Practice Quiz 5 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, FALL 2023

NAME: Solutions A#: ____________

Problem 1. Chapter 6.5b Use the substitution appropriate to Bernoulli equations to find a general solutionto the following:

$$\frac{dy}{dx} - \frac{3}{x}\,y = \left(\frac{y}{x}\right)^2$$

Solution:

First, write

$$\frac{dy}{dx} - \frac{3}{x}y = \frac{1}{x^2}y^2$$

$n=2 \Rightarrow r = 1-n = -1$

$u = y^{1-n} = y^{-1}$

$\Rightarrow y = u^{-1} \Rightarrow \frac{dy}{dx} = -u^{-2}\frac{du}{dx}$

$$-u^{-2}\frac{du}{dx} - \frac{3}{x}u^{-1} = \frac{1}{x^2}(u^{-1})^2$$

$$\Rightarrow -u^{-2}\frac{du}{dx} - \frac{3}{x}u^{-1} = \frac{1}{x^2}u^{-2}$$

$$\Rightarrow \frac{du}{dx} + \frac{3}{x}u = \frac{1}{x^2}$$

$$p(x) = 3/x \Rightarrow \mu = e^{\int 3/x\,dx} = e^{3\ln|x|} = e^{\ln x^3} = x^3$$

$$\Rightarrow x^3\frac{du}{dx} + 3x^2u = x$$

$$\Rightarrow \frac{d}{dx}\left[x^3u\right] = x$$

$$\Rightarrow x^3u = \frac{1}{2}x^2 + C$$

$$\Rightarrow u = \frac{1}{2}x^{-1} + Cx^{-3}$$

$$\Rightarrow y^{-1} = \frac{1}{2}x^{-1} + Cx^{-3}$$

$$y = \left(\frac{1}{2}x^{-1} + Cx^{-3}\right)^{-1}$$

Problem 2. Chapter 7.4a (10 points) The following differential equation is in exact form. Find a corresponding potential function and then find a general solutionto the differential equation using that potential equation (even if it can be solved by simpler means).

$$2\,x\,y\;+y^2+[2\,x\,y+x^2]\,\frac{dy}{dx}=0$$

Solution:

$M(x,y) = 2xy + y^2 \longrightarrow \frac{\partial M}{\partial y} = 2x + 2y$

$N(x,y) = 2xy + x^2 \longrightarrow \frac{\partial N}{\partial x} = 2y + 2x$ equal

$\frac{\partial \phi}{\partial x} = 2xy + y^2 \Rightarrow \phi(x,y) = x^2y + xy^2 + p(y)$

$\Rightarrow \frac{\partial \phi}{\partial y} = x^2 + 2xy + p'(y)$

$\Rightarrow \cancel{2xy} + x^2 = x^2 + \cancel{2xy} + p'(y)$

$\Rightarrow p'(y) = 0 \Rightarrow p(y) = C_1$

So, $\phi(x,y) = x^2y + xy^2 = C$

$\Rightarrow xy^2 + x^2y - C = 0$

$\Rightarrow y = \dfrac{-x^2 \pm \sqrt{x^4 + 4xC}}{2x}$