The Phillips Curve

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Last time

The New Keynesian model

- Static RBC model
+ Monopolistic competition
+ Prices

Monetary non-neutrality

- In equilibrium with flexible prices, money is neutral
- Small price adjustment frictions may allow changes in money to have real effects

Agenda

Rational expectations

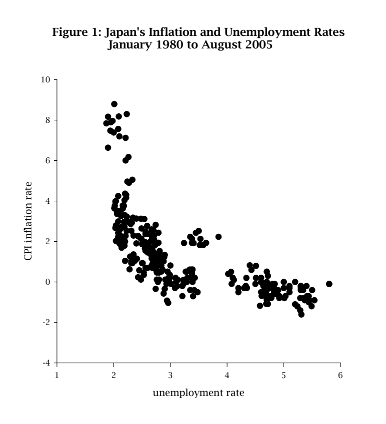
- Expectations of optimizing agents
- Law of iterated expectations

The Phillips curve

- The impact of rigid prices
- Identification of the Phillips curve

The Phillips Curve

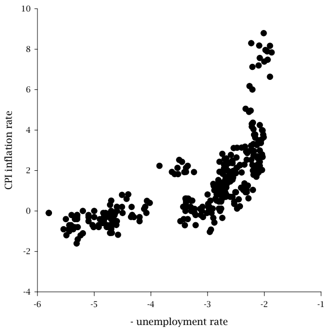
Japan's Phillips Curve



The Phillips Curve

Japan's Phillips Curve

Figure 2: Japan's Inflation Rate and (Minus) Unemployment Rate
January 1980 to August 2005



The Phillips Curve

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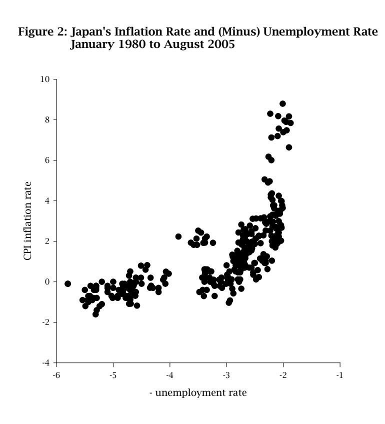
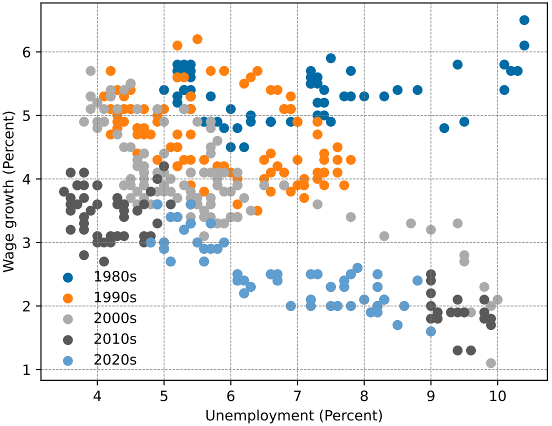


US Phillips Curve



The Phillips Curve and economic policy

Robust relationship in the data

- When inflation is high, unemployment is low

Very attractive for policy makers

- Just drive up inflation and unemployment will fall!

Helmut Schmidt: "Rather 5% inflation than 5% unemployment"

Robert E. Lucas

- Nobel Laureate in 1995 "for having developed and applied the hypothesis of rational expectations, and thereby having transformed macroeconomic analysis and deepened our understanding of economic policy"
- Lucas critique: We cannot predict the effects of changes in policy based on historical data
- Printing money will not solve unemployment if people expect money to be printed!

Forecasting by optimizing agents

In the static environment, we assume agents optimize their choices.
What does that mean in the dynamic context?

- How should agents optimize in the presence of uncertainty?
- What information is known, which is used?

Agents form rational expectations

- They know the structure of the economy
- They use all available information

Agents do not make systematic forecast errors

Rational expectations

If some variable in our economy behaves stochastically, then agents form the expectation

$$\mathbb{E}_t[X_{t+1}] = \mathbb{E}_t[X_{t+1}|I_t]$$

where X is some economic variable, e.g., output, and I_t is the information set available to the agent.

Example: Efficient market hypothesis implies that all information I_t is priced into the stock price X_t

Note: If information sets differ, not everyone needs to form the same expectations.

Very controversial at the time. No more animal spirits (Keynes), only rational agents (Lucas, Sargeant).

The Law of Iterated Expectations

What do you think the rate of inflation will be in December 2025?

$$\mathbb{E}_t[\pi_{t+12}|I_t]?$$

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$$\mathbb{E}_t\left[\mathbb{E}_{t+1}[\pi_{t+12}|I_{t+1}]|I_t\right]?$$

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What do you think the rate of inflation will be in December 2025?

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$$\mathbb{E}_t\left[\mathbb{E}_{t+1}[\pi_{t+12}|I_{t+1}]|I_t\right]?$$

What do you think **your neighbor** thinks the rate of inflation will be in December 2025?

$$\mathbb{E}_t\left[\mathbb{E}_t[\pi_{t+12}]|I_t\right]?$$

Expectational difference equations (EDEs)

Current economic conditions may depend on what we expect in the future

$$y_t = a\mathbb{E}_t[y_{t+1}|I_t] + cx_t$$

- The current endogenous variable y_t depends on exogenous variable x_t and it's own expected future value
- Rational expectations imply that agents know I_t
- Importantly, agents know all past values of y_t and x_t and the model itself

e.g. Inflation may depend on expected inflation and output

Expectational difference equations (EDEs)

Agents can solve the equation forward

$$
\begin{aligned}
y_t &= a\mathbb{E}_t[y_{t+1}|I_t] + cx_t \\
&= a\mathbb{E}_t[a\mathbb{E}_{t+1}[y_{t+2}|I_{t+1}] + cx_{t+1}|I_t] + cx_t \\
&= a^2 \underbrace{\mathbb{E}_t[\mathbb{E}_{t+1}[y_{t+2}|I_{t+1}]|I_t]}_{\text{Apply law of iterated expectations!}} + ac\mathbb{E}_t[x_{t+1}|I_t] + cx_t \\
&= a^2\mathbb{E}_t[y_{t+2}|I_t] + ac\mathbb{E}_t[x_{t+1}|I_t] + cx_t \\
&= a^2\mathbb{E}_t[y_{t+2}] + ac\mathbb{E}_t[x_{t+1}] + cx_t \\
&= a^3\mathbb{E}_t[y_{t+3}] + a^2c\mathbb{E}_t[x_{t+2}] + ac\mathbb{E}_t[x_{t+1}] + cx_t
\end{aligned}
$$

- A pattern emerges: y_t depends on exogenous variables and distant expectations of y

Expectational difference equations (EDEs)

Repeat this procedure T times:

$$y_t = a^T \mathbb{E}_t[y_{t+T}] + c \sum_{i=0}^{T} a^i \mathbb{E}_t[x_{t+i}]$$

- Usually assume that $a < 0$, or more generally $\lim_{T\to\infty} a^T \mathbb{E}_t[y_{t+T}] = 0$

$$y_t = c \sum_{i=0}^{\infty} a^i \mathbb{E}_t[x_{t+i}]$$

- y_t only depends on the expected value of exogenous shocks
- Example: Stock prices depend on the value of the dividends they are expected to pay

Example

Assume that x_t follows the AR(1) process

$$x_{t+1} = \rho\ x_t + \varepsilon_{t+1} \text{ with } \mathbb{E}[\varepsilon_{t+i}|I_t] = 0$$

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Rational expectation:

$$\mathbb{E}[x_{t+j}|I_t] = \rho^j x_t + \rho^{j-1} \sum_{i=0}^{j} \mathbb{E}[\varepsilon_{t+i}|I_t]$$

- Innovations ε_t are zero in expectation
- Agents **know** that further predictions have larger uncertainty, but the best guess is still given by $\rho^i x_t$.

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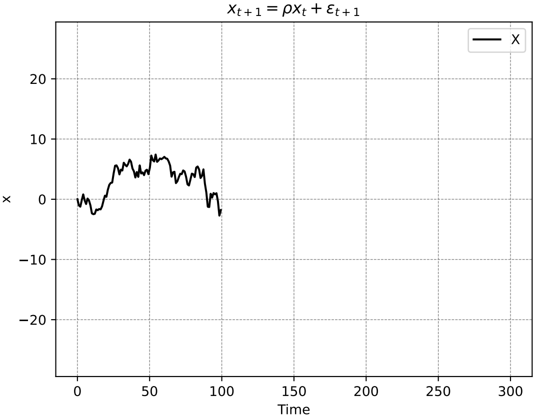
Plug into equation on previous slide to obtain (assume $a\rho < 1$)

$$y_t = c \sum_{i+0}^{\infty} a^i \mathbb{E}_t[x_{t+j}] = c \sum_{i+0}^{\infty} (a\rho)^i x_t$$
$$= \frac{c}{1 - a\rho} x_t$$

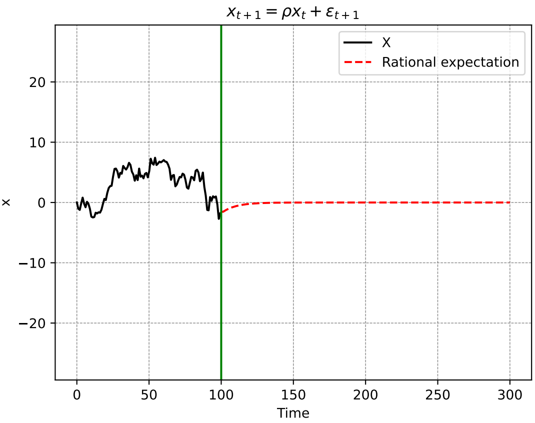
Example in pictures

Exogenous process x



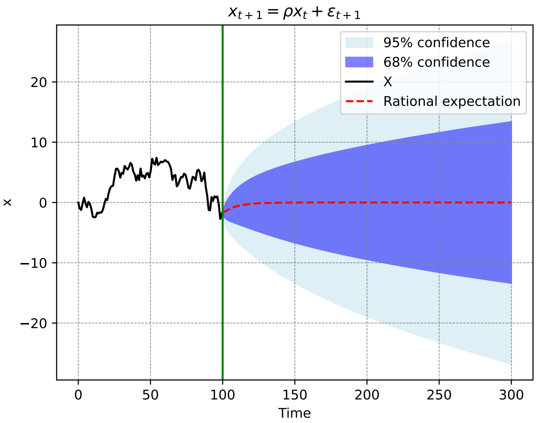
Example in pictures

Rational expectation starting from vertical line



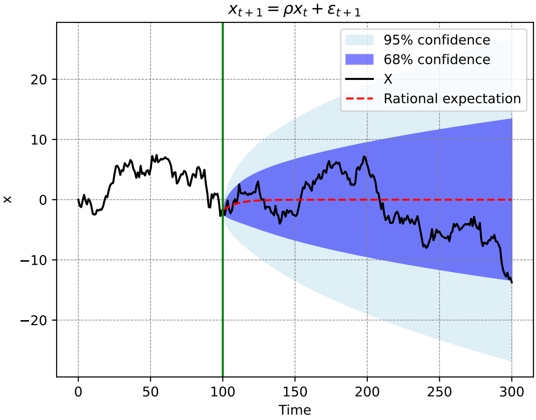
Example in pictures

Rational expectation including uncertainty



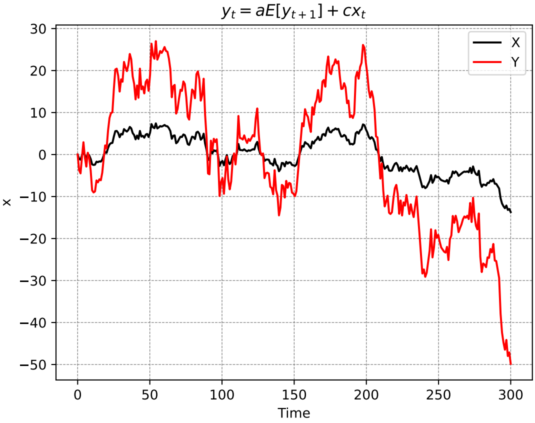
Example in pictures

Process realization



Example in pictures

Exogenous and endogenous variable



Quiz questions

Under the efficient market hypothesis, the value of a stock is simply the present discounted value of expected dividend payments.

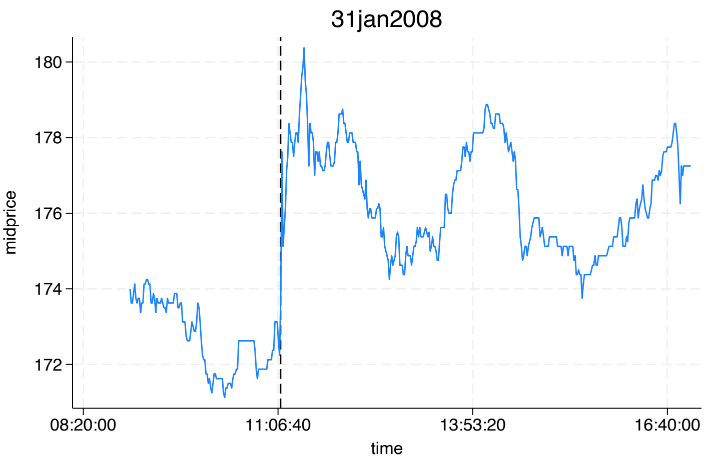
Imagine dividends follow $x_{t+1} = x_t + \varepsilon_{t+1}$, ε is 0 in expectation.

If traders discount the future at rate β, what is the value of the stock price s_t today?

- x_t
- $\lim_{i \to \infty} \beta^i x_t$
- $\frac{1}{1-\beta} x_t$
- $\sum_{i=0}^{\infty} \varepsilon_i$

Quiz questions

Danske Bank makes an earnings announcement once every quarter. Imagine it implies that all future dividends will be slightly higher. What do you expect to happen to the stock price?



A simple model of price setting

Assumptions

- There is a measure one of consumers/households
- All households are alike
- Each household produces one variety of the consumption good
- Each household consumes all varieties according to an aggregator
- Labor is the only input into production (linearly)
- The product market is described by monopolistic competition
- Aggregate demand is given by real money balances

Setup

Household i's utility function

$$U_i = \mathbf{C}_i - \frac{1}{\phi} L_i^{\phi}$$

- $\mathbf{C}$ is a consumption basket, as in previous lecture, composed of C_i
- Households use their labor L to produce output according to $Y_i = L_i$ ($\alpha = 1$)
- P is the aggregate price level, P_i is the price of the household's variety i

Budget constraint

$$\underbrace{P\mathbf{C}_i}_{\text{Expenses}} = \underbrace{P_i Y_i}_{\text{Income}} \implies \mathbf{C}_i = \frac{P_i}{P} Y_i$$

Demand

Demand function for each household's variety

$$Y_i = \left(\frac{P_i}{P}\right)^{-\theta} Y \iff \frac{P_i}{P} = \left(\frac{Y_i}{Y}\right)^{-1/\theta}$$

Remember:

- Each household is a monopolist for their own variety
- Prices are set taking demand into account
- Households make monopoly profits (price > marginal cost)
- θ represents elasticity of substitution across goods i

Optimality condition

Program

$$\max_{Y_i, P_i, L_i} \quad \frac{P_i}{P} Y_i - \frac{1}{\phi} L_i^{\phi}$$

$$\text{substitute} \rightarrow \max_{Y_i} \quad \left(\frac{Y_i}{Y}\right)^{-1/\theta} Y_i - \frac{1}{\phi} Y_i^{\phi}$$

- Households have to finance their expenses (sub for $\mathbf{C}_i$)
- Households take the demand function into account (sub for P_i/P)
- Production is linear in labor (sub for L_i)

First order condition

$$-\frac{1}{\theta}\left(\frac{Y_i}{Y}\right)^{-\frac{1}{\theta}-1} \frac{Y_i}{Y} + \left(\frac{Y_i}{Y}\right)^{-\frac{1}{\theta}} = Y_i^{\phi-1}$$

Rearrange

Optimal output

$$-\frac{1}{\theta}\left(\frac{Y_i}{Y}\right)^{-\frac{1}{\theta}-1}\frac{Y_i}{Y}+\left(\frac{Y_i}{Y}\right)^{-\frac{1}{\theta}}=Y_i^{\phi-1}$$

$$-\frac{1}{\theta}\left(\frac{Y_i}{Y}\right)^{-\frac{1}{\theta}}+\left(\frac{Y_i}{Y}\right)^{-\frac{1}{\theta}}=Y_i^{\phi-1}$$

$$\left(1-\frac{1}{\theta}\right)\left(\frac{Y_i}{Y}\right)^{-\frac{1}{\theta}}=Y_i^{\phi-1}$$

$$\underbrace{\left(\frac{\theta-1}{\theta}\right)}_{\text{Inverse markup}}\left(\frac{P_i}{P}\right)=\underbrace{Y_i^{\phi-1}}_{\text{Marg. disutil. of labor}} \qquad \leftarrow \text{ p/P} = \text{markup} * \text{MC}$$

Last lecture: (when we had wages)

$$\frac{P_i}{P}=\frac{\theta-1}{\theta}\frac{W}{P}$$

Price setting

$$\frac{P_i}{P} = \frac{\theta - 1}{\theta} Y_i^{\phi - 1}$$

Take logs

$$p_i - p = (\phi - 1) y_i - \log\left(\frac{\theta - 1}{\theta}\right)$$
$$p_i - p = (\phi - 1) y_i + \mathcal{M}$$

Monopolistic competition

- Goods are too expensive, relative to marginal cost
- Welfare loss, as in last lecture

"Firm optimality"

Optimal price setting

$$p_i^* - p = (\phi - 1) y_i + \mathcal{M}$$

- Optimal price depends on elasticity of labor supply $\frac{1}{\phi}$ and markup $\mathcal{M}$

Aggregate up (all firms are symmetric $\implies$ make the same choices)

$$p^* - p = (\phi - 1) \underbrace{(m - p)}_{y} + \mathcal{M}$$

$$p^* = (\phi - 1) m + (2 - \phi) p + \mathcal{M}$$

- Note that $p = m$ is not the outcome if $p^* = p$, due to monopolistic competition
- As ϕ rises (labor becomes less elastic) m's effect on p^* increases

Quiz

"Firm optimality"

Optimal price setting

$$p_i^* - p = (\phi - 1)y_i + \mathcal{M}$$

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- Note that $p = m$ is not the outcome if $p^* = p$, due to monopolistic competition
- As ϕ rises (labor becomes less elastic) m's effect on p^* increases

$m \uparrow$ raises demand, which means y_i must rise. If L_i is inelastic, this requires large price movements

Model summary

Key equation

$$p^* = (\phi - 1)(m - p) + p + \mathcal{M}$$

- All firms who are allowed to reset their price set p^*
- In equilibrium, $p > m$ because of the markup
- Relative to perfect competition, output is lower
- If ϕ is high, adjusting output is costly → prices need to change more

Flexible prices

All firms can reset prices continuously

$$p^* = p$$
$$p = m + \frac{1}{\phi - 1}\mathcal{M}$$

$\implies$ any change in m directly translates into p

Output is invariant to changes in m

$$y = -\frac{1}{\phi - 1}\mathcal{M}$$

Note:

- $y \leq 0$ does not imply negative output! y is in logs, so $Y \leq 1$

Predicting the future / Fixed prices

Rigid/Fixed/Sticky prices

New assumption:

- Firms (households) set their prices only at the end of the period
- No resetting until the end of the next period

$\implies$ prices **not fully flexible**, cannot adjust to shocks immediately

Demand shocks:

- Total money supply depends on shocks
- Will define m_t as random variable
- Mean: 0, Variance: σ_m

Introducing expectations

Money supply is random

- Households form expectations about future m to set prices p
- But also about how other price setters will behave

$$p_t^* = (\phi - 1)\mathbb{E}_{t-1}[m_t | I_{t-1}] + (2 - \phi)\mathbb{E}_{t-1}[p_t | I_{t-1}] + \mathcal{M}$$

- Since everyone behaves the same, take expectations of whole expression

$$\mathbb{E}_{t-1}[p_t] = \mathbb{E}_{t-1}\left\{(\phi - 1)\mathbb{E}_{t-1}[m_t] + (2 - \phi)\mathbb{E}_{t-1}[p_t] + \mathcal{M}\right\}$$
$$\mathbb{E}_{t-1}[p_t] = \mathbb{E}_{t-1}[m_t] + \frac{1}{\phi - 1}\mathcal{M}$$

Equilibrium

Prices and output

$$
\begin{aligned}
p_t &= (\phi - 1)\mathbb{E}_{t-1}[m_t] + (2 - \phi)\left(\mathbb{E}_{t-1}[m_t] + \frac{1}{\phi - 1}\mathcal{M}\right) + \mathcal{M} \\
&= \mathbb{E}_{t-1}[m_t] + \frac{1}{\phi - 1}\mathcal{M} \\
y_t &= m_t - \mathbb{E}_{t-1}[m_t] - \frac{1}{\phi - 1}\mathcal{M}
\end{aligned}
$$

- If prices are flexible, $m = \mathbb{E}[m]$
- Only unanticipated movements in aggregate demand ($\mathbb{E}[m] \neq m$) have real effects
- Monopolistic competition leads to lower output and welfare loss
  [Possible to suppress markup, but with Mon.Comp. it's there]

Lucas' critique

People will not be continuously surprised

- If the CB decided to increase m_t^{cb} out of nowhere, y_t would move
- After that, agents wise up and price in CB behavior

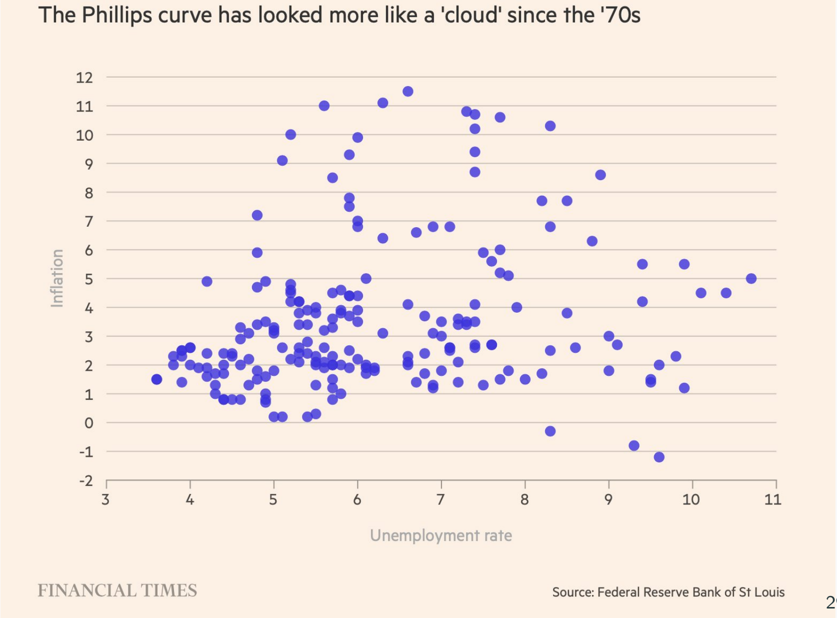
Rational expectations

- If the CB follows any kind of rule, trying to increase output

$\Longrightarrow$ p_t will move more, y_t unaffected

Identifying the Phillips Curve

The modern Phillips Curve



A simple Phillips curve

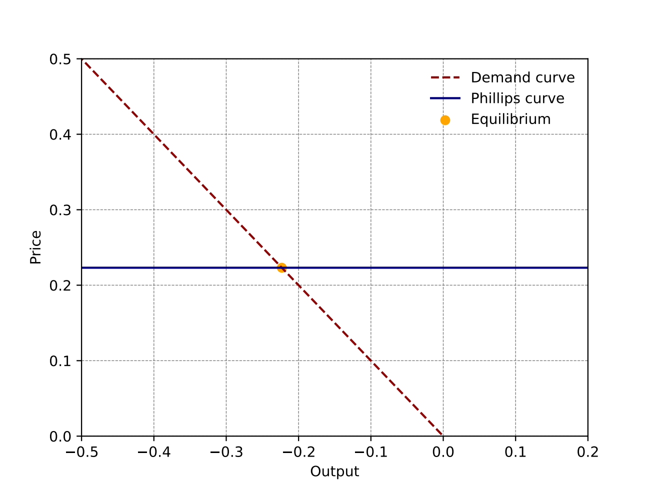
$$p_t = \mathbb{E}_{t-1}[m_t] + \frac{1}{\phi - 1}\mathcal{M} + u_t$$
$$y_t = m_t - \mathbb{E}_{t-1}[m_t] - \frac{1}{\phi - 1}\mathcal{M}$$

- u_t is a cost-push shock: changes prices for no reason
- m_t is the demand shock: changes output for no reason
- Both mean 0, hence $\mathbb{E}_{t-1}[u_t] = 0$, $\mathbb{E}_{t-1}[p_t]$ unchanged

Draw supply-demand diagram

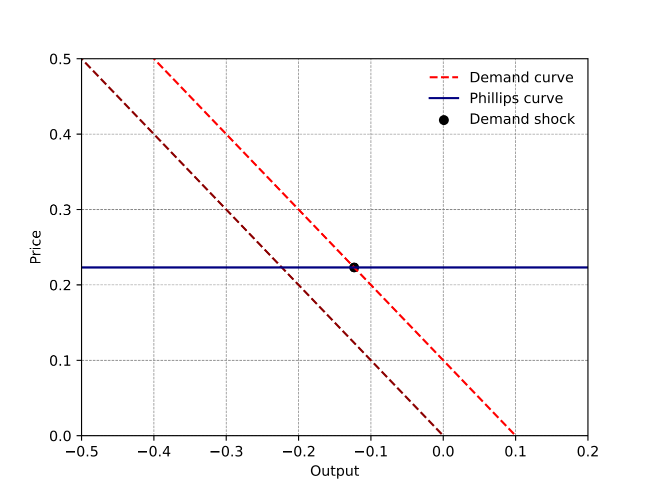
$$p_t = \frac{1}{\phi - 1}\mathcal{M} + u_t \qquad \text{(Supply/Phillips curve)}$$
$$y_t = m_t - p_t \qquad \text{(Demand curve)}$$

The model's Phillips curve



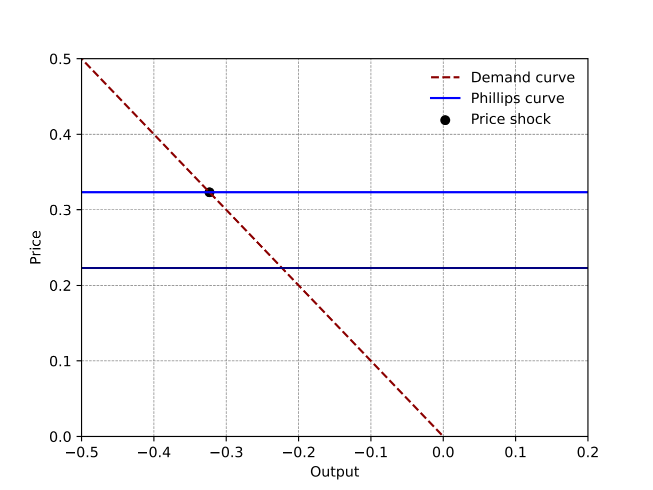
- Phillips curve is flat in the model, will change in the future

The model's Phillips curve



- Demand shocks raise output

The model's Phillips curve

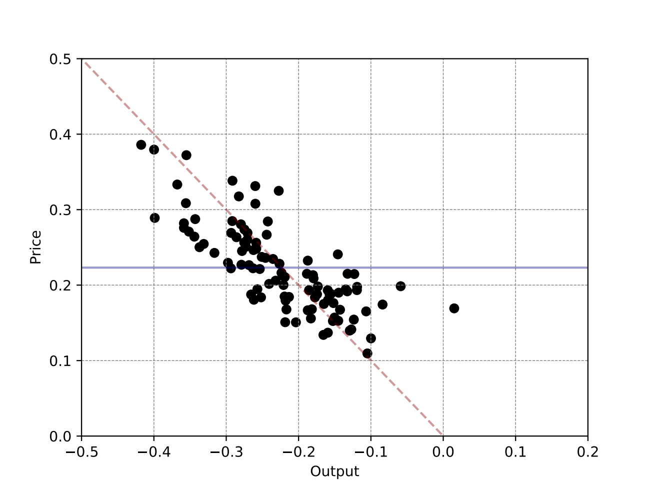


- Price/supply shocks raise prices and decrease output

Question:

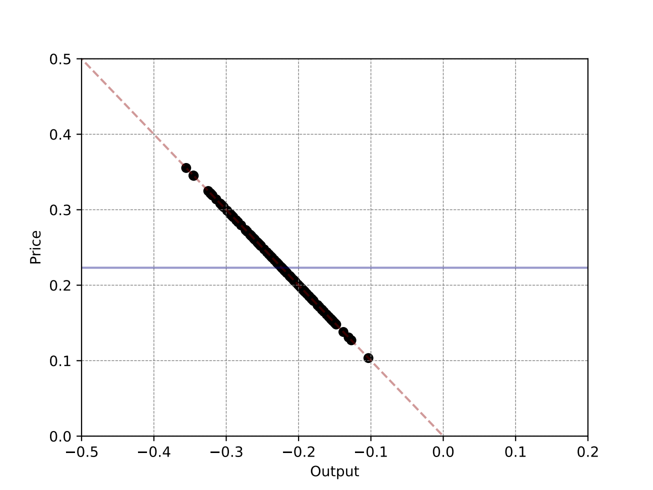
What shocks are needed to identify the Phillips curve?

Simulating the model



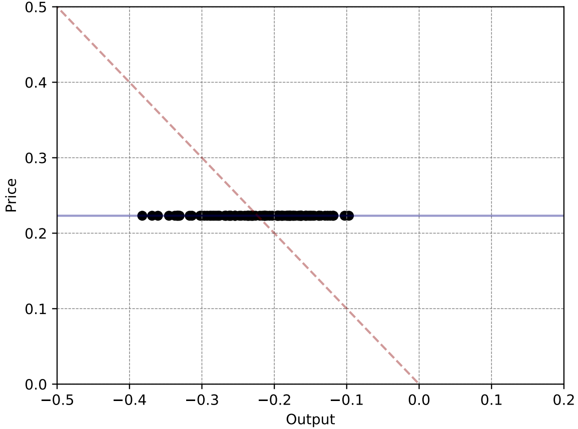
- Draw u_t and m_t from $N(0, \sigma_u)$ and $N(0, \sigma_m)$, plot equilibria

Simulating the model



- Set m_t to 0 → only supply shocks

Simulating the model



- Set u_t to 0 → only demand shocks

The Phillips Curve is a general equilibrium object

A new monetary policy regime

- US inflation was very high in the 70s
- Central banks started to aggressively hike interest rates
- Monetary policy has become more predictable and conservative

Identification is difficult

- The Phillips curve traces the aggregate supply curve
- Can only be identified through *shocks* to aggregate demand
- If central banks work **against** demand shocks, that's difficult

$\Longrightarrow$ No obvious slope anymore

Discussion

Progress

- Another step towards the New Keynesian model

Rigid prices

- With rigid prices, output and prices depend on money supply
- But policy makers cannot exploit this (unless they surprise everyone)
- If central bankers raise money growth unexpectedly, it will only affect output in the first period

Lucas critique

- Don't take an empirical relationship for granted! Agents might reoptimize when we try to exploit it
- Expectations affect the equilibrium
- Physicists do not teach atoms how to behave - "Luigi Zingales"

Next time

More elaborate pricing frictions

- Some empirical results about price changes
- Theoretical predictions of different models
    - Fischer pricing
    - Taylor contracts
- Inflation persistence

Monetary policy

- Demand stabilization