Math 2280 Lecture Notes Day 4

First, huge class of ODEs we can solve

<u>Directly Integrable</u>s

$\frac{dy}{dx} = f(x)$ ⟵ integrate once

<u>OR</u>

$\frac{d^n y}{dx^n} = f(x)$ ⟵ integrate n times

Ex: $f(x) = x\ln(x)$

$\frac{dy}{dx} = f(x) \Rightarrow y(x) = \int x\ln(x)\,dx$ $u = x$ $dv = \ell \ldots$

not correct

$\Rightarrow y(x) = \int x\ln(x)\,dx$ $\begin{matrix} u = \ln(x) \\ dv = x\,dx \end{matrix} \Rightarrow \begin{matrix} du = \frac{1}{x}dx \\ v = \frac{1}{2}x^2 \end{matrix}$

So $y(x) = \frac{1}{2}x^2\ln(x) - \int\left(\frac{1}{2}x^2\right)\cdot\frac{1}{x}\,dx$

$= \frac{1}{2}x^2\ln(x) - \frac{1}{2}\int x\,dx$

$= \frac{1}{2}x^2\ln(x) - \frac{1}{4}x^2 + C$

constant of <u>int.</u>

<u>Ex</u>: $f(x) = x\ln(x)$

$\frac{dy}{dx} = f(x), \quad y(1) = 3$

We can write

$$\int_{x_0}^{x} \frac{dy}{ds}\, ds = \int_{x_0}^{x} \sin(s)$$

Ex: $\frac{dy}{dx} = e^{-x^2}$, $y(0) = 0$

$$\Rightarrow \int_{x_0}^{x} \frac{dy}{ds}\, ds = \int_{x_0}^{x} e^{-s^2}\, ds$$

$$\Rightarrow y(s)\Big|_0^x = \int_0^x e^{-s^2} ds$$

$$\Rightarrow y(x) - y(0) = \int_0^x \frac{2}{\sqrt{\pi}} \cdot \frac{\sqrt{\pi}}{2} e^{-s^2} ds$$

$$= \frac{\sqrt{\pi}}{2} \underbrace{\int_0^x \frac{2}{\sqrt{\pi}} e^{-s^2} ds}_{\text{erf}}$$

$$= \frac{\sqrt{\pi}}{2} \operatorname{erf}(x)$$

Chapter 3: First-Order Basics

3.1 Algebraically Solve for $\frac{dy}{dx}$

Ex $$x^2 \frac{dy}{dx} - 4x = 6$$

$$\Rightarrow \frac{dy}{dx} = \frac{4x+6}{x^2} = \frac{4}{x} + \frac{6}{x^2}$$

$$\Rightarrow y(x) = 4\ln|x| + \frac{6}{x} + C$$