Practice Quiz 6 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, SPRING 2024

NAME: Solutions A#: _____

Problem 1. Exercise 13.2e (10 points) For the following, determine if the given equation contains y. If it does not, solve it using the substitution $v = y'$ as described in Section 3.1.

$$x\,y'' - y' = 6\,x^5$$

Solution:

This equation does not involve (explicitly) y.

$$\begin{cases} v = y' \\ v' = y'' \end{cases}$$

$$\Rightarrow x\,v' - v = 6x^5$$

$$\Rightarrow x\,\frac{dv}{dx} - v = 6x^5$$

$$\Rightarrow \frac{dv}{dx} - \frac{1}{x}v = 6x^4$$

$p(x) = -\frac{1}{x} \Rightarrow \mu = e^{\int -1/x} = e^{-\ln|x|} = e^{\ln x^{-1}} = x^{-1}$

$$\Rightarrow \frac{d}{dx}\left[x^{-1}v\right] = 6\cdot x^3$$

$$\Rightarrow x^{-1}v = \frac{6}{4}x^4 + C_1$$

$$\Rightarrow v = \frac{6}{4}x^5 + C_1 x$$

$$\Rightarrow y' = \frac{6}{4}x^5 + C_1 x$$

$$\Rightarrow y = \frac{1}{4}x^6 + \frac{1}{2}C_1 x^2 + C_2$$

$$= \frac{1}{4}x^6 + Ax^2 + B$$

Problem 2. Exercise 13.5c (10 points) For the following, determine if the given differential equation is autonomous. If it is, then solve it using the substitution $v = y'$ as described in Section 3.2.

$$y'\, y'' = 1$$

Recall: $\frac{d^2y}{dx^2} = v\,\frac{dv}{dy}$.

Solution:

The equation is autonomous

$$\begin{cases} y' = v \\ y'' = v\frac{dv}{dy} \end{cases}$$

$$\Rightarrow v\cdot\left(v\frac{dv}{dy}\right) = 1$$

$$\Rightarrow v^2\frac{dv}{dy} = 1$$

$$\Rightarrow v^2\,dv = dy$$

$$\Rightarrow \int v^2\,dv = \int dy$$

$$\Rightarrow \frac{1}{3}v^3 = y + C_1$$

$$\Rightarrow v^3 = (3y + C_2)$$

$$\Rightarrow v = (3y + C_2)^{1/3}$$

$$\Rightarrow y' = (3y + C_2)^{1/3}$$

$$\Rightarrow \frac{1}{(3y+C_2)^{1/3}}\,dy = dx$$

$$\Rightarrow \int (3y + C_2)^{-1/3}\,dy = x + C_3$$

$$\Rightarrow \frac{3}{2}(3y + C_2)^{2/3}\left(\frac{1}{3}\right) = x + C_3$$

$$\Rightarrow \frac{1}{2}(3y + C_2)^{2/3} = x + C_3$$

$$\Rightarrow (3y + C_2)^{2/3} = 2x + C_4$$

$$\Rightarrow 3y + C_2 = \pm(2x + C_4)^{3/2}$$

$$\Rightarrow 3y = C_2 \pm (2x + C_4)^{3/2}$$

$$\Rightarrow y = C_5 \pm \frac{1}{3}(2x + C_4)^{3/2}$$