Quiz 6

NAME: Solution b   A#: _____

Problem 1. Exercise 13.1 (10 points) The following second-order differential equation does not contain y. Solve the differential equation using the substitution $v = y'$ as described in Section 3.1.

$$y'' = y'$$

Solution:

Let, $y' = v$ and $y'' = v'$

$$\Rightarrow v' = v \qquad v' = 0 \text{ when } v = 0 \Rightarrow y' = 0 \Rightarrow y = C_1$$

Then

$$\frac{dv}{dx} = v$$

$$\hookrightarrow \frac{1}{v} dv = dx$$

$$\hookrightarrow \ln|v| = x + C_1$$

$$\hookrightarrow v = e^{x+C} = Ae^x$$

$$\hookrightarrow \frac{dy}{dx} = Ae^x$$

$$\Rightarrow y = Ae^x + B$$

Problem 2. Exercise 13.4c (10 points) The following second-order equation is autonomous. Solve using the substitution as described in Section 3.2.

$$3\, y\, y'' - 2\, (y')^2$$

Solution:

$$\begin{cases} y'' = v\,\dfrac{dv}{dy} \\[2mm] y' = v \end{cases}$$

$$\Rightarrow\ 3y\left(v\,\frac{dv}{dy}\right) = 2v^2$$

$$\Rightarrow\ 3v\,\frac{dv}{dy} = \frac{2}{y}\,v^2$$

$$\Rightarrow\ 3\,\frac{v}{v^2}\,\frac{dv}{dy} = \frac{2}{y}$$

$$\Rightarrow\ \frac{1}{v}\,\frac{dv}{dy} = \frac{2}{3}\,y^{-1}$$

$$\Rightarrow\ \frac{1}{v}\,dv = \frac{2}{3y}\,dy$$

$$\Rightarrow\ \ln|v| = \frac{2}{3}\ln|y| + c_1$$

$$\Rightarrow\ v = A e^{\frac{2}{3}\ln|y|}$$

$$\Rightarrow\ v = A e^{\ln\left(y^{2/3}\right)} = A\,y^{2/3}$$

$$\Rightarrow\ \frac{dy}{dx} = A\,y^{2/3}$$

$$\Rightarrow\ \int y^{-\frac{2}{3}}\,dy = \int A\,dx$$

$$\Rightarrow\ y^{1/3} = Ax + B$$

$$\Rightarrow\ y = (Ax + B)^3$$