Math 2280 Homework Solutions

2.2a The ODE is

$$\frac{dy}{dx} = 3 - \sin(x) = F(x)$$

There is no explicit dependence on the right hand side. So, the equation is directly integrable.

2.2c We can write

$$\frac{dy}{dx} = -4y + e^{2x} = F(x,y)$$

Since $F(x,y)$ is dependent on x and y, the equation is not directly integrable

2.2e $y\frac{dy}{dx} = 2x$

$$\Rightarrow \frac{dy}{dx} = \frac{2x}{y} = F(x,y)$$

Since $F(x,y)$ depends on both x and y the equation is not directly integrable.

2.2g $x^2\frac{d^2y}{dx^2} = 1$

$$\Rightarrow \frac{d^2y}{dx^2} = 1/x^2 = F(x,y) = F(x)$$

Since $F(x)$ is only dependent on x, the equation is directly integrable.

2.3b $\frac{dy}{dx} = 20e^{-4x}$

$$\Rightarrow \int \frac{dy}{dx} = y(x) + c_1 = \int 20e^{-4x}dx = 20\left(-\frac{1}{4}e^{-4x}\right) = -5e^{-4x} + c_2$$

$$\Rightarrow \boxed{y(x) = -5e^{-4x} + c}$$

2.3d $\sqrt{x+4}\ \frac{dy}{dx} = 1$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{x+4}} = (x+4)^{-1/2}$$

$$\Rightarrow y(x) = \int (x+4)^{-1/2}\,dx$$

$$= 2(x+4)^{1/2} + C$$

$$= 2\sqrt{x+4} + C$$

2.3g $x = (x^2-9)\frac{dy}{dx}$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{x^2-9}$$

$$\Rightarrow y(x) = \int \frac{x}{x^2-9}\,dx$$

$u = x^2-9$

$du = 2x\,dx \Rightarrow x\,dx = \frac{1}{2}du$

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln|u| + C$$

$$= \ln|u^{1/2}| + C = \ln\left|\sqrt{x^2-9}\right| + C$$

2.3h $1 = (x^2-9)\frac{dy}{dx}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x^2-9} = \frac{1}{(x-3)(x+3)}$$

$$\frac{1}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3}$$

$$\Rightarrow 1 = A(x+3) + B(x-3)$$

$$\Rightarrow x = -3 \Rightarrow 1 = -6B \Rightarrow B = -\frac{1}{6}$$

$$\Rightarrow x = 3 \Rightarrow 1 = 6A \Rightarrow A = 1/6$$

$$= \frac{1}{6(x-3)} - \frac{1}{6(x+3)}$$

$$\Rightarrow y(x) = \int \frac{1}{6(x-3)}\,dx - \int \frac{1}{6(x+3)}\,dx$$

$$= \frac{1}{6}\ln|x-3| - \frac{1}{6}\ln|x+3| + C$$

$$= \frac{1}{6}\left(\ln|x-3| - \ln|x+3|\right) + C$$

$$= \frac{1}{6}\left(\ln\left|\frac{x-3}{x+3}\right|\right) + C$$

$$= \ln\left|\frac{x-3}{x+3}\right|^{1/6} + C$$

2.3k

$$\frac{d^2y}{dx^2} - 3 = x$$

$$\Rightarrow \frac{d^2y}{dx^2} = x + 3$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}x^2 + 3x + C_1$$

$$\Rightarrow y(x) = \frac{1}{6}x^3 + \frac{3}{2}x^2 + C_1 x + C_2$$

2.4b $\sqrt[3]{x+6}\ \frac{dy}{dx} = 1$, $y(2) = 10$

$$\Rightarrow \frac{dy}{dx} = (x+6)^{-1/3}$$

$$\Rightarrow y = \frac{3}{2}(x+6)^{2/3} + C_1$$

For $y(2) = 10$

$$y(2) = 10 = \frac{3}{2}(2+6)^{2/3} + C_1$$

$$\Rightarrow \frac{3}{2}(8)^{2/3} + C_1 = 10$$

$$\Rightarrow \frac{3}{2}\cdot(4) + C_1 = 10$$

$$\Rightarrow +6 + C_1 = 10 \Rightarrow C_1 = 4$$

$$\Rightarrow y(x) = \frac{3}{2}(x+6)^{2/3} + 4 \rightarrow x \in (-\infty, +\infty)$$

2.4c $\frac{dy}{dx} = \frac{x-1}{x+1}$ $\quad y(0) = 8$

$$\Rightarrow \frac{dy}{dx} = \frac{x+1-1-1}{x+1} = \frac{(x+1)-2}{(x+1)} = 1 - \frac{2}{x+1}$$

$$\Rightarrow y = x - 2\ln|x+1| + C$$

So $y(0) = 8 = 0 - 2\cancel{\ln(1)}^{\to 0} + C \Rightarrow \underline{C = 8}$

$$y(x) = x - 2\ln|x+1| + 8 \rightarrow \boxed{x > -1} \text{ or } x \in (-1, +\infty)$$

2.4.f $(x^2+1)\frac{dy}{dx} = 1 \qquad y(0)=3$

$\Rightarrow \frac{dy}{dx} = \frac{1}{1+x^2}$

$\Rightarrow y(x) = \int \frac{1}{1+x^2}\,dx$

$= \arctan(x) + C$

$\Rightarrow y(0) = 3 = \arctan(0) + c$

$\Rightarrow C = 3 - \cancel{\arctan(0)}^{\,0} \Rightarrow C = 3$

So, $y = \arctan(x) - 3$.

2.6a $\frac{dy}{dx} = 3\sqrt{x+3}$, $y(1) =$ a given value

a.) We can write

$\int_1^x \frac{dy}{ds}\,ds = \int_1^x 3\sqrt{s+3}\,ds$

$\Rightarrow y(s)\Big|_1^x = 3\left(\frac{2}{3}\right)(s+3)^{3/2}\Big|_1^x$

$\Rightarrow y(x) - y(1) = 2(x+3)^{3/2} - 2(1+3)^{3/2}$

$\Rightarrow y(x) - y(1) = 2(x+3)^{3/2} - 2(4)^{3/2}$

$= 2(x+3)^{3/2} - 16$

$\Rightarrow y(x) = y(1) + 2(x+3)^{3/2} - 16$

b. i) when $y(1) = 16$

$\Rightarrow y(x) = \cancel{16} + 2(x+3)^{3/2} \cancel{-16} \Rightarrow y(x) = 2(x+3)^{3/2}$

$\Rightarrow y(6) = 2(9)^{3/2} = 54$

ii) If $y(1) = 20$, then

$$y(x) = 20 + 2(x+3)^{3/2} - 16 = 4 + 2(x+3)^{3/2}$$

$$y(6) = 4 + (54) = 58.$$

iii) If $y(1) = 0$, then

$$y(x) = 0 + 2(x+3)^{3/2} - 16$$

$$\Rightarrow y(-2) = 2(1)^{3/2} - 16 = -14$$