Quiz 9

NAME: $\quad$ Solution $\qquad$ A#: $\qquad$

Problem 1. Ex. 17.4.b Solve the following initial value problem.

$$y'' - 8\,y' + 16\,y = 0, \quad y(0) = 0, \ y'(0) = 1$$

Solution:

First write the characteristic polynomial is

$$r^2 - 8r + 16 = 0$$
$$\Rightarrow (r-4)^2 = 0$$
$$\Rightarrow r_1 = 4, \ r_2 = 4 \quad (\text{repeated})$$
$$y_1 = e^{4x}, \ y_2 = xe^{4x}$$

So,

$$y = c_1 y_1 + c_2 y_2$$
$$= c_1 e^{4x} + c_2 x e^{4x}$$

$$y(0) = c_1 e^0 + c_2 \cdot 0 \cdot e^0 \quad c_1 = 0 \Rightarrow c_1 = 0$$

$$y'(x) = 4 c_1 e^{4x} + c_2 (e^{4x} + 4 x e^{4x})$$

$$y'(0) = 4 \cdot 0 \cdot e^0 + c_2 (e^0 + 0)$$
$$= c_2 = 1$$

So, $y(x) = 0 \cdot e^{4x} + (1) x e^{4x}$

$$\underline{= x e^{4x}}$$

Problem 2. Ex 19.1.c (10 points) Using clever factoring, of the characteristic polynomial find the general solution of the following:

$$y^{(4)} - 34\,y'' + 225\,y = 0$$

Solution:

The characteristic equation is

$$r^4 - 34r^2 + 225 = 0$$

$$p(1) = 1^4 - 34(1)^2 + 225 \neq 0$$

$$p(3) = 3^4 - 34(3)^2 + 225$$
$$= 81 - 306 + 225 = 0 \checkmark$$

try $1, 3, 5, 9, \ldots$

$$\longrightarrow (r-3)(r^3 + 3r^2 - 25r - 75) = 0$$

$$
\begin{array}{r}
r^3 + 3r^2 - 25r - 75 \\
(r-3)\,\overline{\smash{)}\,r^4 \quad -34r^2 \quad +225} \\
\underline{r^4 - 3r^3} \\
3r^3 - 34r^2 \qquad +225 \\
\underline{3r^3 - 9r^2} \\
-25r^2 \qquad +225 \\
\underline{= 25r^2 + 75r} \\
-75r + 225 \\
\underline{-75r + 225}
\end{array}
$$

Another way $\quad S = r^2$

$$r^4 - 34r^2 + 225 = S^2 - 34S + 225 = 0$$
$$= (S - 9)(S - 25)$$
$$= (r^2 - 9)(r^2 - 25)$$
$$= (r - 3)(r + 3)(r - 5)(r + 5)$$
$$\Rightarrow r_1 = 3, \ r_2 = -3, \ r_3 = 5, \ r_4 = -5$$
$$\Rightarrow y_1 = e^{3x}, \ y_2 = e^{-3x}, \ y_3 = e^{5x}, \ y_4 = e^{-5x}$$

$$y = C_1 e^{3x} + C_2 e^{-3x} + C_3 e^{5x} + C_4 e^{-5x}$$