Practice Quiz 7 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, FALL 2023

NAME: Solutions A#: _____

Problem 1. Chapter 13 Ex. 13.4c Solve the following initial value problem.

$$y'' = y'$$

with $y(0) = 8$ and $y'(0) = 5$.

Solution:

Set $v = y' \Rightarrow v' = y''$

So $v' = v \Rightarrow \frac{dv}{dx} = v$

$\hookrightarrow \frac{1}{v}\frac{dv}{dx} = 1$

$\hookrightarrow \frac{1}{v}\,dv = dx$

$\hookrightarrow \int \frac{1}{v}\,dv = \int dx$

$\hookrightarrow \ln|v| = x + C_1$

$\hookrightarrow v = e^{x+C_1} = Ae^x$

$\Rightarrow y' = Ae^x$

$\Rightarrow y = Ae^x + C_2$

$y(0) = 8 \Rightarrow 8 = Ae^0 + C_2 = A + C_2$

$y'(0) = 5 \Rightarrow 5 = Ae^0 = A$

$\Rightarrow 8 = 5 + C_2 \Rightarrow C_2 = 3$

So $y(x) = 5e^x + 3$

Problem 2. Chapter 14.2d (10 points) For the following, first verify that y_1 is a solution to the differential equation then find the general solution using y_1 with the method of reduction of order.

$$2x^2 y'' - x y' + y = 0, \quad x > 0, \quad y_1(x) = x$$

Solution:

$y_1' = 1$

$y_1'' = 0$

$\Rightarrow \quad 2x^2(0) - x(1) + x = 0$ ✓

Then define $y = y_1 u = xu$

$y' = u + xu'$

$y'' = u' + u' + xu'' = 2u' + xu''$

Substitute:

$2x^2(2u' + xu'') - x(u + xu') + xu$

$= 4x^2u' + 2x^3u'' - xu - x^2u' + xu$

$= (4x^2 - x^2)u' + 2x^3u''$

$= 3x^2u' + 2x^3u'' = 0$

$\Rightarrow \quad 2x^3\left(u'' + \frac{3}{2}\frac{1}{x}u'\right) = 0 \qquad x \neq 0$

$\Rightarrow \quad u'' + \frac{3}{2}\frac{1}{x}u' = 0$

$\Rightarrow \quad v' + \frac{3}{2}\frac{1}{x}v = 0$

$\mu = e^{\int \frac{3}{2}\frac{1}{x}\,dx} = e^{\frac{3}{2}\ln|x|} = x^{3/2}$

$\Rightarrow \quad \frac{d}{dx}\left[x^{3/2}v\right] = 0$

$\Rightarrow \quad x^{3/2}v = c_1$

$v = c_1 x^{-3/2}$

$u' = c_1 x^{-3/2}$

$u = -2c_1 x^{-1/2} + c_2$

$= c_2 - 2c_1 x^{-1/2}$

$y = y_1 u$

$= x \cdot (c_2 - 2c_1 x^{-1/2})$

$= c_2 x - 2c_1 x^{1/2}$