Math 2280 Lecture notes Day 15

Spatial Term'

* Chapter 9 - Quiz, Not on Exam or final
* Chapter 10 - Skip → Math 4620 is better
* Chapter 11 - Modeling

The models are for/of a measurable quantity.

- Rabbits in a field
- velocity of a falling duck
- temperature of a cup of coffee

Notation:

t - time

$Q(t)$ = the amount of the measurable quantity at time t.

$\Rightarrow \frac{dQ}{dt}$ = rate of change of the measurable quantity at time t.

If we identify what controls this rate, we can write a first order IVP for many problems

$$\begin{cases} \frac{dQ}{dt} = F(t, Q) \\ Q(0) = Q_0 \end{cases}$$

Ex. Better falling body model

$$m\frac{dy}{dt} = -mg \qquad \Rightarrow \qquad m\frac{d^2y}{dt^2} = -mg - r\frac{dy}{dt}$$

$$y(0) = 0 \qquad\qquad y(0) = y_0,\ y'(0) = v_0$$

Rabbit Ranching Model

t - number of months since the rabbits were released

$R(t)$ - the number of rabbits at time t.

$R(0) = 2$ - at least 1 male & 1 female.

$\Rightarrow \frac{dR}{dt}$ = change in the number of rabbits per month.

= number of births per month - number of deaths per month

$\rightarrow$ number of births per month

= (number of births per female rabbit per month) $\times$ (number of female rabbits)

= number of births per female rabbit per month $\times$ ($\frac{1}{2}R$)

Assumption: The average number of births per female rabbit per month is a constant. For convenience set

$\beta = \frac{1}{2} \times$ number of births per female per month.

This is the "monthly birth rate per rabbit."

$\rightarrow$ # of births per month = βR

Assumptions: Each female has 6 litters of 5 bunnies per year (12 month)

So,

$\beta = (\frac{1}{2}) \times$ {number of births per female rabbit per month

$= (\frac{1}{2})(\frac{1}{12})(6)(5)$

$= \frac{5}{4}$

For the deaths: 10 year life span. So the death rate may be blown off
(120) months

$\Rightarrow$ deaths / month $= 0$)

$\Rightarrow \frac{dR}{dt} = \beta R - 0 = \beta R$

IVP

$\begin{cases} \frac{dR}{dt} = \frac{5}{4} R \\ R(0) = 2 \end{cases} \Rightarrow \frac{1}{R}\, dR = \frac{5}{4}\, dt \Rightarrow \ln|R| = \frac{5}{4} t + C$

$\Rightarrow R(t) = Ae^{\frac{5}{4}t}$

$= 2 \cdot Ae^{\frac{5}{4}t}$

$\underline{R(60) = \ldots}$

Use the model

mass of earth $= 6 * 10^{24}$ kg

$\underline{\text{mass of sun} = 2 \times 10^{30}}$

$R(60) = 7.47 * 10^{32}$

The rabbits weigh about 3kg more than the sun!

$\Rightarrow$ mass $= 2.2 * 10^{33}$ kg

Exponential models

1. Population ⟶ $P(t) = P_0 e^{\alpha t}$
2. Radioactive Materials ⟶ $A(t) = A_0 e^{-\alpha t}$

Rabbit Ranching

$$\begin{cases} \dfrac{dR}{dt} = \beta_0 R - \gamma R^2 \\ R(0) = 0 \end{cases}$$ ⟵ Constant Solution

Solve this

Chapter 12 – Skip

Part 2

Chapter 13

$y'' + y = 0$

$y'' + 2xy' - 5\sin(x)y = 30e^{2x}$

$(y+1)y'' = (y')^2$

$y^{(83)} + 2y^3 y^{(63)} = x^2 y^2$

Use what we know at first

One example:

If the ODE explicitly involves x, y', and y'' and not y, we can substitute,

$$\frac{dy}{dx} = v \Rightarrow \frac{d^2y}{dx^2} = \frac{dv}{dx}$$

Ex: $$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 30e^{3x}$$

$$v = \frac{dy}{dx}$$

$$\frac{dv}{dx} = \frac{d^2y}{dx^2}$$

$$\frac{dv}{dx} + 2v = 30e^{3x}$$

⋮

first order linear

$$\mu = e^{\int 2} = e^{2x}$$

$$\Rightarrow \frac{d}{dx}\left[e^{2x}v\right] = e^{2x}\left(30e^{3x}\right) = 30e^{5x}$$

$$\Rightarrow e^{2x}v = \frac{30}{5}e^{5x} + c_1$$

$$\Rightarrow v = 6e^{3x} + c_1e^{-2x}$$

$$\Rightarrow \frac{dy}{dx} = 6e^{3x} + c_1e^{-2x}$$

$$y(x) = \frac{6e^{3x}}{3} - \frac{c_1}{2}e^{-2x} + c_2$$