

Web-Based Supporting Materials for  
“RNN-Based Counterfactual Time-Series Prediction” by  
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# 1 RNNs architecture

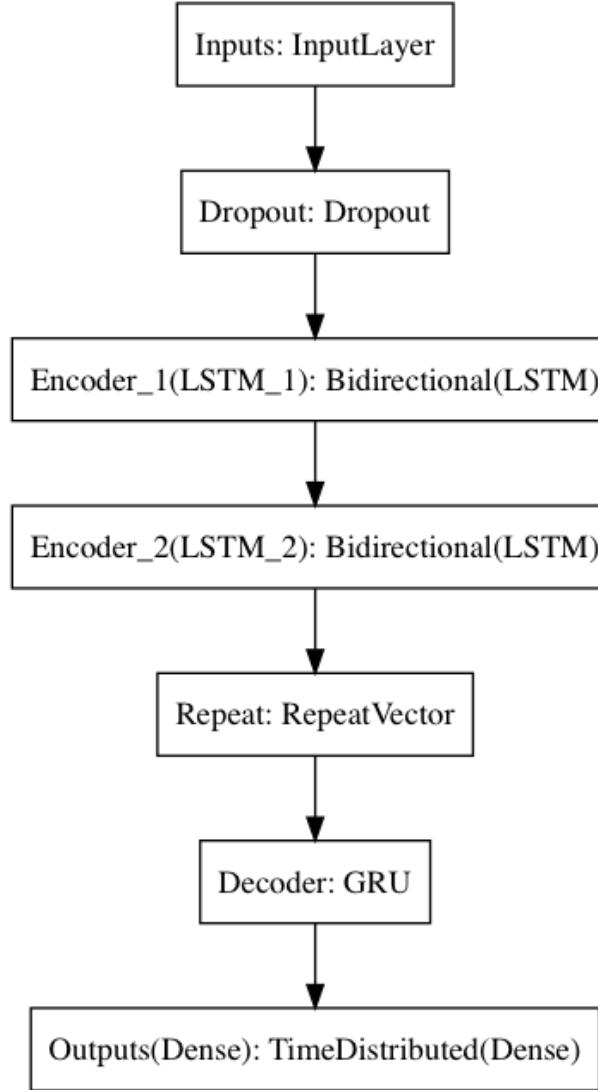


Figure 1: Encoder-decoder networks architecture. Dropout is applied to the visible input sequences, which are then fed to a two-layer bidirectional LSTM encoder. The encoder encodes the input sequences into a single vector that contains information about the entire sequence. The output of the encoder is repeated  $t$  times and fed to the single-layer GRU decoder, which translates the encoded sequence into the predicted sequence. Finally, a dense layer is applied to the decoder output to generate predictions.

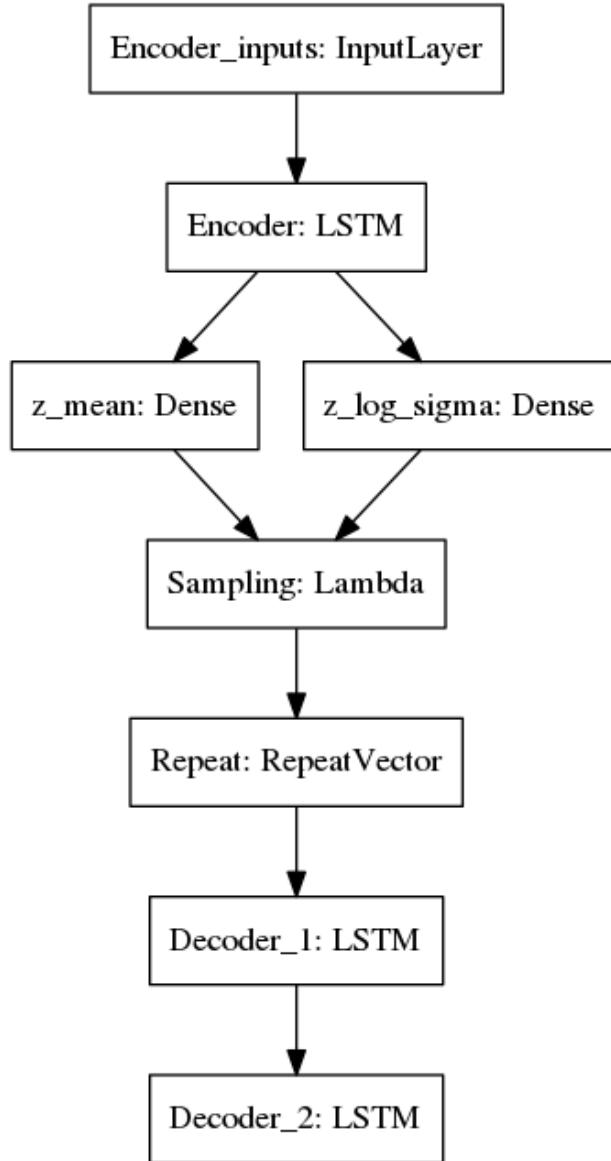


Figure 2: Recurrent VAE architecture. First, an LSTM encoder turns the input samples into two parameters in a latent space. Latent space points are randomly sampled from the latent distribution that is assumed to generate the data. The sampling output is repeated  $t$  times and fed to the decoder LSTM, which maps the latent space points back to the original input data.

## 2 RNNs training history: SCM datasets

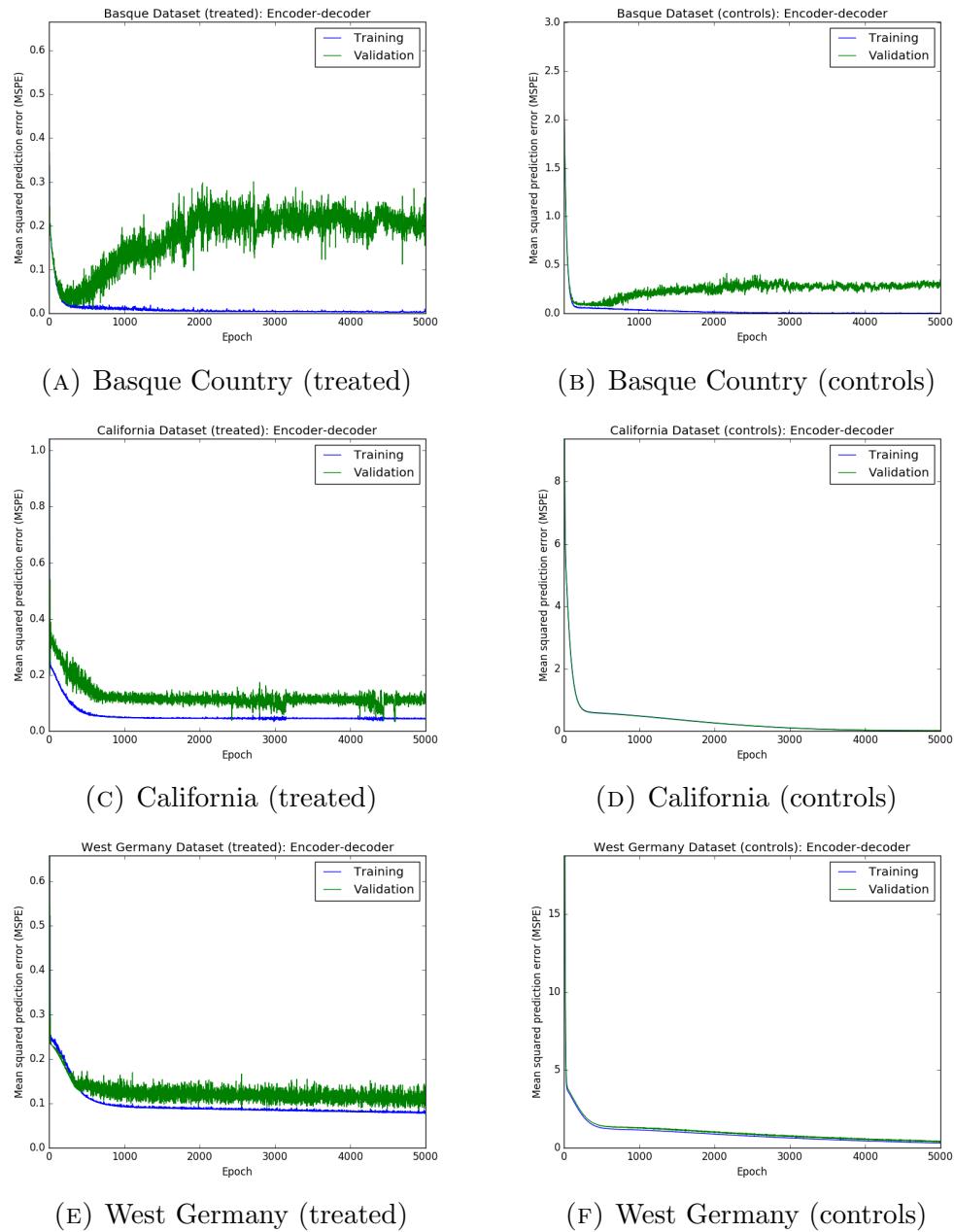


Figure 3: Evolution of encoder-decoder networks training and validation loss in terms of MSPE.

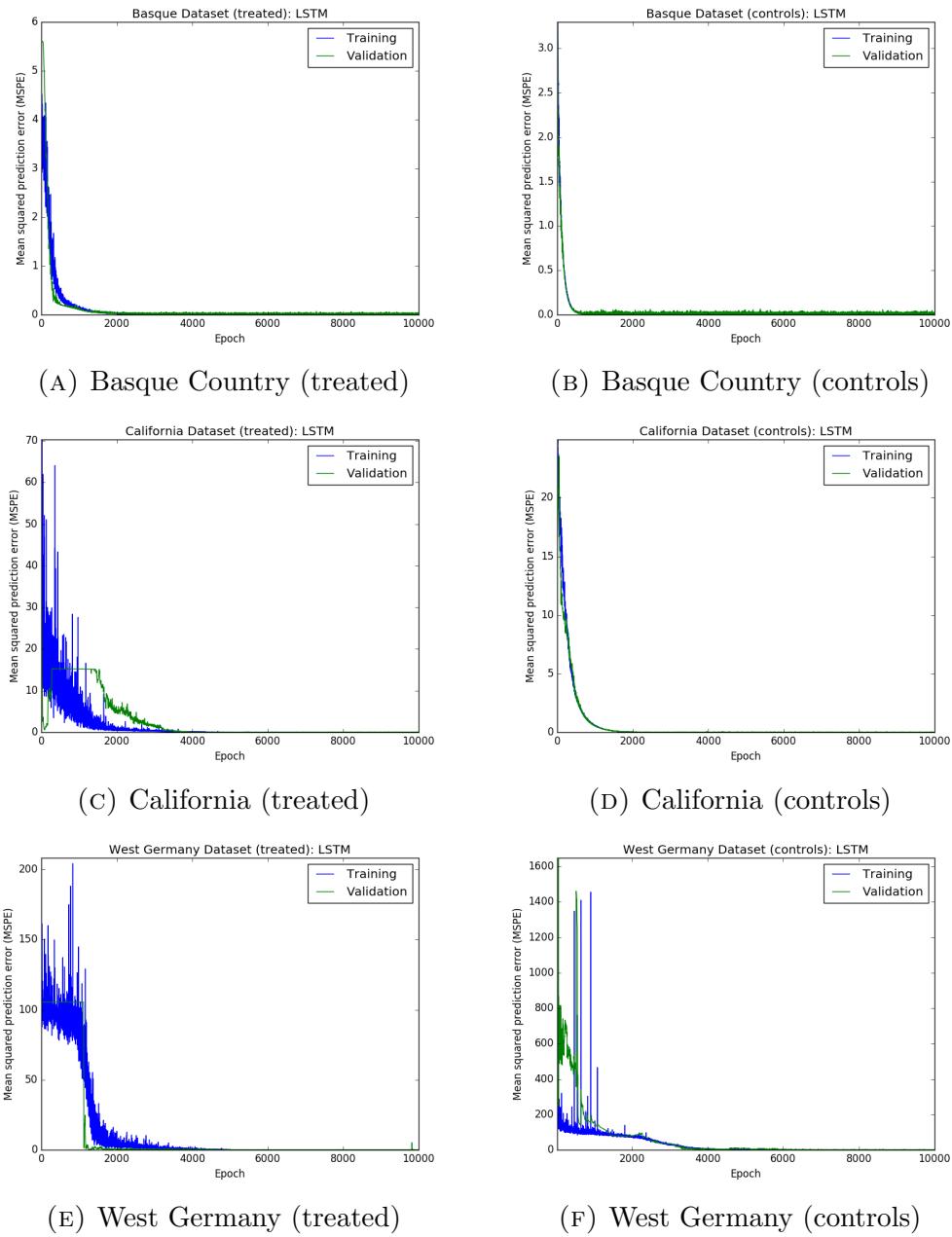


Figure 4: Evolution of baseline LSTM training and validation loss in terms of MSPE.

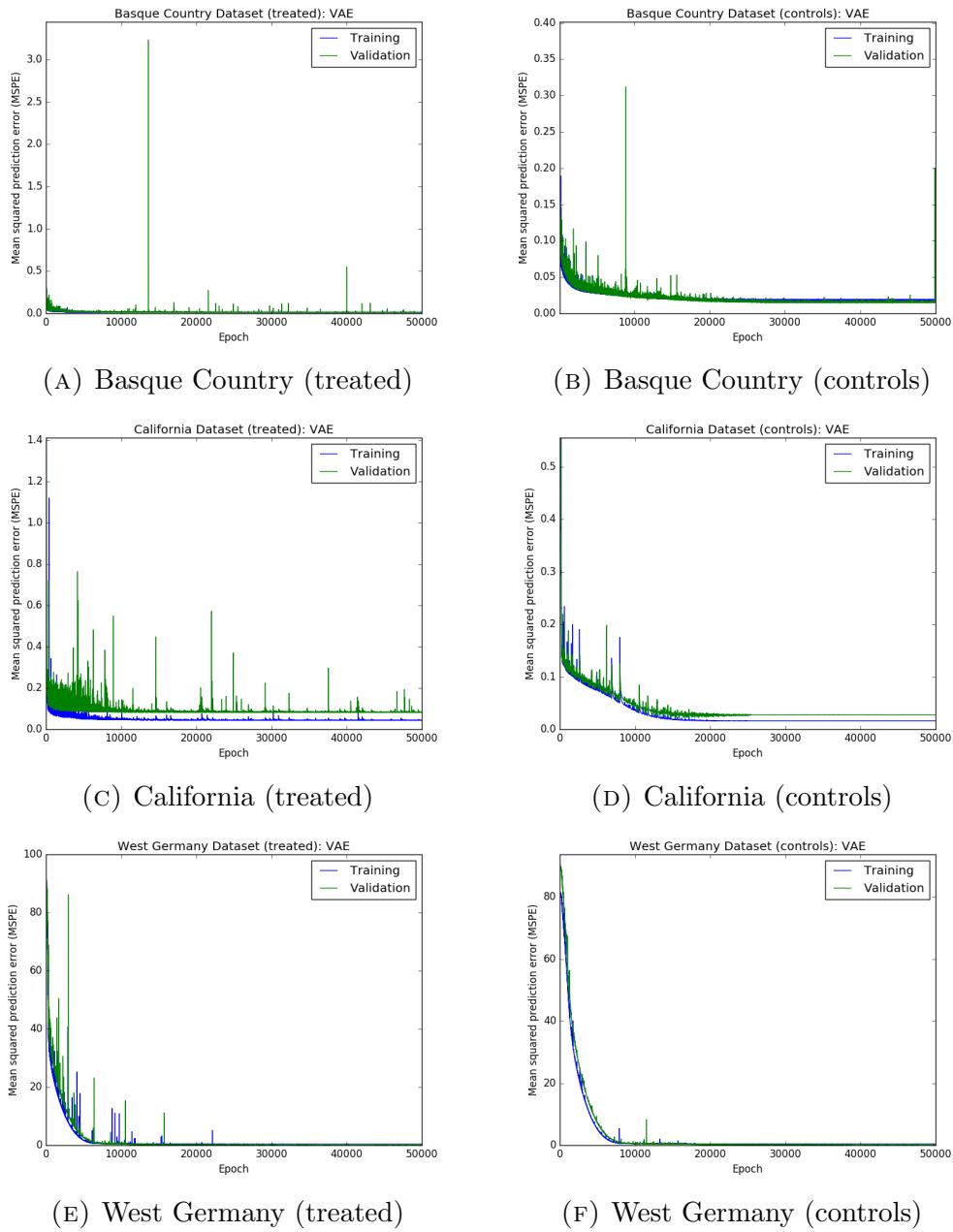


Figure 5: Evolution of VAE training and validation loss in terms of MSPE.

### 3 Estimates on Basque Country data

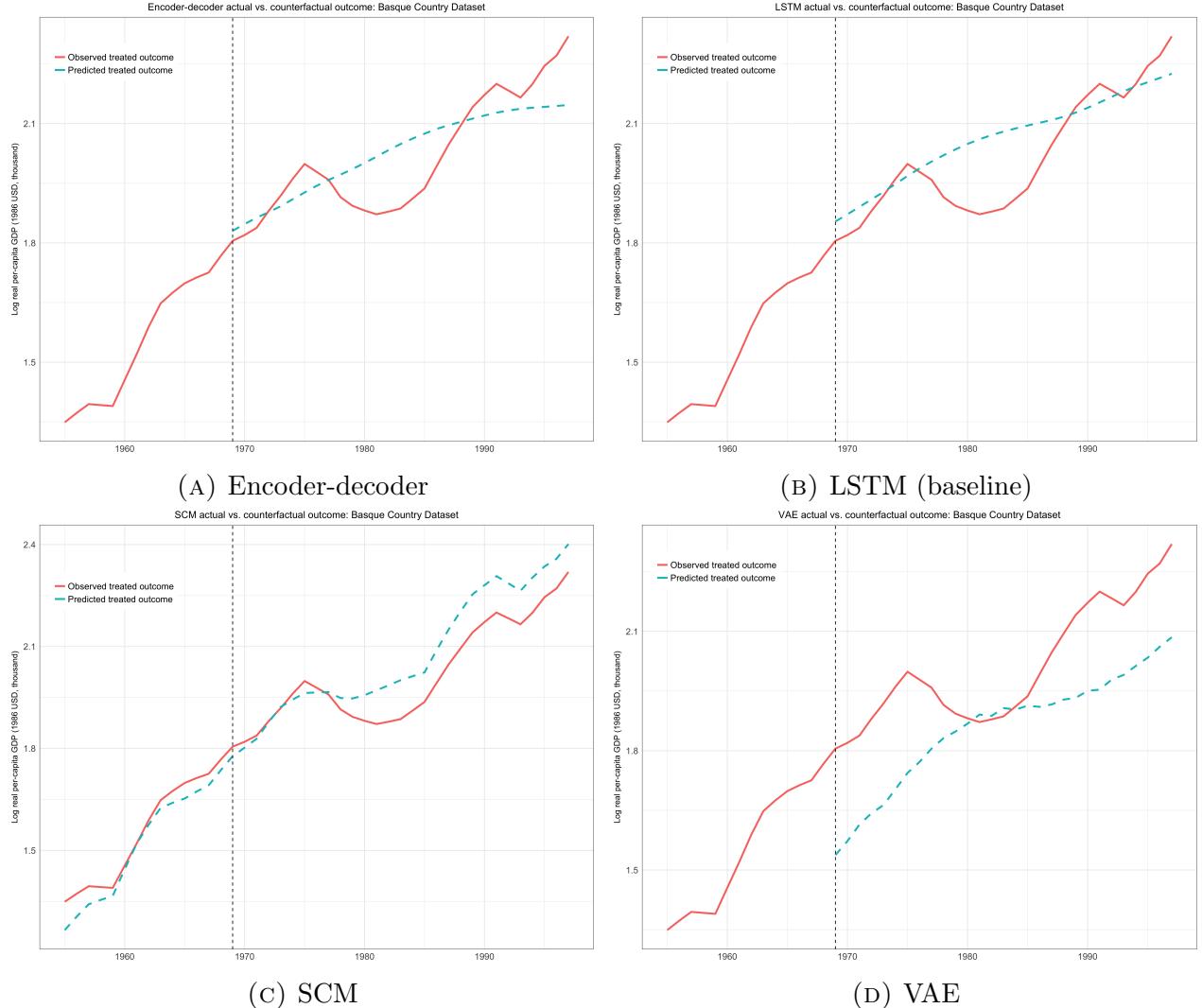


Figure 6: Observed and counterfactual predicted outcomes for treated unit in Basque Country dataset.

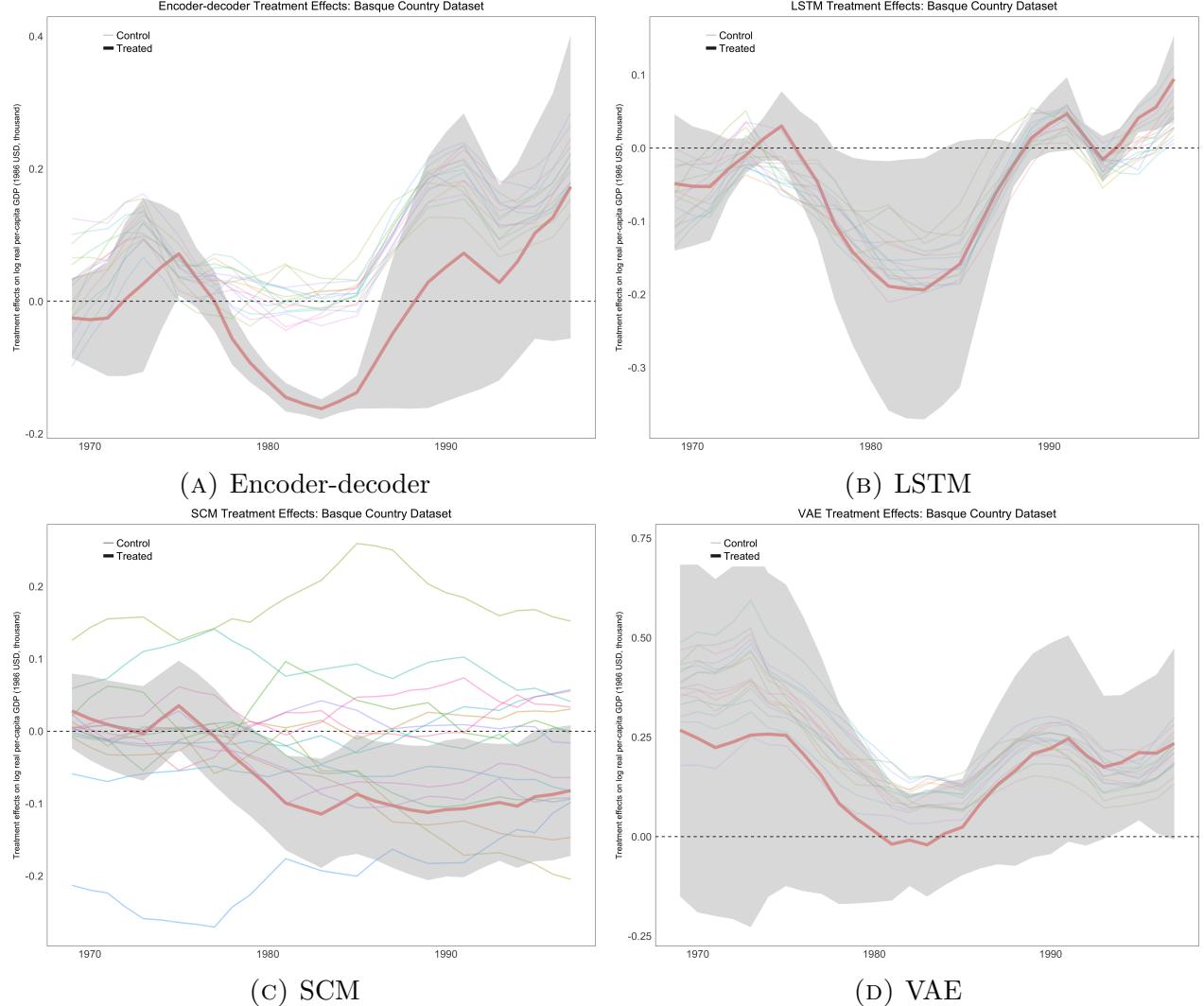


Figure 7: Time-series of post-period treatment effects in Basque Country dataset. Darker line represents the effect on the actual treated unit and each lighter line represents the effects on control units. Shaded regions represent 95% randomization confidence intervals.

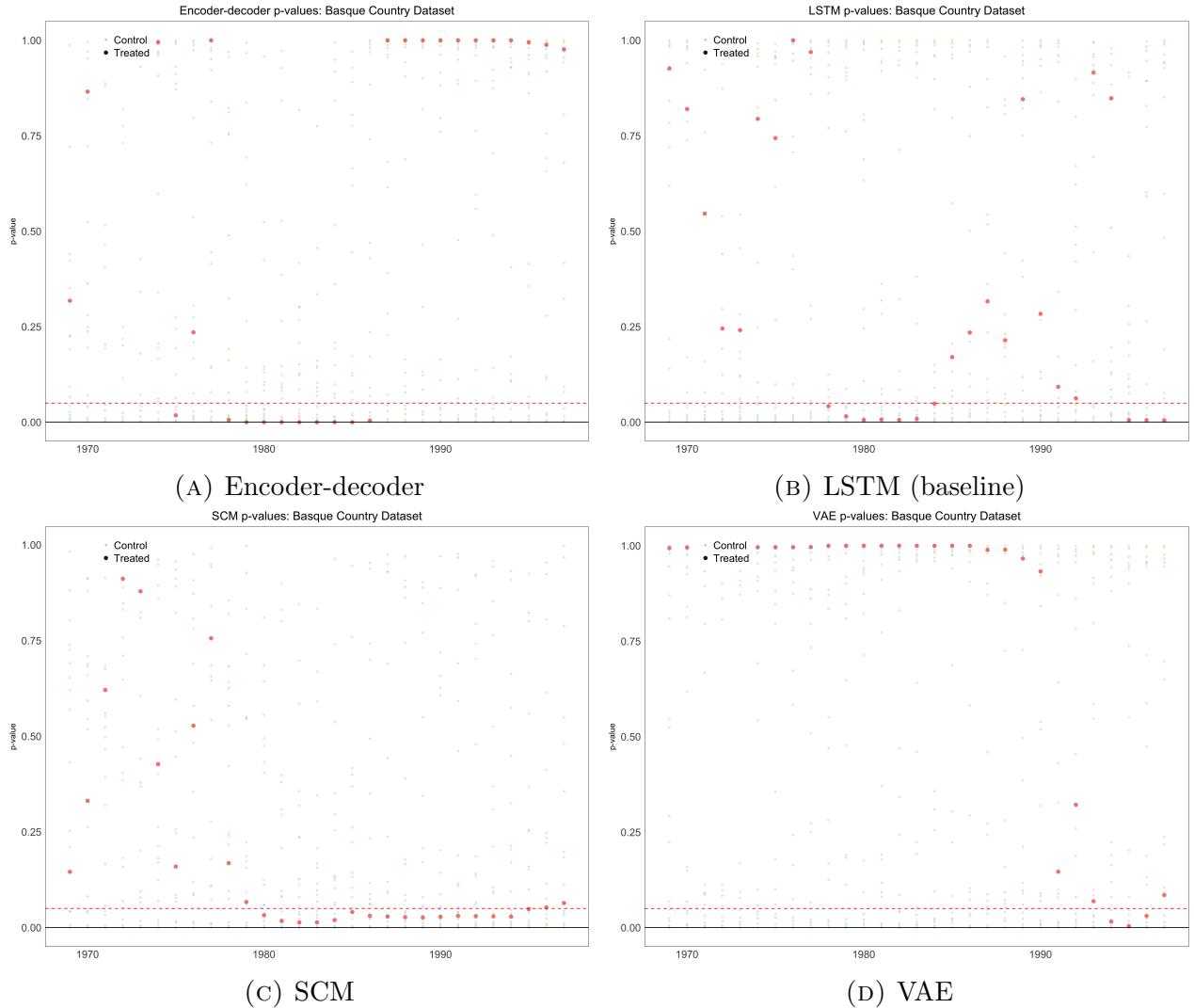


Figure 8: Per-period randomization  $p$ -values corresponding to treatment effects on treated and control units in Basque Country dataset.

## 4 Estimates on California data

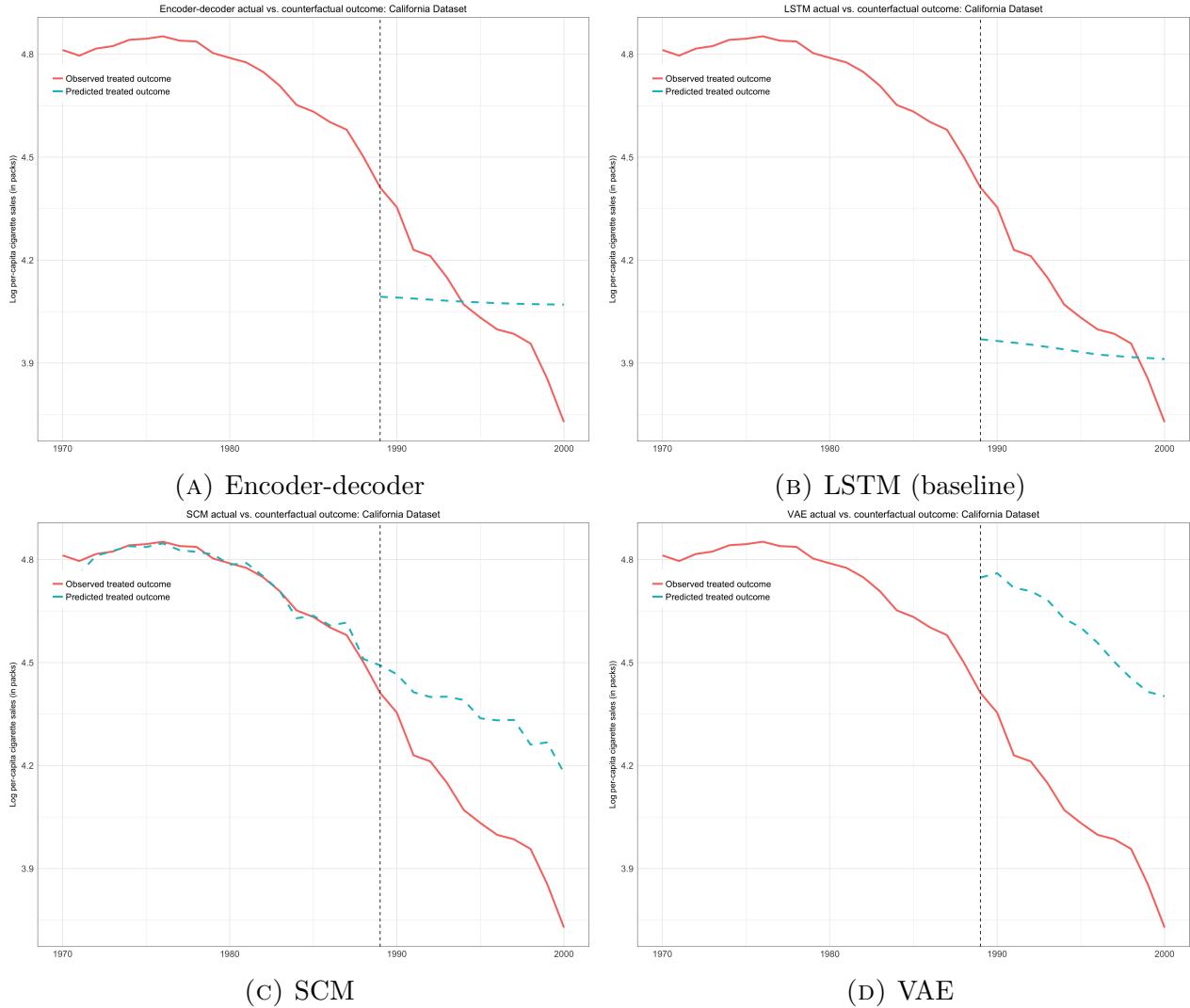


Figure 9: Observed and counterfactual predicted outcomes for treated unit in California dataset.

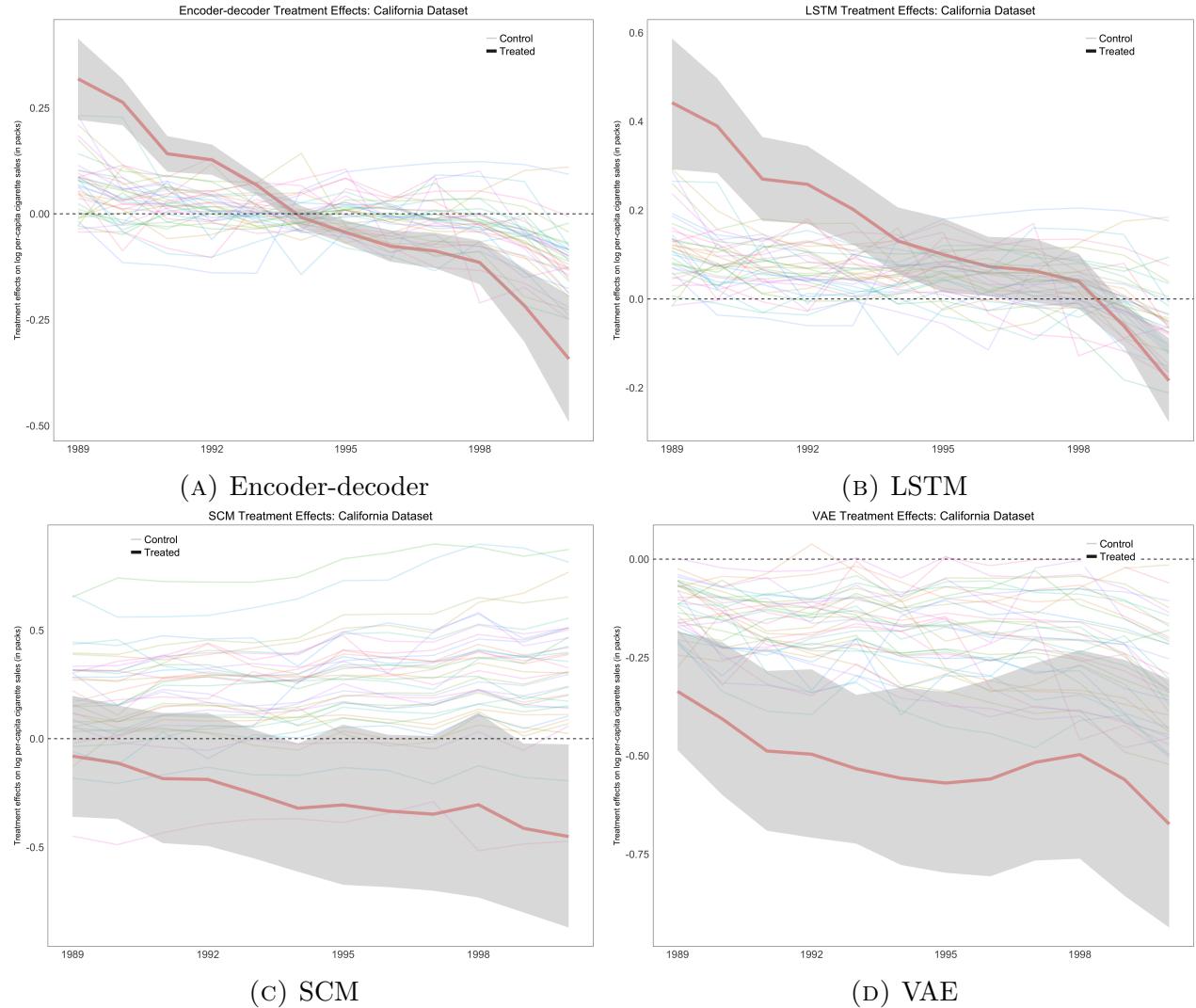


Figure 10: Time-series of post-period treatment effects in California dataset. See notes to SM-Fig. 7.

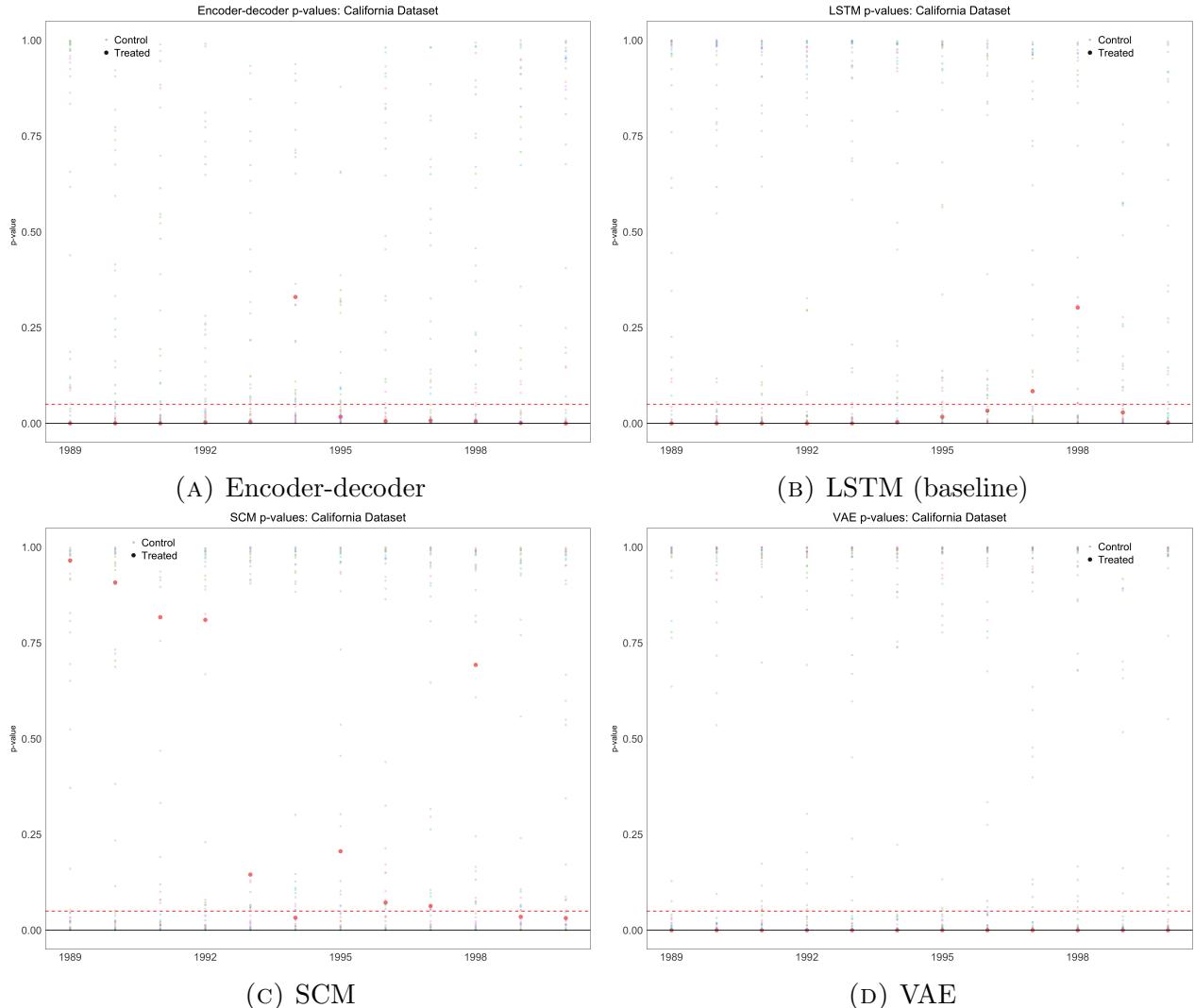


Figure 11: Per-period randomization  $p$ -values corresponding to treatment effects on treated and control units in California dataset.

## 5 Estimates on West Germany data

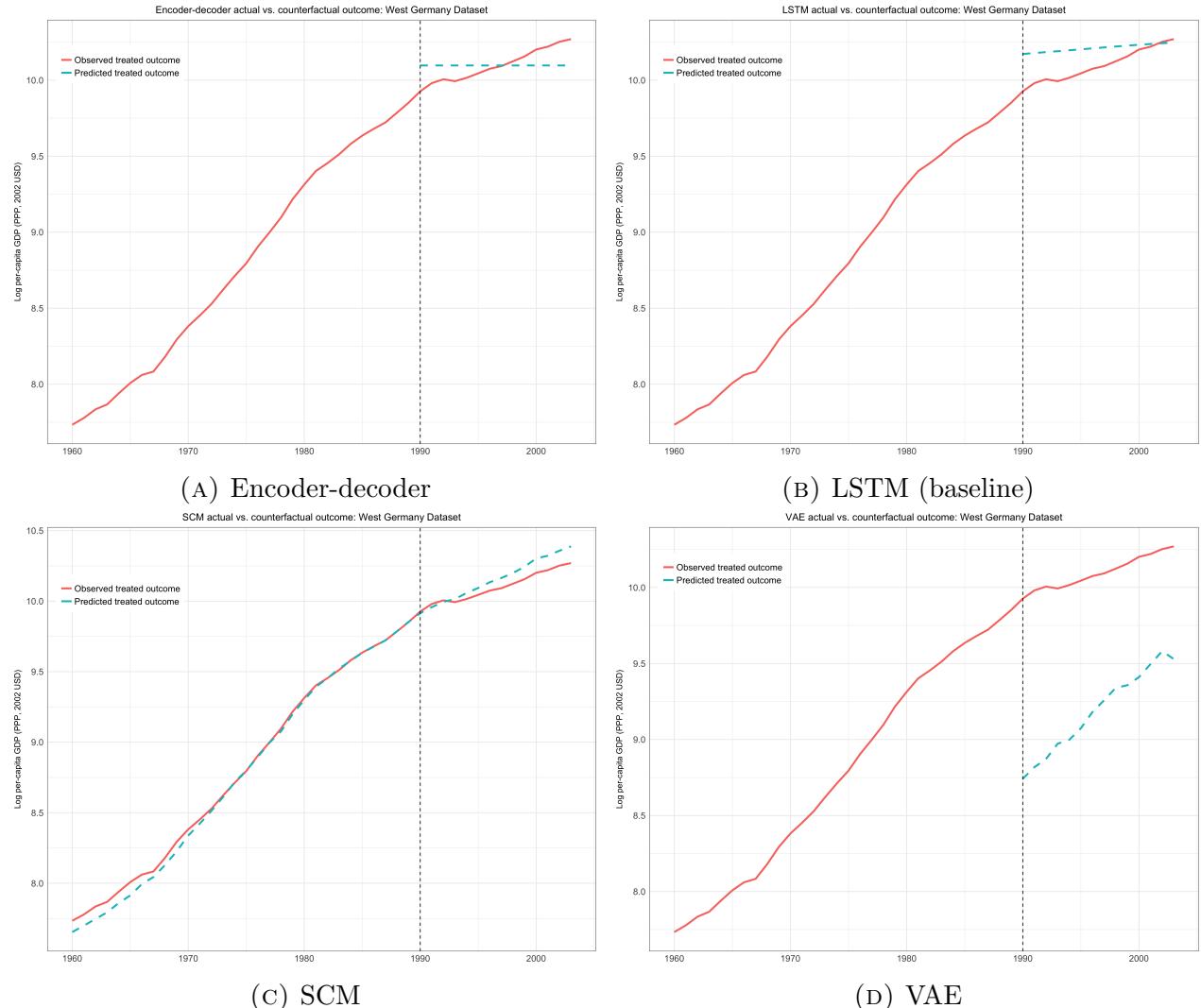


Figure 12: Observed and counterfactual predicted outcomes for treated unit in West Germany dataset.

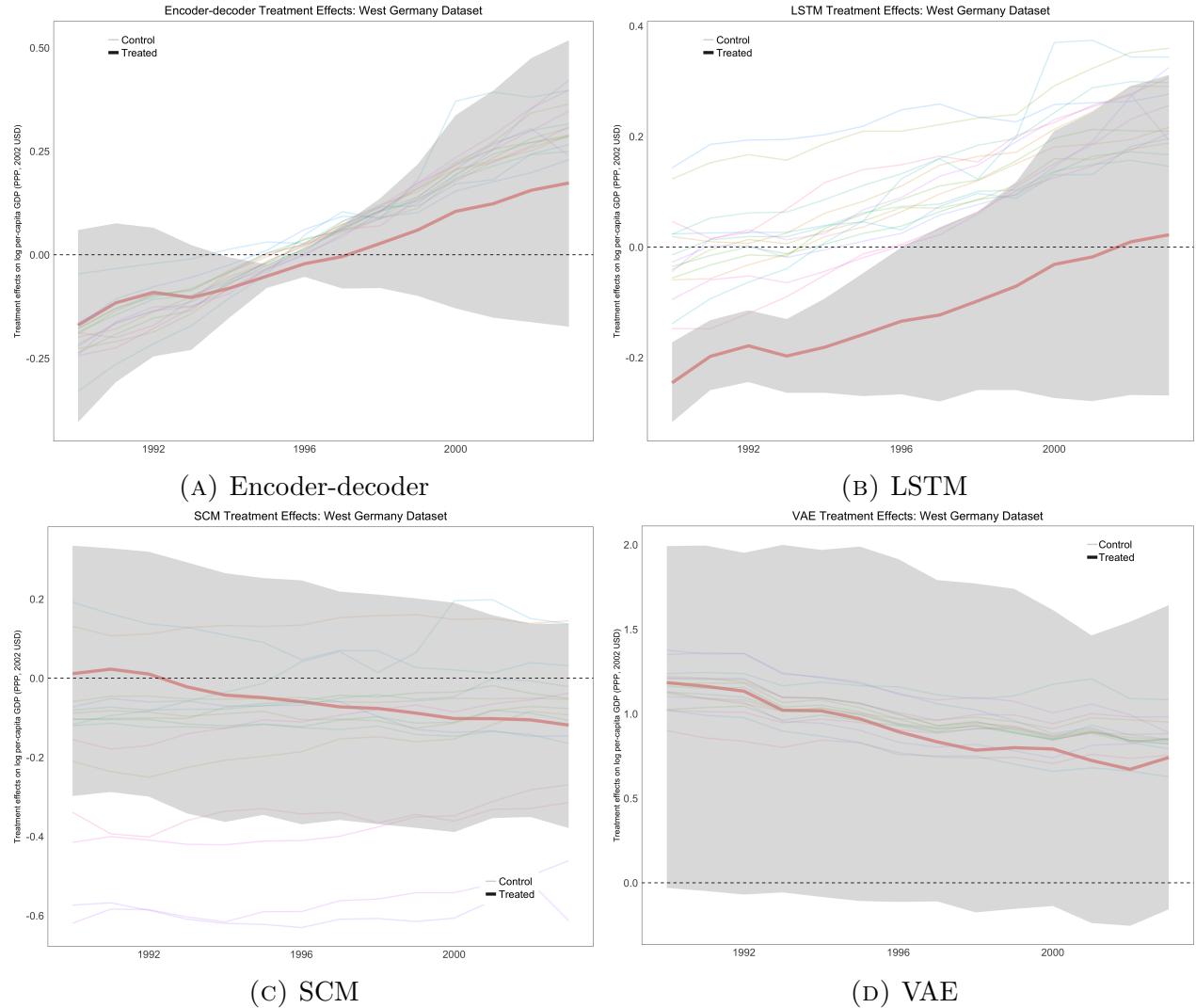


Figure 13: Time-series of post-period treatment effects in West Germany dataset. See notes to Fig. SM-7.

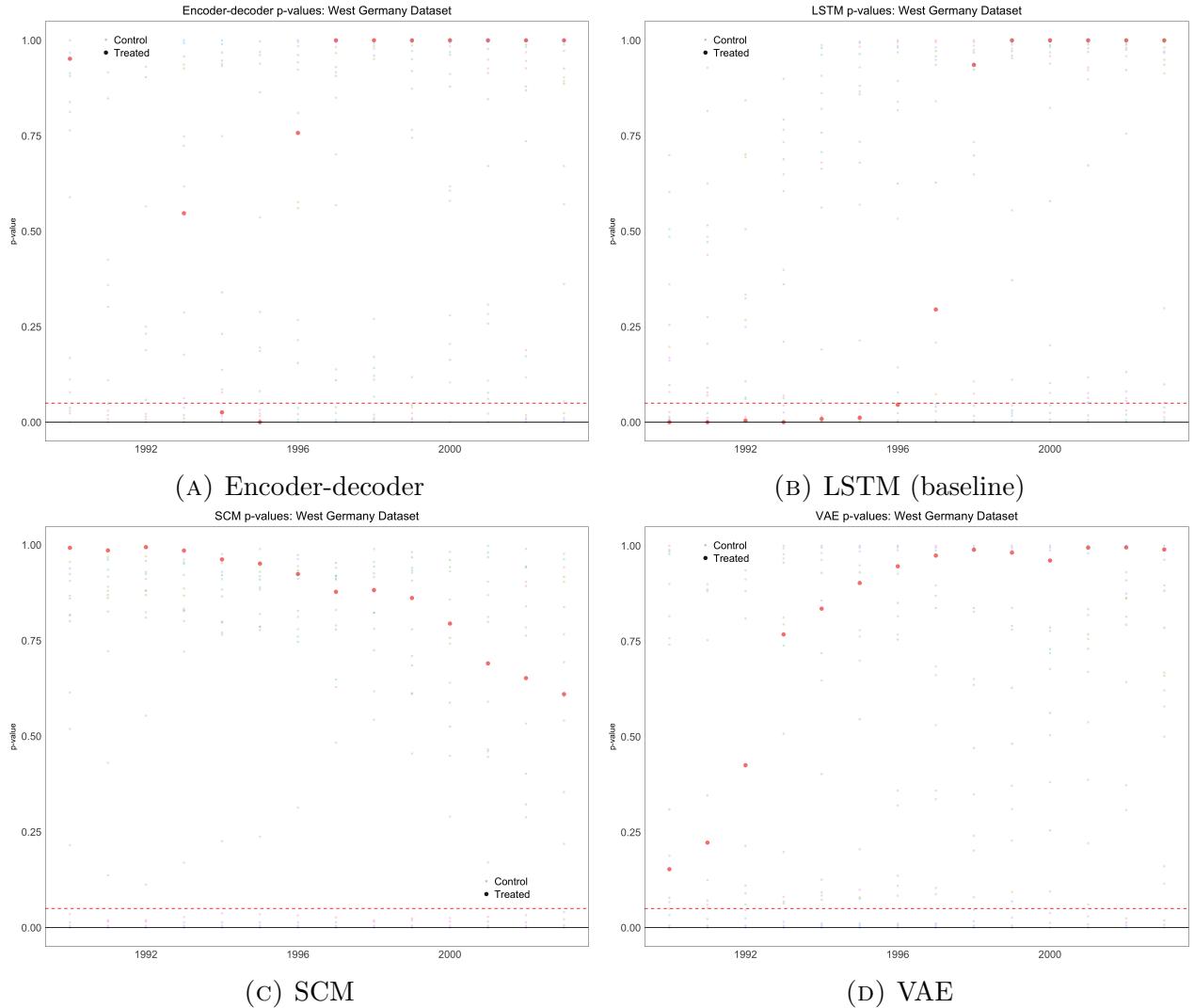


Figure 14: Per-period randomization  $p$ -values corresponding to treatment effects on treated and control units in West Germany dataset.

## 6 RNNs training history: Education spending data

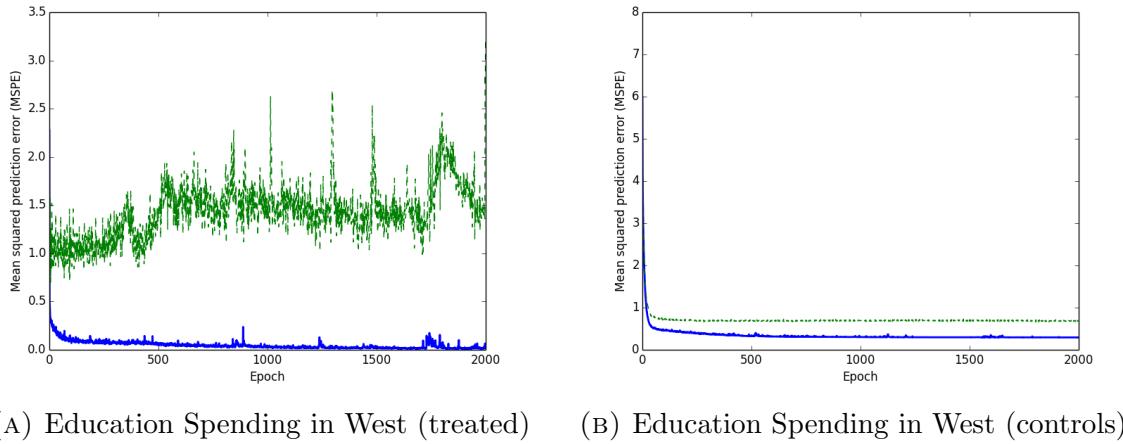


Figure 15: Encoder-decoder networks training (solid line) and validation loss (dashed line).

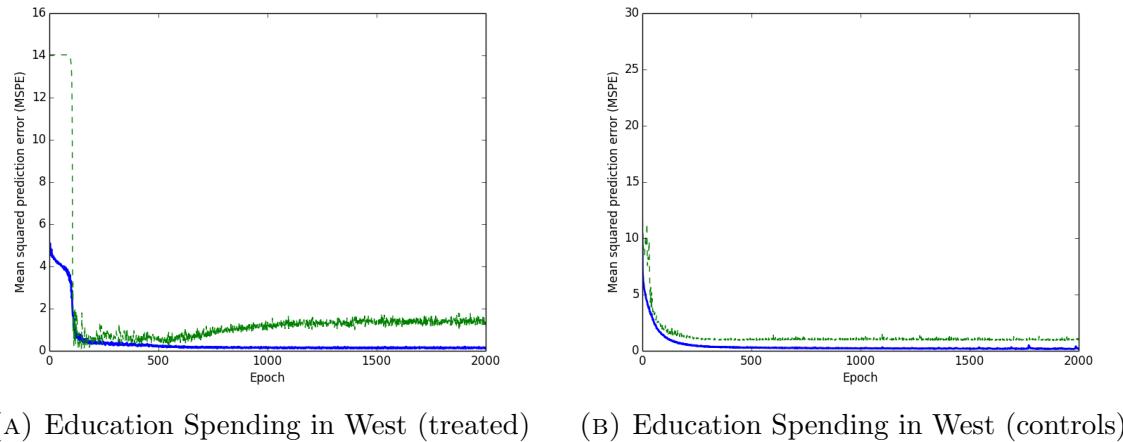


Figure 16: LSTM training (solid line) and validation loss (dashed line).

## 7 Estimates on education spending data

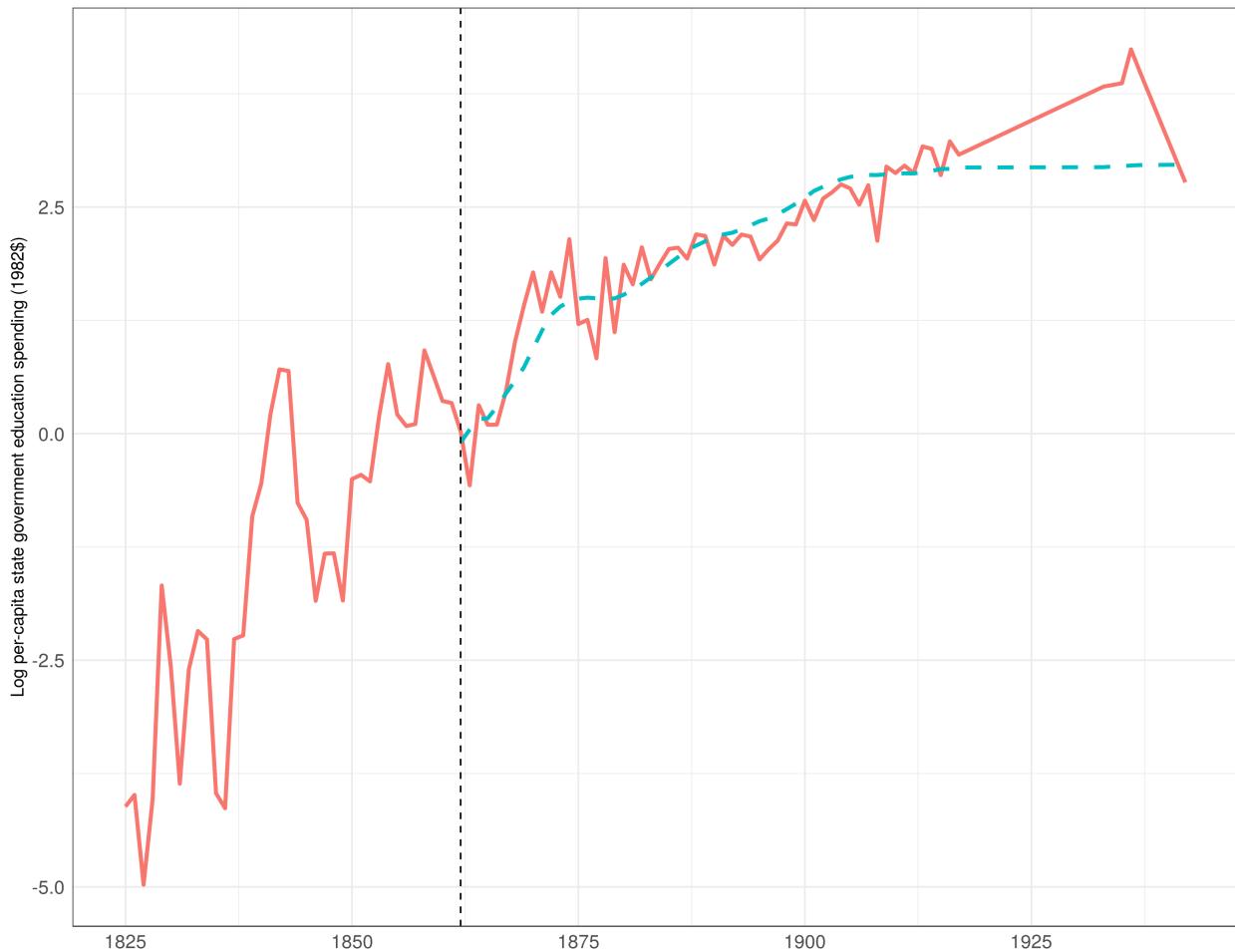


Figure 17: Observed (solid line) and counterfactual predicted (dashed line) outcomes for treated unit. Dashed vertical line represents intervention year.

Table 1: Encoder-decoder FPR and MSPE on state capacity placebo tests.

Outcome \ Measure	MSPE		FPR	
	South	West	South	West
Education spending	$0.44 \pm 0.59$	$0.37 \pm 0.53$	0.23	0.23
Expenditure	$0.75 \pm 0.2$	$0.47 \pm 0.13$	0.31	0.31
Revenue	$0.8 \pm 0.2$	$0.48 \pm 0.2$	0.28	0.28

Note: Errors represent  $\pm$  one standard deviation from the MSPE.

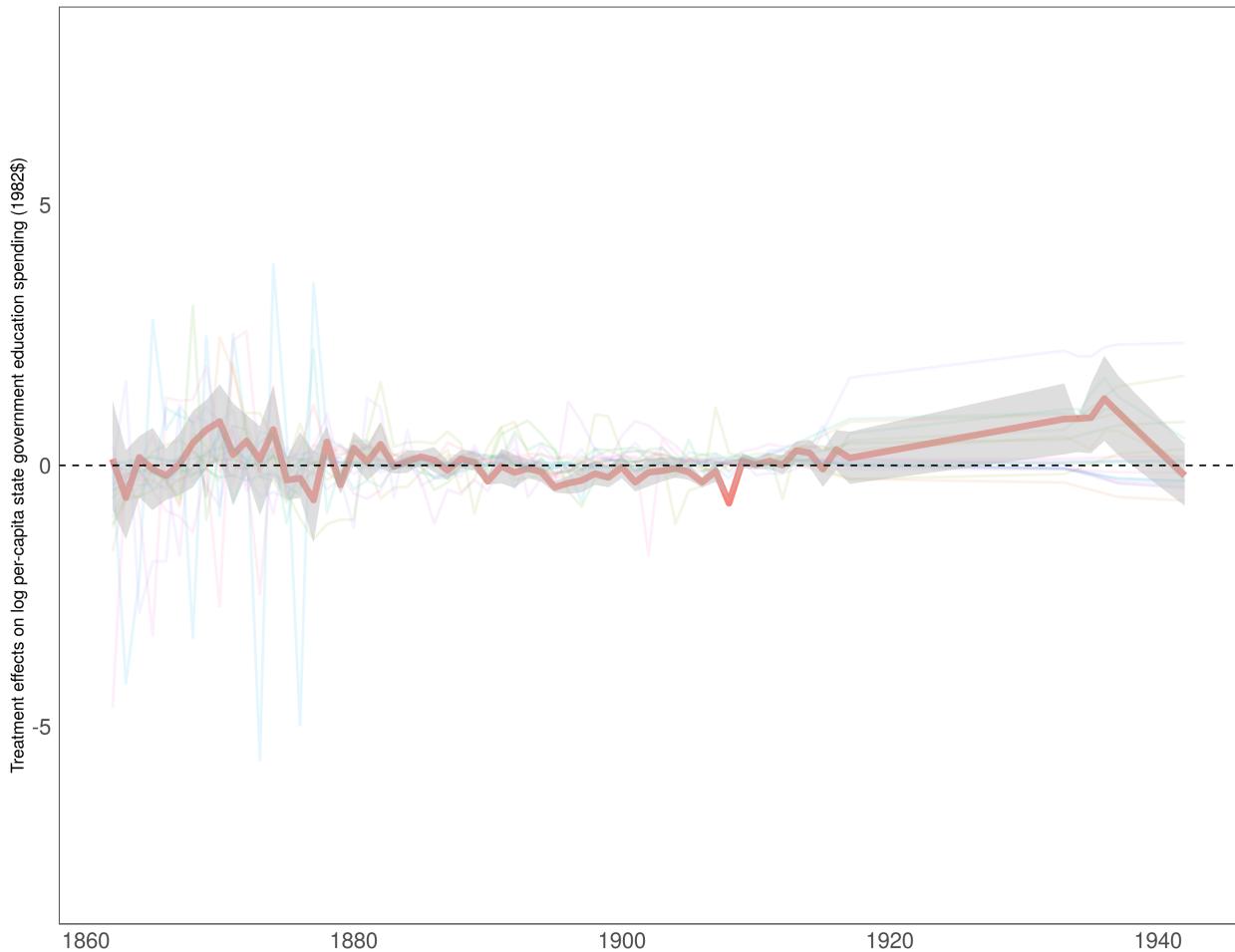


Figure 18: Time-series of post-period treatment effects in state capacity datasets. Darker line represents the effect on the actual treated unit and each lighter line represents the effects on placebo treated units. Shaded regions represent 95% randomization confidence intervals.

Table 2: LSTM FPR and MSPE on state capacity placebo tests.

Outcome \ Measure	MSPE		FPR	
	South	West	South	West
Education spending	$0.55 \pm 0.65$	$0.56 \pm 0.83$	0.22	0.19
Expenditure	$6.66 \pm 5.77$	$0.73 \pm 0.23$	0.36	0.27
Revenue	$1.5 \pm 3.37$	$1.68 \pm 3.72$	0.09	0.15

Note: Errors represent  $\pm$  one standard deviation from the MSPE.

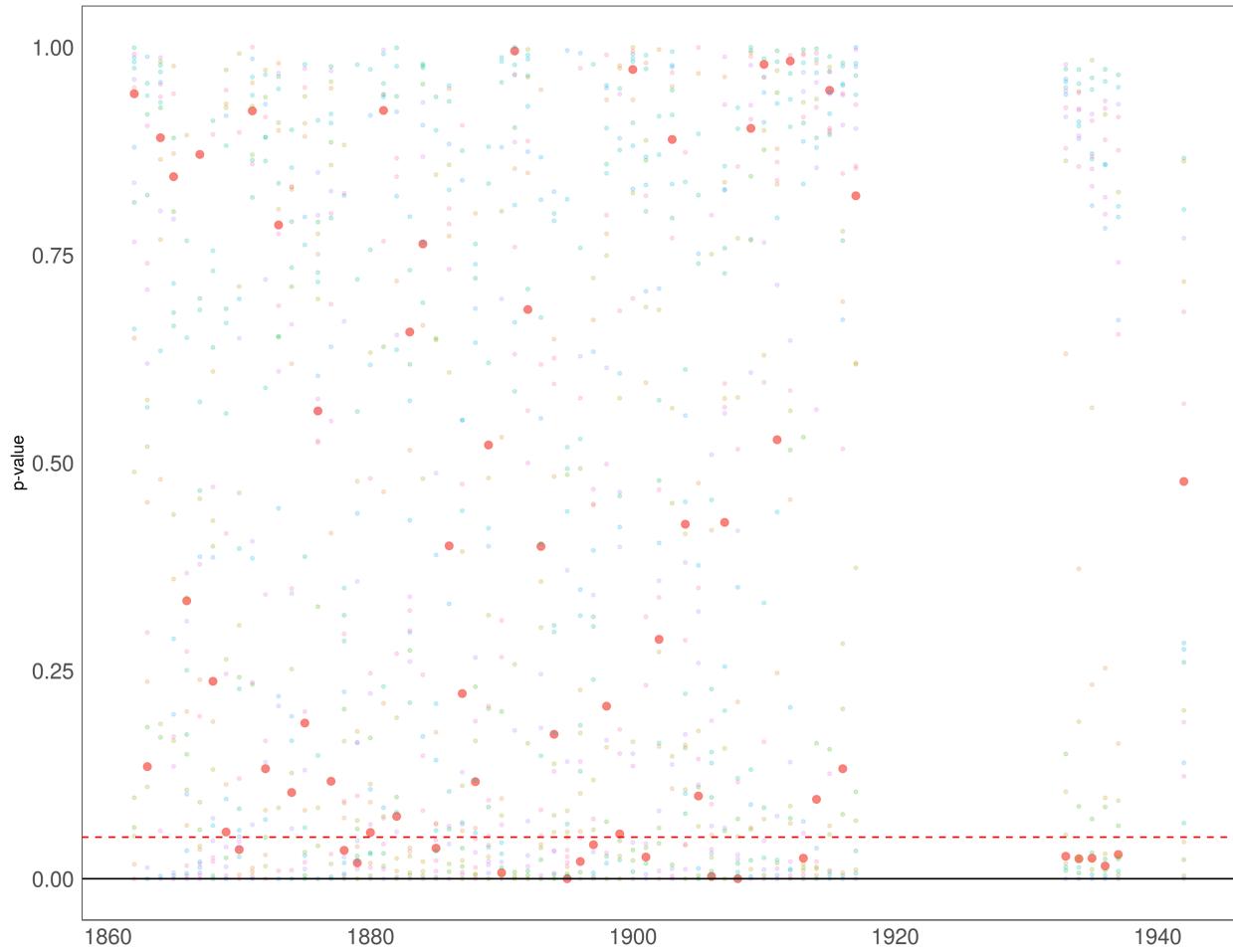


Figure 19: Encoder-decoder networks: Per-period randomization  $p$ -values corresponding to treatment effects on treated and control units in state capacity datasets. Darker dot represents  $p$ -values associated with treatment effects on the actual treated unit and lighter dots represent  $p$ -values associated with the effects on control units