Rigid prices

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Last time

A simple model of rigid prices

- Rational expectations
- The model produces a positive relationship between output and prices
- But policy makers cannot exploit it (unless they surprise everyone)

Lucas critique

- Don't take an empirical relationship for granted! Agents might reoptimize when we try to exploit it
- Expectations affect the equilibrium
- "Physicists do not teach atoms how to behave" - Luigi Zingales

Today

Some Empirics

A New Keynesian model – light

- **Dynamic** model with monopolistic competition
- Main message remains intact: only unexpected shocks have effects

Price rigidities

- Fischer contracts
- Taylor contracts

Optimal policy

- What should central banks do against demand shocks?

Empirics about price changes

Important research

Jon Steinsson & Emi Nakamura

- Five Facts about Prices: A Reevaluation of Menu Cost Models (2008)
- Price Rigidity: Microeconomic Evidence and Macroeconomic Implications (2013)

→ Need to know how we should model price rigidity

Data about price changes

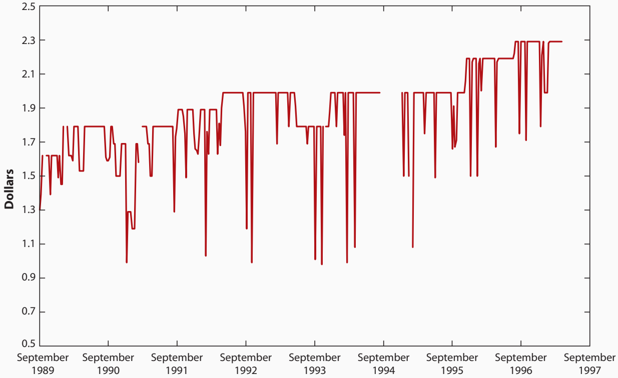


Figure 2

Price series of Nabisco Premium Saltines (16 oz) at a Dominick's Finer Foods store in Chicago.

- Is a sale a price change?

Data about price changes

Table 2 Transience of temporary sales

	Fraction return after one-period sales	Frequency of regular price change	Frequency of price change during one-period sales	Average duration of sales
Processed food	78.5	10.5	11.4	2.0
Unprocessed food	60.0	25.0	22.5	1.8
Household furnishings	78.2	6.0	11.6	2.3
Apparel	86.3	3.6	7.1	2.1

The sample period is 1998–2005. The first data column gives the median fraction of prices that return to their original level after one-period sales. The second is the median frequency of price changes excluding sales. The third lists the median monthly frequency of regular price change during sales that past one month. The monthly frequency is calculated as $1 - (1 - f)^{0.5}$, where f is the fraction of prices that return to their original levels after one-period sales. The fourth data column gives the weighted average duration of sale periods in months. Data taken from Nakamura & Steinsson (2008).

- Most prices return to original level (Nakamura & Steinsson, 2013)
- Still an important question → more work needs to be done

Data about price changes

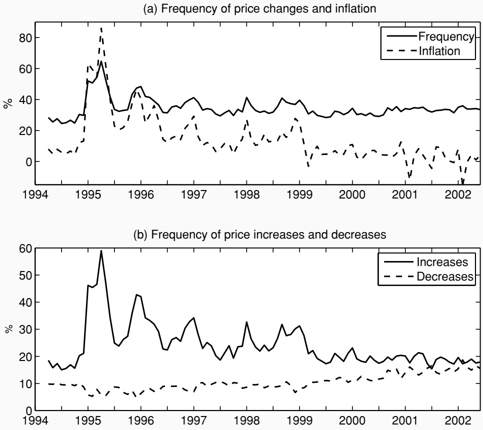


FIGURE III
Monthly Frequency of Price Changes (Nonregulated Goods)
All statistics in the figure, including inflation, are computed using the sample of nonregulated goods.

- In Mexico, high inflation means more adjustment (Gagnon, 2009)
- Frequency is flat for low inflation period → state dependence

Data about price changes

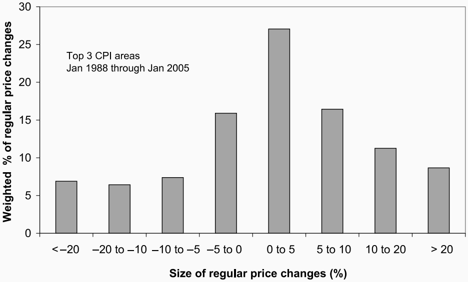


FIGURE II
Weighted Distribution of Regular Price Changes

- Average price change is positive and big $> 5\%$
- Many price changes are very small (Klenow & Krystov, 2008)

Data about price changes

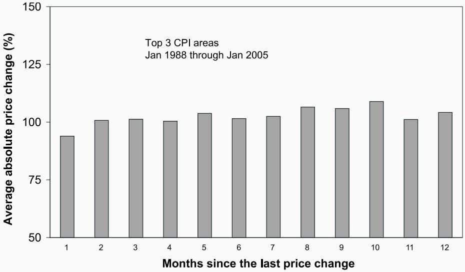


FIGURE VIII
Size of Regular Price Changes by Age vs. Decile Fixed Effects

- Price change doesn't seem to depend on when price was last set
- Klenow & Krystov (2008)

Empirical summary

Measuring price changes is complicated

- Sales
- New goods
- Better quality products

Price changes

- Higher inflation leads to higher and more frequent price changes
- In low inflation periods, most price changes are small
- Price change doesn't depend on how old the price is

NK Model – light

The New Keynesian model – light

Today's model is almost the New Keynesian model

- As last time: cut out the middle man

→ No workers and firms, every household produces one good and consumes all goods

→ No wages, just disutility of work/utility of leisure

- Limited dynamics

Other crucial features

- Monopolistic competition and pricing power

→ **Keynesian**

Setup – same as last lecture

Representative household

$$U_i = \mathbf{C}_i - \frac{1}{\phi} L_i^{\phi}$$

- $\mathbf{C}$ is a consumption basket, as in previous lectures, composed of C_i
- Households use their labor L to produce output according to $Y_i = L_i$
- P is the aggregate price level, P_i is the price of the household's variety i

Budget constraint

$$P\mathbf{C}_i = P_i Y_i \implies \mathbf{C}_i = \frac{P_i}{P} Y_i$$

Demand function

$$Y_i = \left(\frac{P_i}{P}\right)^{-\theta} Y \iff \frac{P_i}{P} = \left(\frac{Y_i}{Y}\right)^{-1/\theta}$$

Rearrange

Optimal output

$$\left(1-\frac{1}{\theta}\right)\left(\frac{Y_i}{Y}\right)^{-\frac{1}{\theta}}=Y_i^{\phi-1}$$

$$\underbrace{\left(\frac{\theta-1}{\theta}\right)}_{\text{Inverse markup}}\left(\frac{P_i}{P}\right)=Y_i^{\phi-1} \leftarrow \text{ p/P} = \text{markup} * \text{“marginal cost”}$$

Take logs

$$p_i-p=(\phi-1)y_i-\log\left(\frac{\theta-1}{\theta}\right)$$

$$p_i-p=(\phi-1)y_i+\mathcal{M}$$

Price setting

Optimal price setting

$$p_i^* - p = (\phi - 1)y_i + \mathcal{M}$$

- Optimal price depends on elasticity of labor supply $\frac{1}{\phi}$ and markup $\mathcal{M}$

Aggregate up (all firms are symmetric $\implies$ make the same choices)

$$p^* - p = (\phi - 1)\underbrace{(m - p)}_{y} + \mathcal{M}$$

$$p^* = (\phi - 1)m + (2 - \phi)p + \mathcal{M}$$

- Note that $p = m$ is not the outcome if $p^* = p$, due to monopolistic competition. Prices are higher in this model, compared to Lucas'
- As ϕ rises (labor becomes less elastic) m's effect on p^* increases

Introducing expectations

Demand (money supply) is not deterministic

- Households must form expectations
- General expression! **How** m and p vary is irrelevant

$$p_t^* = (\phi - 1)\mathbb{E}_t[m_{t+1}|I_t] + (2 - \phi)\mathbb{E}_t[p_{t+1}|I_t] + \mathcal{M}$$

- Households form expectations about price level

$$\mathbb{E}_{t-1}[p_t] = \mathbb{E}_{t-1}\left\{(\phi - 1)\mathbb{E}_{t-1}[m_t] + (2 - \phi)\mathbb{E}_{t-1}[p_t] + \mathcal{M}\right\}$$
$$\mathbb{E}_{t-1}[p_t] = \mathbb{E}_{t-1}[m_t] + \frac{1}{\phi - 1}\mathcal{M}$$

Equilibrium

Prices and output

$$p = (\phi - 1)\mathbb{E}[m|I] + (2-\phi)\left(\mathbb{E}[m|I] + \frac{1}{\phi - 1}\mathcal{M}\right) + \mathcal{M}$$
$$= \mathbb{E}[m|I] + \frac{1}{\phi - 1}\mathcal{M}$$
$$y = m - \mathbb{E}[m|I] - \frac{1}{\phi - 1}\mathcal{M}$$

- Only unanticipated movements in aggregate demand ($\mathbb{E}[m] \neq m$) have real effects
- Monopolistic competition leads to lower output and welfare loss
- Sometimes, we suppress the markup, but with Mon.Comp. it's there

Rigid/sticky prices

Two different approaches today

- Fischer contracts: Set price **schedule** in advance
- Taylor pricing: Fix prices for a certain time
- Calvo fairy: Fixed probability of adjusting

Not here, but still interesting

- Menu costs: pay fixed price to change a price

Fischer contracts (DR 7.2)

Environment

- Firms set price schedules in advance and stick to them
- Only some fraction of firms renews their schedules each period
- Everyone has to fulfil demand $\Longrightarrow$ work more if prices too low

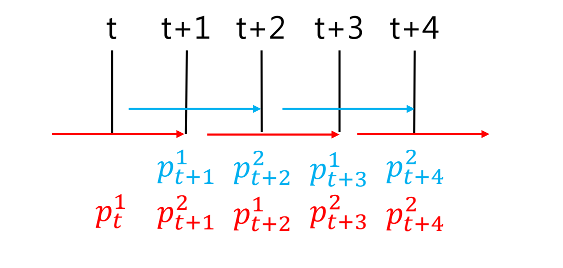
Operationalization

- Each price-setter sets prices for two periods – potentially different
- Assume that half of producers set prices in even periods, rest in odd
- Rational expectations: everyone knows the environment

Timing

- Those who reset do so right before the end of period $t-1$, setting prices for t and $t+1$
- They **do not** know the shocks in t

Timing and notation



- Price schedules are set with the information set of the previous period
- Prices can change over time
- Subscript: period for which prices were set
- Superscript: how many periods ago prices were set

Price level with Fischer contracts

Price level

$$p_t = \frac{1}{2}\left(p_t^1 + p_t^2\right)$$
$$p_t^* = (\phi - 1)m + (2 - \phi)p + \mathcal{M}$$

Optimal price setting in expectation

$$p_t^1 = \mathbb{E}_{t-1}[p_t^*] = (\phi - 1)\mathbb{E}_{t-1}[m_t] + (2 - \phi)\frac{1}{2}\left(p_t^1 + p_t^2\right) + \mathcal{M}$$
$$p_t^2 = \mathbb{E}_{t-2}[p_t^*] = (\phi - 1)\mathbb{E}_{t-2}[m_t] + (2 - \phi)\frac{1}{2}\left(\mathbb{E}_{t-2}[p_t^1] + p_t^2\right) + \mathcal{M}$$

- p_t^2 is observed in period $t-1$, so no expectation needed
- Price setters don't need expectations about their own prices

Rearrange and solve

$$p_t^1 = \frac{2(\phi-1)}{\phi}\mathbb{E}_{t-1}[m_t] + \frac{(2-\phi)}{\phi}p_t^2 + \frac{2}{\phi}\mathcal{M}$$
$$p_t^2 = \frac{2(\phi-1)}{\phi}\mathbb{E}_{t-2}[m_t] + \frac{(2-\phi)}{\phi}\mathbb{E}_{t-2}[p_t^1] + \frac{2}{\phi}\mathcal{M}$$

- Rational expectations imply that resetters 2 periods ago knew the other's policy function $\implies$ plug in + some tedious algebra

$$p_t^2 = \mathbb{E}_{t-2}[m_t] + \frac{1}{(\phi-1)}\mathcal{M}$$

- Exactly the same as before: base prices on expected demand
- Use this expression to solve for p_t^2

$$p_t^1 = \mathbb{E}_{t-1}[m_t] + \frac{\phi-2}{\phi}\underbrace{\left(\mathbb{E}_{t-1}[m_t] - \mathbb{E}_{t-2}[m_t]\right)}_{\text{Updated information set}} + \frac{1}{(\phi-1)}\mathcal{M}$$

Equilibrium with Fischer contracts

$$
\begin{aligned}
p_t &= \frac{1}{2}\left(p_t^1 + p_t^2\right) \\
&= \frac{1}{2}\left(\left[\mathbb{E}_{t-1}[m_t] + \frac{\phi-2}{\phi}\left(\mathbb{E}_{t-1}[m_t] - \mathbb{E}_{t-2}[m_t]\right)\right] + \left[\mathbb{E}_{t-2}[m_t]\right]\right) + \frac{1}{(\phi-1)}\mathcal{M} \\
&= \mathbb{E}_{t-1}[m_t] - \frac{1}{\phi}\left(\mathbb{E}_{t-1}[m_t] - \mathbb{E}_{t-2}[m_t]\right) + \frac{1}{(\phi-1)}\mathcal{M} \\
y_t &= m_t - \mathbb{E}_{t-1}[m_t] + \frac{1}{\phi}\left(\mathbb{E}_{t-1}[m_t] - \mathbb{E}_{t-2}[m_t]\right) - \frac{1}{(\phi-1)}\mathcal{M} \\
&= \varepsilon_t + \frac{1}{\phi}\varepsilon_{t-1} - \frac{1}{(\phi-1)}\mathcal{M}
\end{aligned}
$$

Consider a surprising increase in m_t, announced after p_t^2 was set

Equilibrium with Fischer contracts

$$
\begin{aligned}
p_t &= \frac{1}{2}\left(p_t^1 + p_t^2\right) \\
&= \frac{1}{2}\left(\left[\mathbb{E}_{t-1}[m_t] + \frac{\phi-2}{\phi}\left(\mathbb{E}_{t-1}[m_t] - \mathbb{E}_{t-2}[m_t]\right)\right] + \left[\mathbb{E}_{t-2}[m_t]\right]\right) + \frac{1}{(\phi-1)}\mathcal{M} \\
&= \mathbb{E}_{t-1}[m_t] - \frac{1}{\phi}\left(\mathbb{E}_{t-1}[m_t] - \mathbb{E}_{t-2}[m_t]\right) + \frac{1}{(\phi-1)}\mathcal{M} \\
y_t &= m_t - \mathbb{E}_{t-1}[m_t] + \frac{1}{\phi}\left(\mathbb{E}_{t-1}[m_t] - \mathbb{E}_{t-2}[m_t]\right) - \frac{1}{(\phi-1)}\mathcal{M} \\
&= \varepsilon_t + \frac{1}{\phi}\varepsilon_{t-1} - \frac{1}{(\phi-1)}\mathcal{M}
\end{aligned}
$$

Consider a surprising increase in m_t, announced after p_t^2 was set

- Prices are **lower** compared to flexprice: p^2s were locked in in $t-1$ at too low level
- Output is **higher** compared to flexprice
- If labor is inelastic the costs of fixing prices are lower

Monetary policy

Thinking of m as completely random seems strange

- Central banks are not trying to surprise anybody
- They try to "lean against the wind" and work against demand shocks

New definition of aggregate demand

- Postulate the following relationship:

$$y_t = m_t^{cb} - p_t + v_t$$

- New definition: v_t represents shocks to aggregate demand
- Think of m_t^{cb} as (potentially active) monetary policy
- Important: monetary policy does not know more than the market

New equilibrium

Note: m_t^{cb} and v_t always enter as a sum $\implies$ substitute for m_t

$$p_t = \mathbb{E}_{t-1}[m_t^{cb} + v_t] - \frac{1}{\phi}\left(\mathbb{E}_{t-1}[m_t^{cb} + v_t] - \mathbb{E}_{t-2}[m_t^{cb} + v_t]\right) + \frac{1}{(\phi - 1)}\mathcal{M}$$

$$y_t = m_t^{cb} + v_t - \mathbb{E}_{t-1}[m_t^{cb} + v_t] + \frac{1}{\phi}\left(\mathbb{E}_{t-1}[m_t^{cb} + v_t] - \mathbb{E}_{t-2}[m_t^{cb} + v_t]\right) - \frac{1}{(\phi - 1)}\mathcal{M}$$

Question

- If v_t is random, but m_t^{cb} can be controlled, what should policy makers do?
- What's the **optimal** monetary policy to stabilize output?

Fischer contracts with demand stabilization

Environment

- Assume v_t follows a random walk: $v_t = v_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$
- Assume monetary policy is given by

$$m_t^{cb} = a_1 \varepsilon_{t-1} + a_2 \varepsilon_{t-2} + \cdots$$

- Policy makers know all past information
- Policy rule is perfectly credible but cannot react immediately
- The policy rule only contains linear terms – this turns out to be enough, but we will return to this issue.
- Question: What should be the weight on each a_x term?

Fischer contracts with demand stabilization II

Rewriting

- Plugging in the information on the previous slide gives

$$m_t^{cb} + v_t = \underbrace{a_1\varepsilon_{t-1} + a_2\varepsilon_{t-2} + \cdots}_{m_t} + \underbrace{v_{t-1} + \varepsilon_t}_{v_t}$$

- This is a model with rational expectations, hence the policy rule enters expectations

$$\begin{aligned} y_t =& m_t^{cb} + v_t - \mathbb{E}_{t-1}[m_t^{cb} + v_t] \\ &+ \frac{1}{\phi}\left(\mathbb{E}_{t-1}[m_t^{cb} + v_t] - \mathbb{E}_{t-2}[m_t^{cb} + v_t]\right) - \frac{1}{(\phi-1)}\mathcal{M} \end{aligned}$$

Fischer contracts with demand stabilization III

$$
\begin{aligned}
y_t =& m_t^{cb} + v_t - \mathbb{E}_{t-1}[m_t^{cb} + v_t] \\
& + \frac{1}{\phi}\left(\mathbb{E}_{t-1}[m_t^{cb} + v_t] - \mathbb{E}_{t-2}[m_t^{cb} + v_t]\right) - \frac{1}{(\phi - 1)}\mathcal{M}
\end{aligned}
$$

Future can be expressed in terms of the past

$$
\begin{aligned}
\mathbb{E}_{t-1}[m_t^{cb} + v_t] &= \mathbb{E}_{t-1}[a_1\varepsilon_{t-1} + a_2\varepsilon_{t-2} + a_3\varepsilon_{t-3} + \cdots + v_{t-1} + \varepsilon_t] \\
&= m_t^{cb} + v_{t-1} + \mathbb{E}_{t-1}[\varepsilon_t] \\
&= m_t^{cb} + v_{t-1} \\
\mathbb{E}_{t-2}[m_t^{cb} + v_t] &= \mathbb{E}_{t-2}[a_1\varepsilon_{t-1} + a_2\varepsilon_{t-2} + a_3\varepsilon_{t-3} + \cdots + v_{t-2} + \varepsilon_{t-1} + \varepsilon_t] \\
&= \mathbb{E}_{t-2}[a_1\varepsilon_{t-1} + a_2\varepsilon_{t-2} + a_3\varepsilon_{t-3} + \cdots + \varepsilon_{t-1} + \varepsilon_t] + v_{t-2} \\
&= \mathbb{E}_{t-2}[a_1\varepsilon_{t-1} + \varepsilon_{t-1} + \varepsilon_t] + a_2\varepsilon_{t-2} + a_3\varepsilon_{t-3} + \cdots + v_{t-2} \\
&= a_2\varepsilon_{t-2} + a_3\varepsilon_{t-3} + \cdots + v_{t-2}
\end{aligned}
$$

Fischer contracts with demand stabilization IV

$$y_t = m_t^{cb} + v_t - \mathbb{E}_{t-1}[m_t^{cb} + v_t] + \frac{1}{\phi}\left(\mathbb{E}_{t-1}[m_t^{cb} + v_t] - \mathbb{E}_{t-2}[m_t^{cb} + v_t]\right) - \frac{1}{(\phi - 1)}\mathcal{M}$$

$$\mathbb{E}_{t-1}[m_t^{cb} + v_t] = m_t^{cb} + v_{t-1}$$

$$\mathbb{E}_{t-2}[m_t^{cb} + v_t] = a_2\varepsilon_{t-2} + a_3\varepsilon_{t-3} + \cdots + v_{t-2}$$

Deriving output with monetary policy

$$\begin{aligned} \mathbb{E}_{t-1}[m_t^{cb} + v_t] - \mathbb{E}_{t-2}[m_t^{cb} + v_t] &= m_t^{cb} - a_2\varepsilon_{t-2} - a_3\varepsilon_{t-3} - \cdots + v_{t-1} - v_{t-2} \\ &= a_1\varepsilon_{t-1} + \varepsilon_{t-1} = (1 + a_1)\varepsilon_{t-1} \\ m_t^{cb} + v_t - \mathbb{E}_{t-1}[m_t^{cb} + v_t] &= m_t^{cb} + v_t - m_t^{cb} - v_{t-1} \\ &= v_t - v_{t-1} = \varepsilon_t \end{aligned}$$

$$y_t = \varepsilon_t + \frac{1 + a_1}{\phi}\varepsilon_{t-1} - \frac{1}{(\phi - 1)}\mathcal{M}$$

Optimal monetary policy

The central bank can affect output with a_1

$$y_t = \varepsilon_t + \frac{1 + a_1}{\phi}\varepsilon_{t-1} - \frac{1}{(\phi - 1)}\mathcal{M}$$

Other terms are irrelevant

- Contracts are only set for two periods
- The central bank (CB) cannot see what will happen tomorrow

$\Longrightarrow$ CB can eliminate effects of unanticipated shocks

Minimize variance of output

$$\begin{aligned}\mathrm{Var}(y_t) &= \mathrm{Var}(\varepsilon_t) + \frac{(1 + a_1)^2}{\phi^2}\mathrm{Var}(\varepsilon_{t-1}) \\ &= \sigma_\varepsilon^2 + \frac{(1 + a_1)^2}{\phi^2}\sigma_\varepsilon^2\end{aligned}$$

Optimal monetary policy

The central bank can affect output with a_1

$$y_t = \varepsilon_t + \frac{1 + a_1}{\phi}\varepsilon_{t-1} - \frac{1}{(\phi - 1)}\mathcal{M}$$

Other terms are irrelevant

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Minimize variance of output

$$\begin{aligned}
\mathrm{Var}(y_t) &= \mathrm{Var}(\varepsilon_t) + \frac{(1 + a_1)^2}{\phi^2}\mathrm{Var}(\varepsilon_{t-1}) \\
&= \sigma_\varepsilon^2 + \frac{(1 + a_1)^2}{\phi^2}\sigma_\varepsilon^2 \\
&= \sigma_\varepsilon^2 \text{ with } a_1 = -1
\end{aligned}$$

Central bankers save firms from frictions

The central bank can return the economy equilibrium with less frictions

- It neutralizes all unanticipated **demand** shocks it observes
- Firms anticipate the actions of the CB
- Demand shocks still have effects, but only for one period

$\Longrightarrow$ as if contract length was only 1 period

Optimal policy

- If the policy goal is to minimize output volatility, a linear rule is enough
- "Lean against the wind": If aggregate demand is high, decrease money supply

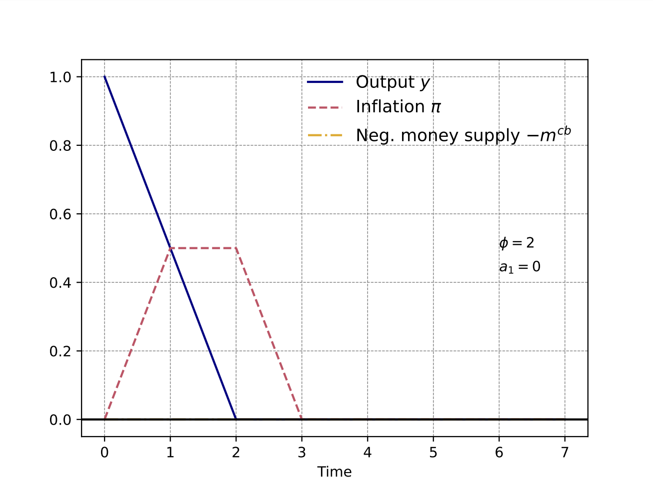
Summary – persistence of shocks

Fischer Contracts – m_t is random walk (restrict $a_{>1} = 0$)

$$
\begin{aligned}
p_t &= \mathbb{E}_{t-1}[m_t^{cb} + v_t] - \frac{1}{\phi}\left(\mathbb{E}_{t-1}[m_t^{cb} + v_t] - \mathbb{E}_{t-2}[m_t^{cb} + v_t]\right) \\
&= a_1\varepsilon_{t-1} + v_{t-2} + \varepsilon_{t-1} - \frac{1}{\phi}(1 + a_1)\varepsilon_{t-1} \\
&= v_{t-2} + \frac{\phi - 1}{\phi}(1 + a_1)\varepsilon_{t-1} \\
\implies p_t - p_{t-1} &= v_{t-2} + \frac{\phi - 1}{\phi}(1 + a_1)\varepsilon_{t-1} - \left(v_{t-3} + \frac{\phi - 1}{\phi}(1 + a_1)\varepsilon_{t-2}\right) \\
\pi_t &= \frac{\phi - 1}{\phi}(1 + a_1)\varepsilon_{t-1} - \frac{\phi - 1}{\phi}(1 + a_1)\varepsilon_{t-2} + \varepsilon_{t-2} \\
y_t &= \varepsilon_t + \frac{1 + a_1}{\phi}\varepsilon_{t-1} - \frac{1}{(\phi - 1)}\mathcal{M}
\end{aligned}
$$

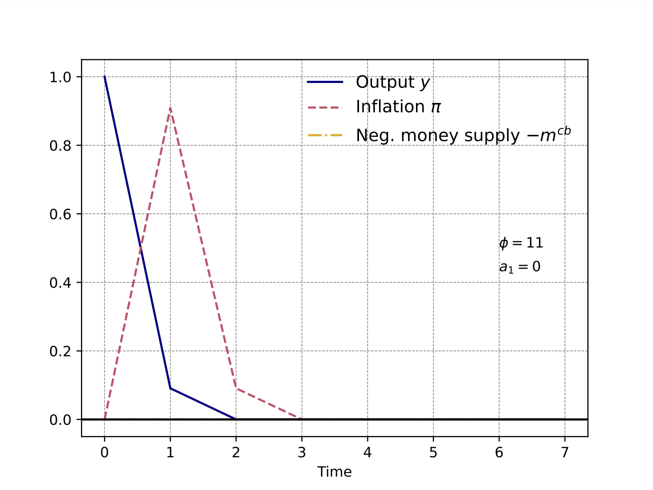
- Remember: prices are reset in $t-1$ for t and $t+1$ $\implies$ shock lasts two periods
- As labor becomes inelastic ($\phi \uparrow$), y_t moves less, π_t more

Optimal monetary policy in a picture



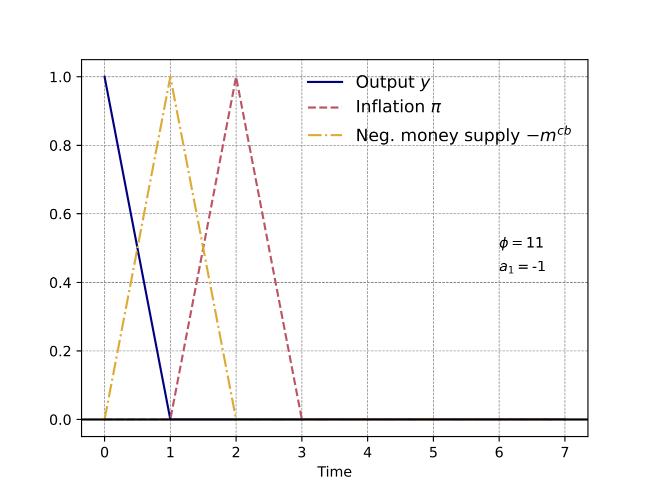
- $v_0 = 1 \implies$ demand $m = m^{cb} + v$ is **permanently higher** $(\rho = 1)$
- Since $y = m^{cb} + v - p$, we need $\sum \pi_t = 1$ for $y = 0$

Optimal monetary policy in a picture



- Main action in period 1, when first firms reset prices
- Prices absorb bigger share of shock with inelastic labor

Optimal monetary policy in a picture



- Monetary policy can suppress demand back to steady state
- Buys time for firms locked in, p changes later

Taylor contracts

Taylor contracts (DR 7.3)

Environment

- Firms set prices in advance and stick to them
- Only some fraction of firms resets every period

Operationalization

- Each price-setter sets prices for two periods (same price this time)
- Assume that half of all producers set prices in even periods, the rest in odd
- Rational expectations: everyone knows the environment

Timing

- Those who reset do so at the beginning of period t, setting the price for t and $t+1$
- They **know** the shocks in t (change! – makes algebra easier)

Taylor contracts – Firm decisions

Let x_t be the optimal price for firms firms who set in t

$$\begin{aligned} x_t &= \frac{1}{2}(p^*_{i,t} + \mathbb{E}_t[p^*_{i,t+1}]) \\ &= \frac{1}{2}\left\{(\phi-1)m_t + (2-\phi)p_t + (\phi-1)\mathbb{E}_t[m_{t+1}] + (2-\phi)\mathbb{E}_t[p_{t+1}]\right\} \\ &= \frac{1}{2}\left\{(\phi-1)m_t + (2-\phi)p_t + (\phi-1)m_t + (2-\phi)\mathbb{E}_t[p_{t+1}]\right\} \end{aligned}$$

The realized price level each period is

$$p_t = \frac{1}{2}(x_t + x_{t-1})$$

Combining the two:

$$x_t = \frac{(2-\phi)}{4}\left\{(x_t + x_{t-1}) + \mathbb{E}_t[x_{t+1} + x_t]\right\} + (\phi-1)m_t$$

Taylor contracts – Optimal prices

Solve for the optimal reset price

$$
\begin{aligned}
x_t &= \frac{(2-\phi)}{4}\left\{(x_t + x_{t-1}) + \mathbb{E}_t[x_{t+1} + x_t]\right\} + (\phi - 1)m_t \\
&= 2\left(\frac{\phi - 1}{\phi}\right)m_t + \underbrace{\frac{1}{2}\left(\frac{2-\phi}{\phi}\right)}_{A}\left[x_{t-1} + \mathbb{E}_t[x_{t+1}]\right] \\
&= (1 - 2A)m_t + A\left[x_{t-1} + \mathbb{E}_t[x_{t+1}]\right]
\end{aligned}
$$

- Both past and future matter for reset price
- Difficult to solve (don't just plug in, it's futile!)

Guess that x_t follows some process

$$x_t = \mu + \lambda x_{t-1} + \upsilon m_t$$

Method of undetermined coefficients I

Guess that x_t follows some process

$$x_t = \mu + \lambda x_{t-1} + v m_t$$

- Could just plug in, but if it is true, this equation must always hold
- Even if there are no shocks, i.e., $x_t = x_{t-1} = m_t \implies \mu = 0; v = 1 - \lambda$
- Then $x_t = \lambda x_{t-1} + (1-\lambda) m_t$. Now plug in!

$$\begin{aligned}
\mathbb{E}_t[x_{t+1}] &= \mathbb{E}_t[\lambda x_t + (1-\lambda) m_{t+1}] \\
&= \lambda\left(\lambda x_{t-1} + (1-\lambda) m_t\right) + (1-\lambda) m_t \\
&= \lambda^2 x_{t-1} + (1-\lambda^2) m_t
\end{aligned}$$

From before:

$$\begin{aligned}
x_t &= (1-2A) m_t + A\left[x_{t-1} + \mathbb{E}_t[x_{t+1}]\right] \\
&= (1-2A) m_t + A\left[x_{t-1} + \lambda^2 x_{t-1} + (1-\lambda^2) m_t\right] \\
&= (1 - 2A + A(1-\lambda^2)) m_t + A(1+\lambda^2) x_{t-1}
\end{aligned}$$

Method of undetermined coefficients II

Guess that x_t follows some process

$$
\begin{aligned}
& x_t = \lambda x_{t-1} + (1-\lambda) m_t \\
& x_t = (1 - 2A + A(1-\lambda^2)) m_t + A(1+\lambda^2) x_{t-1} \\
\implies & \lambda = A(1+\lambda^2) \\
\implies & \lambda = \frac{\phi \pm 2\sqrt{\phi - 1}}{2 - \phi} \quad \text{only one } \lambda : -1 < \lambda < 0! \text{ (-)}
\end{aligned}
$$

Take aways

- $|\lambda_+| > 1$, which means it would never converge back to steady state, even after only a single shock
- This slide pins down the evolution of the optimal price, output is next

Output dynamics under Taylor Contracts

Using this result for the process of x_t, we can solve for output

$$
\begin{aligned}
y_t &= m_t - \frac{1}{2}\left(x_{t-1} + x_t\right) \\
&= m_t - \frac{1}{2}\left(\lambda x_{t-2} + (1-\lambda)m_{t-1} + \lambda x_{t-1} + (1-\lambda)m_t\right) \\
&= m_t - \frac{1}{2}(1-\lambda)(m_t + m_{t-1}) - \lambda \underbrace{\frac{1}{2}(x_{t-2} + x_{t-1})}_{p_{t-1}} \\
&= \lambda y_{t-1} + \frac{1+\lambda}{2}\varepsilon_t
\end{aligned}
$$

- As $\phi \to \infty$, $\lambda \to -1$. A lower labor supply elasticity means that output oscillates around the steady state. For $\phi < 2$, output converges slowly back to the steady state.

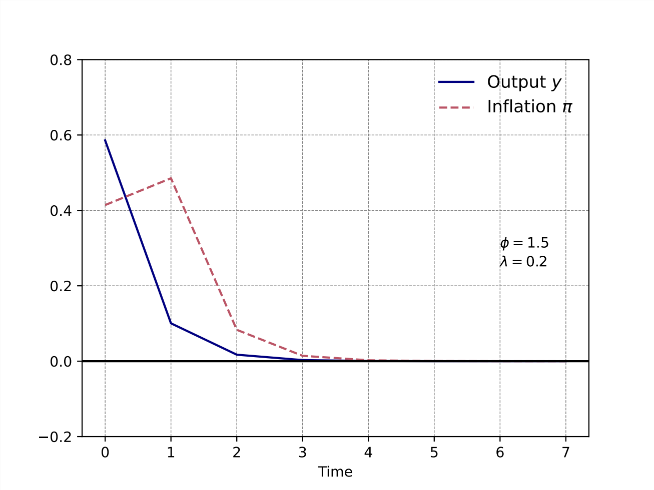
Persistence of inflation II

Taylor Contracts (ignoring markup) – m_t is random walk

$$
\begin{aligned}
p_t &= \frac{1}{2}\left(x_t + x_{t-1}\right) \\
&= \frac{1}{2}(1-\lambda)(m_t + m_{t-1}) + \lambda \underbrace{\frac{1}{2}(x_{t-2} + x_{t-1})}_{p_{t-1}} \\
&= \frac{1}{2}(1-\lambda)(2m_{t-1} + \varepsilon_t) + \lambda p_{t-1} \\
&= (1-\lambda)m_{t-1} + \frac{1-\lambda}{2}\varepsilon_t + \lambda p_{t-1} \\
p_t - p_{t-1} &= \frac{1-\lambda}{2}\varepsilon_t + (1-\lambda)m_{t-1} + (\lambda - 1)p_{t-1} \\
&= \frac{1-\lambda}{2}\varepsilon_t + (1-\lambda)y_{t-1}
\end{aligned}
$$

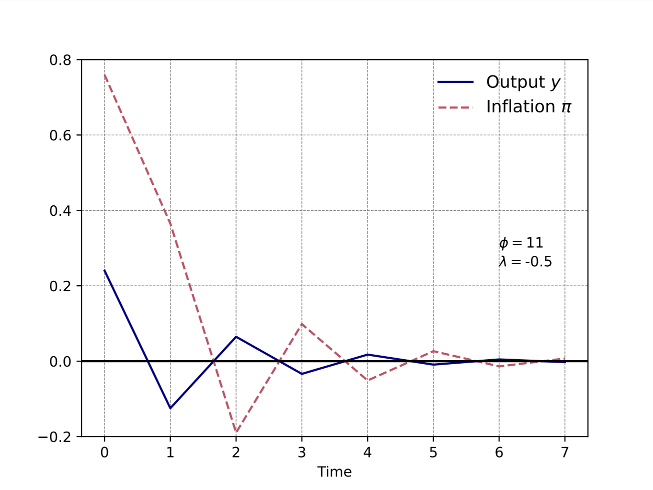
- Inflation depends on **past output** $\implies$ shocks last longer
- Resetters know that the competition is locked in → change prices just enough to capture some market share → sluggishness

Inflation impulse responses



- Again, demand is **permanently higher**, hence $\sum \pi_t = 1$
- Nice dynamics with **very** elastic labor supply

Inflation impulse responses



- Strange dynamics with inelastic labor, aversion to changing y
- Overreact to m, but know others will underreact to x_{t-1}

Punchline

Pricing frictions

- Romer's conclusion carries through: without frictions, anticipated shocks have no effect on output
- If agents cannot adjust prices freely, however, demand shocks have real effects
- Monetary policy can help stabilize output

Different forms of rigidity

- Fischer contracts: set price schedule and stick to it
- Taylor contracts: set prices for fixed number of periods
- Taylor has longer lasting effects

Next time

- Third (and most influential) form of price rigidity: the Calvo fairy