

# Optimal monetary policy

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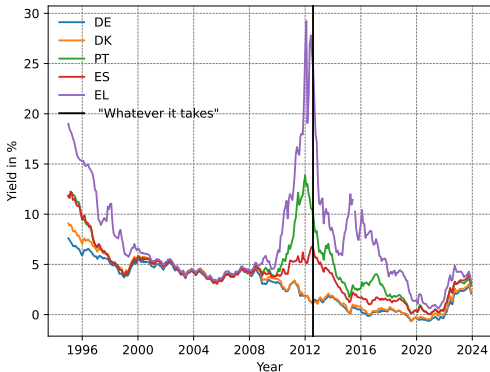
November 2024

UNIVERSITY OF COPENHAGEN



# Words have consequences

Mario Draghi [Video](#)



- "Within our mandate, the ECB is ready to do whatever it takes to preserve the euro. And believe me, it will be enough."

## Central bankers are (rational) people, too

- Policy makers have goal functions, but where do they come from?
- Can central bankers be expected to adhere to rules?

## Policy and politics

- Assuming some exogenous rule for monetary policy is too simple
- The policy itself is an outcome of its environment: it is endogenous
- In this context, credibility and reputation will play key roles

## Optimal Monetary policy

- Agents and the central bank play a non-cooperative "game" (  $\implies$  Nash)
- Outcome depends on the ability of the central bank to **commit** to a plan

# Monetary policy makers care about their credibility

## Isabel Schnabel

- "Instead, for monetary policy to remain credible in the current environment, it must not be an inflationary source itself."

## Christine Lagarde

- "What became evident [during 2012] is that the perceived commitment of policymakers was a crucial variable in effective policymaking."

## Janet Yellen

- "My remarks today will focus on the issue of credibility—in particular on the Federal Reserve's credibility regarding its announced commitment to maintaining price stability."

## Optimal monetary policy

- So far, all of our analyses have been **positive**
- Given the initial assumptions, they contained no value judgements, only descriptive conclusions
- Optimal monetary policy (i.e., what the central bank **should** do) requires us to do **normative** analysis

## Rational expectations

- How trustworthy/predictable the central bank's actions are is important
- Policy outcomes differ based on commitment/discretion

# The model

## Setup (Persson & Tabellini, Ch 15)

- The model is as simple as possible to isolate the channels we care about: the influence of central bank policy on output and inflation
- There is a relationship between output and inflation (PC) and an IS curve (reduced form)

The goal is to

- identify a rule for monetary policy that optimizes a loss function
- analyze how the economy's aggregates change under different assumption on credibility

The insights carry over to the New Keynesian model!

## Unions

- In the Persson-Tabellini model, labor unions negotiate for some wage growth  $w$  such that

$$w = \omega + \pi^e$$

- Output growth, in turn, depends on the negotiated real wage:

$$x = \gamma - (w - \pi) - \varepsilon$$

- $\gamma$  is a parameter, if wages are too high, output is too low
- supply shocks  $\varepsilon$  lower domestic output

## Resulting output

$$x = \underbrace{(\gamma - \omega)}_{\theta} + (\pi - \pi^e) - \varepsilon$$

# Equations

## Demand Curve

$$\pi_t = m_t + \underbrace{v_t}_{\text{Demand shock}} + \underbrace{\mu_t}_{\text{MP shock}}$$

## Phillips Curve

$$x_t = \underbrace{\theta_t}_{\text{natural rate of output}} + (\pi_t - \pi_t^e) - \underbrace{\varepsilon_t}_{\text{Supply shock}}$$

- Inflation depends on money growth, unexpected demand and monetary policy mistakes
- Output depends on unexpected inflation, supply and its natural rate
- Expected inflation is  $\mathbb{E}_t[\pi_t] \equiv \pi_t^e$
- All shocks are independent and 0 in expectation

## Survey



# Commitment

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# Timing assumptions

## Perfect commitment

1. Announcement of monetary rule
2. Everyone observes the natural level of output  $\theta$
3. Expectations  $\pi^e$  are formed, given the information about  $\theta$
4. Everyone observes  $v$  and  $\varepsilon$
5. The central bank decides the money supply  $m$
6.  $\mu$  is realized, pinning down output  $x$  and inflation  $\pi$

## Consequences

- The central bank has an informational advantage (expectations are pinned down before the money supply is set)

## Monetary policy can move after expectations are formed

- This is a reduced form way to make monetary policy powerful
  - $\implies$  it can stabilize output against shocks
  - $\implies$  it can save the agents from themselves
- Monetary policy is decided every six weeks, wages are only renegotiated at longer intervals

## Lucas again

- After  $\theta$  realizes, only unexpected changes in monetary policy have an effect
- Moves in  $m$  can stabilize shocks to  $v$  and  $\mu$

$$\mathbb{E}_t[\pi_t|\theta] = \mathbb{E}_t[m_t|\theta]$$
$$x_t = \theta + \underbrace{(m_t + v_t + \mu_t - \mathbb{E}_t[m_t|\theta])}_{\pi_t} + \varepsilon_t$$

## Quadratic loss function

$$\mathcal{L} = \frac{1}{2} [a(\pi - \bar{\pi})^2 + \lambda(x - \bar{x})^2]$$

- Loss function implies that the central bank dislikes deviations from some inflation benchmark  $\bar{\pi}$ , and deviations from some output target  $\bar{x}$
- The degree of "pain" such deviations cause the banker are governed by the parameters  $a$  and  $\lambda$
- The parameters  $a$  and  $\lambda$  are known to all agents in the economy

# Policy rule

## Linear policy rule

- With a quadratic objective and linear shock processes, it can be shown that a policy rule which is linear in the shocks is optimal
- It can achieve the minimization of the loss function given the realizations of the shocks

## Assumes the rule

$$m = \varphi + \varphi_{\theta}\theta + \varphi_v v + \varphi_{\varepsilon}\varepsilon$$

- The central bank reacts to shocks to natural output  $\theta$ , demand shocks  $v$  and productivity shocks  $\varepsilon$
- By definition, it cannot do anything about  $\mu$
- Recall:  $\theta$  is observed before expectations are formed,  $v$  and  $\varepsilon$  realize after

# Perfect credibility

## Credible policy rule

- If agents know the rule and it is perfectly credible, they will include it into their expectations
- Strong assumption: central bankers may have an incentive to deviate (more on that later)

## Expectations

$$\mathbb{E}_t[m_t|\theta] = \varphi + \varphi_\theta \mathbb{E}[\theta|\theta] + \varphi_v \mathbb{E}[v|\theta] + \varphi_\varepsilon \mathbb{E}[\varepsilon|\theta]$$

$$\mathbb{E}_t[\pi_t|\theta] = \mathbb{E}[m_t|\theta] = \varphi + \varphi_\theta \theta$$

- The shocks are independent  $\implies$  conditional expectations don't help
- Expected inflation only depends on the realization of  $\theta$
- Other shocks are 0 in expectation

$$\mathbb{E}_t[\pi_t] = \varphi + \varphi_\theta \theta$$

Realized inflation

$$\begin{aligned}\pi_t &= \underbrace{\varphi + \varphi_\theta \theta + \varphi_v v + \varphi_\varepsilon \varepsilon + v + \mu}_{m_t} \\ &= \varphi + \varphi_\theta \theta + (1 + \varphi_v)v + \varphi_\varepsilon \varepsilon + \mu\end{aligned}$$

Realized output

$$\begin{aligned}x &= \theta + \underbrace{(\varphi + \varphi_\theta \theta + (1 + \varphi_v)v + \varphi_\varepsilon \varepsilon + \mu)}_{\pi_t} - \underbrace{(\varphi + \varphi_\theta \theta)}_{\pi_t^e} - \varepsilon \\ &= \theta + (1 + \varphi_v)v + (\varphi_\varepsilon - 1)\varepsilon + \mu\end{aligned}$$

# What is the optimal policy?

## Ex-ante optimality

- What parameters should be set for the policy rule **ex-ante**?
- CB ties its hands: **could** move after private sector but doesn't
- Crucially: Implies optimal policy **in expectation**

## Minimize the loss function

- If the rule is credible, then output and inflation will behave as on the previous slide
- Plug into the loss function
- Minimize **the expectation**



$$\begin{aligned}\mathbb{E}[\mathcal{L}] &= \frac{1}{2} \mathbb{E} \left[ a(\pi - \bar{\pi})^2 + \lambda(x - \bar{x})^2 \right] \\ &= \frac{1}{2} \mathbb{E} \left[ a \underbrace{(\varphi + \varphi_{\theta}\theta + (1 + \varphi_v)v + \varphi_{\varepsilon}\varepsilon + \mu - \bar{\pi})}_{\pi}^2 \right. \\ &\quad \left. + \lambda \underbrace{(\theta + (1 + \varphi_v)v + (\varphi_{\varepsilon} - 1)\varepsilon + \mu - \bar{x})}_{x}^2 \right] \\ &= \frac{1}{2} \mathbb{E} [a(A) + \lambda(B)]\end{aligned}$$

- Want to minimize  $\implies$  take derivatives
- But: Expectation of square term is complicated, better multiply out first (next slide)

$$\begin{aligned}
 A &= (\varphi + \varphi_\theta \theta + (1 + \varphi_v)v + \varphi_\varepsilon \varepsilon + \mu - \bar{\pi})^2 \\
 &= \varphi^2 + \varphi \varphi_\theta \theta + \varphi(1 + \varphi_v)v + \varphi \varphi_\varepsilon \varepsilon + \varphi \mu - \varphi \bar{\pi} + \varphi \varphi_\theta \theta + \varphi_\theta^2 \theta^2 + (1 + \varphi_v)v \varphi_\theta \theta \\
 &\quad + \varphi_\varepsilon \varepsilon \varphi_\theta \theta + \mu \varphi_\theta \theta - \bar{\pi} \varphi_\theta \theta + \varphi(1 + \varphi_v)v + \varphi_\theta \theta (1 + \varphi_v)v + (1 + \varphi_v)^2 v^2 \\
 &\quad + \varphi_\varepsilon \varepsilon (1 + \varphi_v)v + \mu(1 + \varphi_v)v - \bar{\pi}(1 + \varphi_v)v + \varphi_\varepsilon \varepsilon \varphi + \varphi_\theta \varphi_\varepsilon \varepsilon \theta + (1 + \varphi_v) \varphi_\varepsilon \varepsilon v \\
 &\quad + \varphi_\varepsilon^2 \varepsilon^2 + \varphi_\varepsilon \varepsilon \mu - \varphi_\varepsilon \varepsilon \bar{\pi} + \varphi \mu + \varphi_\theta \mu \theta + (1 + \varphi_v) \mu v + \varphi_\varepsilon \mu \varepsilon + \mu^2 - \mu \bar{\pi} \\
 &\quad + \varphi \bar{\pi} + \varphi_\theta \bar{\pi} \theta + (1 + \varphi_v) \bar{\pi} v + \varphi_\varepsilon \bar{\pi} \varepsilon + \bar{\pi} \mu - \bar{\pi} \bar{\pi}
 \end{aligned}$$

All shocks are independent!

- In expectation, shock terms multiplied by constants are zero, e.g.

$$\mathbb{E}[\varphi \varphi_\theta \theta] = \varphi \varphi_\theta \mathbb{E}[\theta] = 0$$

- In expectation, cross-terms are zero:

$$\mathbb{E}[\varphi_\varepsilon \varepsilon \varphi_\theta \theta] = \varphi_\varepsilon \varphi_\theta \mathbb{E}[\varepsilon \theta] = \varphi_\varepsilon \varphi_\theta \mathbb{E}[\varepsilon] \mathbb{E}[\theta] = 0$$

$$\begin{aligned}\mathbb{E}[A] &= \mathbb{E}[(\varphi + \varphi_\theta \theta + (1 + \varphi_v)v + \varphi_\varepsilon \varepsilon + \mu - \bar{\pi})^2] \\ &= \varphi^2 - \varphi \bar{\pi} + \varphi_\theta^2 \mathbb{E}[\theta^2] + (1 + \varphi_v)^2 \mathbb{E}[v^2] + \varphi_\varepsilon^2 \mathbb{E}[\varepsilon^2] + \mathbb{E}[\mu^2] - \varphi \bar{\pi} + \bar{\pi}^2 \\ &= \varphi^2 - \varphi \bar{\pi} + \varphi_\theta^2 \sigma_\theta^2 + (1 + \varphi_v)^2 \sigma_v^2 + \varphi_\varepsilon^2 \sigma_\varepsilon^2 + \sigma_\mu^2 - \varphi \bar{\pi} + \bar{\pi}^2\end{aligned}$$

## Expectations depend on variances of shocks

- $\sigma_q^2 = \text{Var}(q) = \mathbb{E}[(q - \bar{q})^2]$  – If mean of random variable is 0, the expectation of its square is the variance
- The zero-mean and independence assumptions are doing **a lot** of heavy lifting for us

Apply the same principle to the square variable  $B$

$$\begin{aligned}\mathbb{E}[B] &= \mathbb{E}[(\theta + (1 + \varphi_v)v + (\varphi_\varepsilon - 1)\varepsilon + \mu - \bar{x})^2] \\ &= \mathbb{E}[\theta^2] + (\varphi_v + 1)^2 \mathbb{E}[v^2] + (1 - \varphi_\varepsilon)^2 \mathbb{E}[\varepsilon^2] + \mathbb{E}[\mu^2] + \bar{x}^2 \\ &= \sigma_\theta^2 + (\varphi_v + 1)^2 \sigma_v^2 + (1 - \varphi_\varepsilon)^2 \sigma_\varepsilon^2 + \sigma_\mu^2 + \bar{x}^2\end{aligned}$$

- Now we have all the ingredients to fill in the expectation of the loss function

# Minimize expected loss

Plugging in from the previous slides:

$$\begin{aligned} \min_{\varphi, \varphi_\theta, \varphi_v, \varphi_\varepsilon} \mathbb{E}[\mathcal{L}] = & \frac{1}{2}a \left( \varphi^2 - \varphi\bar{\pi} + \varphi_\theta^2\sigma_\theta^2 + (1 + \varphi_v)^2\sigma_v^2 + \varphi_\varepsilon^2\sigma_\varepsilon^2 + \sigma_\mu^2 - \varphi\bar{\pi} + \bar{\pi}^2 \right) \\ & + \frac{1}{2}\lambda \left( \sigma_\theta^2 + (\varphi_v + 1)^2\sigma_v^2 + (1 - \varphi_\varepsilon)^2\sigma_\varepsilon^2 + \sigma_\mu^2 + \bar{x}^2 \right) \end{aligned}$$

- A hypothetical social planner wants to set the rule (i.e., the parameters in the central bank's response function) to minimize this loss
- The rule is in place forever  $\implies$  minimizing single period expectation of loss is the same as discounted infinite sum of all future periods' losses
- Take the derivatives w.r.t.  $\varphi, \varphi_\theta, \varphi_v, \varphi_\varepsilon$

# Minimum expected loss

$$\frac{\partial \mathbb{E}[\mathcal{L}]}{\partial \varphi} : a(\varphi - \bar{\pi}) = 0 \implies \varphi = \bar{\pi}$$

$$\frac{\partial \mathbb{E}[\mathcal{L}]}{\partial \varphi_{\theta}} : a v_{\theta} \sigma_{\theta}^2 = 0 \implies \varphi_{\theta} = 0$$

$$\frac{\partial \mathbb{E}[\mathcal{L}]}{\partial \varphi_v} : \sigma_v^2 (a + \lambda)(1 + \varphi_v) = 0 \implies \varphi_v = -1$$

$$\frac{\partial \mathbb{E}[\mathcal{L}]}{\partial \varphi_{\varepsilon}} : \sigma_{\varepsilon}^2 (a \varphi_{\varepsilon} - \lambda(1 - \varphi_{\varepsilon})) = 0 \implies \varphi_{\varepsilon} = \frac{\lambda}{a + \lambda}$$

## Implications

- Anchor inflation where society wants it
- Shocks that are priced into expectations need no reaction ( $\theta$ )
- Neutralize demand shocks
- Supply shocks: it depends. Countering supply shocks causes less deviations from  $\bar{x}$ , but at the cost of deviations from  $\bar{\pi} \implies \text{tradeoff}_{20/100}$

# Equilibrium under commitment

## Optimal rule

$$m_t = \bar{\pi} - v_t + \frac{\lambda}{a + \lambda} \varepsilon$$

## Equilibrium inflation

$$\pi^C = \bar{\pi} + \frac{\lambda}{a + \lambda} \varepsilon + \mu$$

## Equilibrium output

$$x^C = \theta - \frac{a}{a + \lambda} \varepsilon + \mu$$

- Output may fluctuate due to changes in the natural rate, supply shocks or policy errors
- Inflation only changes due to supply shocks and policy errors
- Depending on preferences, supply shocks will feed more into output, or more into inflation

## Benefits

- If the policy maker can commit to a rule, inflation and output are stable around their natural levels
- As we will see, this is the best possible outcome

## Simplify the problem

- Demand shocks are neutralized  $\implies$  we can ignore them
- Policy errors  $\mu$  are not interesting to study because there is little we can do about them  $\implies$  ignore for now
- **The only important shocks left are  $\theta$  and  $\varepsilon$**



## Problems with rules

- Central bankers are not computers. They may want to exploit their informational advantage
- Once expectations are locked in, it's possible to decrease societal losses even further
- The rule may not be credible if bankers have **discretion** (i.e., ability) to deviate

## Discretion

- It's more realistic to assume policy makers don't stick to a rule
- This feeds back into agents (rational) expectations
- Equilibrium outcomes are different without commitment

# Discretion

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## Discretion/Non-credible rule

1. ~~Announcement of monetary rule~~
2. Everyone observes the natural level of output  $\theta$
3. Expectations  $\pi^e$  are formed, given the information about  $\theta$
4. Everyone observes  $\varepsilon$
5. The central bank decides the money supply  $m$
6. Output  $x$  and inflation  $\pi$  are pinned down

## Implications

- Without a (credible) rule, the central bank is free to do what it wants each period

## Ex-post optimality

- When the CB could commit to a credible rule, that rule was **ex-ante** optimal:  $\frac{\partial \mathbb{E}[\mathcal{L}]}{\partial m} = 0$
- Without a rule, policy will be ex-post optimal:  $\frac{\partial \mathcal{L}}{\partial m} = 0$
- What seems like a small difference has big consequences

## Nash-equilibrium

- Central bank and consumers play a game. In equilibrium nobody wants to deviate from decision
- Solve by backwards induction

## Central bank optimum under discretion (second stage)

$$\mathcal{L} = \frac{1}{2} \left[ a(\pi_t - \bar{\pi})^2 + \lambda(x_t - \bar{x})^2 \right]$$

$$\pi_t = m_t \quad \text{remember: } v \text{ and } \mu \text{ set to } 0$$

$$x_t = \theta + (\pi_t - \pi_t^e) - \varepsilon_t$$

- Since  $\pi_t = m_t$ , just assume that the CB sets  $\pi_t$  directly
- Recall that the CB takes  $\pi_t^e$  as given

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \pi_t} : \quad & a(\pi_t - \bar{\pi}) + \lambda(\theta + (\pi_t - \pi_t^e) - \varepsilon_t - \bar{x}) = 0 \\ \implies \quad & \pi = \frac{a}{a + \lambda} \bar{\pi} + \frac{\lambda}{a + \lambda} (\pi_t^e - \theta + \varepsilon + \bar{x}) \end{aligned}$$

- Note: If we plug in the result from the commitment equilibrium  $\pi = \pi^e = \bar{\pi}$ ,  $\frac{\partial \mathcal{L}}{\partial \pi_t} > 0 \implies$  CB can do better!

# Consumer expectation under discretion (first stage)

Take expectation of central banks decision function

$$\begin{aligned}\mathbb{E}[\pi|\theta] &= \frac{a}{a+\lambda}\bar{\pi} + \frac{\lambda}{a+\lambda}\mathbb{E}[(\mathbb{E}[\pi|\theta] - \theta + \varepsilon + \bar{x})|\theta] \\ &= \bar{\pi} + \underbrace{\frac{\lambda}{a}(\bar{x} - \theta)}_{\text{Inflation bias}}\end{aligned}$$

- Expected inflation is higher than in the commitment case
- Because  $\theta$  is known to consumers, they know what the CB will do.  
If  $\theta < \bar{x}$ : increase  $m$ , if  $\theta > \bar{x}$ : decrease  $m$
- However, these actions are pointless, because prices adjust. As always, if higher  $m$  is expected,  $p$  (and therefore  $\pi$ ) adjusts, and  $x$  stays constant

# Realized values of inflation and output

## Output

$$x^D = \theta - \frac{a}{a + \lambda} \varepsilon$$

- Output is the same as under commitment!

## Inflation

$$\pi^D = \bar{\pi} + \frac{\lambda}{a} (\bar{x} - \theta) + \frac{\lambda}{a + \lambda} \varepsilon$$

- Inflation is higher and more volatile

Giving central banks discretion leaves output constant, but  $\mathcal{L}$  is actually higher than it could be under commitment

# Reputation

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## Single period

- The commitment and discretion cases before are single-period games
- In reality, central banks make decisions all the time

⇒ Current decisions affect future reputation

## Multi-period game

- The central bank makes decisions every period, proclaiming a rule
- Consumers decide whether they trust the bank or not
- Trust can never be rebuilt (strong assumption)

# Longer run optimality

## Infinite loss function

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \mathcal{L}(\pi_{t+j}, x_{t+j})$$

- The central bank now cares about all future periods

## Simplifying assumption

$$\mathcal{L}(\pi_t, x_t) = \frac{\pi_t^2}{2} - \lambda x$$
$$\implies \pi^C = 0, \quad \pi^D = \lambda, \quad x^C = x^D = \theta - \varepsilon$$

- As always: this contains the most important intuition
- Using the double square loss function is much more messy
- Inflation volatility is costly  $\implies$  CB let's  $\varepsilon$  only affect output

## Inflation expectations

$$\pi_t^e = 0 \text{ if } \pi_{t-1} = \pi_{t-1}^e$$

$$\pi_t^e = \lambda \text{ otherwise}$$

- If realized inflation was in line with the agents expectations yesterday, the bank has not deviated from its rule
- In this case: keep trusting the central bank
- In any other case the bank has deviated  $\implies$  don't trust the CB ever again

# The central bank's problem

## Adhere to rule or break trust?

- Each period, the CB faces a choice
- If it deviates, it can decrease its loss function today
- But at the cost of never being able to do so ever again

## Determinants of decision

- Because the CB is a rational agent, it computes the one-time benefits of deviating and compares the to the future costs
- Whichever is more attractive is the equilibrium outcome

# Contemporary benefit of deviating

## Loss in case of exploitation

$$\pi_t = \lambda \quad (\text{inflation rises})$$

$$x_t = \theta + \lambda - \varepsilon \quad (\text{output rises})$$

$$\mathcal{L}(\lambda, \theta + \lambda - \varepsilon) = +\frac{1}{2}\lambda^2 - \lambda(\theta + \lambda - \varepsilon)$$

## Loss in case of continuous commitment

$$\pi_t = 0$$

$$x_t = \theta - \varepsilon$$

$$\mathcal{L}(0, \theta - \varepsilon) = -\lambda(\theta - \varepsilon)$$

## One-time loss from deviating

$$\mathcal{L}(\lambda, \theta + \lambda - \varepsilon) - \mathcal{L}(0, \theta - \varepsilon) = -\frac{1}{2}\lambda^2 \quad (\text{loss is lower})$$

# Long-run cost of deviating

Loss in case of deviating (starting at period  $t = s + 1$ —tomorrow)

$$\pi_s = \lambda, \quad x_s = \theta - \varepsilon$$
$$\mathbb{E} \left[ \sum_{t=s+1}^{s+T} \beta^{t-s} \mathcal{L}(\lambda, \theta - \varepsilon) \right] = \sum_{t=s+1}^{s+T} \beta^{t-s} \left( \frac{1}{2} \lambda^2 - \lambda \mathbb{E}[(\theta - \varepsilon)] \right)$$

Loss in case of continuous commitment

$$\pi_s = 0, \quad x_s = \theta - \varepsilon$$
$$\mathbb{E} \left[ \sum_{t=s+1}^{s+T} \beta^{t-s} \mathcal{L}(0, \theta - \varepsilon) \right] = - \sum_{t=s+1}^{s+T} \beta^{t-s} \lambda \mathbb{E}[(\theta - \varepsilon)]$$

Long-run loss from deviating Algebra

$$\mathbb{E} \left[ \sum_{t=s+1}^{s+T} \beta^{t-s} \mathcal{L}(\lambda, \theta - \varepsilon) \right] - \mathbb{E} \left[ \sum_{t=s+1}^{s+T} \beta^{t-s} \mathcal{L}(0, \theta - \varepsilon) \right] = \beta \frac{1}{2} \frac{(1 - \beta^{T-1})}{1 - \beta} \lambda^2$$

# Overall cost-benefit analysis

Add up single-period and long-run losses from deviating

$$Q = \beta \frac{1}{2} \frac{(1 - \beta^{T-1})}{1 - \beta} \lambda^2 - \frac{1}{2} \lambda^2$$

- If  $Q < 0$ , the effect of deviating on the loss function (contemporaneous + long-run) is negative  $\implies$  desirable! Smaller loss means gain
- If  $Q > 0$ , the loss is positive (that's bad) and the CB does not want to deviate
- The central bank will deviate if:

$$\frac{1}{2} \lambda^2 \left( \beta \frac{(1 - \beta^{T-1})}{1 - \beta} - 1 \right) < 0 \iff \beta \frac{(1 - \beta^{T-1})}{1 - \beta} < 1$$

$$\beta \frac{(1 - \beta^{T-1})}{1 - \beta} < 1$$

## Special cases

- If the world end tomorrow ( $T = 1$ ),  $0 < 1$  implies that the central bank will deviate with certainty
- If the world never ends, we need  $\beta < 0.5$  for the CB to find deviating attractive
- If the discount factor is low (0.5 is very low), the CB doesn't care about the future and will deviate

## Implications

- The repeated game nature of this example, together with the threat of higher inflation forever, keep the central bank honest
- Once the CB has deviated, the economy can never go back



# Institutions

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## European Central Bank

- The primary objective of the European System of Central Banks (hereinafter referred to as 'the ESCB') shall be to maintain price stability. (Article 127, TFEU)
- In pursuing price stability, the ECB seeks to hold inflation below but close to 2 percent over a medium-term horizon.
- (...) support the general economic policies in the Union with a view to contributing to the achievement of the objectives of the Union as laid down in Article 3 of the Treaty on European Union.

## Federal Reserve

- (...) so as to promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates.

# Central bank appointments

## Guidance

- Both ECB and Fed have been given clear guidelines on what to focus their policies on
- ECB: Price stability – everything else is secondary
- Fed: Dual mandate – more in line with the formulas above

## Doves or Hawks

- However, it is impossible for a central bank to credibly commit to a rule
- Still, governments can at least appoint the right person to head the central bank
- Who are they?

## Finding the right central banker

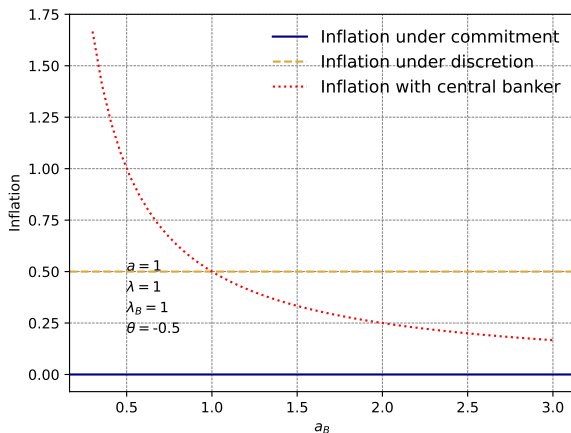
$$x_B^D = \theta - \frac{a_B}{a_B + \lambda_B} \varepsilon$$
$$\pi_B^D = \bar{\pi} + \frac{\lambda_B}{a_B} (\bar{x} - \theta) + \frac{\lambda_B}{a_B + \lambda_B} \varepsilon$$

- Each central banker has their own  $a_B$  and  $\lambda_B$
- Which one should be chosen to make decisions?

⇒ analyze conditional loss function

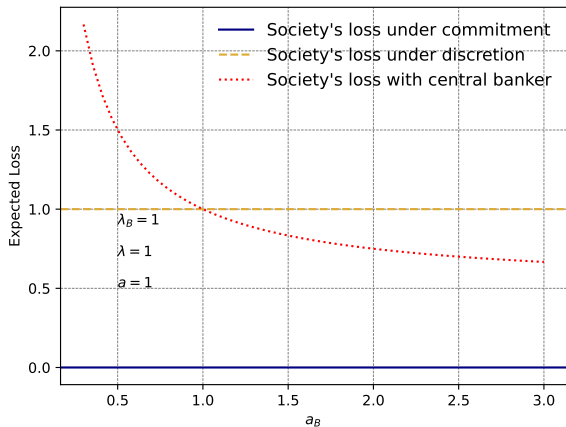
$$\mathbb{E} [\mathcal{L}(x_B^D, \pi_B^D)] = \mathbb{E} \left[ \frac{1}{2} a \left( \frac{\lambda_B}{a_B} (\bar{x} - \theta) + \frac{\lambda_B}{a_B + \lambda_B} \varepsilon \right)^2 + \frac{1}{2} \lambda \left( \theta - \frac{a_B}{a_B + \lambda_B} \varepsilon - \bar{x} \right)^2 \right]$$

# Finding the right central banker



- Inflation hawks keep inflation low after hypothetical  $\theta$

# Finding the right central banker



- Inflation hawks minimize the **expected** loss function

## Commitment and Discretion

- Commitment to a rule leads to lowest inflation
- Discretion creates an inflationary bias—output is unchanged

⇒ Credibility is important

## Repeated game

- Interaction across many periods can keep the central bank in check
- Future costs of deviating make optimum more attractive

## The ideal central banker

- Inflation hawks lead to a lower loss function
- Can approach commitment optimum

# Appendix

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$$\begin{aligned}
& \sum_{t=s+1}^{s+T} \beta^{t-s} \left( \frac{1}{2} \lambda^2 - \lambda \mathbb{E}[(\theta - \varepsilon)] \right) + \sum_{t=s+1}^{s+T} \beta^{t-s} \lambda \mathbb{E}[(\theta - \varepsilon)] \\
&= \sum_{t=s+1}^{s+T} \beta^{t-s} \frac{1}{2} \lambda^2 = \frac{1}{2} \lambda^2 \sum_{t=s+1}^{s+T} \beta^{t-s} \\
&= \frac{1}{2} \lambda^2 \sum_{t=s+1}^{s+T} \beta^{t-s} = \frac{1}{2} \lambda^2 \beta \sum_{t=s}^{s+T-1} \beta^{t-s} = \frac{1}{2} \lambda^2 \beta \sum_{j=0}^{T-1} \beta^j \\
&= \frac{1}{2} \lambda^2 \beta \left( \sum_{j=0}^{\infty} \beta^j - \sum_{j=T-1}^{\infty} \beta^j \right) = \frac{1}{2} \lambda^2 \beta \left( \frac{1}{1-\beta} - \sum_{j=T-1}^{\infty} \beta^j \right) \\
&= \frac{1}{2} \lambda^2 \beta \left( \frac{1}{1-\beta} - \beta^{T-1} \sum_{j=0}^{\infty} \beta^j \right) \\
&= \frac{1}{2} \lambda^2 \beta \frac{1 - \beta^{T-1}}{1 - \beta}
\end{aligned}$$