

# RNN-based counterfactual time-series prediction (Online Appendix)

April 27, 2018

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# 1 Encoder-decoder architecture

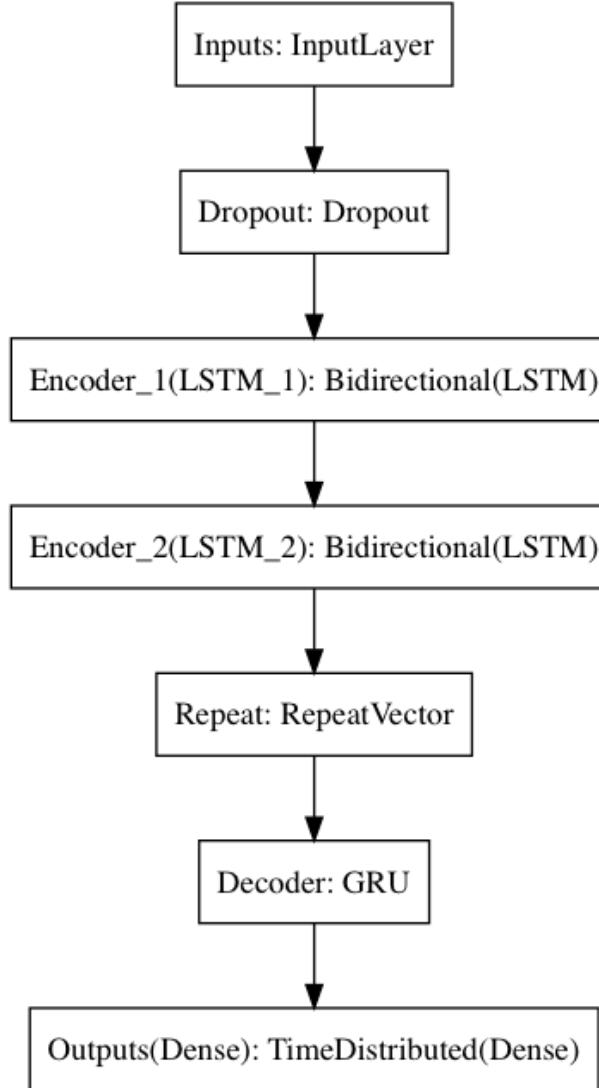


Figure 1: Encoder-decoder networks architecture. Dropout is applied to the visible input sequences, which are then fed to a two-layer bidirectional LSTM encoder. The encoder encodes the input sequences into a single vector that contains information about the entire sequence. The output of the encoder is repeated  $t$  times and fed to the single-layer GRU decoder, which translates the encoded sequence into the predicted sequence. Finally, a dense layer is applied to the decoder output to generate predictions.

## 2 VAE architecture

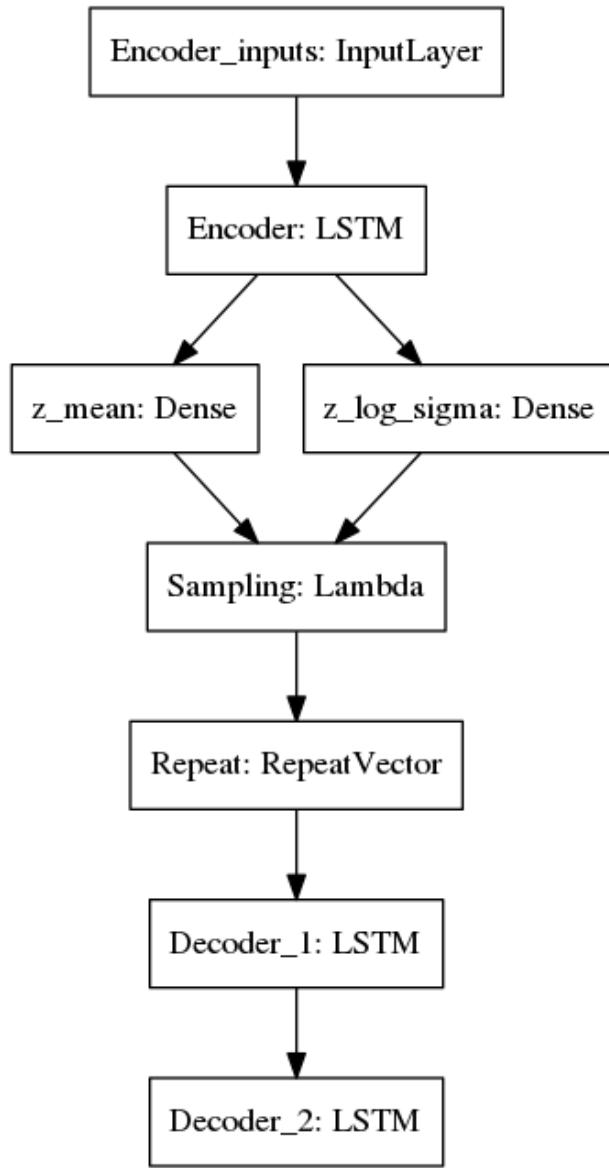


Figure 2: Recurrent VAE architecture. First, an LSTM encoder turns the input samples into two parameters in a latent space. Latent space points are randomly sampled from the latent distribution that is assumed to generate the data. The sampling output is repeated  $t$  times and fed to the decoder LSTM, which maps the latent space points back to the original input data.

### 3 Training loss

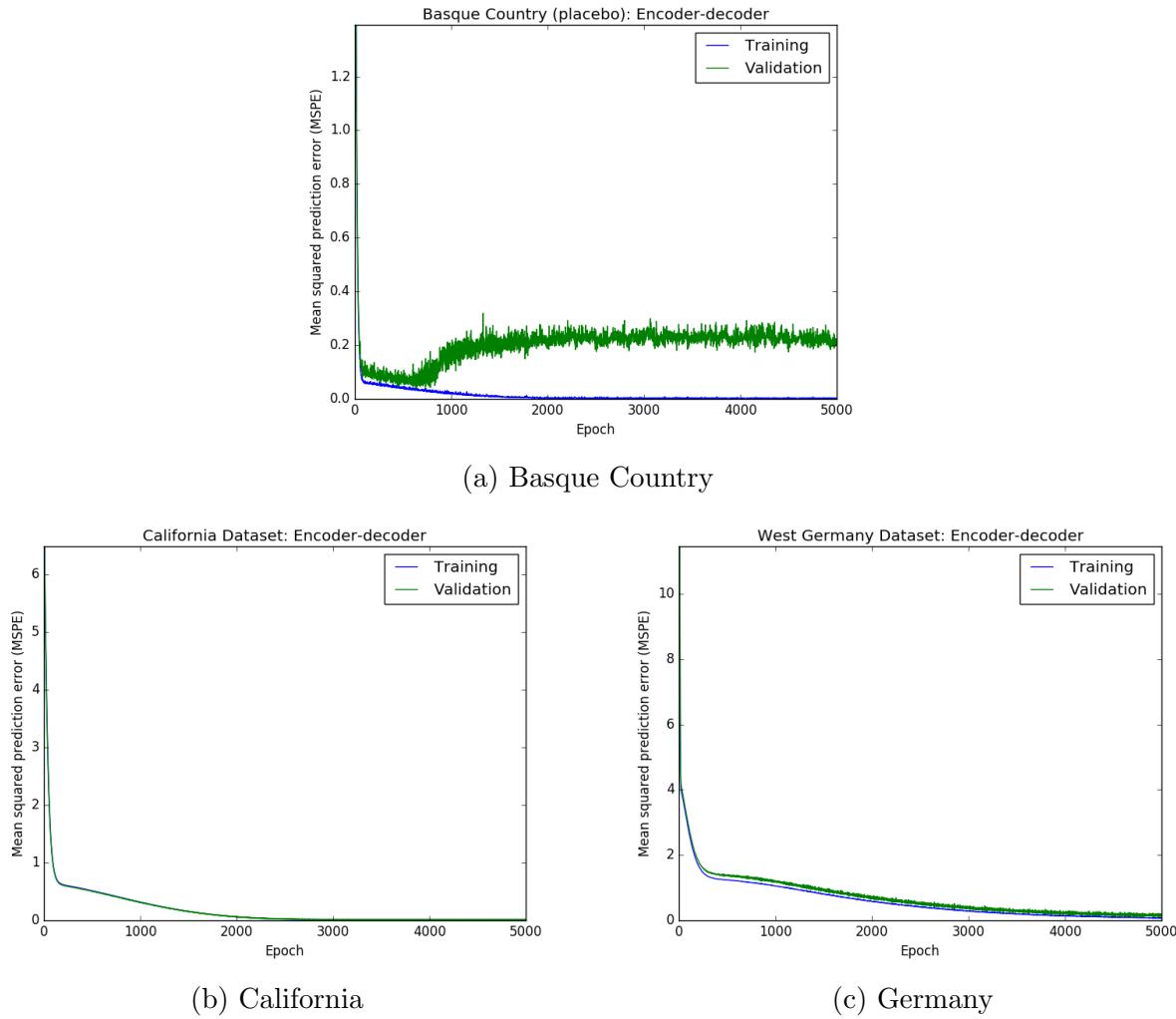
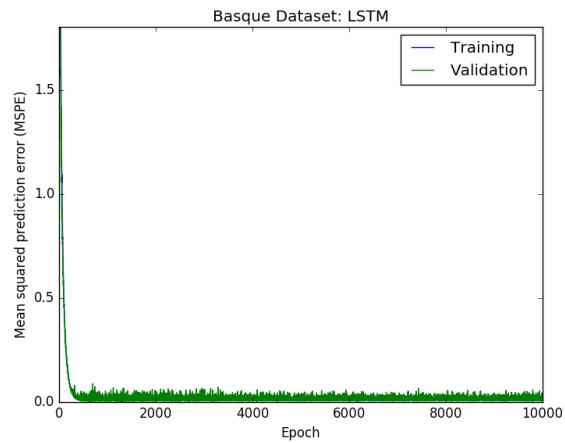
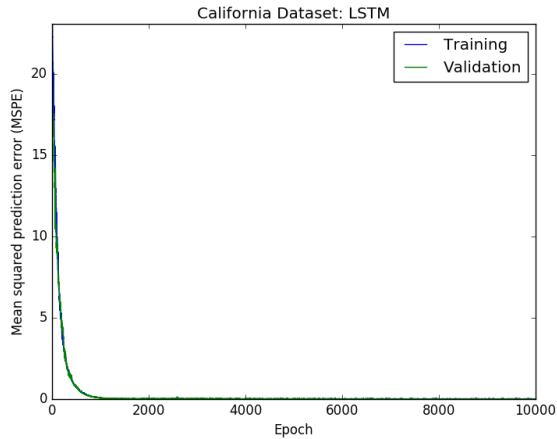


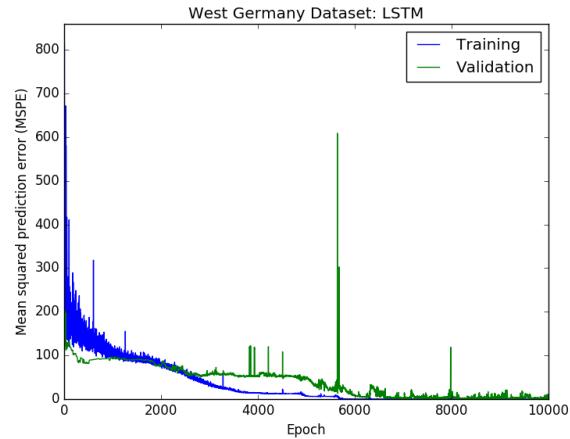
Figure 3: Evolution of encoder-decoder networks training and validation loss in terms of MSPE.



(a) Basque Country

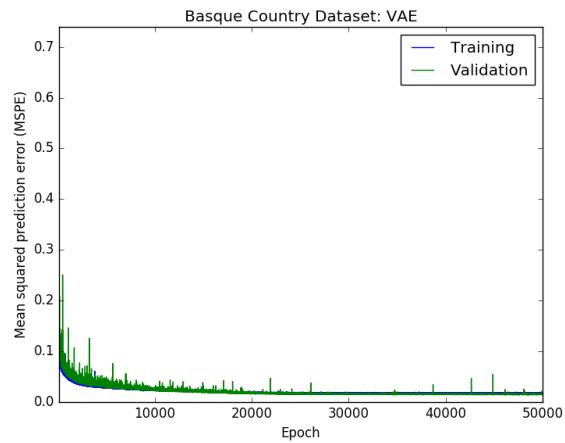


(b) California

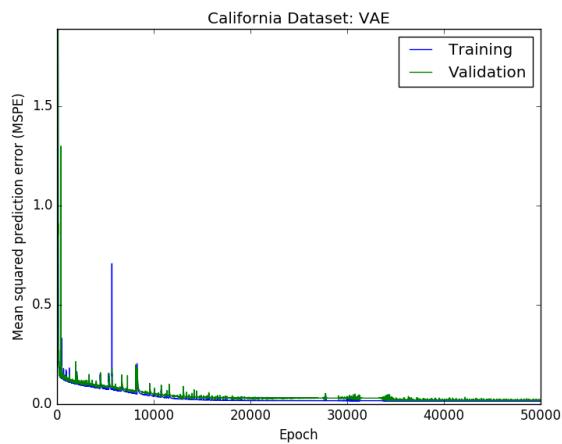


(c) Germany

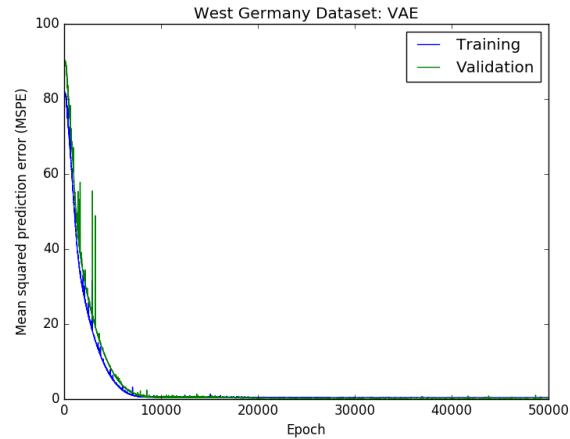
Figure 4: Evolution of LSTM training and validation loss in terms of MSPE.



(a) Basque Country



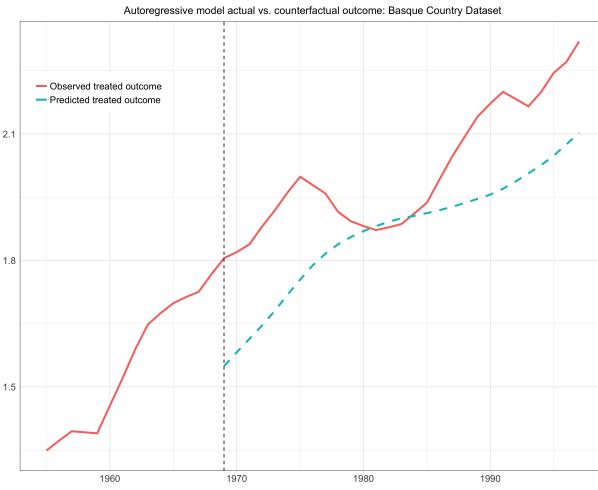
(b) California



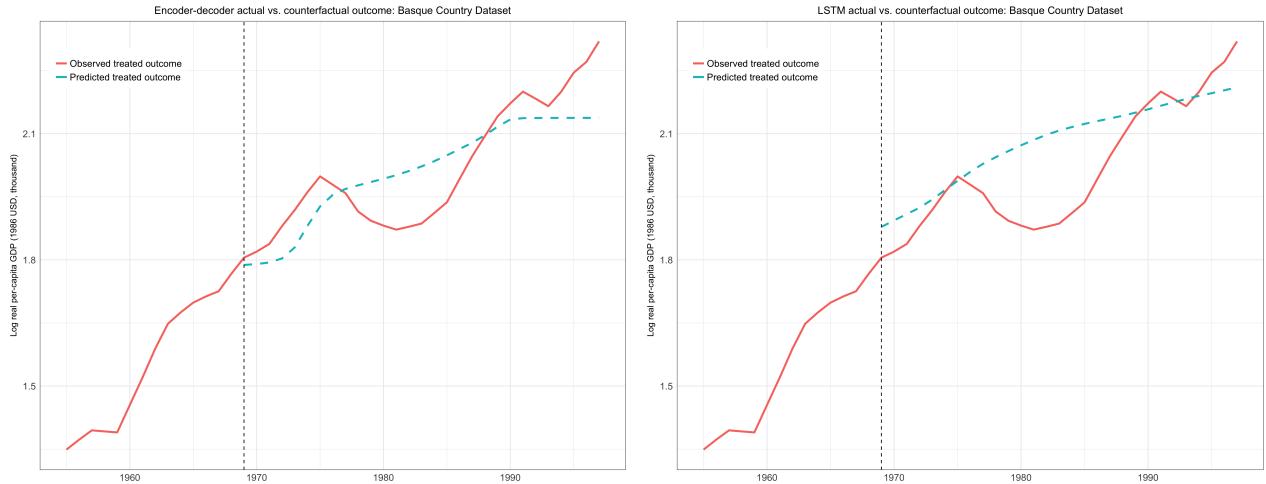
(c) Germany

Figure 5: Evolution of VAE training and validation loss in terms of MSPE.

## 4 Treatment effect estimates on Basque Country dataset

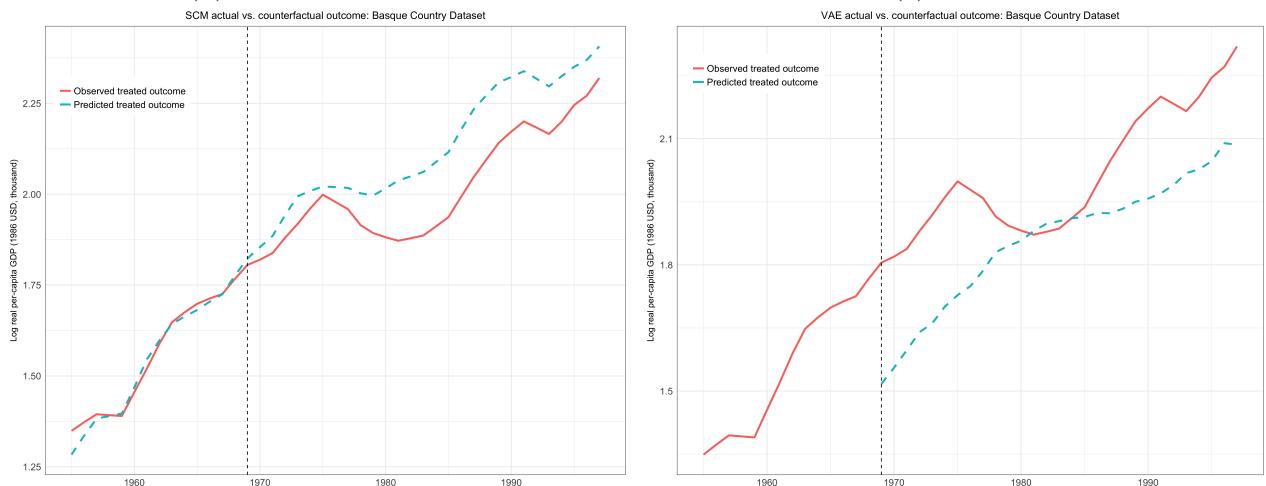


(a) Autoregressive model



(b) Encoder-decoder

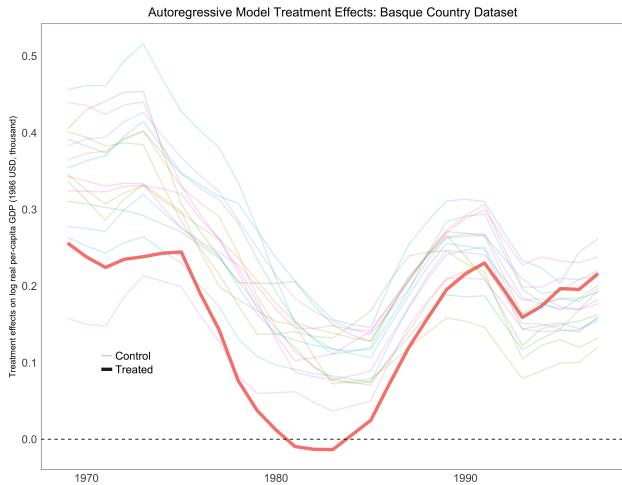
(c) LSTM



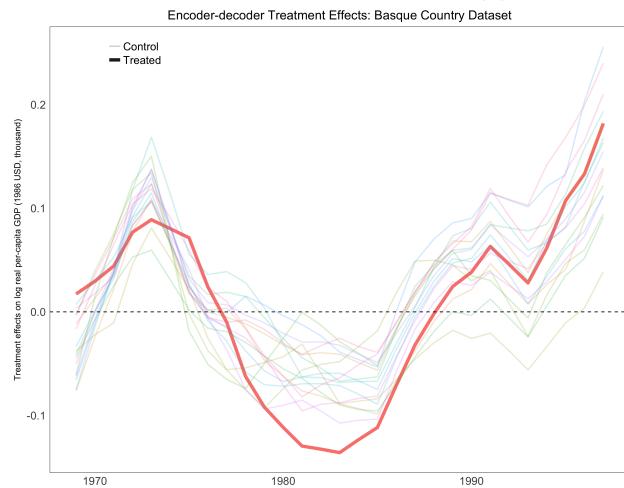
(d) SCM

(e) VAE

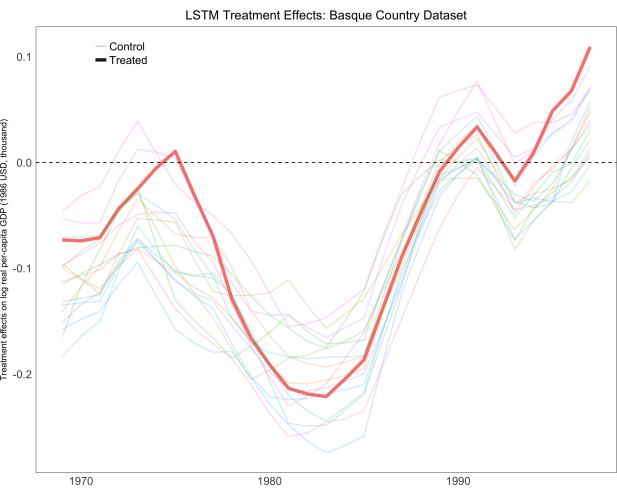
Figure 6: Observed and counterfactual predicted outcomes for treated unit in Basque Country dataset.



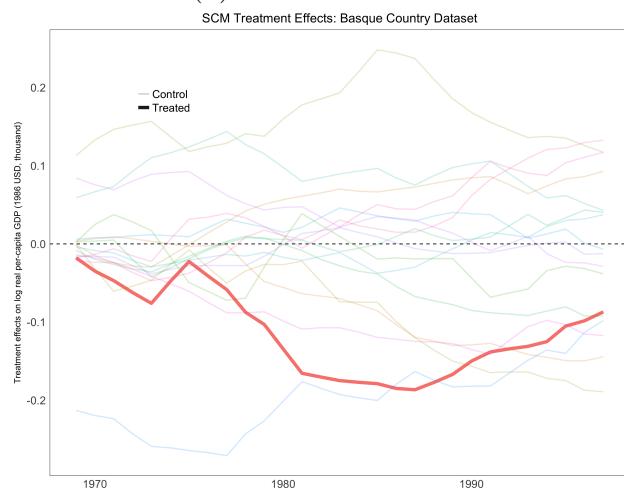
(a) Autoregressive model



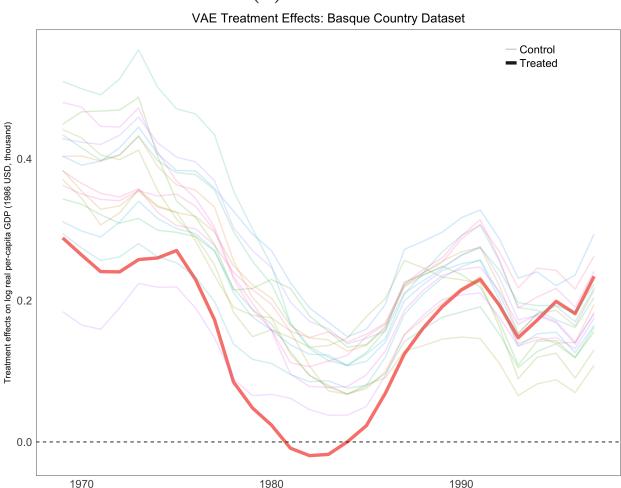
(b) Encoder-decoder



(c) LSTM

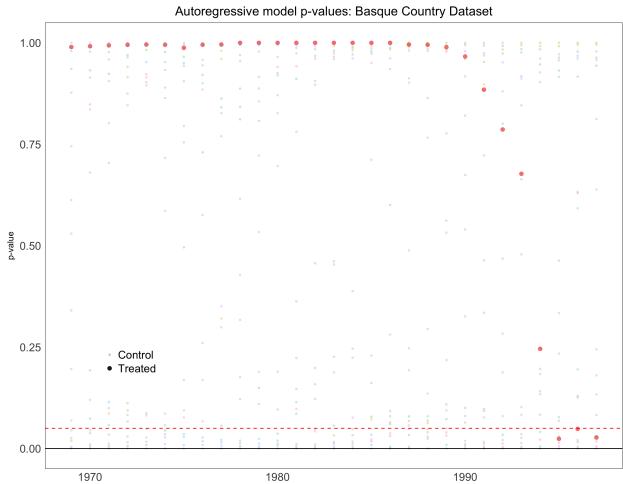


(d) SCM

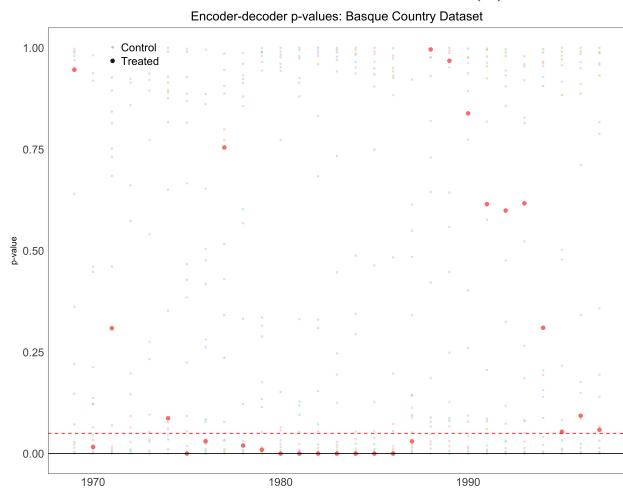


(e) VAE

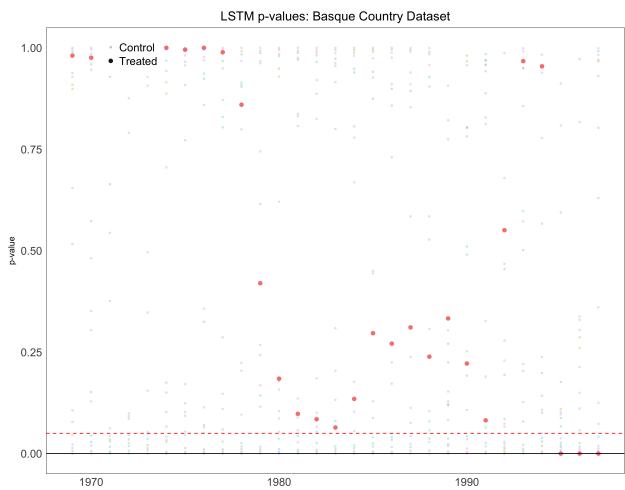
Figure 7: Time-series of post-period treatment effects in Basque Country dataset. Darker line represents the effect on the actual treated unit (i.e., Basque Country) and each lighter line represents the effects on control units.



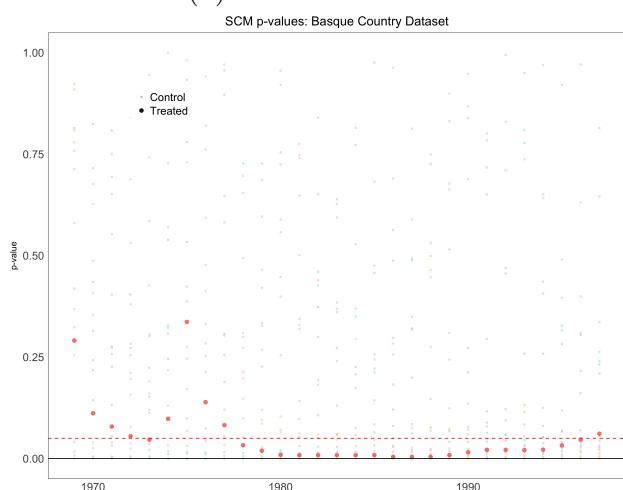
(a) Autoregressive model



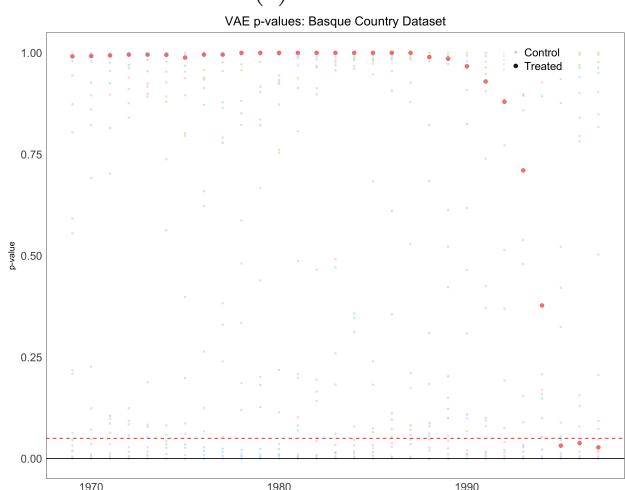
(b) Encoder-decoder



(c) LSTM



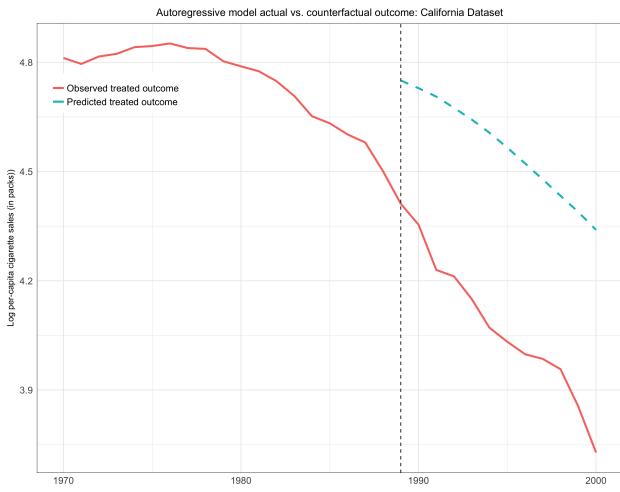
(d) SCM



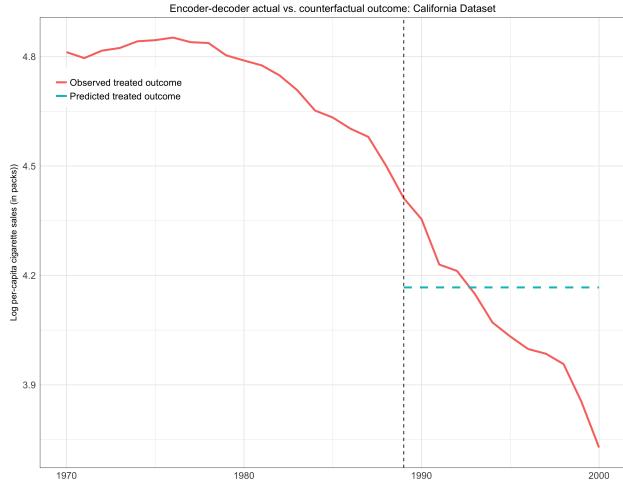
(e) VAE

Figure 8: Per-period randomization  $p$ -values corresponding to treatment effects on treated and control units in Basque Country dataset.

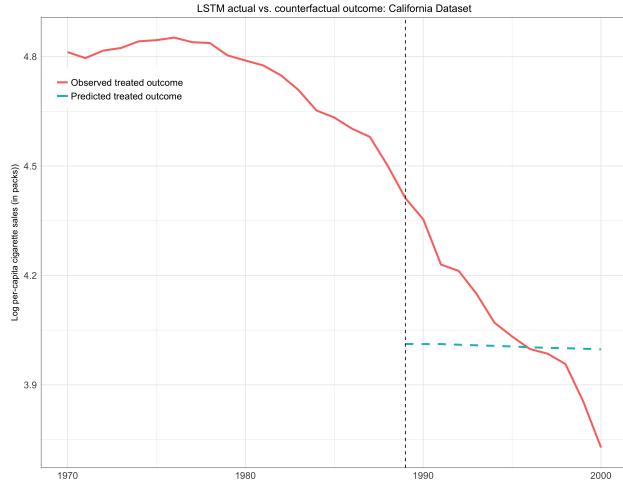
## 5 Treatment effect estimates on California dataset



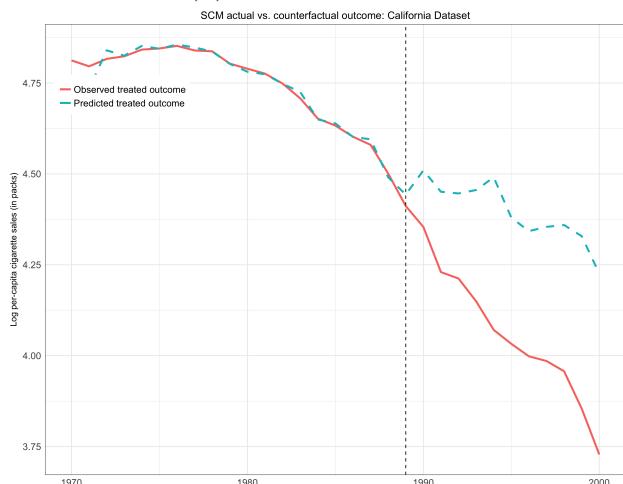
(a) Autoregressive model



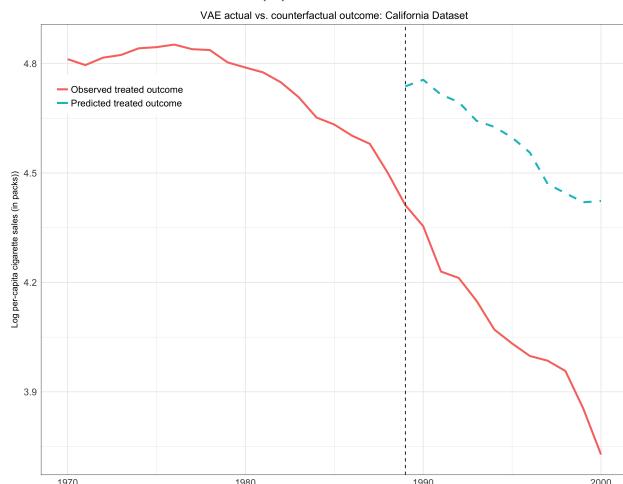
(b) Encoder-decoder



(c) LSTM

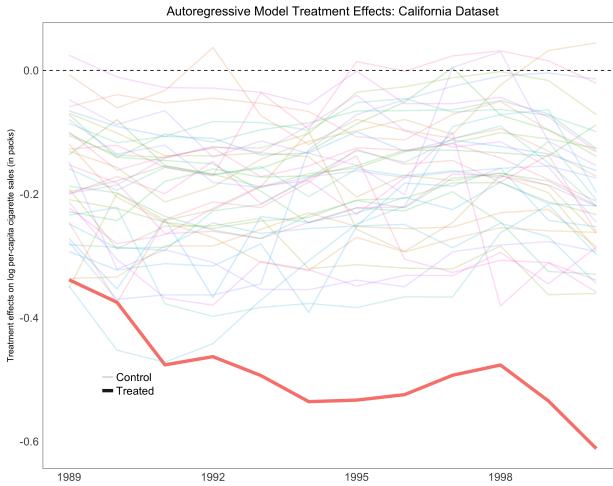


(d) SCM

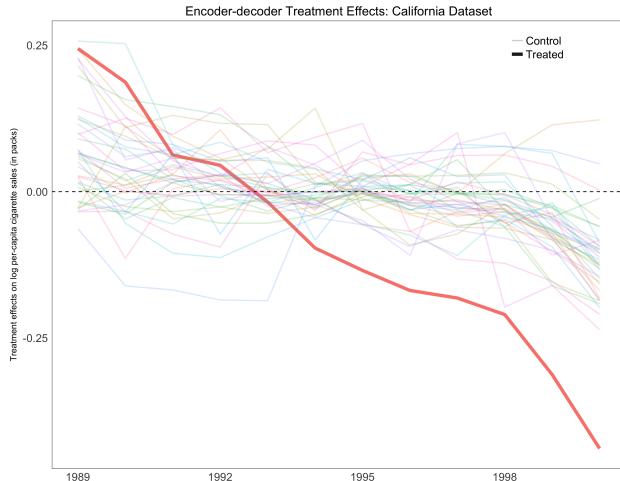


(e) VAE

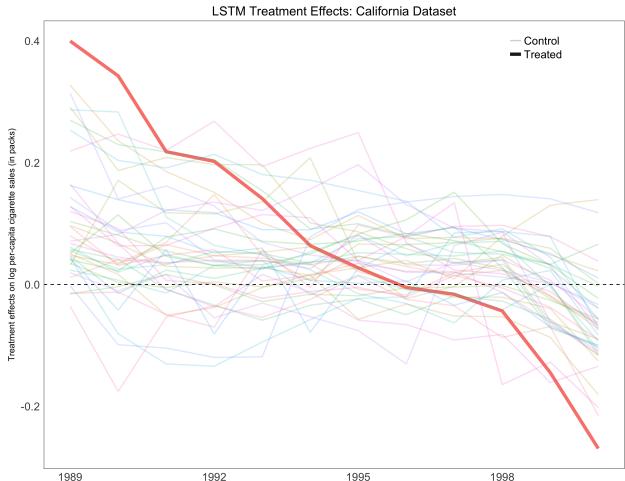
Figure 9: Actual and counterfactual predicted outcomes for treated unit in California dataset.



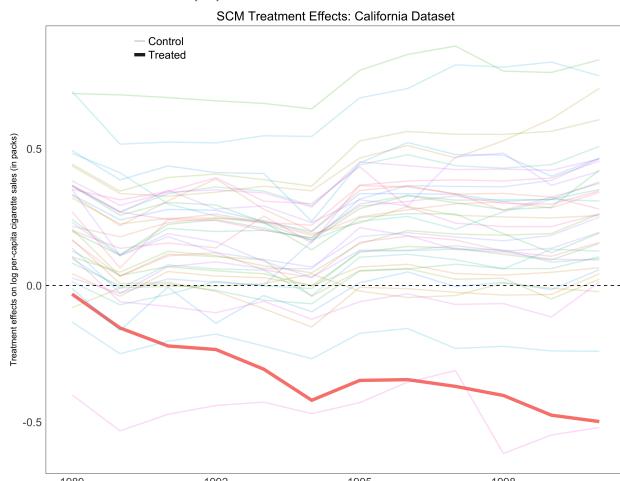
(a) Autoregressive model



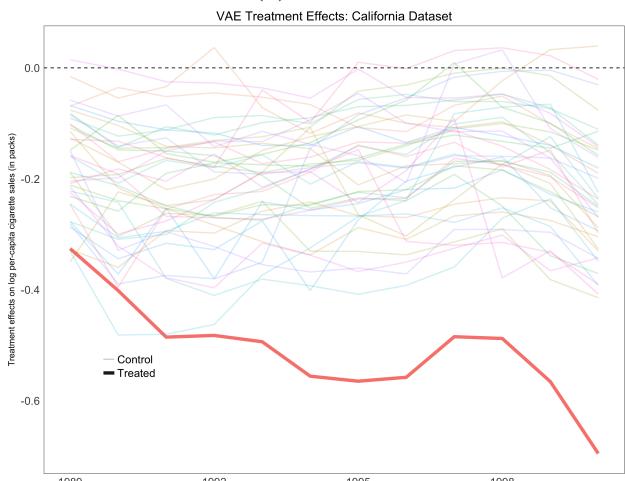
(b) Encoder-decoder



(c) LSTM

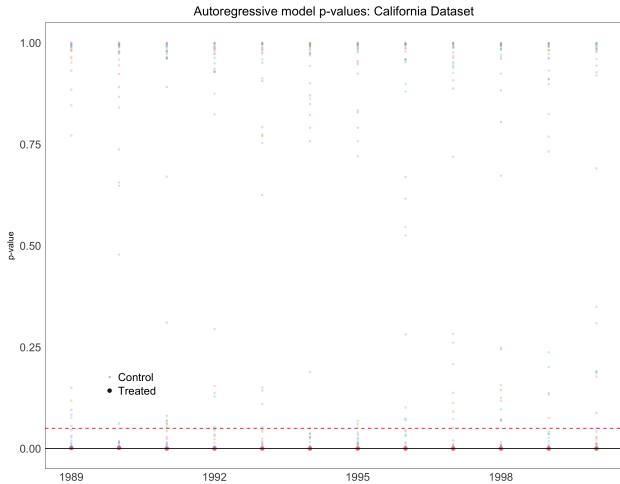


(d) SCM

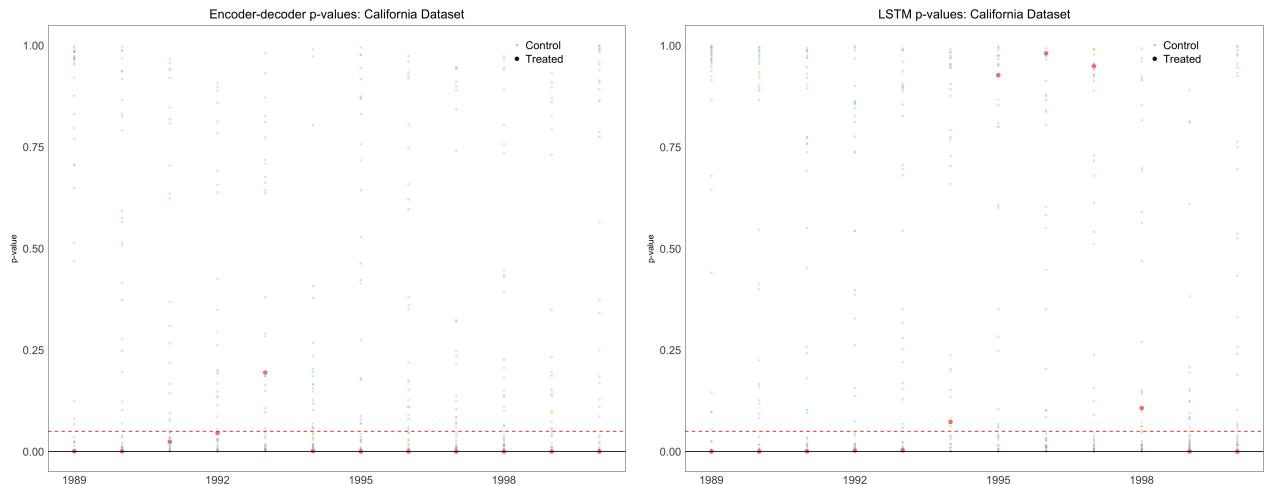


(e) VAE

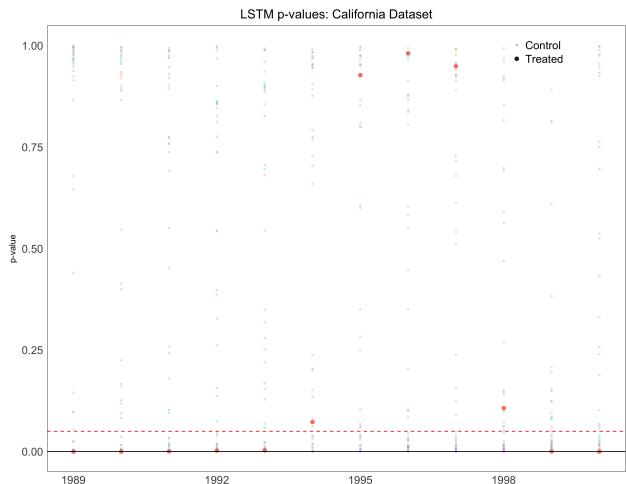
Figure 10: Time-series of post-period treatment effects in California dataset using counterfactual predictions from encoder-decoder networks. Darker line represents the effect on the actual treated unit (i.e., California) and each lighter line represents the effects on control units.



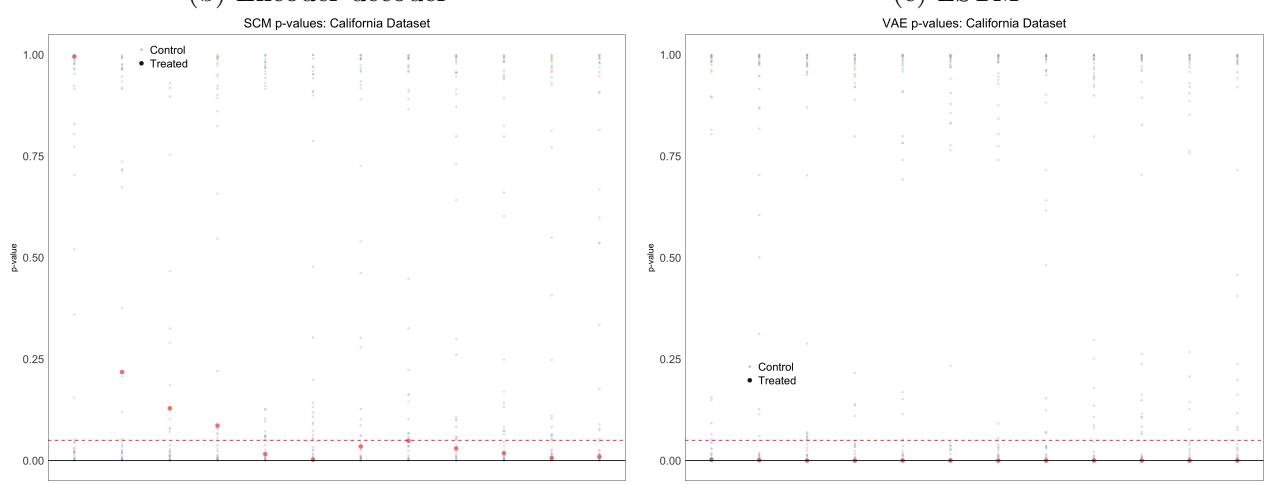
(a) Autoregressive model



(b) Encoder-decoder



(c) LSTM

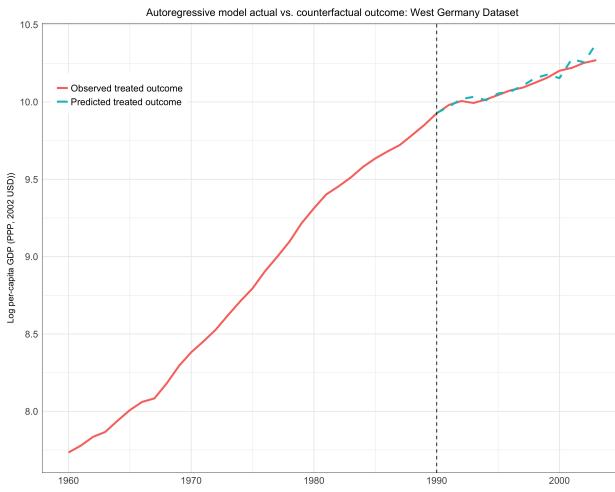


(d) SCM

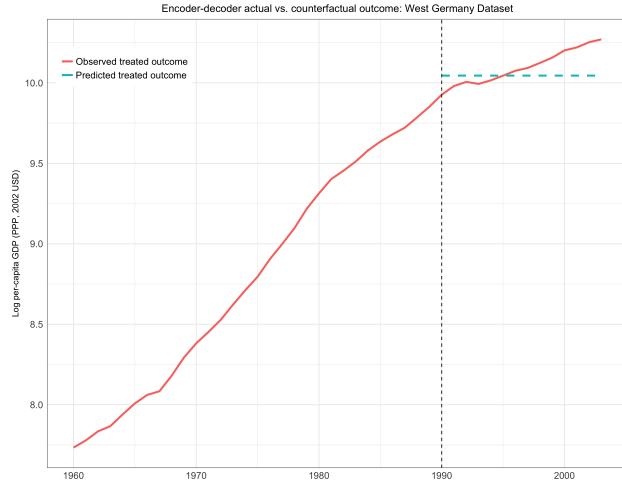
(e) VAE

Figure 11: Per-period randomization  $p$ -values corresponding to treatment effects on treated and control units in California dataset.

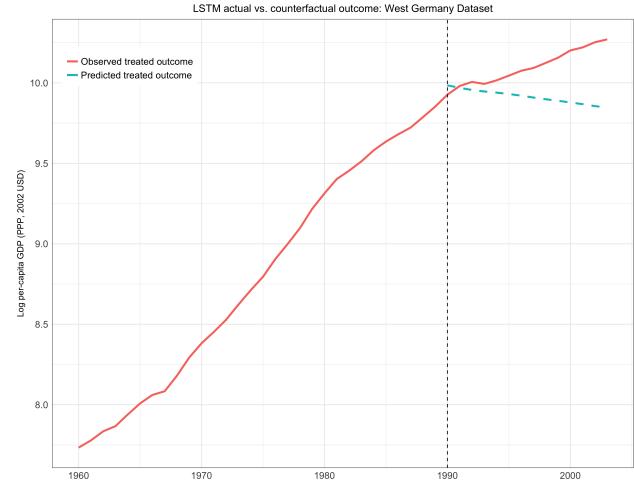
## 6 Treatment effect estimates on West Germany data



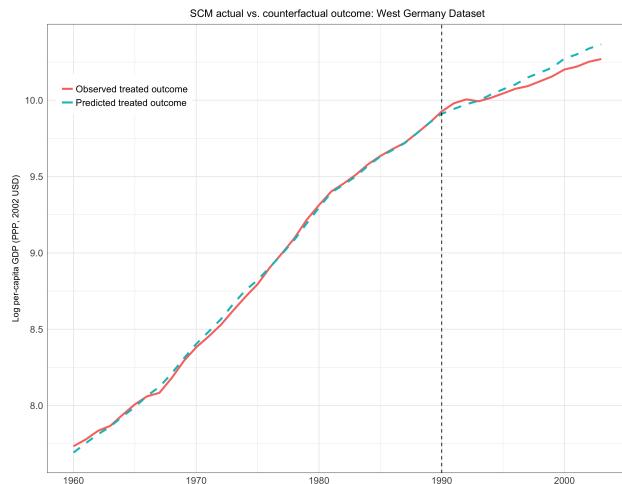
(a) Autoregressive model



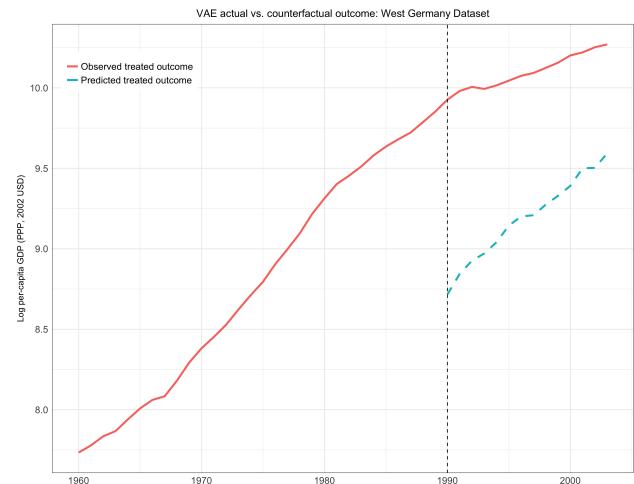
(b) Encoder-decoder



(c) LSTM

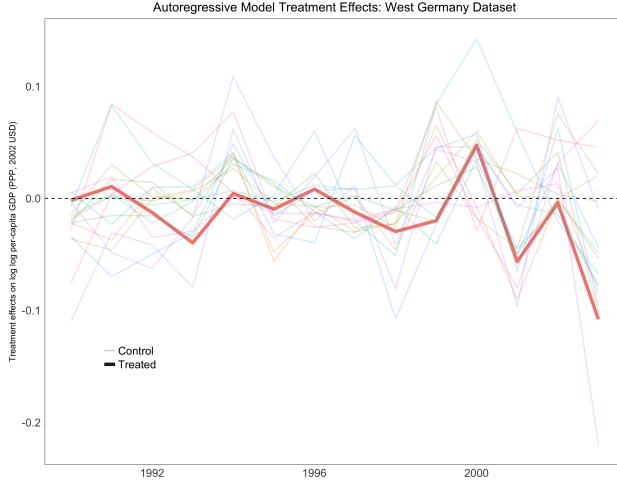


(d) SCM

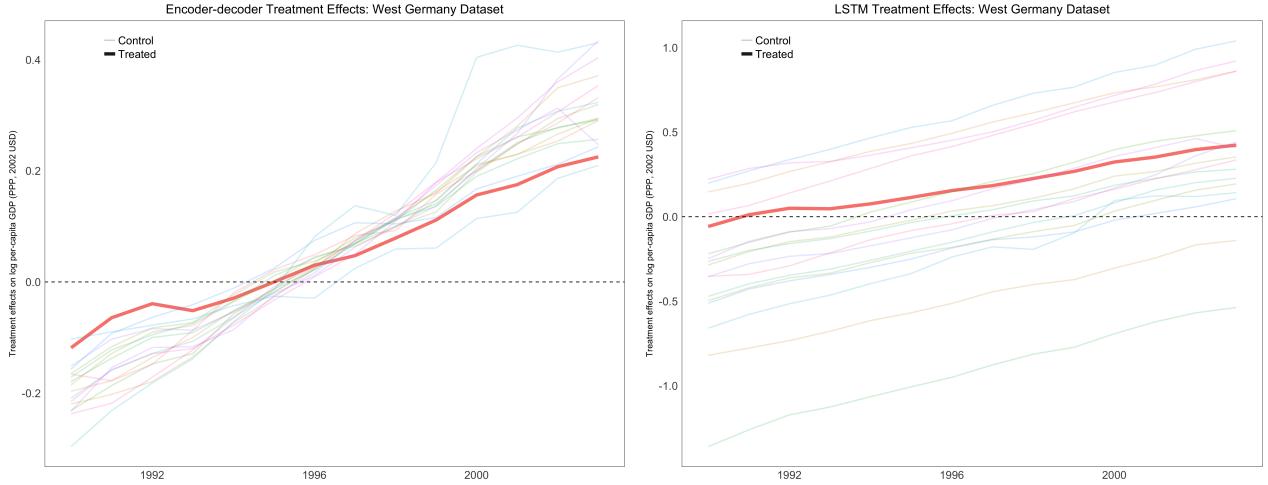


(e) VAE

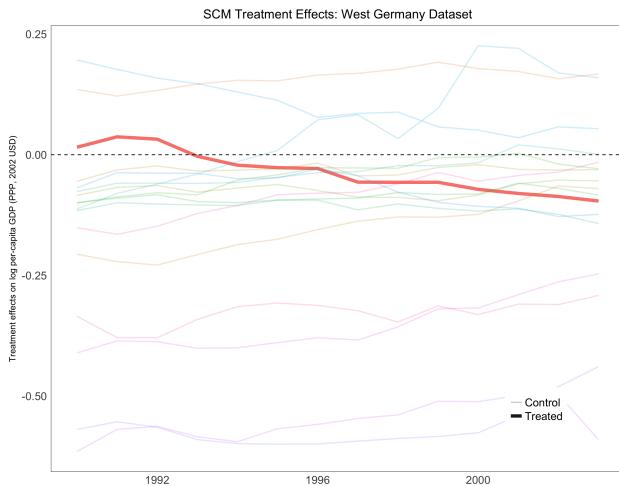
Figure 12: Actual and counterfactual predicted outcomes for treated unit in West Germany dataset.



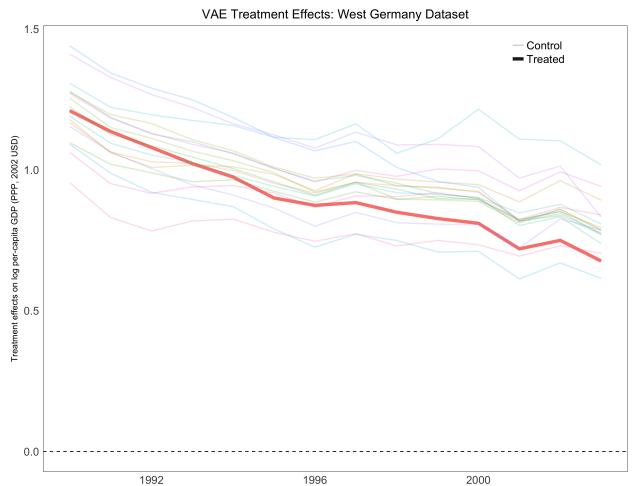
(a) Autoregressive model



(b) Encoder-decoder

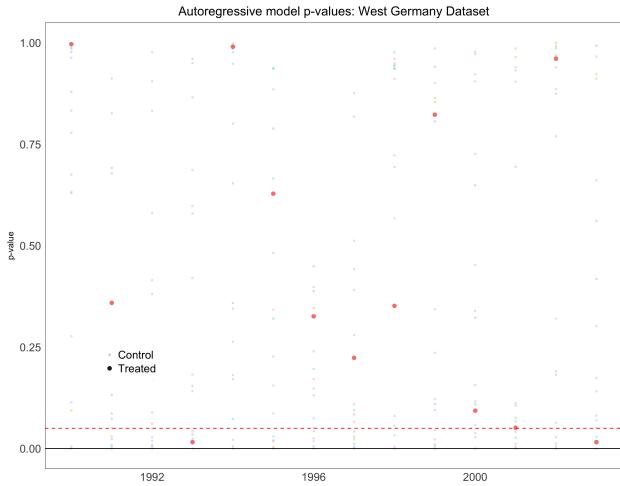


(d) SCM

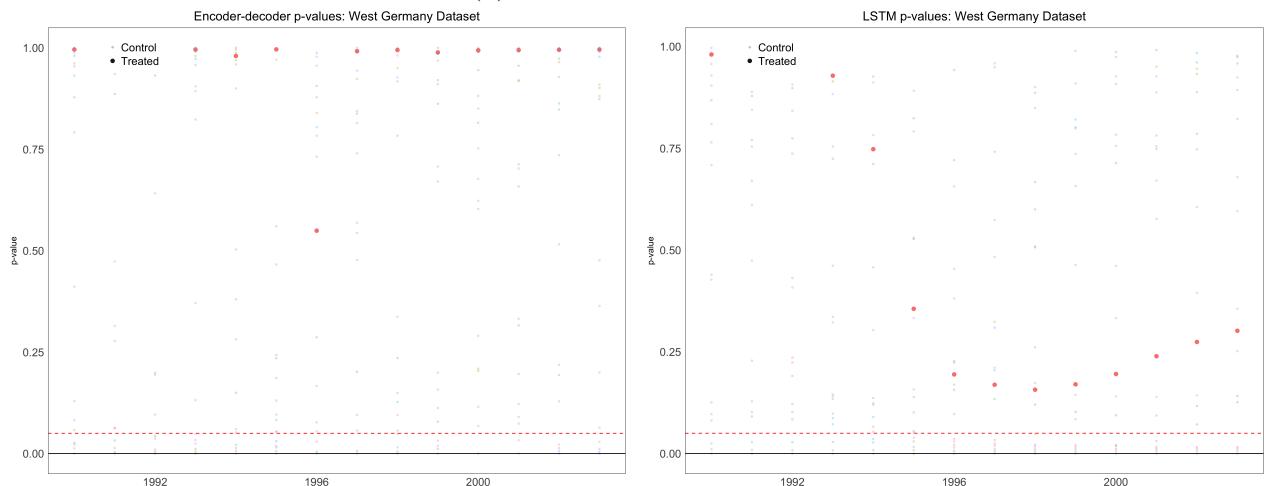


(e) VAE

Figure 13: Time-series post-period treatment effects in West Germany dataset using counterfactual predictions from encoder-decoder networks. Darker line represents the effect on the actual treated unit (i.e., West Germany) and each lighter line represents the effects on control units.

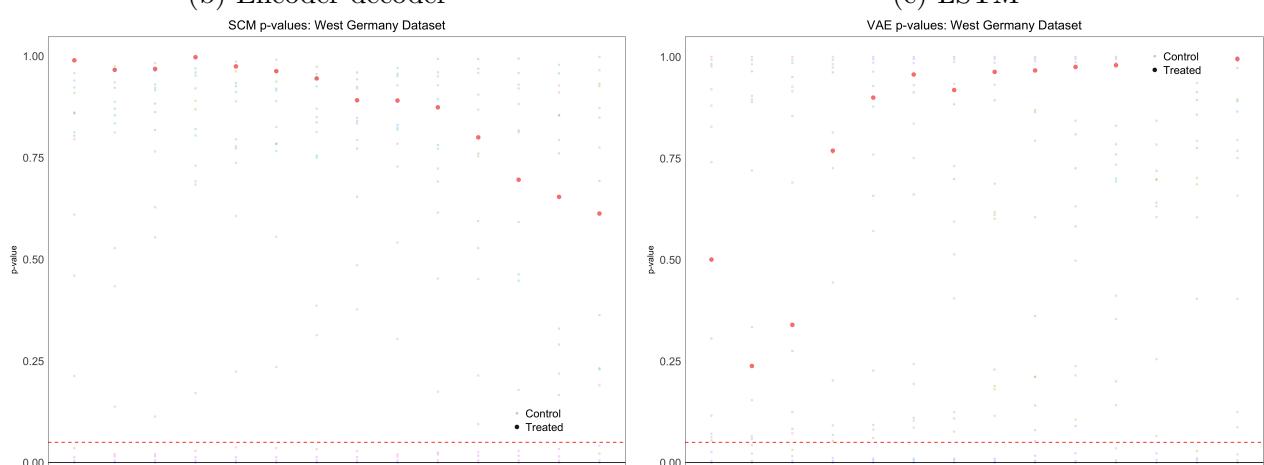


(a) Autoregressive model



(b) Encoder-decoder

(c) LSTM



(d) SCM

(e) VAE

Figure 14: Per-period randomization  $p$ -values corresponding to treatment effects on treated and control units in West Germany dataset.