The Ramsey model

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Agenda

Consumption theory

- Two-period model recap
- Marginal propensities to consume
- Extension to arbitrary periods

The Ramsey model

- Derivation
- Steady state
- Dynamics
- Welfare

Two-period model recap

Preferences are given by

$$U = u(c_0) + \beta u(c_1)$$

- Individuals live for two periods
- Future utility is discounted at rate β

Dynamic budget constraints

$$\begin{aligned} c_0 + a_1 &= y_0 \\ c_1 &= y_1 + (1+r)a_1 \end{aligned}$$

- Individuals receive endowments (income) each period
- Income can be reallocated across periods using saving/borrowing

Two-period model solution

Euler equation

$$u'(c_0) = \beta(1+r)u'(c_1)$$

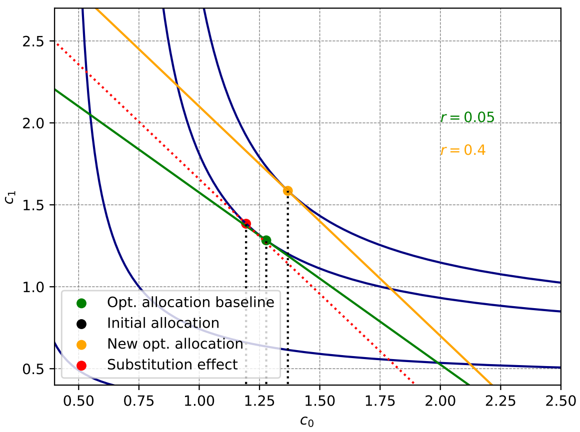
- Tradeoff between today and tomorrow is governed by β and r
- Present value of consumption = present value of endowment
- Higher r always increases c_1/c_0, but c_0 doesn't always fall

c_0 with CRRA utility $\left(u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}\right)$

$$\left(\frac{c_1}{c_0}\right)^{\sigma} = \beta(1+r) \text{ and } c_0 + \frac{c_1}{1+r} = y_0 + \frac{y_1}{1+r}$$

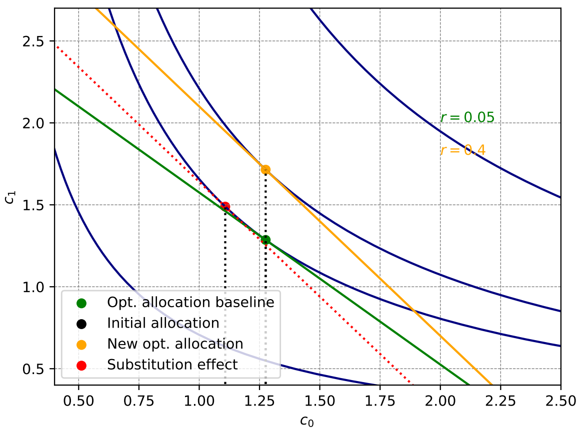
$$\implies c_0\left(1 + \frac{(\beta(1+r))^{\frac{1}{\sigma}}}{1+r}\right) = y_0 + \frac{y_1}{1+r}$$

Income and substitution effects



- Substitution effect: $\partial c_0/\partial r < 0$ because of higher return
- Income effect: $\partial c_0/\partial r > 0$ because agent is richer

Income and substitution effects



- With log-utility ($\sigma = 1$) and no wealth effect ($y_1 = 0$), effects cancel
- Interest rate changes have no impact on consumption **in period 0**

Multiple periods

Two-periods is not enough

- Individuals have longer horizons
- Macroeconomic phenomena may happen on long time scales
- The Solow model has an infinite horizon

Agents with **very** long life-spans

- Intergenerational altruism
- Time-invariant survival probability
- Mathematical simplicity

Finite horizon

Arbitrary (but finite) horizon

Preferences are given by

$$U = u(c_0) + \beta u(c_1) + \beta^2 u(c_2) + \cdots = \sum_{t=0}^{T} \beta^t u(c_t)$$

- Future utility is discounted at rate β

Dynamic budget constraints

$$\begin{aligned} c_0 + a_1 &= y_0 + (1 + r_0)a_0 \\ c_1 + a_2 &= y_1 + (1 + r_1)a_1 \\ &\dots \\ c_t + a_{t+1} &= y_t + (1 + r_t)a_t \\ a_{T+1} &\geq 0 \end{aligned}$$

- a_0 is given – initial wealth
- cannot die with debt (a_{T+1})

Solution method

Problem is given by

$$\max_{c_t, a_{t+1} \forall t} U = \sum_{t=0}^{T} \beta^t u(c_t)$$

$$\text{subject to} \quad c_t + a_{t+1} = y_t + (1 + r_t)a_t; a_T \geq 0; a_0 = 0$$

Lagrangian

$$\mathcal{L} = \sum_{t=0}^{T} \beta^t u(c_t) + \sum_{t=0}^{T} \lambda_t \left[y_t + (1 + r_t)a_t - c_t - a_{t+1} \right] + \mu a_{T+1}$$

- T utilities and budget constraints to optimize
- Final period a_{T+1} must be chosen separately (although trivial)

Optimization

Zoom in to period t (highly questionable notation)

$$\mathcal{L} = \cdots + \beta^t u(c_t) + \cdots + \lambda_t \left[y_t + (1 + r_t) a_t - c_t - a_{t+1} \right] \\ + \lambda_{t+1} \left[y_{t+1} + (1 + r_{t+1}) a_{t+1} - c_{t+1} - a_{t+2} \right]$$

- The asset choice today affects tomorrow's budget!
- a_{t+1} links periods t and $t+1$ – appears twice!

First order conditions $\implies$ Euler equation

$$\frac{\partial \mathcal{L}}{\partial c_t} : \qquad \beta^t u'(c_t) = \lambda_t$$

$$\frac{\partial \mathcal{L}}{\partial a_{t+1}} : \qquad \lambda_t = (1 + r_{t+1}) \lambda_{t+1}$$

$$\implies u'(c_t) = \beta (1 + r_{t+1}) u'(c_{t+1})$$

- Euler equation exactly the same as in 2-period model
- Also, agents will always choose $a_{T+1} = 0$ due to $u'(c) > 0$

Infinite horizon

Infinite horizon

Problem is given by

$$\max_{c_t, a_{t+1} \forall t} U = \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{subject to} \quad c_t + a_{t+1} = y_t + (1 + r_t)a_t$$

$$\lim_{T \to \infty} \left(\prod_{t=0}^{T} \frac{1}{1 + r_t} \right) a_{T+1} \geq 0; a_0 = 0$$

- New addition: No-Ponzi game condition $\implies$ cannot accumulate infinite debt (this is a constraint we impose)
- The present value of "final period" savings must be positive
- Importance becomes clear when using intertemporal budget constraint

Two ways of obtaining an Euler equation

Optimizing using the dynamic budget constraints

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c_t) + \sum_{t=0}^{\infty} \lambda_t \left[y_t + (1+r_t)a_t - c_t - a_{t+1} \right]$$

$$\frac{\partial \mathcal{L}}{\partial c_t} : \quad \beta^t u'(c_t) = \lambda_t$$

$$\frac{\partial \mathcal{L}}{\partial a_{t+1}} : \quad \lambda_t = (1+r)\lambda_{t+1}$$

- As before, combining the two yields $u'(c_t) = \beta(1+r_{t+1})u'(c_{t+1})$

The intertemporal budget constraint

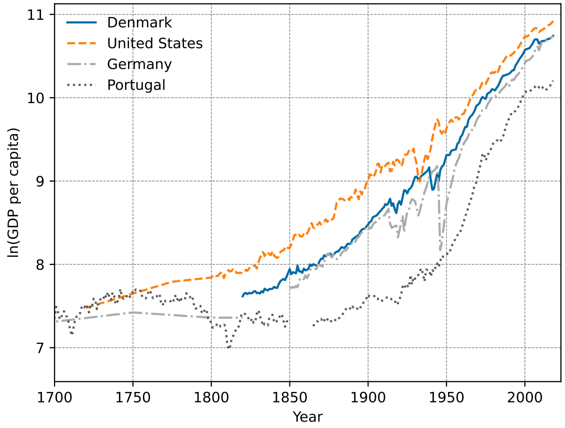
No-Ponzi pins down the path of c_t (by not allowing infinite consumption)

$$
\begin{aligned}
\frac{1}{1+r_0}a_1 &= \frac{1}{1+r_0}(y_0 - c_0) + a_0 \\
\frac{1}{1+r_1}a_2 &= \frac{1}{1+r_1}(y_1 - c_1) + a_1 \\
\implies \frac{1}{1+r_1}a_2 &= \frac{1}{1+r_1}(y_1 - c_1) + (y_0 - c_0) + (1+r_0)a_0 \\
\implies \frac{1}{1+r_1}\frac{1}{1+r_0}a_2 &= \frac{1}{1+r_1}\frac{1}{1+r_0}(y_1 - c_1) + \frac{1}{1+r_1}(y_0 - c_0) + a_0 \\
&\vdots \\
\implies \underbrace{\left(\prod_{t=0}^{T}\frac{1}{1+r_t}\right)a_{T+1}}_{\text{No-Ponzi as } T\to\infty} &= \sum_{t=0}^{T}\left(\prod_{s=0}^{t}\frac{1}{1+r_s}\right)(y_t - c_t) + a_0
\end{aligned}
$$

- Just as in 2 periods: PV of consumption equals PV of income

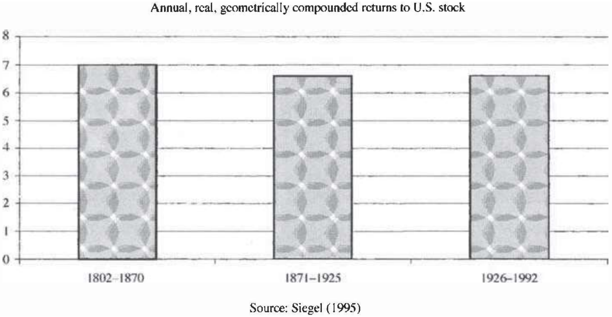
The Ramsey model

Constant growth rate



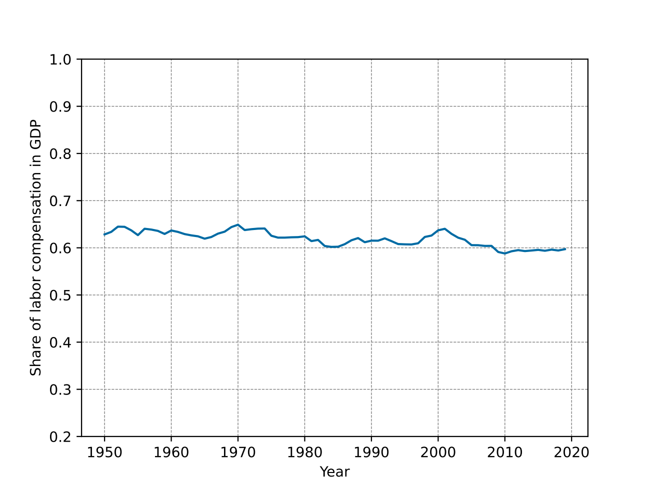
- US long-run growth has been constant for 200 years

Constant return on capital



- The return on capital (r) has been constant for 200 years

Constant factor shares



- The labor income share (wL/Y) is constant (Source: Fred)

Constant capital output ratio

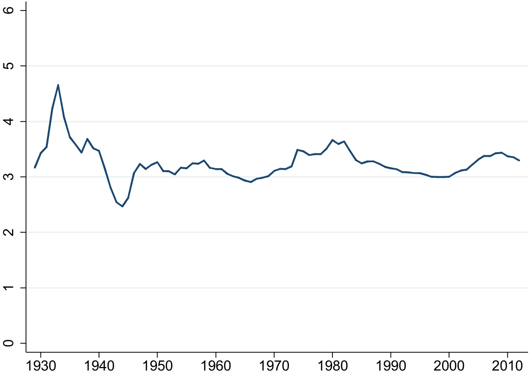


Figure: **Capital-Output ratio in the U.S.**

Source: NIPA table 1.1. The figure plots the ratio between fixed capital and consumer durables relative to the GDP.

- The capital-output ratio (K/Y) in the US hovers around 3 Danish data

Importance of microfoundations

Quick Solow-model recap

$$
\begin{aligned}
k_{t+1} &= i_t + (1-\delta)k_t \\
y_t &= F(k_t, l_t) \\
y_t &= c_t + i_t = c_t + s y_t \implies c_t = (1-s)y_t
\end{aligned}
$$

- The savings rate is not microfounded, just a parameter

Next steps:

- Combine optimal consumption choice with Solow to get Ramsey
- Add firm sector and asset market clearing
- Assume representative agent (infinitely many identical HHs)
- Assume labor is supplied inelastically at $l_t = 1 \forall t$

Firms

Representative firm

$$\max_{K_t, L_t} AK_t^{\alpha} L_t^{1-\alpha} - r_t K_t - w_t L_t$$

- Assume Cobb-Douglas specification for production function
- Production function $F(.)$ turns K and L into Y
- Firms are perfectly competitive
- They take prices r_t, w_t as given

First order conditions

$$f_k'(k_t, 1) = r_t \text{ where } k_t = K_t/L_t$$
$$f_l'(k_t, 1) = w_t$$

Labor income share is constant (prove it!)

$$\frac{w_t L_t}{AK_t^{\alpha} L_t^{1-\alpha}} = 1 - \alpha$$

Households optimality per capita

Modified first order conditions from before

$$u'(c_t) = \beta(1 + r_{t+1} - \delta)u'(c_{t+1})$$
$$c_t + a_{t+1} = w_t + (1 + r_t - \delta)a_t + \underbrace{\Pi_t}_{\text{Profits}}$$

- Households earn wage w_t (similar to endowment, since labor is supplied inelastically)
- Households lend to the firms at rate r_t
- δ depreciates each period

Transversality conditions

$$\lim_{T\to\infty} \beta^T u'(c_T)a_{T+1} = 0; \quad \lim_{T\to\infty} \beta^T u'(c_T)k_{T+1} = 0$$

- Not the same as no-Ponzi condition! No-Ponzi prevents too much debt (constraint)
- Transversality prevents too much capital/assets $\implies$ different sign (optimality)

General Equilibrium

- General equilibrium implies that all markets clear & agents optimize

Market clearing

Capital and labor market clearing

$$l_t = 1$$
$$k_t = a_t$$

Resource constraint (same as in the Solow model)

$$\begin{aligned} c_t + a_{t+1} &= w_t + (1 + r_t - \delta)a_t + \Pi_t \\ c_t + k_{t+1} &= (1-\delta)k_t + r_t k_t + w_t + f(k_t) - r_t k_t - w_t \\ k_{t+1} &= (1-\delta)k_t + f(k_t) - c_t \end{aligned}$$

Capital tomorrow (k_{t+1}) is

- Leftover capital from today (after depreciation)
- Production less consumption

Laws of motion

Ramsey – described by path for c **and** k

$$k_{t+1} = (1-\delta)k_t + f(k_t) - c_t \qquad \text{(Resource constraint)}$$
$$u'(c_t) = \beta(1 + r_{t+1} - \delta)u'(c_{t+1}) \qquad \text{(Euler equation)}$$

Solow – described by path of k, for given k, c is known

$$k_{t+1} = (1-\delta)k_t + f(k_t) - c_t$$
$$c_t = (1-s)f(k_t)$$

- The savings rate s is now endogenized through optimally chosen consumption
- One fewer ad-hoc parameter

Steady state I

In the model's steady state, c and k must be constant

Steady state of capital accumulation

$$
\begin{aligned}
k &= (1-\delta)k + f(k) - c \\
\implies c &= f(k) - \delta k
\end{aligned}
$$

- For any k, there is a specific level of c that keeps k constant

Steady state in the Euler equation

$$
\begin{aligned}
u'(c) &= \beta(1+r-\delta)u'(c) \\
u'(c) &= \beta(1+f'(k)-\delta)u'(c) \\
f'(k) &= \frac{1}{\beta} + \delta - 1
\end{aligned}
$$

- Consumption remains constant at a specific level of capital k
- Functional form of $u(c)$ doesn't matter for steady state k

Steady state II

Steady state of the model

$$f'(k) = \frac{1}{\beta} + \delta - 1$$
$$c = f(k) - \delta k$$

- Both equations must hold at the same time $\implies$ intersection

Assume functional forms

$$F(K) = K^{\alpha} L^{1-\alpha} \implies k^{\alpha} \text{ if } L = 1 \qquad \text{(Cobb-Douglas production)}$$
$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma} \qquad \text{(CRRA utility)}$$

Unique steady state value of capital

$$k = \left(\frac{\alpha}{\frac{1}{\beta} + \delta - 1} \right)^{\frac{1}{1-\alpha}}$$

Dynamics

How do we get to the steady state?

- Does every starting point converge to the steady state?
- How fast is the speed of convergence?
- What are the dynamics of the model far away from steady state?

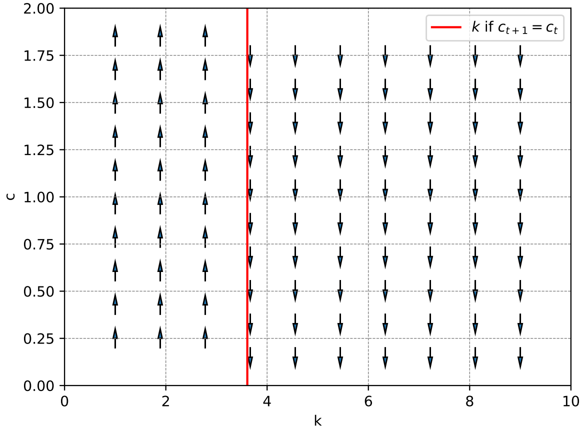
Phase diagram

- Describe the dynamics graphically

Plug in some numbers!

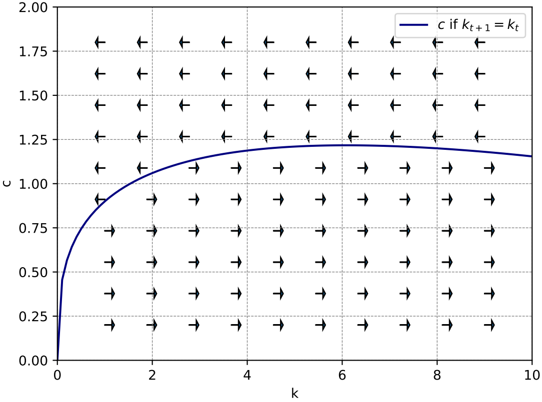
- $\beta = 0.96$
- $\delta = 0.1$
- $\alpha = 0.3$
- $\sigma = 1$

Dynamics along c dimension – think of arrows as c growth



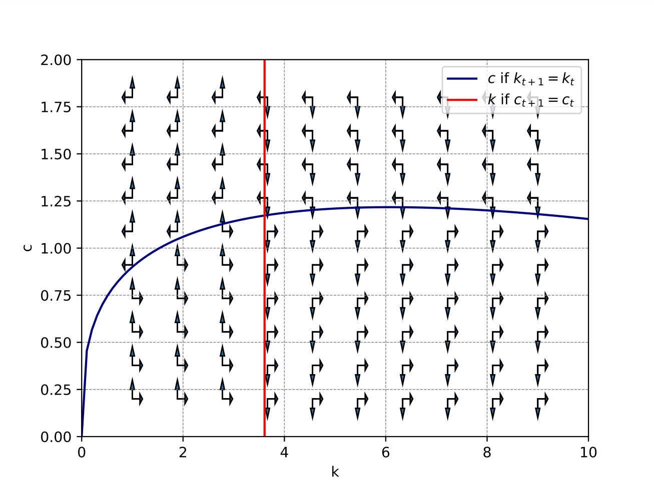
- $u'(c_t) = \beta(1 + r_t - \delta)u'(c_{t+1})$
- High $k_t \to$ low marginal product $f'(k_t) \to$ low $r_t \to \frac{c_{t+1}}{c_t} \downarrow$ today

Dynamics along k dimension – think of arrows as k growth



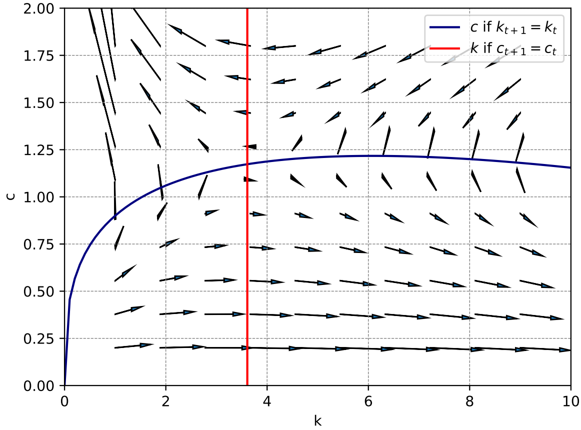
- $k_{t+1} = (1 - \delta)k_t + f(k_t) - c_t$
- High $c_t \rightarrow$ little left to invest in $k_{t+1} \rightarrow$ capital falls
- $f(k)$ is concave $\rightarrow$ as k grows, output grows by less and less

Full dynamics – Phase diagram



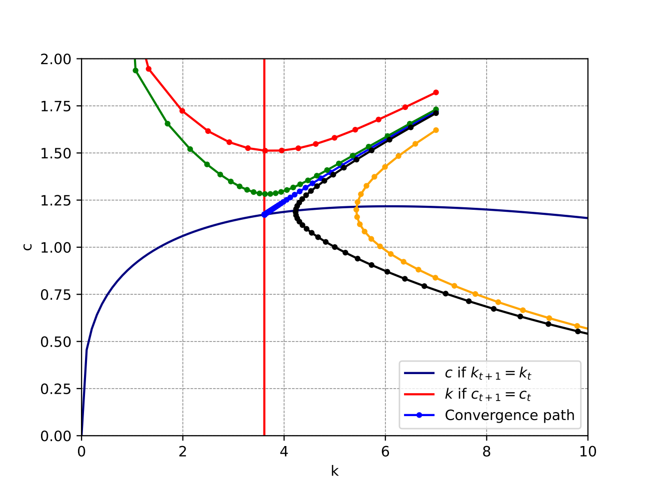
- Steady state where the two lines cross
- c^* and k^* define the balanced growth path (after transition)

Full dynamics – Phase diagram



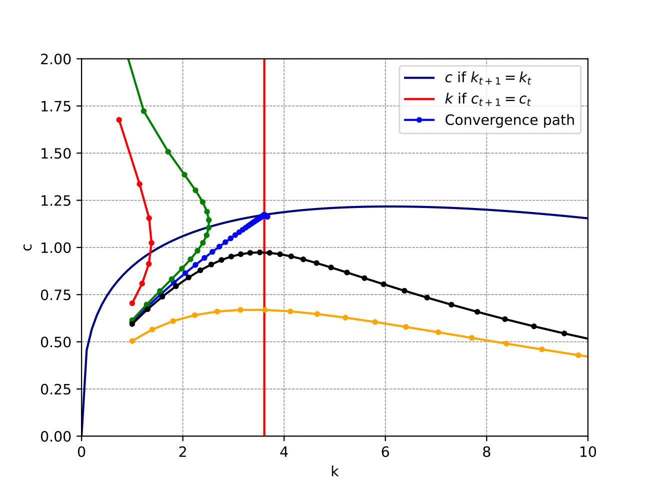
- Steady state where the two lines cross
- c^* and k^* define the balanced growth path (after transition)

Dynamics towards convergence



- Only very specific starting conditions converge

Dynamics towards convergence



- For each k_0, only one value of c_0 leads to convergence

Solving the Ramsey model

For each k_0, only one c_0 is eligible!

- Remember the no-ponzi condition: too much debt (c) is not permitted (top left)
- Remember the transversality condition: too much wealth (k) is not permitted (bottom right)

$$\sum_{t=0}^{T} \left(\prod_{s=0}^{t} \frac{1}{1 - \delta + r_s} \right) (w_t - c_t) = 0$$

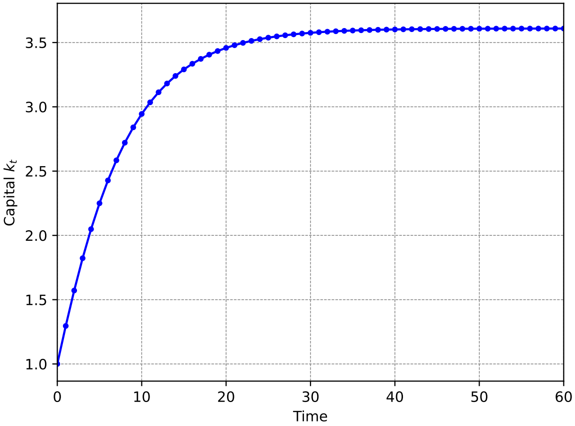
- There is no analytical solution for c_0

Solution

- Guess c_0, iterate forward using Euler eq. and resource constr.
- Narrow search c_0 using, e.g., bisection search

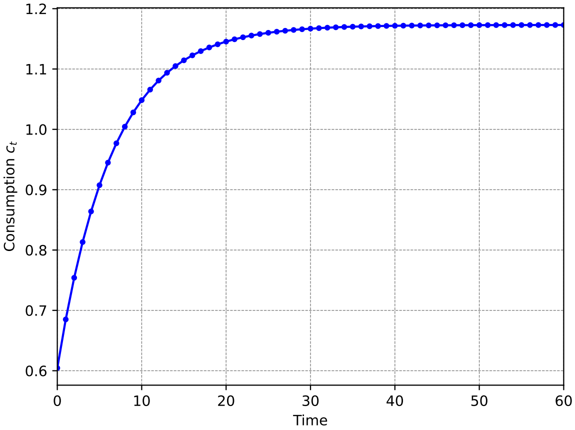
Convergence paths

Capital



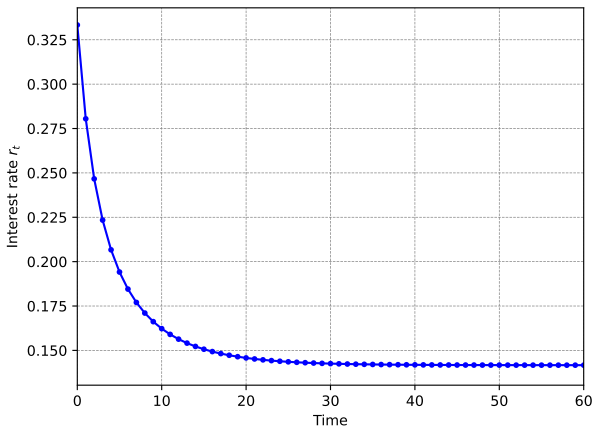
Convergence paths

Consumption



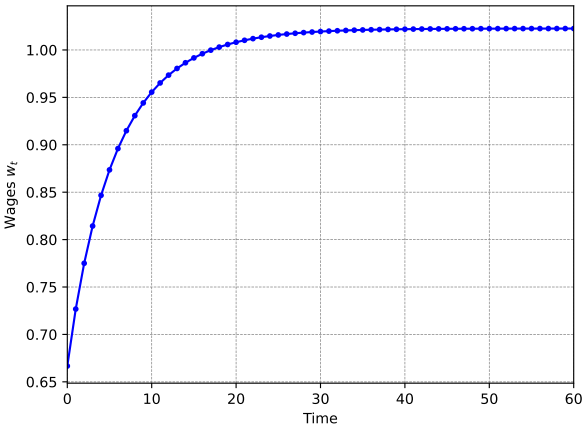
Convergence paths

Rental rate of capital



Convergence paths

Wage

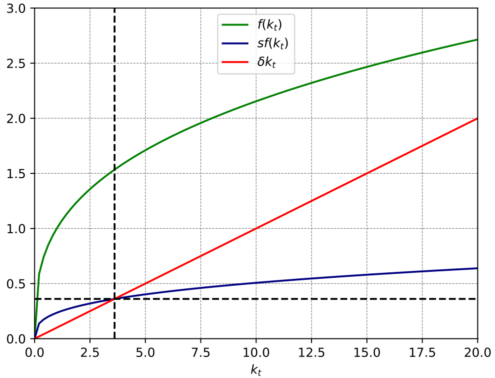


Solow comparison

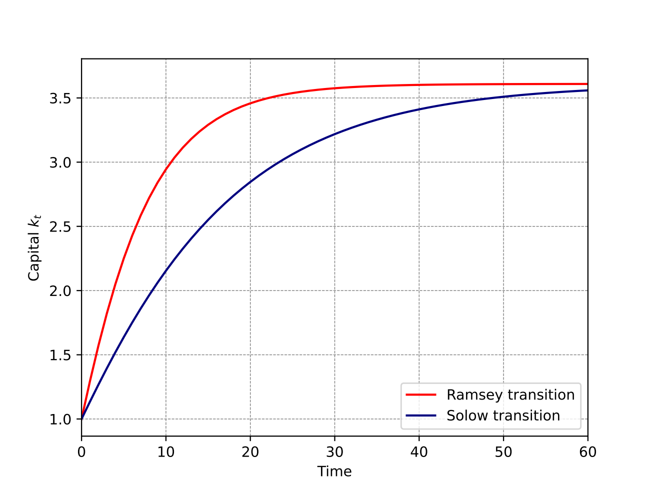
Law of motion in Solow (only depends on k)

$$k_{t+1} = (1-\delta)k_t + sf(k_t) \implies k_{ss} = \frac{s}{\delta}k_{ss}^{\alpha} \implies k_{ss} = \left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}$$

- Set s to match Ramsey steady state capital, $k \approx 3.6$

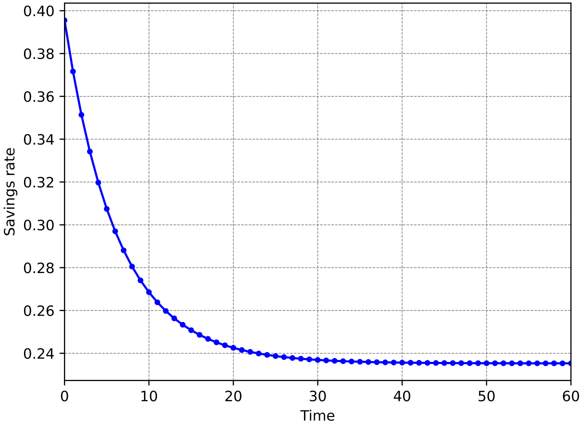


Transition comparison from same k_0



- Ramsey model converges much faster. Why?

Savings rate is endogenous



- Savings rate in Ramsey is very high initially (Why?)

Welfare

Welfare in the Solow model

Maximize steady state consumption subject to the resource constraint

$$\max_k f(k) - \delta k \implies f'(k^{gr}) = \delta$$

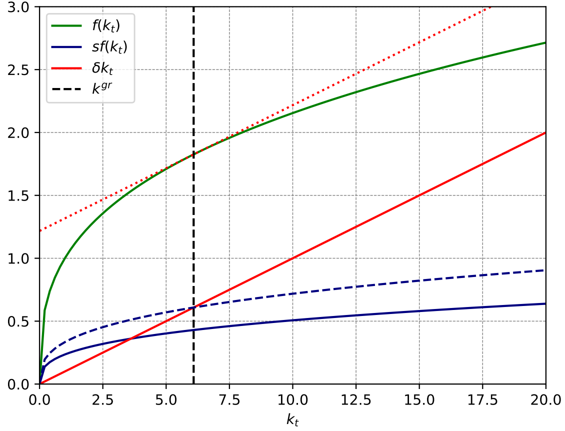
- The maximum possible consumption is attained when $\delta = \text{MPK}$

Optimal savings rate

$$k^{gr} = \left(\frac{s^{gr}}{\delta}\right)^{\frac{1}{1-\alpha}} \implies s^{gr} = \delta (k^{gr})^{1-\alpha}$$

- In Solow's model, the savings rate is an exogenous parameter
- If policy can manipulate s, it can attain optimal consumption

Golden rule capital accumulation



- Largest distance between $sf(k^{gr})$ and $f(k^{gr})$, given δ slope

Welfare in the Ramsey model I

Planner's problem

$$\max_{c_t, k_{t+1} \forall t} U = \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to $k_{t+1} = (1-\delta)k_t + f(k_t) - c_t; \quad k_{t+1} > 0; \quad k_0$ given

- Similar to the household's optimization problem HH problem
- Note: resource constraint, not budget constraint
- w_t and r_t not taken as given

Solution – **same as agent's solution!**

$$u'(c_t) = \beta(1 + r_{t+1} - \delta)u'(c_{t+1})$$
$$k_{t+1} = (1-\delta)k_t + f(k_t) - c_t$$

Welfare in the Ramsey model II

Modified golden rule

$$u'(c_t) = \beta(1 + r_{t+1} - \delta)u'(c_{t+1})$$
$$\implies f'(k^{ss}) = \delta + \frac{1}{\beta} - 1$$

- β enters welfare maximizing capital level
- Preferences matter

Comparison

$$f'(k^{ss}) = \delta + \frac{1}{\beta} - 1 > \delta = f'(k^{gr})$$

- Optimal capital in Ramsey always below Solow
- Return on capital (to consumers) must equal MPK
- Not necessary in Solow (since s is exogenous)

Welfare theorem

First welfare theorem

- Economy is perfectly competitive
- No inefficiencies, frictions or externalities
- The utility function exhibits the usual properties

$\Longrightarrow$ The equilibrium is Pareto efficient!

Growth model?

Baseline model without growth

$$F(K_t, L_t) = AK_t^{\alpha} L_t^{1-\alpha}$$

$$K_{t+1} = K_t(1-\delta) + F(K_t, L_t) - C_t$$

$$U = \sum_{t=0}^{\infty} \beta^t u(C_t) \quad \text{s.t.} \quad C_t + B_{t+1} = B_t(1-\delta+R_t) + L_t w_t$$

$$w_t = F'_L(K_t, L_t); \quad r_t = F'_K(K_t, L_t)$$

- We set $L_t = 1$, which simplified things
- What if there is population growth in the economy?
- What if there is technological progress?
- Can this model be used to understand growth in steady state?

Normalization I

Population growth

- The model accomodates population growth
- Assume that L_t implies that the whole labor force works full time
- Assume the labor force growth at $L_{t+1} = (1+n)L_t$
- Need to assume new per-worker utility function $u(c_t)$

Ramsey model normalized for population growth

$$\frac{F(K_t, L_t)}{L_t} = \qquad f(k_t) = Ak_t^{\alpha}$$

$$\frac{L_{t+1}}{L_t}k_{t+1} = \qquad (1+n)k_{t+1} = k_t(1-\delta) + f(k_t) - c_t$$

$$U = \sum_{t=0}^{\infty}(\beta(1+n))^t u(c_t) \qquad \text{s.t.} \qquad c_t + (1+n)b_{t+1} = b_t(1-\delta+r_t) + w_t$$

Normalization II

Technological progress

- The model also accomodates (exogenous) technological progress
- Assume that $A_{t+1} = (1+\gamma)A_t$
- Normalize the model by dividing by $A_t L_t$
- Express everything in efficiency units, reformulate $Y_t = K_t^{\alpha}[\hat{A}_t L_t]^{1-\alpha}$
- This only works with the preferences we have assumed!

Ramsey model normalized for population and technological growth

$$\frac{F(K_t, L_t)}{A_t L_t} = f(k_t) = k_t^{\alpha}$$

$$\frac{L_{t+1} A_{t+1}}{L_t A_t} k_{t+1} = (1+n)(1+\gamma)k_{t+1} = k_t(1-\delta) + f(k_t) - \frac{c_t}{(1+\gamma)^t}$$

$$U = \sum_{t=0}^{\infty} (\widetilde{\beta}(1+n))^t u(c_t) \quad \text{s.t.} \quad c_t + (1+n)b_{t+1} = b_t(1-\delta+r_t) + w_t$$

Appendix

BC math I

Backwards substitution Back

$$
\begin{aligned}
a_{t+1} &= y_t + (1+r_t)a_t - \left[\beta^t (1+r)^t\right]^{\frac{1}{\sigma}} c_0 \\
a_{t+1} &= y_t + (1+r)\left(y_{t-1} + (1+r)a_{t-1} - \left[\beta^{t-1}(1+r)^{t-1}\right]^{\frac{1}{\sigma}} c_0\right) - \left[\beta^t (1+r)^t\right]^{\frac{1}{\sigma}} c_0 \\
a_{t+1} &= \sum_{i=0}^{t} (1+r)^i y_{t-i} + (1+r)^t a_0 - \sum_{i=0}^{t} (1+r)^i \left[\beta^{t-i}(1+r)^{t-i}\right]^{\frac{1}{\sigma}} c_0 \\
a_{t+1} &= (1+r)^t \left[\sum_{i=0}^{t} (1+r)^{i-t} y_{t-i} - \sum_{i=0}^{t} (1+r)^{i-t} \left[\beta^{t-i}(1+r)^{t-i}\right]^{\frac{1}{\sigma}} c_0\right] \\
0 &= \sum_{i=0}^{T} (1+r)^{i-T} y_{T-i} - \sum_{i=0}^{T} (1+r)^{i-T} \left[\beta^{T-i}(1+r)^{T-i}\right]^{\frac{1}{\sigma}} c_0 \\
0 &= \sum_{i=0}^{T} \frac{1}{(1+r)^{T-i}} y_{T-i} - \sum_{i=0}^{T} \frac{1}{(1+r)^{T-i}} \left[\beta^{T-i}(1+r)^{T-i}\right]^{\frac{1}{\sigma}} c_0
\end{aligned}
$$

BC math II

Flip counting of sums to make it easier on the eyes Back

$$
\begin{aligned}
0 &= \sum_{i=0}^{T} \frac{1}{(1+r)^i} y_i - \sum_{i=0}^{T} \frac{1}{(1+r)^i} \left[\beta^i (1+r)^i\right]^{\frac{1}{\sigma}} c_0 \\
\sum_{i=0}^{T} \frac{1}{(1+r)^i} \left[\beta^i (1+r)^i\right]^{\frac{1}{\sigma}} c_0 &= \sum_{i=0}^{T} \frac{1}{(1+r)^i} y_i \\
c_0 &= \frac{\sum_{i=0}^{T} \frac{1}{(1+r)^i} y_i}{\sum_{i=0}^{T} \frac{\left[\beta^i (1+r)^i\right]^{\frac{1}{\sigma}}}{(1+r)^i}}
\end{aligned}
$$

Marginal propensity to consume

How much of an additional dollar would people spend/save in period 0?
Use result from before!

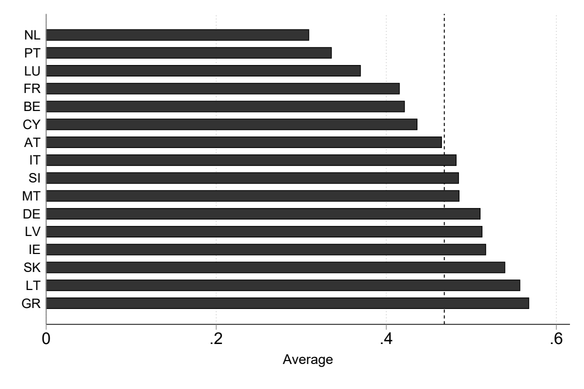
$$c_0 = \frac{\frac{y_1}{1+r} + y_0}{\left(1 + \frac{(\beta(1+r))^{\frac{1}{\sigma}}}{1+r}\right)}$$

Marginal propensity to consume

$$\frac{\partial c_0}{\partial y_0} = \frac{1}{\left(1 + \frac{(\beta(1+r))^{\frac{1}{\sigma}}}{1+r}\right)}$$

- Consumption is exactly proportional to “permanent income”
- With CRRA, the proportionality factor is constant

MPC in the data (Almgren et al, 2022)



- MPC out of unexpected lottery win equal to monthly HH income

Imposing CRRA utility

Period 0 consumption (assuming constant r and using $a_{T+1} = 0$) Algebra

$$c_0 = \frac{\sum_{i=0}^{T} \frac{1}{(1+r)^i} y_i}{\sum_{i=0}^{T} \frac{\left[\beta^i (1+r)^i\right]^{\frac{1}{\sigma}}}{(1+r)^i}}$$

Permanent income

- Consumption depends on a weighted average of all future income
- Only small shares of income increases are consumed

Marginal Propensity to consume

$$\frac{\partial c_0}{\partial y_0} = \frac{1}{\sum_{i=0}^{T} \frac{\left[\beta^i (1+r)^i\right]^{\frac{1}{\sigma}}}{(1+r)^i}}$$

Marginal propensity to consume

c_0 consumption (similar algebra as with T)

$$c_0 = \lim_{T \to 0} \frac{\sum_{i=0}^{T} \frac{1}{(1+r)^i} y_i}{\sum_{i=0}^{T} \frac{[\beta^i (1+r)^i]^{\frac{1}{\sigma}}}{(1+r)^i}} = \left(1 - \frac{[\beta(1+r)]^{\frac{1}{\sigma}}}{1+r}\right) \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} y_i$$

Marginal Propensity to Consume

$$\frac{\partial c_0}{\partial y_0} = \left(1 - \frac{[\beta(1+r)]^{\frac{1}{\sigma}}}{1+r}\right)$$

- Apply "reasonable calibration" ($\sigma = 1$, $\beta = 0.9$)
- MPC $= 0.1 \implies$ agent spends only 10% of transitory income gain
- Unrealistically low, MPCs are closer to 30% in the data