A Path towards the New-Keynesian Model

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Agenda

Second part of the course – Forget supply, Praise demand!

- Price differences – monopolistic competition
- Rational expectations – Lucas model
- Different forms of price setting mechanisms
- Optimal monetary policy

Today

- Introducing money into the consumer's problem
- Allowing firms to charge different prices

Moving beyond the RBC model

The Neoclassical Growth Model is great!

- The model is widely used to study long-run issues
- It solves several problems of the earlier Keynesian analysis (e.g., Lucas critique using microfoundations)

BUT

- It is real model $\implies$ no money, no prices
- It has nothing to say about inflation dynamics
- Central bankers have no purpose
- The world is governed by labor supply and productivity shocks

Inflation matters!

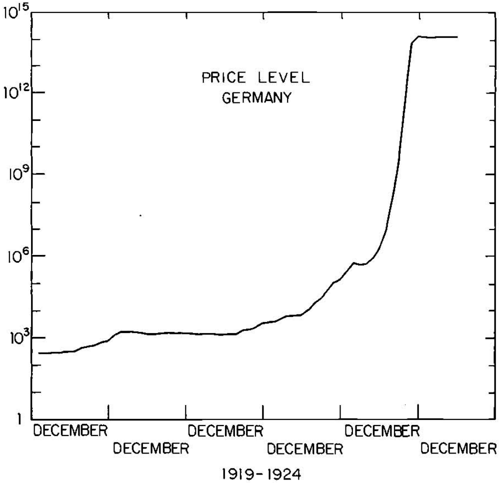


Fig. 2.4 Wholesale prices in Germany.

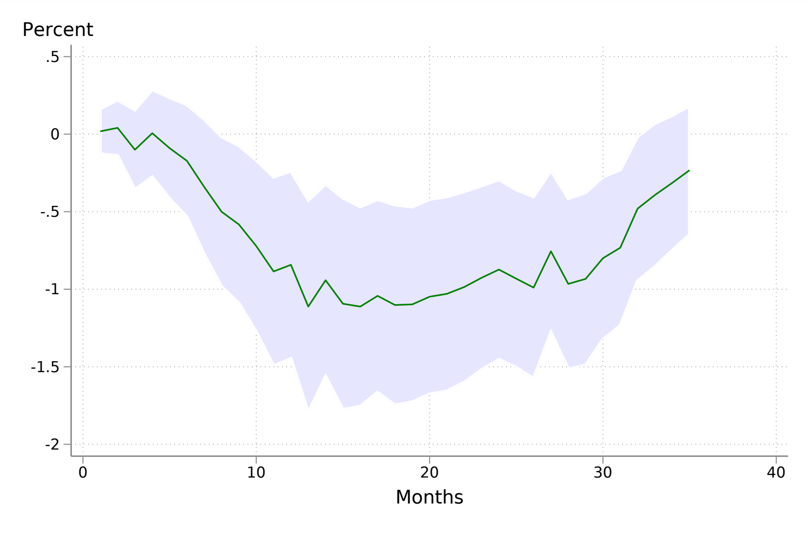
- Price level in Germany during hyperinflation

Inflation matters!

Major group	Weight	Regular prices: Median Freq.	Median Impl. dur.	Mean freq.	Frac. up
Processed food	8.2	10.5	9.0	10.6	65.4
Unprocessed food	5.9	25.0	3.5	25.4	61.2
Household furnishing	5.0	6.0	16.1	6.5	62.9
Apparel	6.5	3.6	27.3	3.6	57.1
Transportation goods	8.3	31.3	2.7	21.3	45.9
Recreation goods	3.6	6.0	16.3	6.1	62.0
Other goods	5.4	15.0	6.1	13.9	73.7
Utilities	5.3	38.1	2.1	49.4	53.1
Vehicle fuel	5.1	87.6	0.5	87.4	53.5
Travel	5.5	41.7	1.9	43.7	52.8
Services (excl. travel)	38.5	6.1	15.8	8.8	79.0
All sectors	100.0	8.7	11.0	21.1	64.8

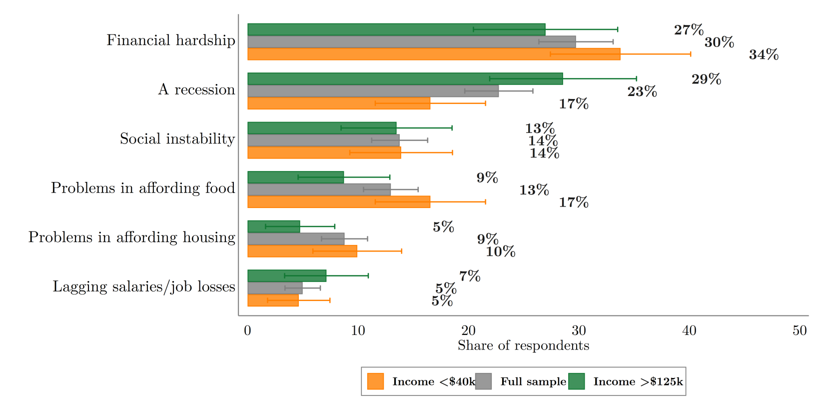
- Data on US price changes from Nakamura & Steinsson (2008)

Inflation matters!



- German output after an interst rate increase by the ECB

Inflation matters!



- If infl. increases, what are you worried about? Stantcheva, 2024

The New Keynesian model

Start from an RBC model

- Microfoundations (model relies on individual optimization)
- Rational expectations

Modifications

- Throw out capital (not important in the short run)
- Introduce rigidities (e.g., price/wage stickiness)
- Abandon perfect competition (today)

Outcome

- **Monetary non-neutrality** ($\implies$ money and interest rates have real effects)

First steps towards rigid prices

Need a way for price differences across firms, without demand collapsing

→ introduce inefficiency

Monopolistic competition

- What happens if every producer is a monopolist?
- Very powerful framework
- Will discuss output and welfare effects of money supply

Rigid prices

- What happens if prices don't change in response to changes in the economy?

Monopolistic Competition

Monopolistic competition

Market structure

- There are many small firms
- Each firm produces a differentiated good
- Consumers have an **inelastic** demand function across **all** goods

$\Longrightarrow$ small price increases don't drive demand to zero

- Firms are profit maximizers and price setters

$\Longrightarrow$ they take into account how price changes affect their demand

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What it does it get us

- Money is still neutral with flexible prices
- Welfare loss relative to perfect competition

BUT: If prices are rigid, changes in money supply affect output

$\Longrightarrow$ Money is not neutral anymore

First steps

Starting point

- Static model, no dynamics (no price changes, inflation or growth)
- Money has no use, people hold it for fun (liquidity services)
- Representative household
- Household consumes many similar (but different) goods

Two step optimization

- Outer layer: **How much** to consume
- Inner layer: **What** to consume, given prices

Representative Household

Household utility

$$U = C^{\gamma}\left(\frac{M}{P}\right)^{1-\gamma} - \frac{1}{\phi}N^{\phi} \quad \text{with } 0 < \gamma < 1, \phi > 1$$

(Nominal) budget constraint

$$\text{Nominal: } PC + M = \underbrace{M_0 + WN + \Pi}_{\text{Endowment } I}$$

- C: **Consumption aggregator**
- P: price index
- M: money holdings
- M_0: initial money holdings
- WN: nominal labor income
- Π: nominal profit income

Representative Household

Household utility

$$U = C^\gamma \left(\frac{M}{P}\right)^{1-\gamma} - \frac{1}{\phi} N^\phi \quad \text{with } 0 < \gamma < 1, \phi > 1$$

(Nominal) budget constraint

$$\text{Real: } C + \frac{M}{P} = \frac{M_0}{P} + \frac{W}{P} N + \frac{\Pi}{P}$$

- C: **Consumption aggregator**
- P: price index
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Solve high level consumer problem

Household Problem: $\max_{C,N,\frac{M}{P}} U \implies$ Lagrangian

$$\mathcal{L} = C^\gamma \left(\frac{M}{P}\right)^{1-\gamma} - \frac{1}{\phi} N^\phi + \lambda \left[\frac{M_0}{P} + \frac{W}{P} N + \frac{\Pi}{P} - C - \frac{M}{P}\right]$$

$$\begin{aligned}
\frac{\partial L}{\partial C}: & \qquad \gamma C^{\gamma-1} \left(\frac{M}{P}\right)^{1-\gamma} = \lambda \\
\frac{\partial L}{\partial M/P}: & \qquad (1-\gamma) C^\gamma \left(\frac{M}{P}\right)^{-\gamma} = \lambda \\
\frac{\partial L}{\partial N}: & \qquad N^{\phi-1} = \lambda \frac{W}{P}
\end{aligned}$$

First order conditions + how much to consume

$$\gamma C^{\gamma-1}\left(\frac{M}{P}\right)^{1-\gamma} = \lambda \tag{1}$$

$$(1-\gamma)C^{\gamma}\left(\frac{M}{P}\right)^{-\gamma} = \lambda \tag{2}$$

$$N^{\phi-1} = \lambda\frac{W}{P} \tag{3}$$

$$\underbrace{\frac{M_0}{P} + \frac{W}{P}N + \frac{\Pi}{P}}_{I/P} = C + \frac{M}{P} \tag{4}$$

Expenditure shares → if we know total income, we know how it's spent between total consumption C and money holdings M/P

$$(1) + (2) \implies \frac{M}{P} = \frac{(1-\gamma)}{\gamma}C$$

$$+(4) \implies C = \gamma I; \quad \frac{M}{P} = (1-\gamma)\frac{I}{P}$$

Remember: many different goods

Consumption aggregator

$$C = \left(\int_0^1 c_i^{\frac{\theta-1}{\theta}} \quad di \right)^{\frac{\theta}{\theta-1}}$$

- θ is elasticity of substitution – interesting special case: $\theta \to \infty$

Remember: many different goods

Consumption aggregator

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Solve for demand of each c_i → given some expenditure Z, maximize C

$$\max_{c_i} \left(\int_0^1 c_i^{\frac{\theta-1}{\theta}} \quad di \right)^{\frac{\theta}{\theta-1}} \text{ s.t. } \quad \int_0^1 p_i c_i di = Z$$

- Take some hypothetical Z as given
- Take p_i as given
- Find the optimal basket of goods to buy

Low level optimization = what to consume

Lagrangian

$$\mathcal{L} = \left(\int_0^1 c_i^{\frac{\theta-1}{\theta}} \quad di \right)^{\frac{\theta}{\theta-1}} + \xi \left(Z - \int_0^1 p_i c_i \quad di \right)$$

First order condition

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_i} : \qquad & \frac{\theta}{\theta-1} \left(\int_0^1 c_i^{\frac{\theta-1}{\theta}} \quad di \right)^{\frac{\theta}{\theta-1}-1} \frac{\theta-1}{\theta} c_i^{\frac{\theta-1}{\theta}-1} - \xi p_i = 0 \\
\Longrightarrow \qquad & \left(\int_0^1 c_i^{\frac{\theta-1}{\theta}} \quad di \right)^{\frac{1}{\theta-1}} c_i^{\frac{-1}{\theta}} = \xi p_i \\
\Longrightarrow \qquad & C^{\frac{1}{\theta}} c_i^{\frac{-1}{\theta}} = \xi p_i
\end{aligned}$$

The price index

Optimal choices

$$\underbrace{Z = \int_0^1 p_i c_i \quad di}_{\text{Budget constraint}} \quad \underbrace{c_i = C\left(\frac{1}{\xi}\right)^{\theta} p_i^{-\theta}}_{\text{First order cond.}}$$

The price index

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What is ξ?

- ξ measures increase in C when constraint is relaxed by 1 unit
- $1/\xi$ measures increase in constraint for increase in C by 1 unit

$\Longrightarrow$ $1/\xi$ is the price of consumption **bundle**

$\Longrightarrow$ $1/\xi = P$

The price index

Optimal choices

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One can (with some algebra) derive a **price index** Algebra

$$P = \left(\int_0^\infty p_i^{1-\theta} \quad di\right)^{\frac{1}{1-\theta}}$$

Consumers demand for goods

Demand function

$$\boxed{c_i = \left(\frac{p_i}{P}\right)^{-\theta} C} \tag{5}$$

- If P rises, demand for all goods rises
- If p_i rises, consumers substitute away with elasticity $\frac{\partial c_i}{\partial p_i}\frac{p_i}{c_i} = -\theta$
- $p_i \neq P$ does not drive demand to zero
- If $\theta \to \infty$, pricing power disappears

Firms take this demand function into account when pricing their goods

- They are monopolists!

Firm problem

Firm problem

Firms maximize profits by choosing their price, hours and output

$$\max_{p_i, y_i, n_i} = \frac{p_i}{P} y_i - \frac{W}{P} n_i$$

- y_i: firm specific output
- W/P: all firms pay the same real wage
- n_i: labor demanded by the firm
- With price-setting frictions, things will get more complicated

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Nothing goes to waste – market clearing at the firm level

$$y_i = c_i = \left(\frac{p_i}{P}\right)^{-\theta} C$$

Firm problem

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Nothing goes to waste – market clearing at the firm level

$$y_i = c_i = \left(\frac{p_i}{P}\right)^{-\theta} C$$

Production function

$$y_i = n_i^{\alpha}$$

Firm optimization

Problem only depends on prices

$$\Pi = \frac{p_i}{P}\left(\frac{p_i}{P}\right)^{-\theta} C - \frac{W}{P}\left[\left(\frac{p_i}{P}\right)^{-\theta} C\right]^{1/\alpha}$$

First order conditions

$$\frac{\partial}{\partial p_i} : (1-\theta) p_i^{-\theta} P^{\theta-1} C + \frac{\theta}{\alpha} p_i^{-\theta/\alpha - 1} P^{\theta/\alpha} C^{1/\alpha} \frac{W}{P} = 0$$

$\vdots$ (tedious algebra)

Firm optimization

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$$\Pi = \frac{p_i}{P}\left(\frac{p_i}{P}\right)^{-\theta} C - \frac{W}{P}\left[\left(\frac{p_i}{P}\right)^{-\theta} C\right]^{1/\alpha}$$

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Optimal pricing Algebra

$$\frac{p_i}{P} = \left[\frac{1}{\alpha}\frac{\theta}{\theta-1}\frac{W}{P} C^{\frac{1-\alpha}{\alpha}}\right]^{\frac{\alpha}{\theta+\alpha(1-\theta)}}$$

- The optimal price pins down output and labor demand of the firm (through consumer demand)

Discussion of pricing policy

Pricing rule (simple case: $\alpha = 1$)

$$\frac{p_i}{P} = \frac{\theta}{\theta - 1} \frac{W}{P}$$

- The firm's price setting policy depends only on its marginal cost W/P
- If $\alpha = 1$ (linear production) firms always set a constant markup above marginal cost

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Markup

- Market power makes goods "too expensive", since $p_i/W = \theta/(\theta - 1) > 1$
- With perfect competition, the markup disappears ($\theta \to \infty$)

General Equilibrium

- Put it all together

Flexible prices, general equilibrium I

Firms and consumers

$$\frac{p_i}{P} = \left[\frac{1}{\alpha}\frac{\theta}{\theta-1}\frac{W}{P}C^{\frac{1-\alpha}{\alpha}}\right]^{\frac{\alpha}{\theta+\alpha(1-\theta)}} \qquad \text{(Firm price setting)}$$

$$y_i = c_i = \left(\frac{p_i}{P}\right)^{-\theta} C \qquad \text{(Consumer demand)}$$

- Without adjustment frictions, all firms set the same price
  $\Longrightarrow p_i = P$

Flexible prices, general equilibrium I

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Goods market equilibrium

$$\frac{W}{P} = \alpha\frac{\theta-1}{\theta}C^{\frac{\alpha-1}{\alpha}}$$

- As output (C) rises, wages fall (lower marginal product of labor)
- If $\alpha = 1$, firms charge a constant markup

Flexible prices, general Equilibrium II

Labor market equilibrium

$$N_D = \int_0^1 y_i^{\frac{1}{\alpha}}\ di = Y^{\frac{1}{\alpha}} = C^{\frac{1}{\alpha}} \qquad \text{(Firm labor demand)}$$

$$N_S = \left[(1-\gamma)^{1-\gamma}\gamma^{\gamma}\frac{W}{P}\right]^{\frac{1}{\phi-1}} \qquad \text{(Labor supply)}$$

- Second equation comes from household optimization see here

Labor market clearing

$$N_D = N_S \iff \frac{W}{P} = \frac{1}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} C^{\frac{(\phi-1)}{\alpha}}$$

- Higher consumption → more labor in production → higher wages

Equilibrium conditions

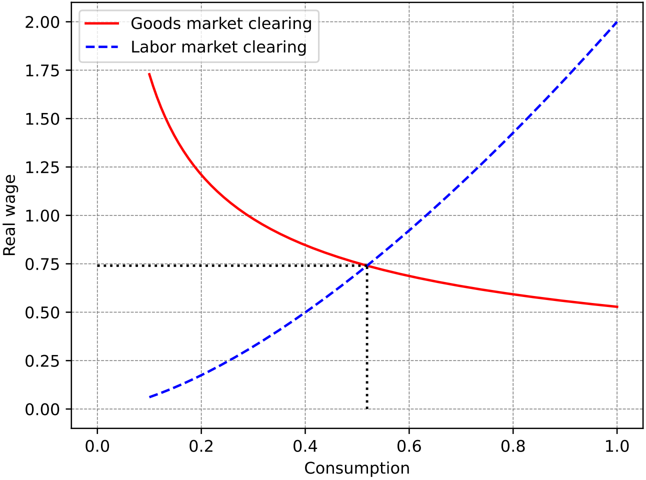
Goods market

$$\frac{W}{P} = \alpha \frac{\theta - 1}{\theta} C^{\frac{\alpha - 1}{\alpha}}$$

Labor market

$$\frac{W}{P} = \frac{1}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} C^{\frac{(\phi - 1)}{\alpha}}$$

Graphical representation



Takeaways

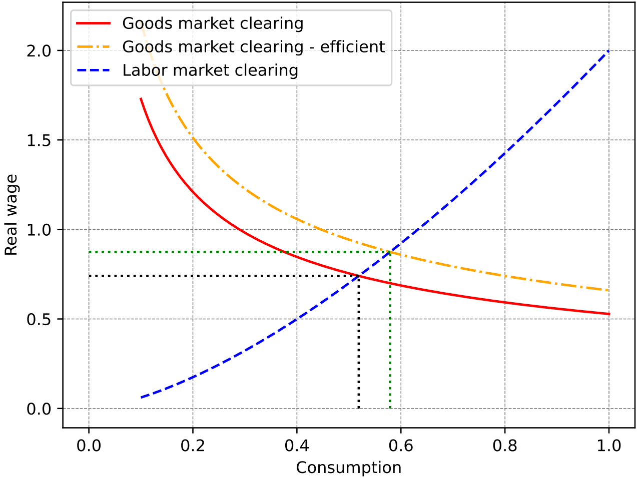
Surprising results

- Money does not matter, it's only in the background $\frac{M}{P} = \frac{(1-\gamma)}{\gamma} C$
- If we issue any amount of money, in equilibrium, the price level P adjusts
- Without adjustment frictions, this is **almost** a normal RBC model

Major innovation

- Firms **could** charge different prices (will be important later)
- More intuitive business cycles: more output = good (RBC model has symmetric costs)

Graphical representation – Efficient equilibrium



Deviations from efficiency

Why does monopolistic competition lead to lower output?

- Firms charge prices that are too high relative to worker productivity

$\Longrightarrow$ wages are too low (decreases incentives for workers)

$\Longrightarrow$ labor supply is too low

$\Longrightarrow$ output is too low

Deviations from efficiency

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Why are wages too low/prices too high?

- There is a coordination failure (externality) among firms
- If **one** firm lowers its price, demand at **all** firms rises
- Firms fail to account for this general equilibrium effect

$$\Pi_i = p_i \left(\frac{(p_i)}{P(p_i)} \right)^{-\theta} C - W \left[\left(\frac{(p_i)}{P(p_i)} \right)^{-\theta} C \right]^{1/\alpha};$$

Aside: Potential remedy for loss of efficiency

Inefficiency can be solved with labor subsidy (take $\alpha = 1$ for simplicity)

$$\frac{W}{P} = \frac{\theta - 1}{\theta}(1 + \tau) = 1$$

- If $\tau = \frac{1}{\theta - 1}$, the economy is restored to its efficient state
- Only works if the subsidy is paid for by lump-sum taxes ($\implies$ no incentive effects on labor supply)

Summary

Assume $\alpha = 1$

- Real wage: $\frac{W}{P} = \frac{\theta-1}{\theta}$ (remember that $N^{\phi-1} = \frac{W}{P}$ and $C = N$)
- Real money balances: $\frac{M}{P} = \left(\frac{\theta-1}{\theta}\right)^{\frac{1}{\phi-1}} \frac{1}{\kappa}$ with $\kappa = \frac{1}{\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}}$
- Output: $C = \kappa \frac{M}{P} = \left(\frac{\theta-1}{\theta}\right)^{\frac{1}{1-\phi}}$
- Real profits: $\frac{\Pi}{P} = \underbrace{\left(1 - \frac{\theta-1}{\theta}\right)}_{\text{Profit share of output}} \left(\frac{\theta-1}{\theta}\right)^{\frac{1}{\phi-1}}$
- Labor earnings: $\frac{\Pi}{P} = \underbrace{\left(\frac{\theta-1}{\theta}\right)}_{\text{Labor share of output}} \left(\frac{\theta-1}{\theta}\right)^{\frac{1}{\phi-1}}$

Price Adjustment Costs

Demand effects in the model

Changes in M

- If firms can freely adjust prices, changes in M are immediately swallowed up by changes in the price level P
- Hence, the nominal money supply is irrelevant

Changes in M with fixed prices

- Imagine all firms' prices are fixed (for whatever reason) at the equilibrium level from above
- In this case, an increase in M can create demand
- This would bring the economy towards the efficient level of output

Price adjustment frictions

Types of frictions

- Menu costs $\implies$ every price change is costly
- Taylor contracts $\implies$ prices can only be changed after t periods
- Calvo fairy $\implies$ constant probability of price adjustment

Today: simple menu cost

- Changing price tags, reprinting menus, etc.
- Renegotiation
- Information-gathering

Price adjustment costs – a thought experiment

How high would adjustment costs need to be?

- Starting at the equilibrium from before, the government increases M
- Complete surprise to firms and consumers
- At what adjustment cost do prices remain constant?

Options for the firm

(1) Keep the price fixed

(2) Change the price (without assuming that anyone else will)

Plan of attack

(1) Derive profit under each option

(2) Quantify difference using numerical solution

Price adjustment frictions – thought experiment

Real firm profits as a function of the money supply

$$\Pi = \left(\frac{p_i}{P} - \underbrace{\left(\kappa \frac{M}{P}\right)^{\phi-1}}_{\text{Real Wage } \frac{W}{P}} \right) \underbrace{\left(\frac{p_i}{P}\right)^{-\theta} \underbrace{\left(\kappa \frac{M}{P}\right)}_{\text{Output } C}}_{\text{Demand}}$$

Option 1 – Keep price fixed (assuming everyone else does)

$$p_i = P$$

$$\Pi_{fix} = \kappa \frac{M}{P} - \left(\kappa \frac{M}{P}\right)^{\phi}$$

Price adjustment frictions – thought experiment

Firm profits as a function of the money supply

$$\Pi = \left(\frac{p_i}{P} - \underbrace{\left(\kappa \frac{M}{P} \right)^{\phi - 1}}_{\text{Real Wage } \frac{W}{P}} \right) \underbrace{\left(\frac{p_i}{P} \right)^{-\theta} \underbrace{\left(\kappa \frac{M}{P} \right)}_{\text{Output } C}}_{\text{Demand}}$$

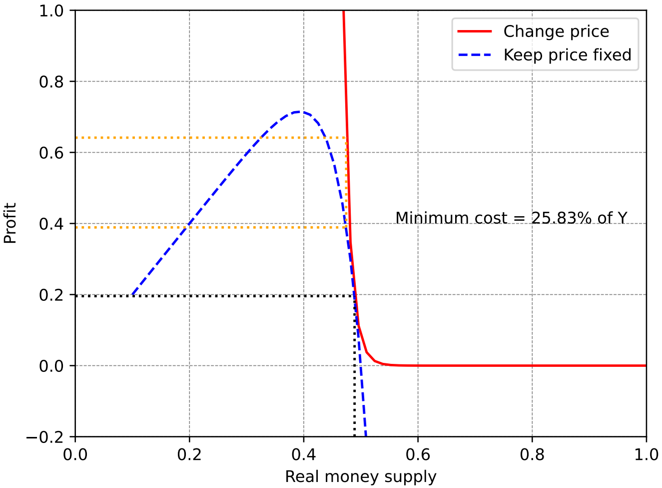
Option 2 – Change price to optimal level (assuming nobody else does)

$$\Pi_{new} = \frac{1}{\theta - 1} \left(\frac{\theta}{\theta - 1} \right)^{-\theta} \left(\kappa \frac{M}{P} \right)^{1-(1-\theta)(\phi-1)} \qquad \text{using } \frac{p_i}{P} = \frac{\theta}{\theta - 1} \frac{W}{P}$$

Note: Option 1 & 2 give the same result at equilibrium levels (check!)

Can price adjustment frictions rationalize sticky prices?

David Romer's calibration: $\theta = 5, \phi = 11$ – labor **very** inelastic



David Romer's take

Required frictions are too large

- Changing price tags does not cost 25% of GDP
- Firms should change prices all the time

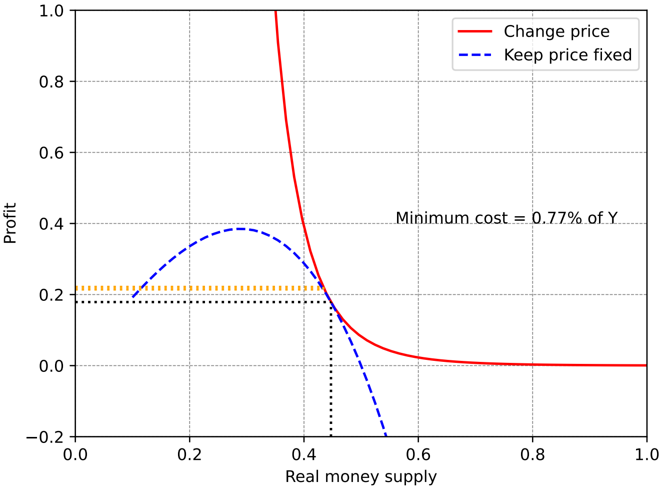
Important

- Inelastic labor supply (large ϕ) means changes in Y have big effects on W/P
- Recall $Y^{\phi-1} = \frac{W}{P}$ – For Y to fall, wages have to adjust
- $\frac{M}{P} \uparrow \implies Y \uparrow \implies \frac{W}{P} \uparrow\uparrow$
- Firms' marginal costs rise, which makes raising prices **very** attractive

What if the labor supply was a little more elastic?

Can price adjustment frictions rationalize sticky prices?

Slightly different calibration: $\phi = 3, \quad \theta = 5$



Menu costs can lead to welfare gains

If prices remain unchanged, more money can bring the economy closer to its efficient state

- Prices are “too high”, more money counteracts this
- Real wages rise

Small adjustment frictions may be enough for firms to not change prices

But expectations will matter, too!

Summary

The New Keynesian model

- The New Keynesian model can rationalize the non-neutrality of money
- Today: first step towards the full framework

Monopolistic competition

- Under monopolistic competition, firms set prices that are too high
- Real wages are low, which leads to low labor supply and lower output
- Welfare is lower in this framework
- Increases in output bring the economy closer to its efficient state (booms are good)

Pricing frictions – Menu costs

- If prices are sticky, more money brings about higher output
- Small price adjustment costs may be enough

Appendix

Derivation of price index

$$
\begin{aligned}
C &= \left(\int_0^1 c_i^{\frac{\theta-1}{\theta}} \quad di\right)^{\frac{\theta}{\theta-1}} = \left(\int_0^1 \left(C\left(\frac{1}{\xi}\right)^{\theta} p_i^{-\theta}\right)^{\frac{\theta-1}{\theta}} \quad di\right)^{\frac{\theta}{\theta-1}} \\
&= \left(\int_0^1 \left(CP^{\theta} p_i^{-\theta}\right)^{\frac{\theta-1}{\theta}} \quad di\right)^{\frac{\theta}{\theta-1}} \\
&= C\left(\int_0^1 P^{\theta-1} \left(p_i^{-\theta}\right)^{\frac{\theta-1}{\theta}} \quad di\right)^{\frac{\theta}{\theta-1}} \\
1 &= P^{\theta}\left(\int_0^1 p_i^{1-\theta} \quad di\right)^{\frac{\theta}{\theta-1}} \\
P^{-\theta} &= \left(\int_0^1 p_i^{1-\theta} \quad di\right)^{\frac{\theta}{\theta-1}} \\
P &= \left(\int_0^1 p_i^{1-\theta} \quad di\right)^{\frac{1}{1-\theta}}
\end{aligned}
$$

Back!

Firm optimization – Algebra

$$
\begin{aligned}
(1-\theta)p_i^{-\theta}P^{\theta-1}C + \frac{\theta}{\alpha}p_i^{-\frac{\theta}{\alpha}-1}P^{\frac{\theta}{\alpha}}C^{\frac{1}{\alpha}}\frac{W}{P} &= 0 \\
\frac{\theta}{\alpha}p_i^{-\frac{\theta}{\alpha}-1}P^{\frac{\theta}{\alpha}}C^{\frac{1}{\alpha}}\frac{W}{P} &= (\theta-1)p_i^{-\theta}P^{\theta-1}C \\
\frac{\theta}{\alpha}p_i^{-\frac{\theta}{\alpha}-1}P^{\frac{\theta}{\alpha}}\frac{W}{P} &= (\theta-1)p_i^{-\theta}P^{\theta-1}C^{\frac{\alpha-1}{\alpha}} \\
\frac{\theta}{\theta-1}\frac{1}{\alpha}p_i^{\theta-\frac{\theta}{\alpha}-1}P^{\frac{\theta}{\alpha}-\theta+1}\frac{W}{P} &= C^{\frac{\alpha-1}{\alpha}} \\
\frac{\theta}{\theta-1}\frac{1}{\alpha}\frac{W}{P}C^{\frac{1-\alpha}{\alpha}} &= \left(\frac{p_i}{P}\right)^{1-\theta+\frac{\theta}{\alpha}} \\
\frac{\theta}{\theta-1}\frac{1}{\alpha}\frac{W}{P}C^{\frac{1-\alpha}{\alpha}} &= \left(\frac{p_i}{P}\right)^{\frac{\theta+\alpha(1-\theta)}{\alpha}} \\
\left[\frac{\theta}{\theta-1}\frac{1}{\alpha}\frac{W}{P}C^{\frac{1-\alpha}{\alpha}}\right]^{\frac{\alpha}{\theta+\alpha(1-\theta)}} &= \left(\frac{p_i}{P}\right)
\end{aligned}
$$

Back!

Labor supply choice

Back!

$$\gamma C^{\gamma-1}\left(\frac{M}{P}\right)^{1-\gamma} = \lambda$$

$$(1-\gamma)C^{\gamma}\left(\frac{M}{P}\right)^{-\gamma} = \lambda$$

$$N^{\phi-1} = \lambda\frac{W}{P}$$

$$\underbrace{\frac{M_0}{P} + \frac{W}{P}N + \frac{\Pi}{P}}_{I/P} = C + \frac{M}{P}$$

Expenditure shares → if we know total income, we know how it's spent between total consumption C and money holdings M/P

$$\implies \frac{M}{P} = \frac{(1-\gamma)}{\gamma}C$$

$$N^{\phi-1} = \gamma C^{\gamma-1}\left(\frac{(1-\gamma)}{\gamma}C\right)^{1-\gamma}\frac{W}{P} = \gamma^{\gamma}(1-\gamma)^{1-\gamma}\frac{W}{P}$$