Practice Quiz 10 MATH 2280, ORDINARY DIFFERENTIAL EQUATIONS, FALL 2023

NAME: Solutions A#: ______

Problem 1. 26.3 Let α be any real number and show that

$$\mathcal{L}[e^{at}]\,|_s = \frac{1}{s-\alpha}, \qquad \alpha < s$$

Solution:

$$\mathcal{L}[e^{\alpha t}] = \int_0^\infty e^{\alpha t} e^{-st}\,dt$$

$$= \int_0^\infty e^{-(s-\alpha)t}\,dt$$

$$= -\frac{1}{s-\alpha}\cdot e^{-(s-\alpha)t}\Big|_0^\infty$$

$$= 0 + \frac{1}{s-\alpha}e^0 = \frac{1}{s-\alpha}$$

Problem 2. Ex. 26.8.e (10 points) Verify the following Laplace transform using integration.

$f(t) = sinh(4t)$

Hint: Use the definition of $sinh(4t)$ in terms of natural exponential functions.

Solution:

$$\mathcal{L}[\sinh(4t)] = \int_0^\infty \sinh(4t)e^{-st}\,dt$$

$$= \int_0^\infty \frac{e^{4t}-e^{-4t}}{2}\cdot e^{-st}\,dt$$

$$= \frac{1}{2}\left(\int_0^\infty e^{4t}e^{-st}\,dt - \int_0^\infty e^{-4t}e^{-st}\,dt\right)$$

$$= \frac{1}{2}\int_0^\infty e^{-(s-4)t}\,dt - \frac{1}{2}\int_0^\infty e^{-(s+4)t}\,dt$$

$$= \frac{1}{2}\cdot\left(\frac{-1}{s-4}e^{-(s-4)t}\Big|_0^\infty\right) - \frac{1}{2}\left(-\frac{1}{s+4}e^{-(s+4)t}\Big|_0^\infty\right)$$

$$= \frac{1}{2}\left(\frac{1}{s-4}\right) - \frac{1}{2}\cdot\left(\frac{1}{s+4}\right)$$

$$= \frac{1}{2}\left(\frac{(s+4)-(s-4)}{(s-4)(s+4)}\right)$$

$$= \frac{1}{2}\cdot\frac{8}{s^2-16} = \frac{4}{s^2-16}$$