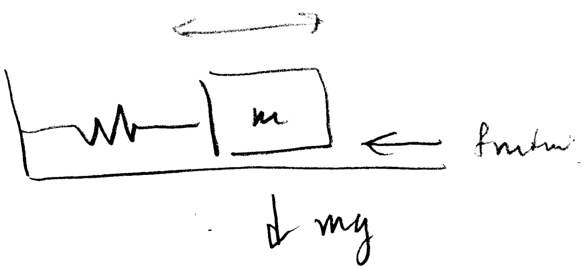
$\hookrightarrow e^{r_1 x} = e^{(3-2i)x} = e^{3x}e^{-2ix}$ ← use $\sin(x)$, $\cos(x)$

So, $y = C_1 e^{ax} e^{ibx} + C_2 e^{ax} e^{-ibx}$

$$= e^{ax}\left(c_1 e^{ibx} + c_2 e^{-ibx}\right)$$

$$= e^{ax}\underbrace{\left(A\cos(bx) + B\sin(bx)\right)}_{\text{real valued}}$$

Do Identities on page 336!



$$F = F_{resis} + F_{spring} + F_{other}$$

$$\sum \text{Forces} = \sum F = ma = m\frac{d^2y}{dt^2}$$

$$\boxed{ma - \gamma \frac{dy}{dt} \quad k\, ky = 0}$$

$$\Rightarrow m\frac{d^2y}{dt^2} - \gamma\frac{dy}{dt} + ky = 0$$