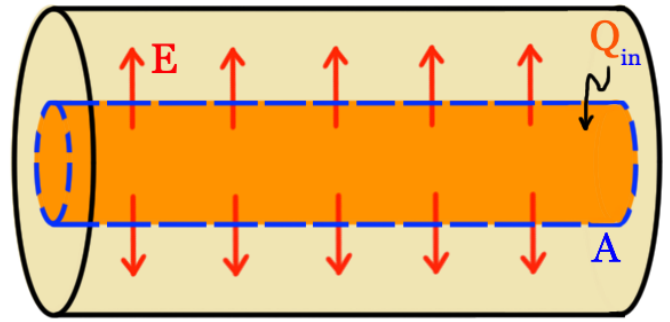


## Gauss' Law

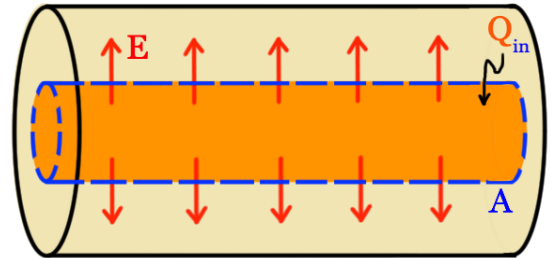
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$



“The **electric flux** through any **closed surface** is equal to the **charge within** that area”

If you choose an **area A** upon which the **electric field E** is uniform, The **E** can be removed from the integral and Gauss' Law becomes..

$$\vec{E} A = \frac{Q_{in}}{\epsilon_0}$$

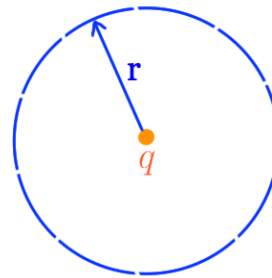


Note: The **dark orange shaded region** is charge contained within the **Gaussian surface**, the **tan shaded region** is charge not contained within the **Gaussian surface**

## Spherical Symmetry

"Point Charge"

$$E(4\pi r^2) = \frac{q}{\epsilon_0}$$

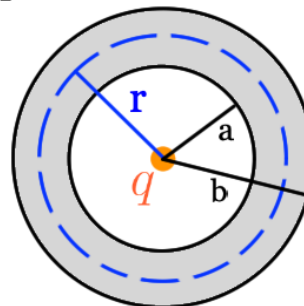


"Point charge in metal shell"

$$E(4\pi r^2) = \frac{q}{\epsilon_0} \quad \text{if } r < a$$

$$E(4\pi r^2) = 0 \quad \text{if } a < r < b$$

$$E(4\pi r^2) = \frac{q}{\epsilon_0} \quad \text{if } r > b$$

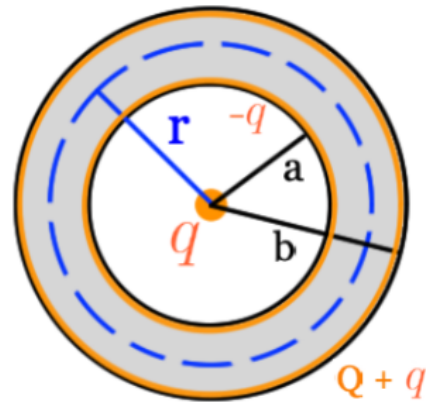


"Metal shell with net charge  $Q$ ,  
surrounding point charge  $q$ "

$$E(4\pi r^2) = \frac{q}{\epsilon_0} \quad \text{if } r < a$$

$$E(4\pi r^2) = 0 \quad \text{if } a < r < b$$

$$E(4\pi r^2) = \frac{q + Q}{\epsilon_0} \quad \text{if } r > b$$

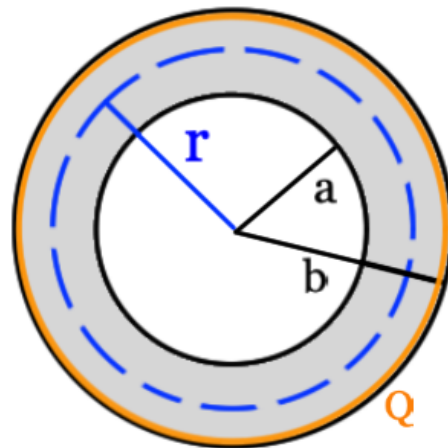


"Metal shell with net charge  $Q$ "

$$E(4\pi r^2) = 0 \quad \text{if } r < a$$

$$E(4\pi r^2) = 0 \quad \text{if } a < r < b$$

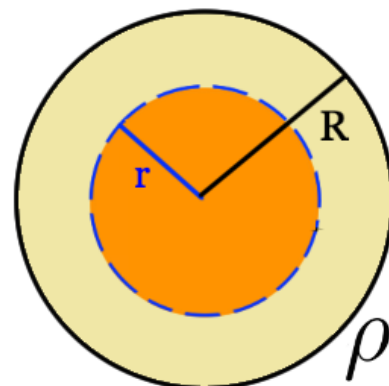
$$E(4\pi r^2) = \frac{Q}{\epsilon_0} \quad \text{if } r > b$$



"Insulating charged sphere of charge density  $\rho$ "

$$E(4\pi r^2) = \frac{\rho \cdot \frac{4}{3}\pi r^3}{\epsilon_0} \quad \text{if } r < R$$

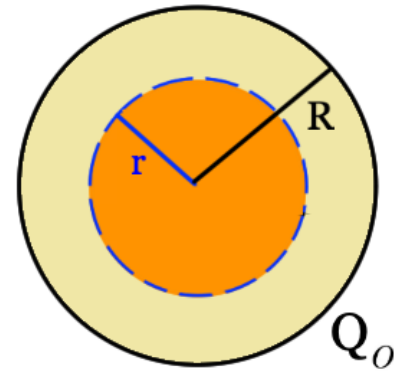
$$E(4\pi r^2) = \frac{\rho \cdot \frac{4}{3}\pi R^3}{\epsilon_0} \quad \text{if } r > R$$



"Insulating charged sphere of total charge  $Q_o$ "

$$E(4\pi r^2) = \frac{\frac{Q_o}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi r^3}{\epsilon_o} \quad \text{if } r < R$$

$$E(4\pi r^2) = \frac{Q_o}{\epsilon_o} \quad \text{if } r > R$$

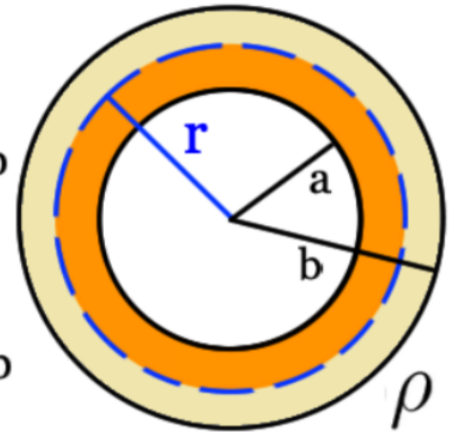


"Insulating charged shell given charge density  $\rho$ "

$$E(4\pi r^2) = 0 \quad \text{if } r < a$$

$$E(4\pi r^2) = \frac{\rho(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3)}{\epsilon_o} \quad \text{if } a < r < b$$

$$E(4\pi r^2) = \frac{\rho(\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3)}{\epsilon_o} \quad \text{if } r > b$$

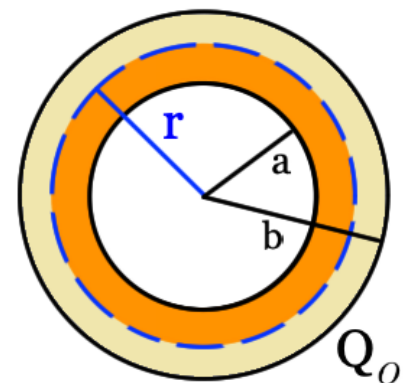


"Insulating charged shell given total charge  $Q_o$ "

$$E(4\pi r^2) = 0 \quad \text{if } r < a$$

$$E(4\pi r^2) = \frac{\frac{Q_o}{\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3} \cdot (\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3)}{\epsilon_o} \quad \text{if } a < r < b$$

$$E(4\pi r^2) = \frac{Q_o}{\epsilon_o} \quad \text{if } r > b$$

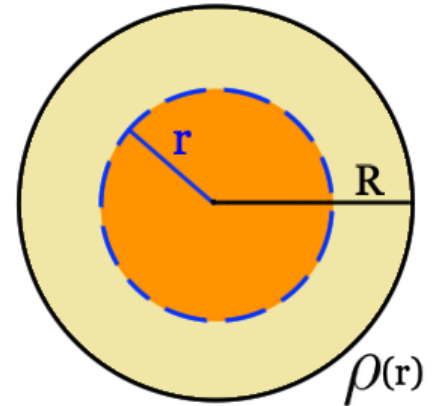


"Insulating sphere with charge density  $\rho(r)$ "

$$E(4\pi r^2) = \frac{\int_0^r \rho(r) 4\pi r^2 dr}{\epsilon_0} \quad r < R$$

$$E(4\pi r^2) = \frac{\int_0^R \rho(r) 4\pi r^2 dr}{\epsilon_0} \quad r > R$$

$$dV_{\text{sphere}} = d\left(\frac{4}{3}\pi r^3\right) = 4\pi r^2 dr$$

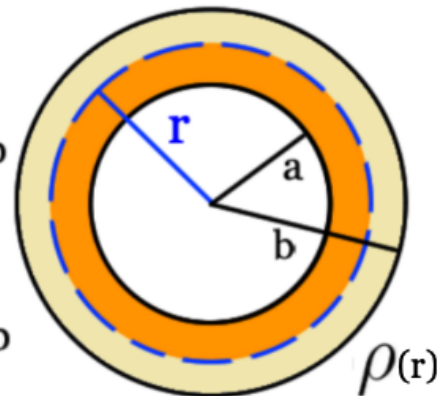


"Insulating charged shell given charge density  $\rho(r)$ "

$$E(4\pi r^2) = 0 \quad \text{if } r < a$$

$$E(4\pi r^2) = \frac{\int_a^r \rho(r) 4\pi r^2 dr}{\epsilon_0} \quad \text{if } a < r < b$$

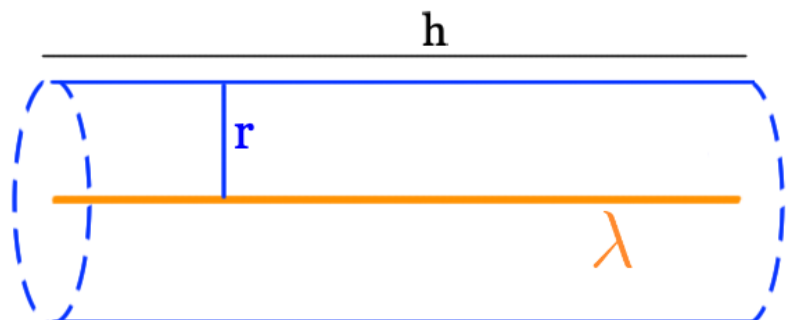
$$E(4\pi r^2) = \frac{\int_a^b \rho(r) 4\pi r^2 dr}{\epsilon_0} \quad \text{if } r > b$$



## Cylindrical Symmetry

"Line of charge"

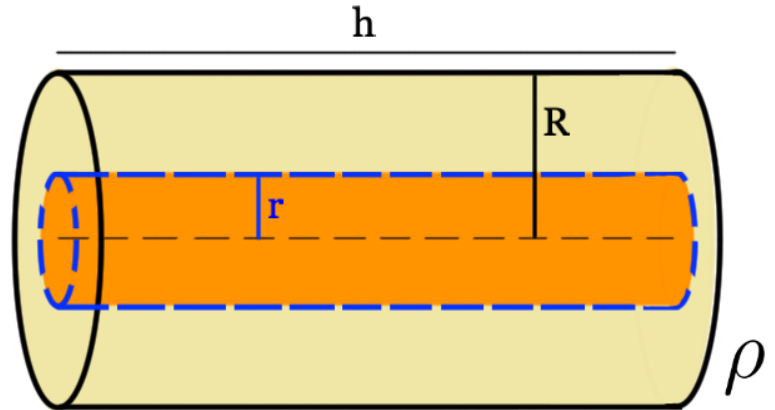
$$E(2\pi r h) = \frac{\lambda h}{\epsilon_0} \quad \text{for all } r$$



"Insulating cylinder with charge density  $\rho$ "

$$E(2\pi rh) = \frac{\rho(\pi r^2 h)}{\epsilon_0} \quad r < R$$

$$E(2\pi rh) = \frac{\rho(\pi R^2 h)}{\epsilon_0} \quad r > R$$

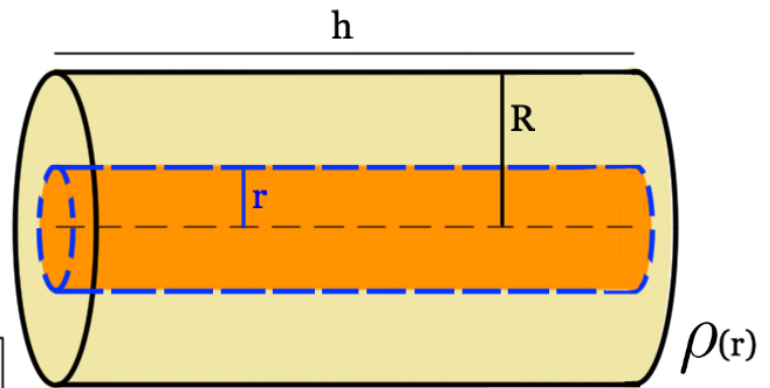


"Insulating cylinder with non-uniform charge density  $\rho(r)$ "

$$E(2\pi rh) = \int_0^r \frac{\rho(r) 2\pi r h dr}{\epsilon_0} \quad r < R$$

$$E(2\pi rh) = \int_0^R \frac{\rho(r) 2\pi r h dr}{\epsilon_0} \quad r > R$$

$$dV_{cylinder} = d(\pi r^2 h) = 2\pi r h dr$$

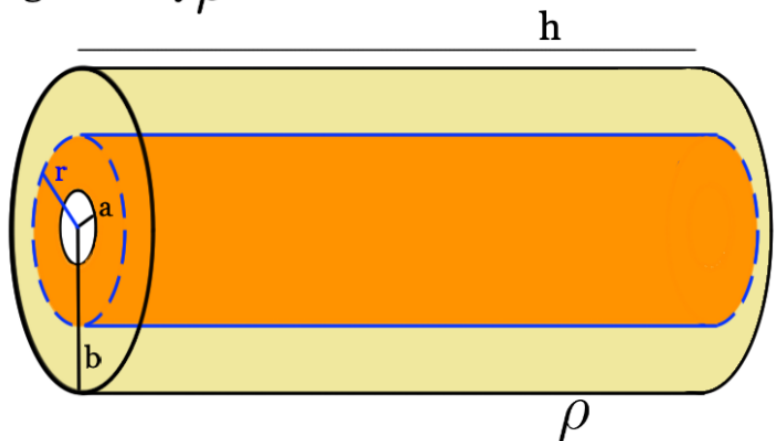


"Insulating hollow cylinder of charge density  $\rho$ "

$$E(2\pi rh) = 0 \quad r < a$$

$$E(2\pi rh) = \frac{\rho(\pi r^2 h - \pi a^2 h)}{\epsilon_0} \quad a < r < b$$

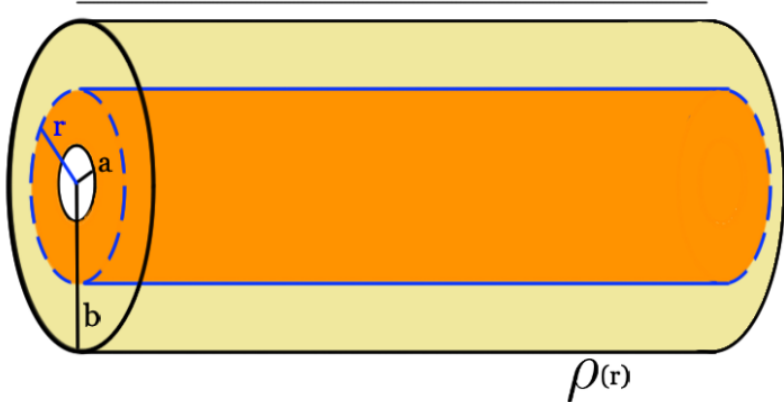
$$E(2\pi rh) = \frac{\rho(\pi b^2 h - \pi a^2 h)}{\epsilon_0} \quad r > b$$



"Insulating hollow cylinder of non-uniform charge density  $\rho(r)$ "

$$E(2\pi r h) = 0 \quad r < a$$

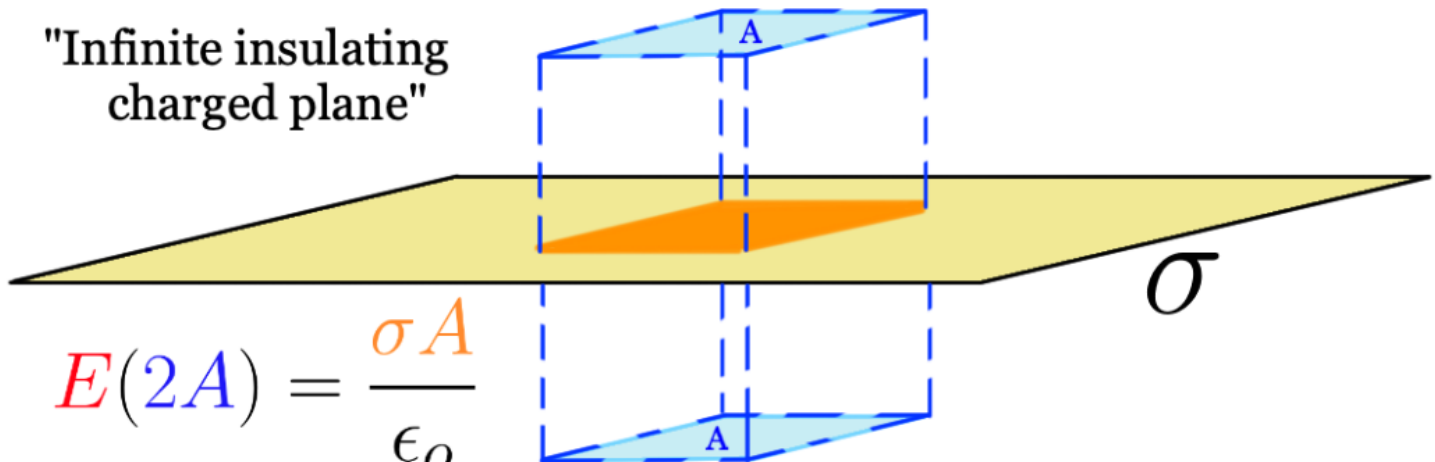
$$E(2\pi r h) = \frac{\int_a^r \rho(r) 2\pi r h dr}{\epsilon_0} \quad b > r > a$$

$$E(2\pi r h) = \frac{\int_a^b \rho(r) 2\pi r h dr}{\epsilon_0} \quad r > b$$


The diagram shows a 3D perspective of a hollow cylinder. The inner radius is labeled 'a' and the outer radius is labeled 'b'. The height is labeled 'h'. A dashed blue line represents a Gaussian cylinder of radius 'r' inside the cylinder wall. The charge density is labeled  $\rho(r)$ .

## Planar Symmetry

"Infinite insulating charged plane"

$$E(2A) = \frac{\sigma A}{\epsilon_0}$$


The diagram shows a yellow plane representing an infinite insulating charged plane with surface charge density  $\sigma$ . A blue dashed pillbox is shown, intersecting the plane. The top and bottom faces of the pillbox are labeled 'A'.