Problem Set II, Macroeconomics III
University of Copenhagen, 2024

John Kramer
Department of Economics, University of Copenhagen

February 2025

1. Consider the social planner problem for the Ramsey economy. This means that the planner owns all capital and can dispose of all production by deciding the allocation of resources into consumption or investment. The planner is assumed to be benevolent, i.e. it wants to maximize households' intertemporal utility. Output is produced with capital and labor and there are constant returns to scale in production. Labor is supplied inelastically. Assume for simplicity that there is no technological growth. We denote by k capital per worker, and $f(k)$ output per capita. Assume that population grows at rate $1+n$ and capital depreciates at rate δ. Thus the planner's problem can be stated as:

$$\max_{\{c_t\}_0^\infty} \sum_0^\infty \beta^t (1+n)^t u\left(c_t\right)$$
$$\text{s.t. } (1+n)k_{t+1} = k_t(1-\delta) + f\left(k_t\right) - c_t$$
$$k_0 > 0 \text{ given}, \quad k_{t+1} \geq 0$$

where c is consumption per capita, and β is the discount rate (households value current consumption more than future consumption). We assume $\beta(1+n) < 1$.

   a) Write the Lagrangian for the optimization problem, find the first order conditions that characterize optimal choices, and from these the Euler equation. Give an economic interpretation to this equation.

   b) Characterize the steady state for this economy. Draw the graphical representation of the equations that represent the dynamics of this economy into a phase diagram (in $c \times k$ space). Describe how the variables move in the different regions and plot the saddle path. How is initial consumption determined for each possible level of k_0 ? Explain.

2. Consider the following version of the Ramsey model with population growing at rate $1+n$. Identical competitive firms maximize the following profit function:

$$\pi\left(K_t, L_t\right) = AK_t^{\alpha} L_t^{1-\alpha} - w_t L_t - r_t K_t$$

where r_t is the economy wide interest rate, and w_t is the wage rate, K_t and L_t denote the quantities of capital and labor employed by the firm. Assume $0 < \alpha < 1$.

A large number of identical households maximize the following intertemporal utility function, that depends on per-capita consumption c_t :

$$U = \sum_{0}^{\infty} \beta^t (1+n)^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

where $\sigma > 0$ is the coefficient of relative risk aversion (for simplicity we assume $\sigma \neq 1$). The maximization is subject to their dynamic budget constraint: 1

$$(1+n)a_{t+1} = a_t R_t + w_t - c_t$$

with $a_0 = k_0 > 0$ given, $a_t = k_t + b_{pt}$ (lower case variables represent quantities in per capita terms) a is wealth, and b_p lending to other households, and $R_t = 1 + r_t - \delta$.

a) Find the first order conditions for the firms' maximization problem. Write the no Ponzi game condition. What is its economic meaning?

b) Write the Lagrangian for the households' optimization problem, find the first order conditions that characterize the behavior of households, and from these the Euler equation (the Keynes Ramsey rule). Give an economic interpretation to this equation.

c) Explain the importance of the no Ponzi game condition and the transversality condition. How do they differ in economic terms?

d) Draw the phase diagram and find the equations that characterize steady state. What is the effect of a permanent increase in n? Find the transition path of the model. Given this, discuss the model's implications for capital accumulation in countries with high or low fertility.