Practice Quiz 4 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, SPRING 2024

NAME: Solutions A#: _____

Problem 1. Exercise 6.2 (10 points) Using a linear substitution, solve the initial value problem.

$$\frac{dy}{dx} = 1 + (y - x)^2$$

with $y(0) = \frac{1}{4}$

Solution:

$u = y - x \Rightarrow y = u + x$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + 1$

no constant solution

$\Rightarrow \frac{du}{dx} + 1 = u^2 + 1$

$\hookrightarrow \frac{du}{dx} = u^2$

$\hookrightarrow \frac{1}{u^2}\frac{du}{dx} = 1$

$\hookrightarrow \frac{1}{u^2}\frac{du}{dx}dx = dx$

$\hookrightarrow \int u^{-2}\,du = \int dx$

$\hookrightarrow -u^{-1} = x + C_1$

$\hookrightarrow u^{-1} = -(x + C_1)$

$\hookrightarrow u = -\frac{1}{x + C_1}$

$\hookrightarrow y - x = -\frac{1}{x + C_1}$

$\hookrightarrow y = x - \frac{1}{x + C_1}$

$y(0) = 0 - \frac{1}{0 + C_1} = -\frac{1}{C_1} = \frac{1}{4}$

$\Rightarrow C_1 = -4$

So, $y(x) = x - \frac{1}{x - 4}$

Problem 2. Exercise 6.7c (10 points) For the following determine a substitution that simplifies the differential equation, and using the substitution, find the general solution.

$$\frac{dy}{dx} + \frac{2}{x}\, y = 4\sqrt{y}$$

Solution:

Bernoulli Eqn

$y=0$ is always a solution for Bernoulli eqn's

$$u = y^r = y^{1-n} = y^{1-\frac{1}{2}} = y^{\frac{1}{2}}$$

$$\Rightarrow \quad y = u^2 \Rightarrow \quad \frac{dy}{dx} = 2u\frac{du}{dx}$$

So

$$2u\frac{du}{dx} + \frac{2}{x}u^2 = 4\left(u^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$\hookrightarrow \quad 2u\frac{du}{dx} + \frac{2}{x}u^2 = 4u \qquad \text{divide by } 2u$$

$$\hookrightarrow \quad \frac{du}{dx} + \frac{1}{x}u = 2$$

$$\Rightarrow \quad p(x) = \frac{1}{x} \Rightarrow \mu = e^{\int p(x)} = e^{\ln|x|} = x$$

$$\rightarrow \quad \frac{d}{dx}\left[x \cdot u\right] = 2x$$

$$\rightarrow \quad x \cdot u = x^2 + C_1$$

$$\rightarrow \quad u(x) = \frac{x^2 + C_1}{x}$$

$$\Rightarrow \quad y^{\frac{1}{2}} = \frac{x^2 + C_1}{x}$$

$$\rightarrow \quad y(x) = \left(\frac{x^2 + C_1}{x}\right)^2$$