

Introduction and intertemporal choices

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Course admin

Lectures every week

- Tuesdays 1000h-1300h
- Office hours on lecture days from 1400h-1500h
- Materials available [here](#)

Problem sets

- Exercise classes every week
- Preparation for these will greatly help with the exam

Assignments

- Pass 2/3 to take the exam

Exam

- Closed book, 3h

What is macroeconomics?

Aggregate quantities

- GDP (growth & fluctuations)
- Inflation
- Unemployment
- ...

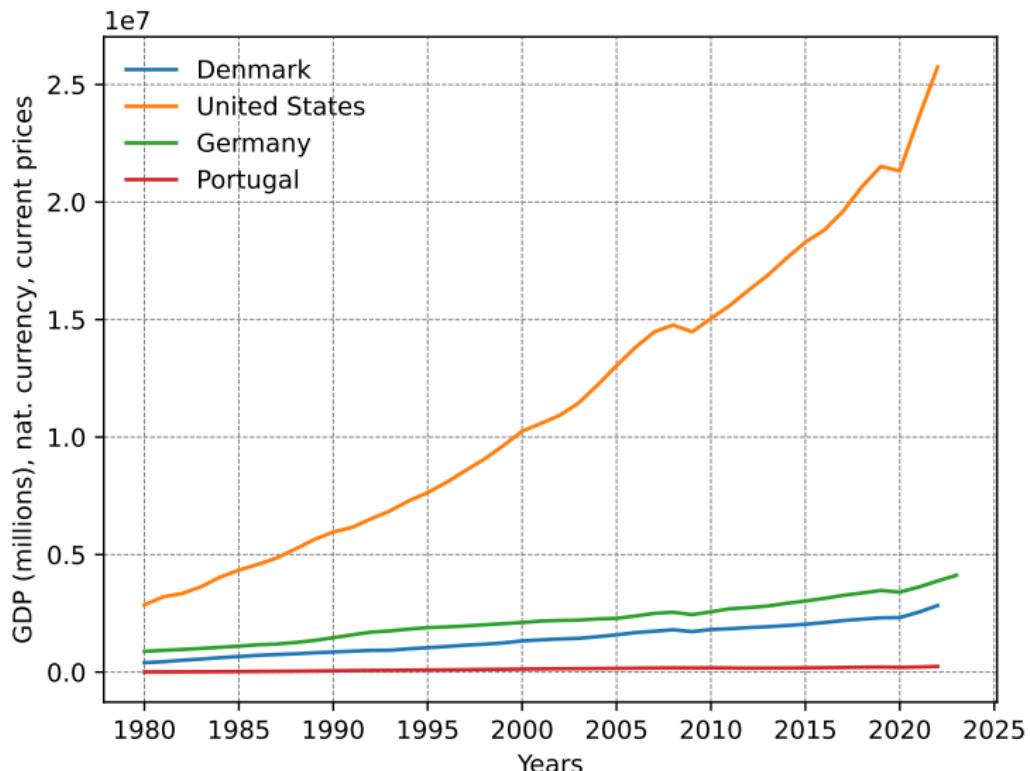
Society-wide phenomena

- Inequality
- Political changes

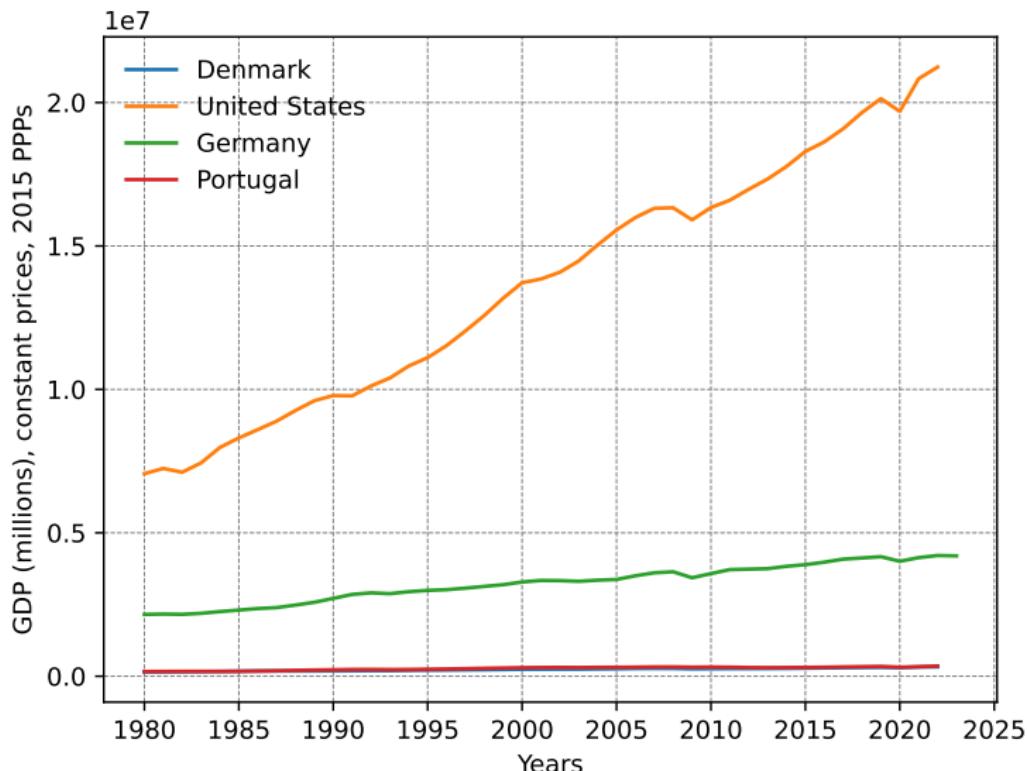
Public policy

- Pension systems
- Unemployment insurance

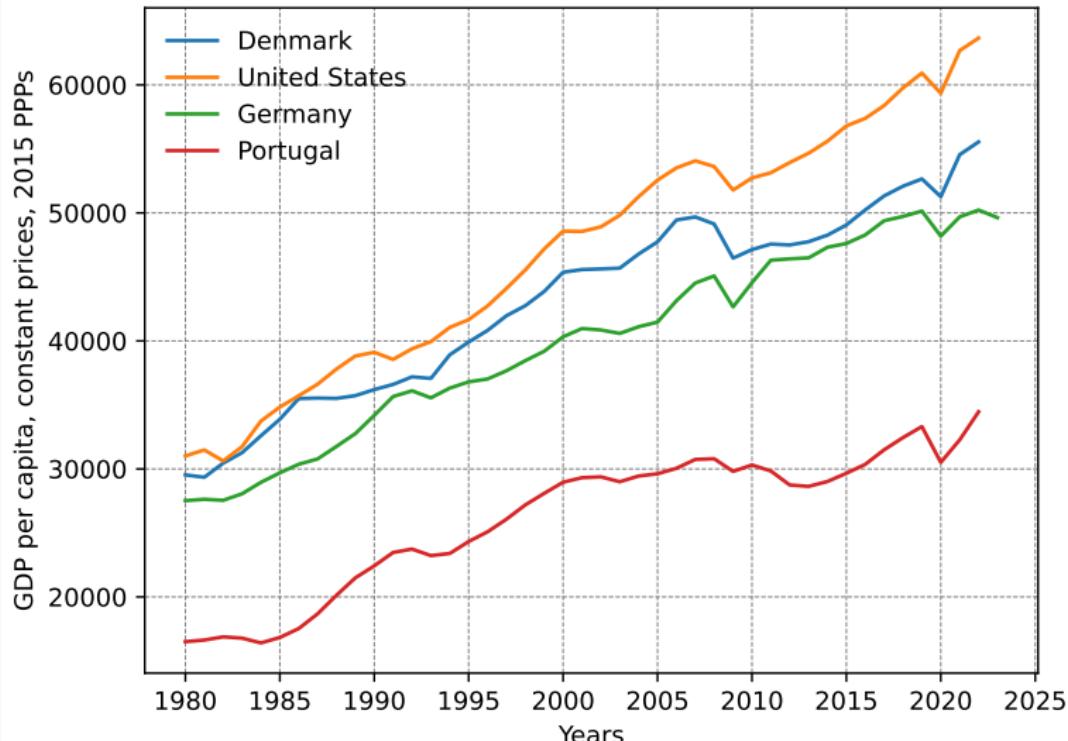
GDP across countries – OECD



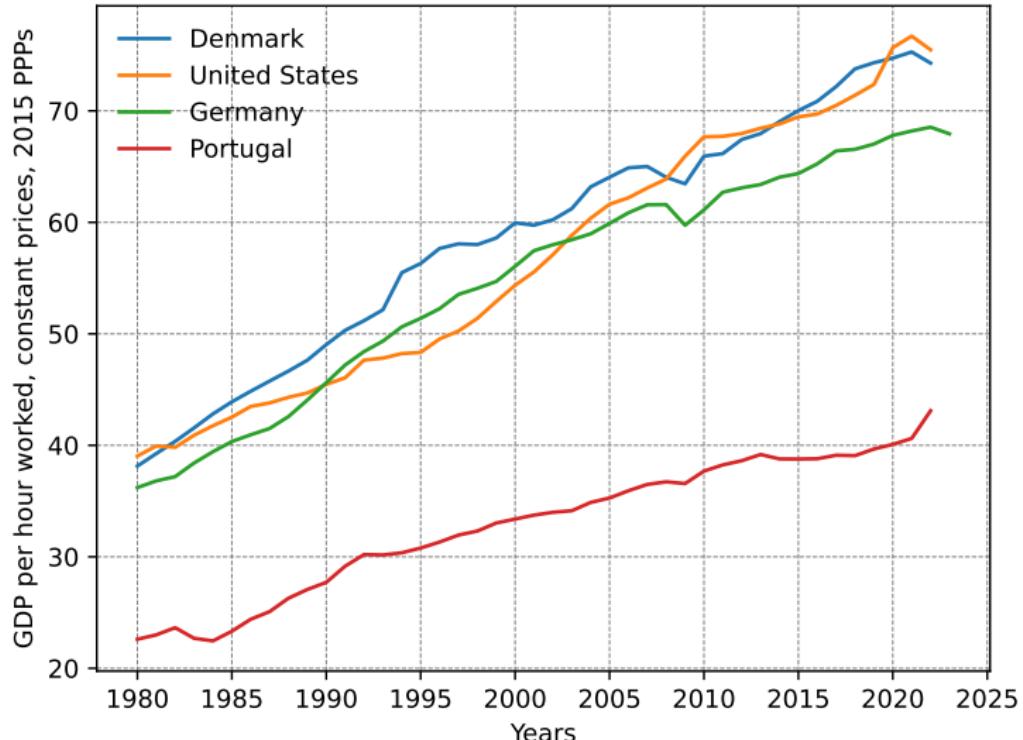
GDP across countries – OECD



GDP across countries – OECD

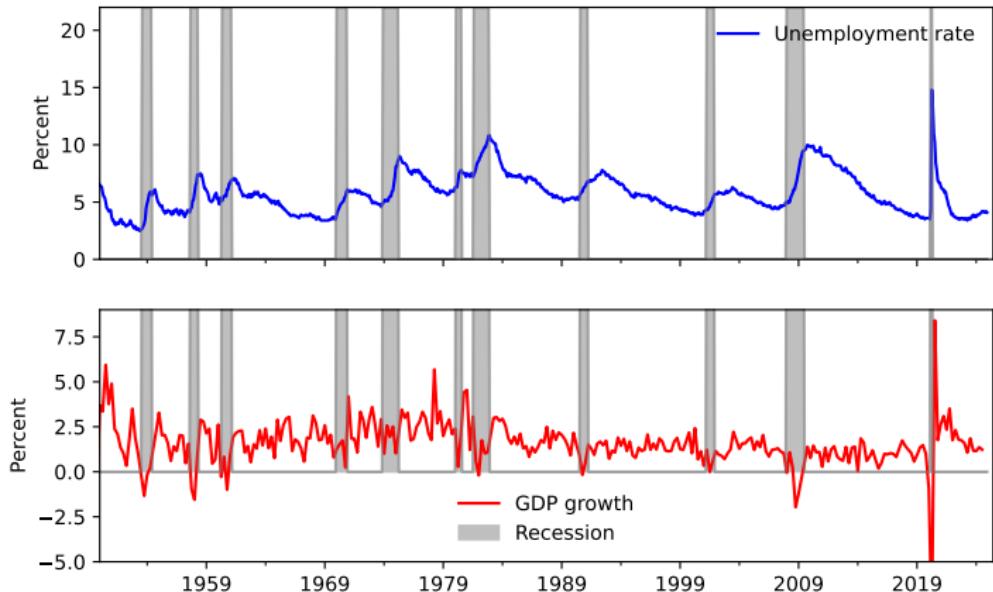


GDP across countries – OECD



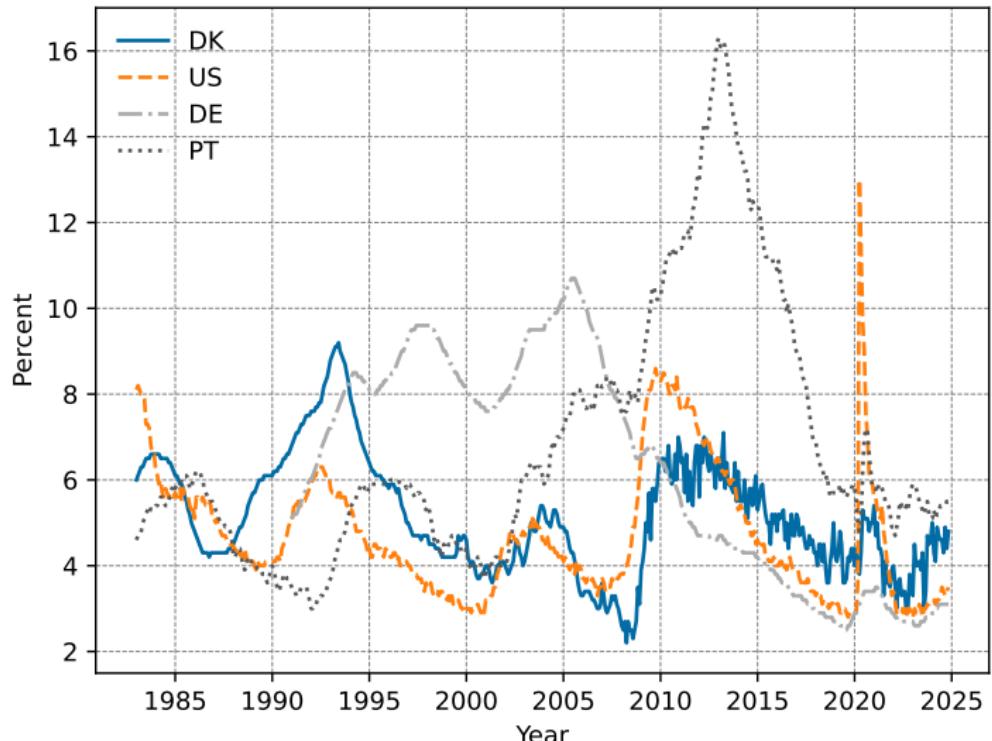
Unemployment and GDP growth in the US

Fred



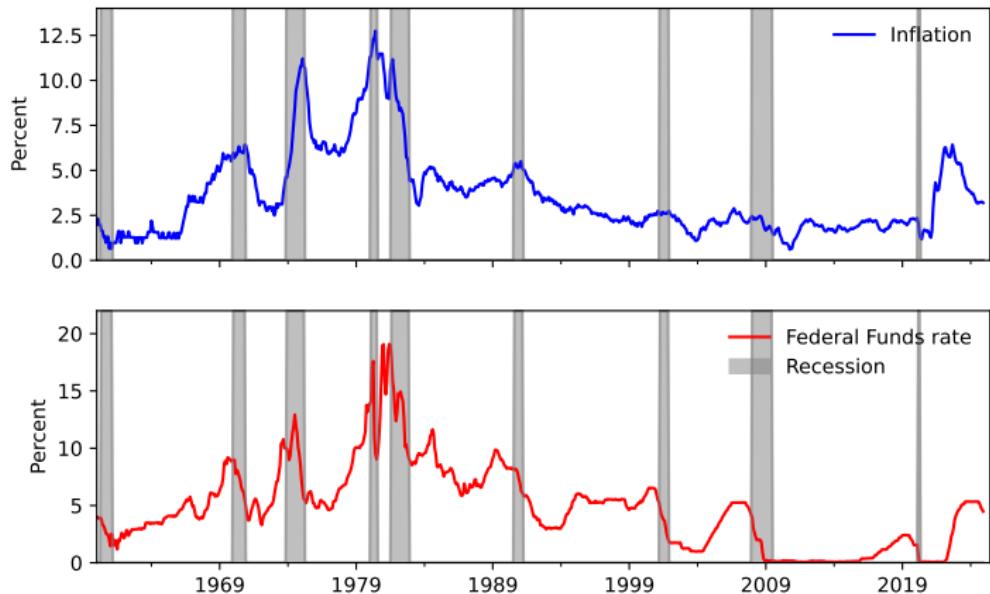
Unemployment in Europe

Eurostat



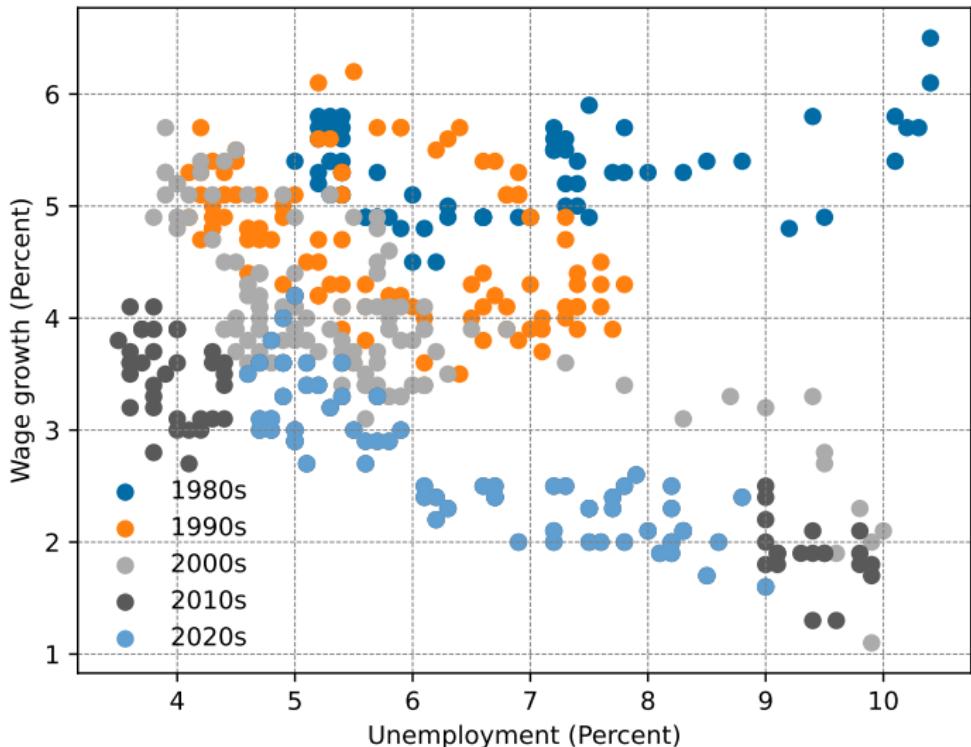
Inflation and interest rates in the US

Fred



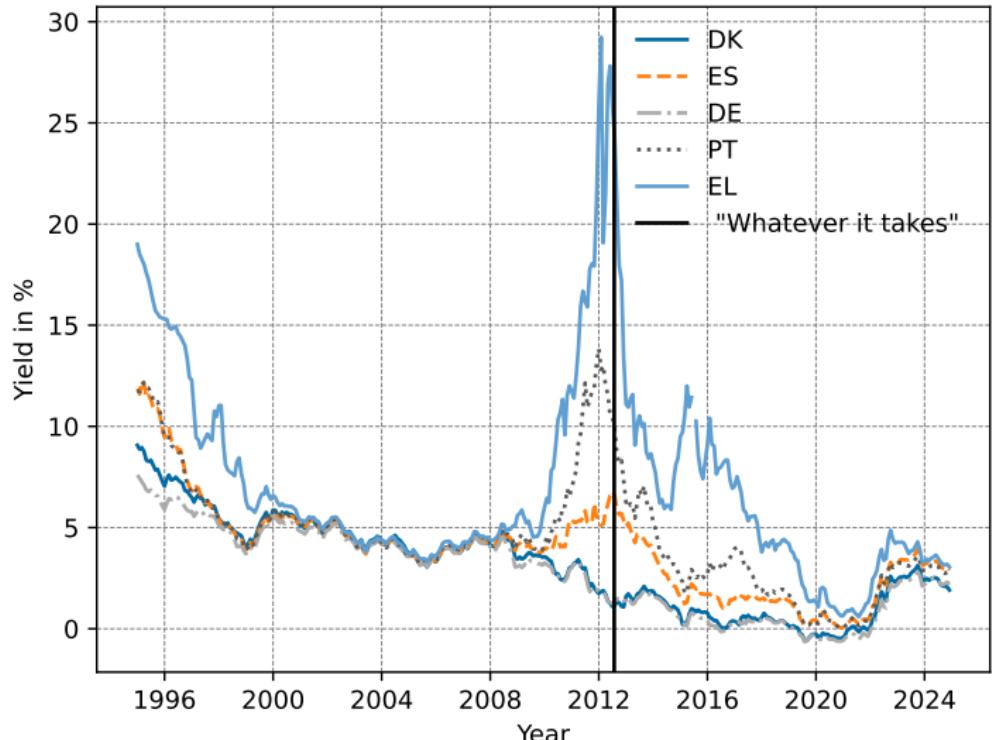
The US Phillips Curve

Fred



The words of a European central banker

Fred



Macroeconomic questions

Very rare to get experiments at the aggregate level (in advance)

- How much more do people work if their wages rise?
- What if European countries form a currency union without a fiscal union?
- How does inflation move after the ECB raises interest rates? (And why?)
- What happens when the US imposes 25% tariffs on its biggest trading partners?

Empirics alone are not enough

- Need frameworks to analyze the present and predict the future
- Absent an event having happened before

A map of ?



A map of ?

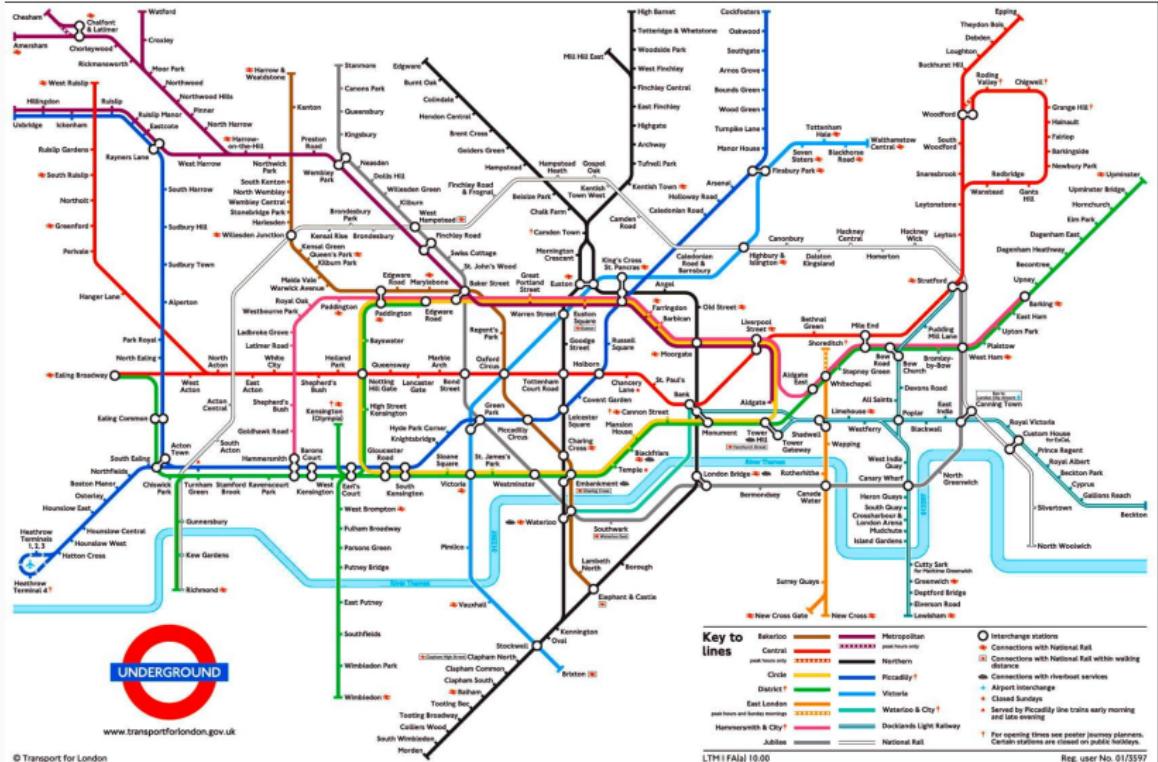


A map of ?



A map of ?

Video



© Transport for London

Understanding/explaining macroeconomic phenomena formally

Models

- Simplified description of the world
- Useful analytical tools

Advantages

- All assumptions clearly stated
- Rigorous representation of the world
- Focus on the most important aspects
- Counterfactuals can easily be understood

Disadvantages

- Simplifications

Main steps from previous courses

Mathematical formalism

- Explain macroeconomic phenomena using common language
- Learn new analytical tools
- Individual optimization

Microfoundations

- Models are built from the ground up
 - Consistent at micro- and macro-level
 - Can be compared to micro-data
- ==> we get the answers right for the right reasons

Main models to be covered

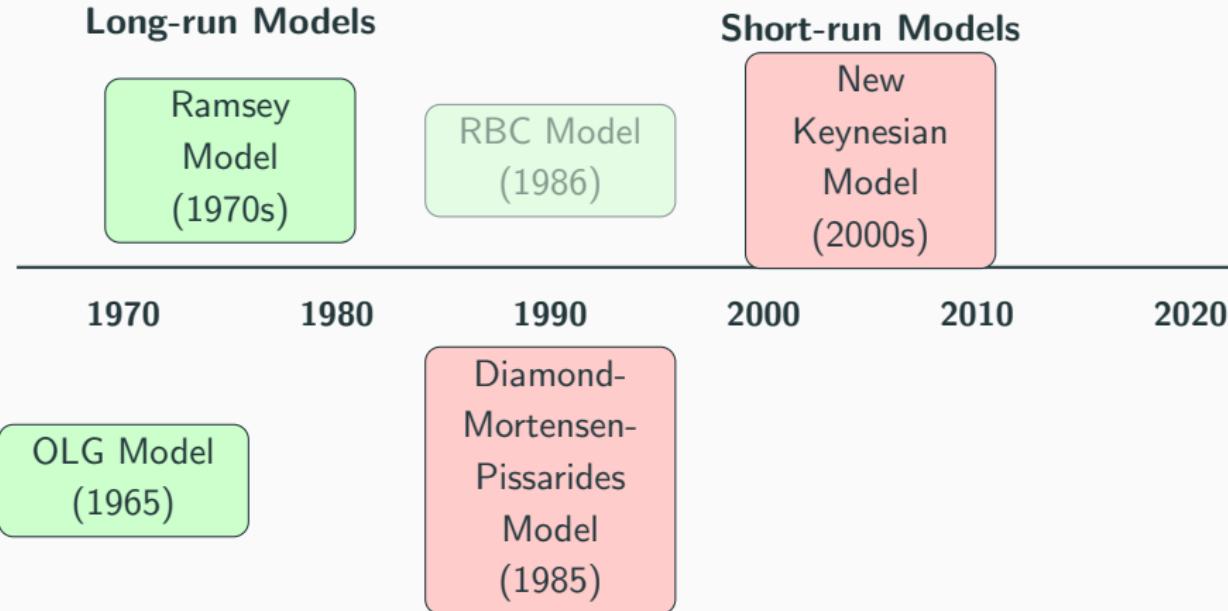
Long-run models

- Ramsey model (aka the neoclassical growth model)
- Overlapping generations model (OLG)

Business cycles

- The real business cycle (RBC) model
 - Open Economy Macro (at the end)
- Diamond-Mortenssen-Pissarides (DMP) model of the labor market
- Nominal rigidities
- The New Keynesian (NK) model of central banking

Main models to be covered



Ramsey model

Frank Ramsey

- English mathematical genius, died early in 1930
- Publishes 'A mathematical theory of saving' in 1928

The model

- Ramsey's theory + Solow model form the neoclassical growth model
- Savings an endogenous object (exogenous in Solow)
- Expectations matter for consumption

Relevance

- Foundation for modern macroeconomics
- What should the role of governments be? What drives growth?
What is the role of fiscal policy?

Overlapping generations model

Peter A. Diamond (student of Solow at MIT)

- Nobel Prize winner in 2010 (for something else)

The model

- Different generations modelled explicitly (heterogeneity)

Relevance

- Social security and pension system design
- Design of educational systems
- Financing of public debt

The RBC model

Finn Kydland (NO) & Edward Prescott (US) (Carnegie Mellon)

- Nobel Prize winners in 2004

The model

- Neo-classical growth model with short-run disturbances
- Business cycles are the result of supply shocks

Relevance

- Economic stabilization policies

The DMP model

Peter Diamond, Dale Mortensen, and Christopher Pissarides

- Nobel Prize winners in 2010

The model

- Moving away from perfectly competitive labor markets
- Search frictions result in involuntary unemployment and wage inequality

Relevance

- Design of unemployment insurance schemes

Nominal rigidities and the New Keynesian model

New Keynesian Economics

- No Nobel Prize winners yet

The model

- Moving away from perfect towards monopolistic competition
- Incorporating rigid prices into the RBC model
- Money is no longer neutral, monetary policy matters

Relevance

- Monetary policy
- Interest rates
- Inflation

Torsten Persson (SE) & Guido Tabellini (IT)

- Not Nobel Prize winners yet

The model

- Game theoretical framework of society
- Expectations, reputation and credibility matter
- Allows prescriptions on optimal policy

Relevance

- Institutional design of monetary policy institutions

Brief history of macroeconomic thought

Keynes' General Theory of empl., interest & money

Beginning of modern macroeconomic theory

- Response to Great Depression
- Attempting to formulate policy responses for the future

Major innovation

- Business cycles are demand driven
- Analytical thinking, but without formalizing in mathematical terms
- Brings together real and nominal quantities for the first time
- Markets may not clear automatically
- Involuntary unemployment

Neoclassical-Keynesian synthesis

Formalization of Keynes' ideas

- Hicks (Nobel 1972) IS-LM
- Samuelson (Nobel 1970) built MIT economics

Major innovation

- Keynes is relevant for the short-run, classical theory is a long-run concept
- Turning Keynes' writings into policy prescriptions
- IS-LM model
- Inclusion of the labor market into the model (\Rightarrow Phillips curve)

(Not really a synthesis at all)

Milton Friedman (University of Chicago)

- Nobel Laureate of 1976

Major innovation

- "Inflation is always and everywhere a monetary phenomenon" (Friedman & Schwartz, 1963)
- Monetary policy can have unintended consequences
- Depression was caused by lack of money supply
- Permanent income hypothesis
- Endogeneity of inflation expectations

The new-classical counter revolution

Fresh water economics

- Lucas (Nobel 1995), Sargent (Nobel 2011), Wallace
- Kydland, Prescott (both Nobel 2004)

Major innovation

- Flexible prices, even in the short-run, Walrasian market clearing
- Rational expectations under uncertainty
- Policy ineffectiveness proposition
- Microfoundations, stochastic modelling of agents, optimization
- Business cycles are supply-driven

The New Keynesian reconstruction

Salt water economics

- Blanchard, Akerloff, Yellen, Rotemberg, Kiyotaki
- Woodford, Gali

Major innovation

- Acceptance of rational expectations and microfoundations
- Monopolistic competition
- Rigid prices
- Short run fluctuations caused by supply **and** demand

Heterogeneity

- Increased computing power allows for more heterogeneity
- Inequality

Data

- Computing power allows for bigger datasets to discipline more complicated models

Non full information rational expectations (FIRE) models

- Behavioral biases, etc

Climate economics

- Synthesis with environmental economics

Microfoundations

Macroeconomic phenomena have to make microeconomic sense

- No more ad-hoc assumptions about macroeconomic relationships
- Optimizing agents make *rational* decisions
- Utility functions and discounting of future consumption
- Aggregate up across individuals

Intertemporal consumption & savings choices (simple)

Assumptions

- Time is discrete (i.e., not continuous)
- There is a representative agent
- The economy ends in $T=1$ (i.e., two periods = $\{0, 1\}$)
- Agents discount the future at rate β
- The agent can save in bonds (a_1) which pay interest rate $R (= 1 + r)$
- Each period, agents receive an endowment y_t , **which is know**
- The utility function is given by $u(c_t)$

No general equilibrium yet

- Assume R is given exogenously ("Partial equilibrium")

Solving a two-period consumption/savings problem

Maximization problem

$$\max_{c_0, c_1} U = u(c_0) + \beta u(c_1)$$

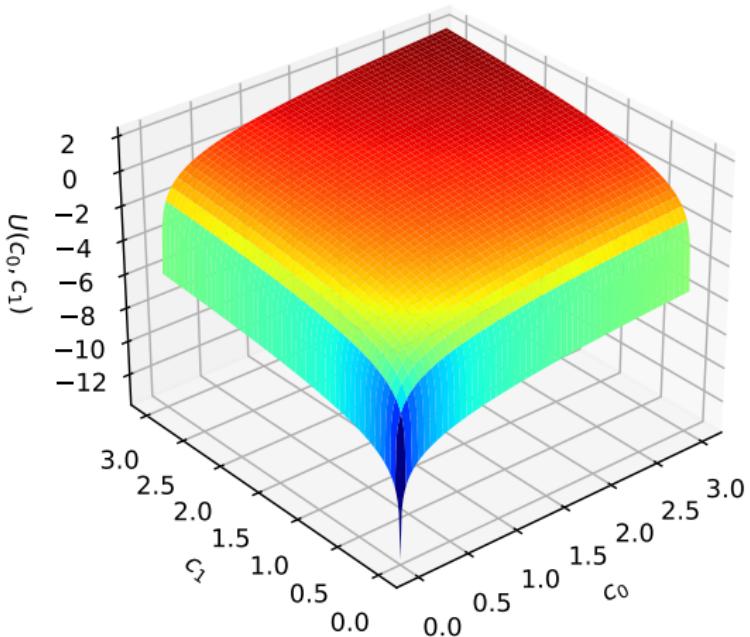
subject to $c_0 + a_1 = y_0$

$$c_1 = y_1 + (1 + r)a_1$$

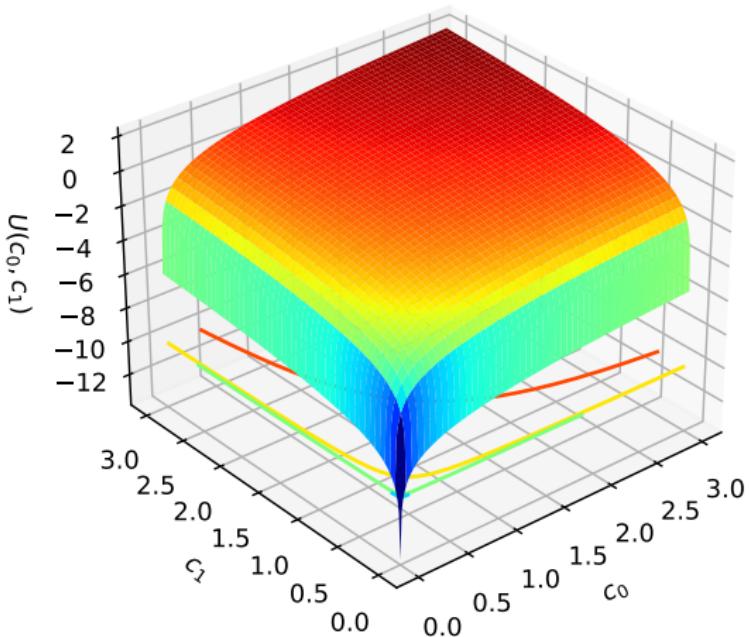
- Budget constraints are "dynamic" \implies influence each other
- Solution: combine and substitute

$$c_0 = y_0 - \frac{c_1 - y_1}{1 + r}$$

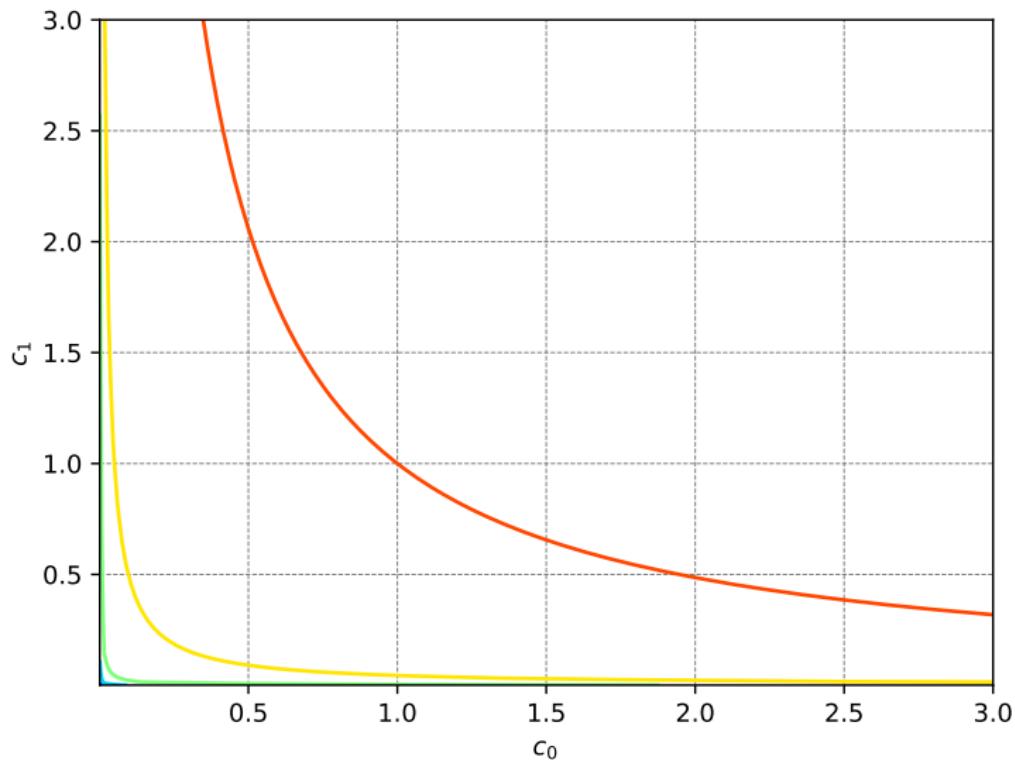
Preferences – graphical



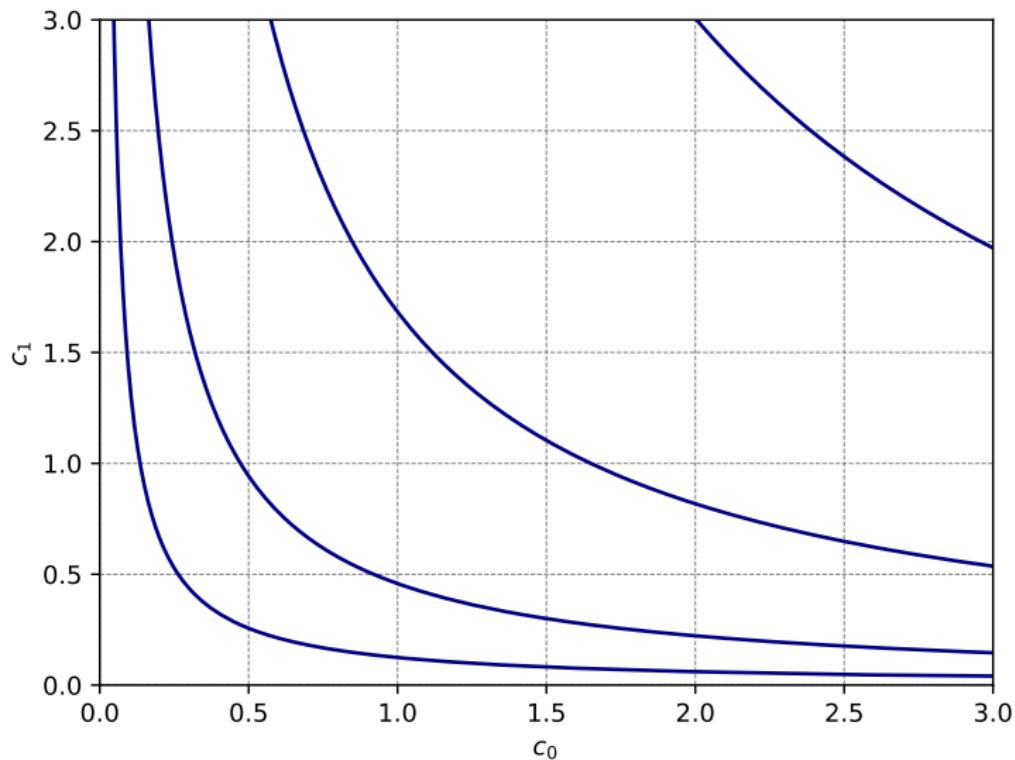
Preferences – graphical



Preferences – graphical



Preferences – graphical



Constrained optimization

Maximization problem

$$\max_{c_0, c_1} U = u(c_0) + \beta u(c_1)$$

subject to $c_0 + a_1 = y_0$

$$c_1 = y_1 + (1+r)a_1$$

\downarrow

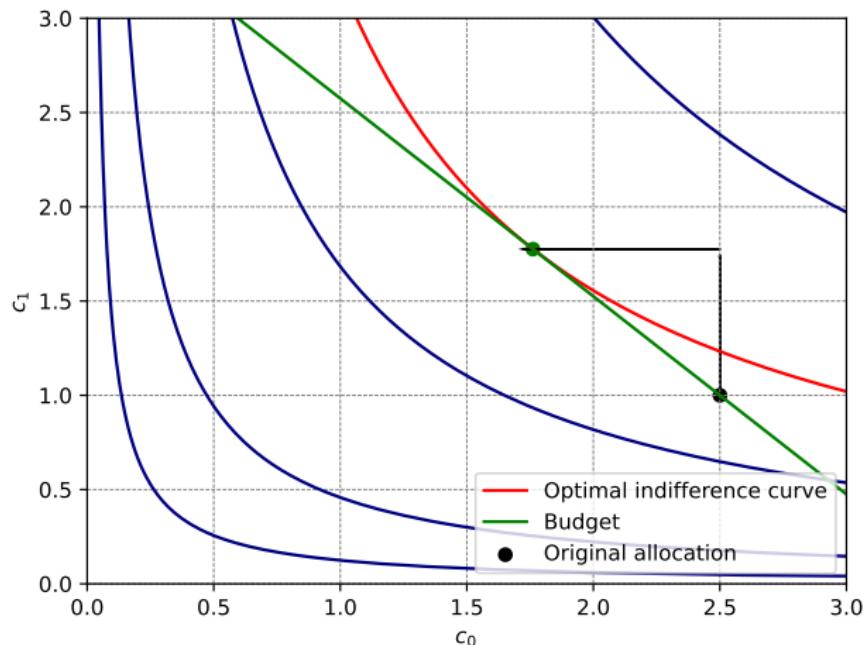
$$\max_{c_1} U = u\left(y_0 - \frac{c_1 - y_1}{1+r}\right) + \beta u(c_1)$$

First order condition for optimality

$$\beta u'(c_1) = \frac{1}{1+r} u'\left(\underbrace{y_0 - \frac{c_1 - y_1}{1+r}}_{c_0}\right)$$

$$\underbrace{u'(c_0) = \beta(1+r)u'(c_1)}_{\text{Euler equation}} \quad \text{and} \quad \underbrace{c_1 = y_1 + (1+r)(y_0 - c_0)}_{\text{Budget constraint}}$$

Solution – Graphical (increasing consumption path)



- Agents save to smooth consumption

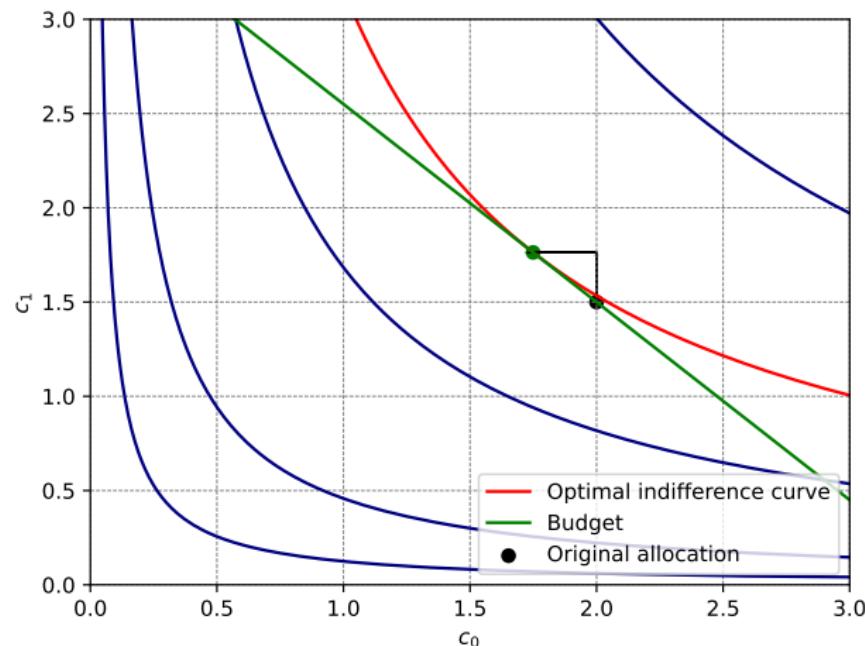
Euler equation

$$u'(c_0) = \beta(1+r)u'(c_1) \iff \frac{u'(c_0)}{\beta u'(c_1)} = (1+r)$$

Tradeoff between marginal utility today and tomorrow is governed by

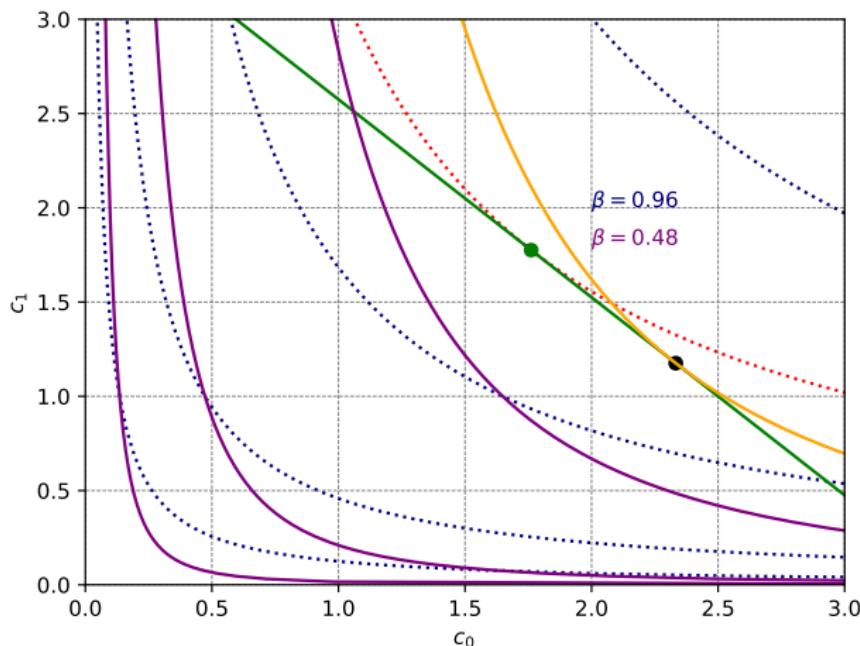
- Endowment path y_t
- Patience β
- Shape of the utility function
- Interest rate r

Solution – Graphical (decreasing consumption path)



- In the face of decreasing endowments, the agent saves

Patience vs impatience – $u'(c_0) = \beta(1+r)u'(c_1)$



- Impatient consumers allocate to consumption today c_0

Utility function

Crucial assumptions

- Continuously differentiable and displays positive but decreasing marginal returns: $\lim_{c \rightarrow \infty} u'(c) = 0$
- Consumption is essential: $\lim_{c \rightarrow 0} u'(c) = \infty$

The workhorse – Constant Relative Risk Aversion (CRRA)

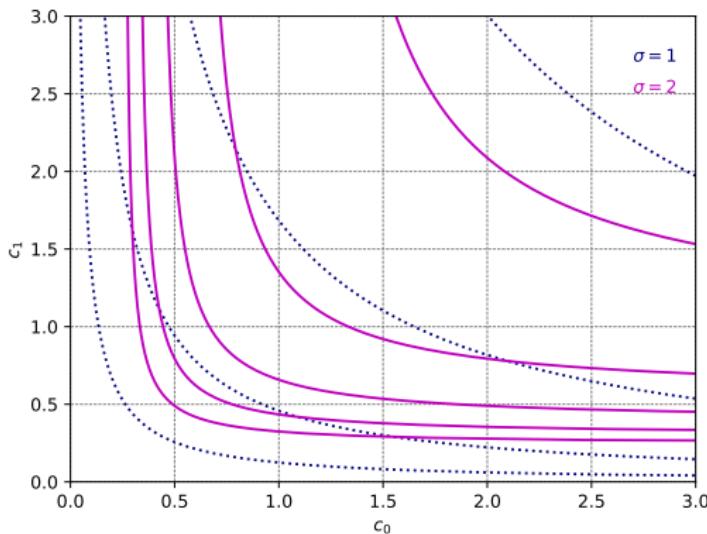
$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}; \quad \sigma > 0; \sigma \neq 1$$

- σ is called the coefficient of relative risk aversion
- $1/\sigma$ is the intertemporal elasticity of substitution between two periods ($\sigma \rightarrow \infty \implies$ Leontief across periods)
- As $\sigma \rightarrow 1$, one can show that $u(c) = \ln(c)$

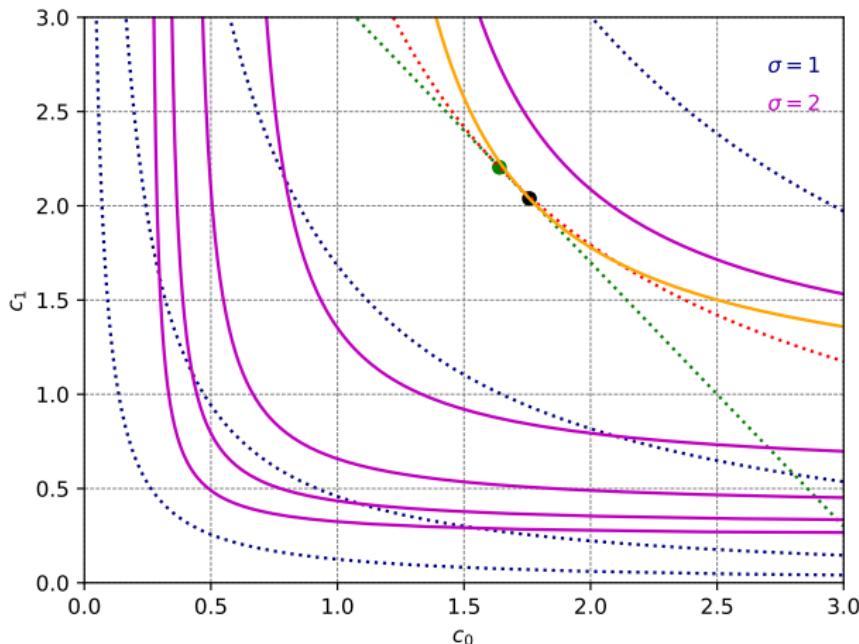
Euler equation

$$c_0^{-\sigma} = \beta(1+r)c_1^{-\sigma} \iff \left(\frac{c_0}{c_1}\right)^{-\sigma} = \beta(1+r)$$

- Differences between c_0 and c_1 become more costly as σ rises

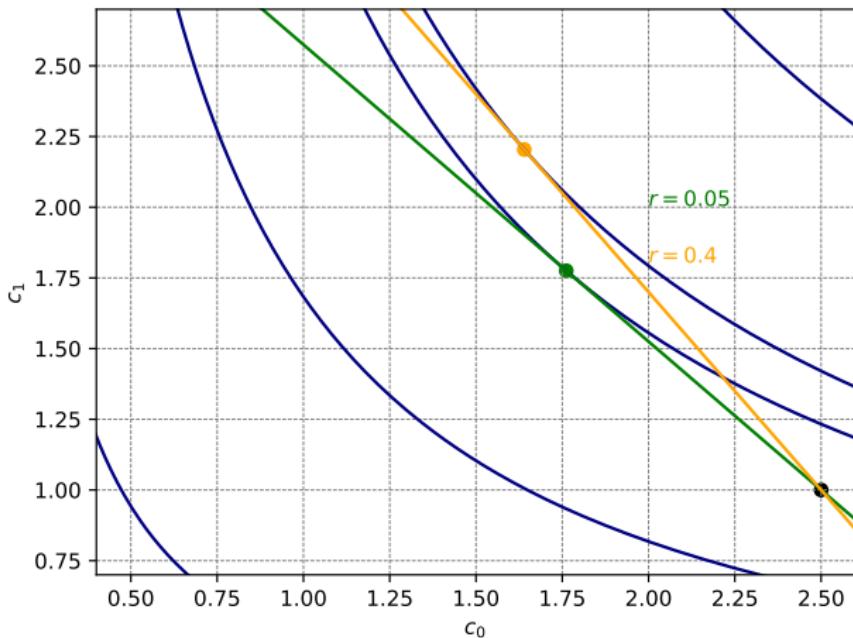


$$\text{Risk aversion} - u'(c_0) = \beta(1+r)u'(c_1)$$



- Risk aversion makes agents smooth more \implies stay away from the edges

$$\text{Interest rates} - u'(c_0) = \beta(1+r)u'(c_1)$$



- Higher interest rates lead to more consumption tomorrow

Income and substitution effects

Remember micro: effect of price changes can be decomposed!

- Interest rate r is the relative price of period 1 vs period 0 utility

Substitution effect

- If relative prices had been the same initially, what bundle would give initial utility level?
- This effect becomes strongest with $\sigma \rightarrow 0$, as agents consume all in one of the periods (corner solutions)

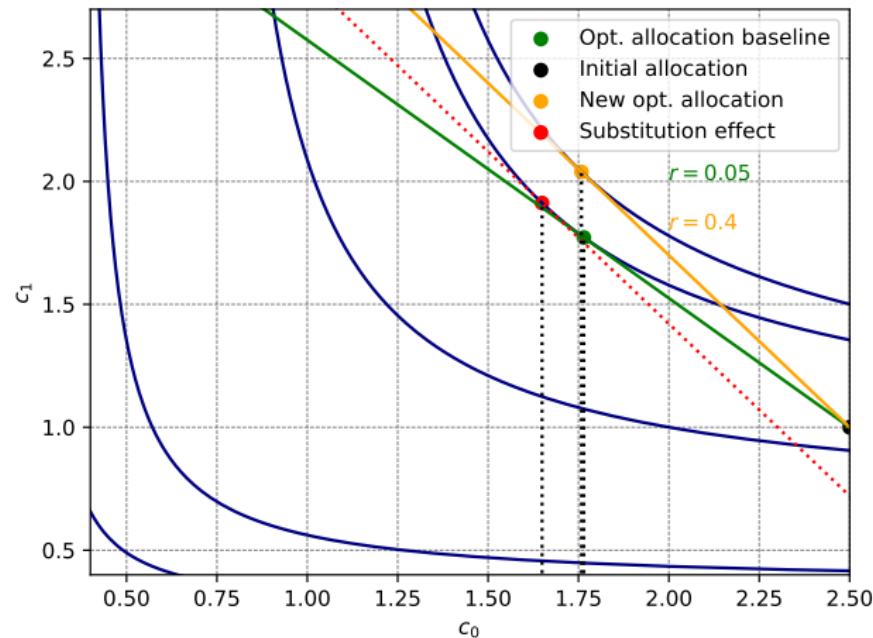
Income effect

- How does having a changed budget affect the bundle?

Danger: Wealth effect

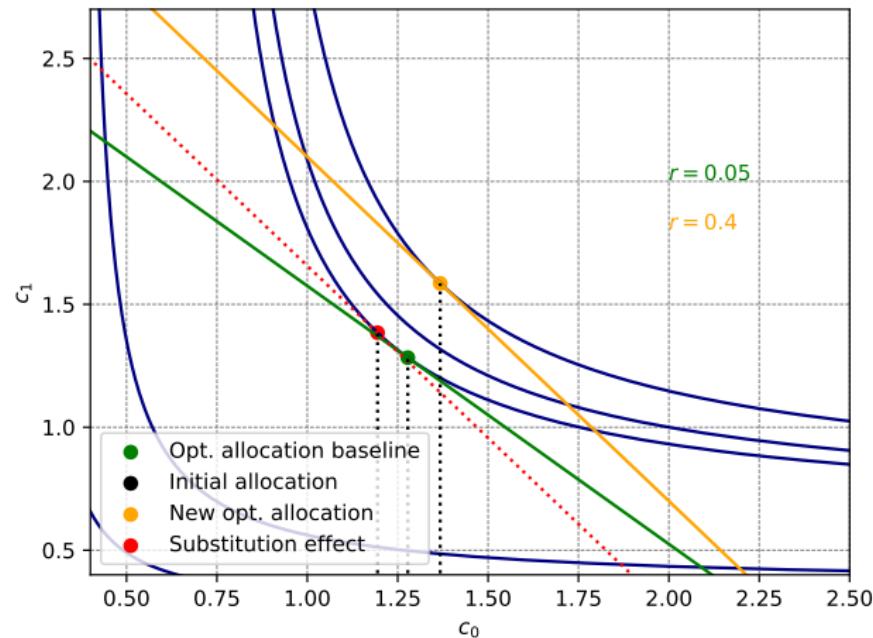
- If r rises, tomorrow's income is worth less today

$$\text{Interest rates} - u'(c_0) = \beta(1+r)u'(c_1)$$



- Substitution, income and wealth effect change choices

$$\text{Interest rates} - u'(c_0) = \beta(1+r)u'(c_1) - y_1 = 0$$



- Substitution effect: $\partial c_0 / \partial r < 0$ because of higher return
- Income effect: $\partial c_0 / \partial r > 0$ because agent is richer

Income and substitution effect

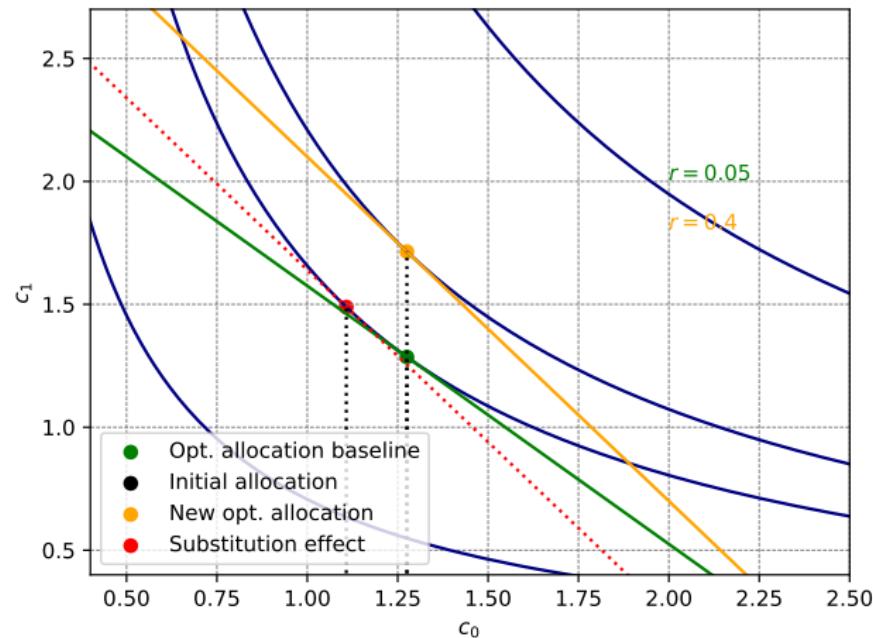
Period 0 consumption as a function of parameters

$$\begin{aligned}c_0^{-\sigma} &= \beta(1+r)c_1^{-\sigma} \text{ and } c_1 = y_1 + (1+r)(y_0 - c_0) \\ \implies c_0 \left(1 + \frac{(\beta(1+r))^{\frac{1}{\sigma}}}{1+r}\right) &= \frac{y_1}{1+r} + y_0\end{aligned}$$

Effects of interest rate changes

- **Wealth effect** (goes to 0 as $y_1 \rightarrow 0$)
- **Income effect**
- **Substitution effect** (goes to 0 as $\sigma \rightarrow \infty$)

$$\text{Interest rates} - u'(c_0) = \beta(1+r)u'(c_1)$$



- With log-utility and $y_1 = 0$ (no wealth effect), the effects cancel