Mathematical representation of the drought decision model - Shiny Version

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1 Model

Year is indexed with t and month is indexed with i. The goal of the player is to maximize the following objective function:

$$\max_{a_t, x_t, y_t} \sum_{t=1}^{T} \pi_t (1+r)^{-t} \tag{1}$$

subject to the following constraints:

$$\theta_t = \theta_{t-1}(1-d_{t-1})(1-y_{t-1}) + \theta_{t-2}\omega_{t-2}(1-x_{t-2}) \tag{2}$$

$$\vec{\alpha}_t = \vec{\alpha}_{t-1}\left(1 - \frac{G_{t-1}}{x}\right), \tag{3}$$

$$\text{where } \max \sum_{i=1}^{12} \alpha_i = 1 \tag{4}$$

$$w_t \in [0, \tilde{w}], \omega_t \in [0, \tilde{\omega}] \tag{5}$$

$$a_t, d_t, x_t, y_t \in [0, 1] \tag{6}$$

$$\lambda_t, p_t, \rho_t, l, \theta_t \geq 0 \tag{7}$$

1.1 Overview of variables:

- Choice Variables

  a_t : Adaptation (%)

  x_t : Percentage of calves sold (%)

  y_t : Percentage of cows sold (%)

- Stock Variables

  θ_t : Herd size (cows, not including calves and yearlings). Herd size in year t is a function of herd size in the previous year, θ_{t-1}, percentage of cows culled in the previous year, z_{t-1}, death rate of cows in the previous year, d_{t-1}, herd size two years prior, θ_{t-2}, weaning percentage two years prior, y_{t-2}, and percentage of calves sold two years prior, m_{t-2}. This implicitly assumes that all yearlings survive to become mature cows.

  α_t : Forage potential (%) (stock variable)

- Response variables

  π_t : Profit in year t ($)

  G_t : Grazing pressure, G_t, depends on the forage production, the herd size, and the level of investment in adaptation. At a sustainable equilibrium, $G_t = 0$.

  w_t : Average weaning weight (pounds) subject to a maximum weight $\tilde{w}$ (default value of $\tilde{w} = 600$ pounds).

  ω_t : Weaning percentage (%) subject to a maximum value of $\tilde{\omega}$ (default value of $\tilde{\omega} = 0.88$).

  λ_t : Forage availability per cow (%), where $\lambda_t = 1$ is sufficient to fully support one cow-calf pair.

- Exogenous variables

  d_t : Death rate of adult cows

  r : Rate of interest

  p_t : Vector of prices for calves, cows, and adaptation

  ρ_t : Precipitation (inches)

  l : Ranch size (acres)

1.2 Description of variables:

1.2.1 Profit

Abstracting away from changes in herd assets, we consider the revenues to be cash flows in a given year and costs to be cash outlays in a given year:

$$\pi_t = R_t - C_t \tag{8}$$

There are four potential sources of revenues: calf sales, cow sales, earnings from interest on cash assets, and indemnities from rainfall-index insurance.

$$R_t = R_{calves,t} + R_{cows,t} + R_{interest,t} + R_{insurance,t} \tag{9}$$

There are four potential sources of costs: normal cow-calf operating costs, drought adaptation costs, interest on negative cash assets, and premiums for rainfall-index insurance.

$$C_t = C_{op,t} + C_{a,t} + C_{interest,t} + C_{insurance,t} \tag{10}$$

1.2.2 Calf Revenues

Calf revenues are a function of the price per pound for calves, $p_{c,t}$, the number of calves in the herd at the end of the growing season, c, the average weaning weight of the calves in the herd, w_t, and the percentage of calves that are chosen to be sold, x_t.

$$R_{calves,t} = p_{c,t} * c_t * w_t * x_t \tag{11}$$

In the simulation, calf prices are held constant at $p_c = \$1.40$.

The number of calves depends on the herd size, θ_t, and the weaning percentage at the end of the season, ω_t:

$$c_t = \theta_t * \omega_t \tag{12}$$

Weaning percentage depends on the health of the herd in year t and year $t-1$, which is assumed to be fully based on total available forage per cow, λ_t and λ_{t-1}. In the simulation, the weaning percentage maximum is 88%, represented by $\tilde{\omega}$. Weaning percentage in year t, ω_t, is given by:

$$\omega_t = \begin{cases} \tilde{\omega} & \text{if } \lambda_t, \lambda_{t-1} \geq 1 \\ \tilde{\omega} * \lambda_t^{(1/4)} & \text{if } \lambda_t < 1 \wedge \lambda_{t-1} \geq 1 \\ \tilde{\omega} * \lambda_{t-1} & \text{if } \lambda_t \geq 1 \wedge \lambda_{t-1} < 1 \\ \tilde{\omega} * \lambda_t^{(1/4)} * \lambda_{t-1} & \text{if } \lambda_t, \lambda_{t-1} < 1 \end{cases} \tag{13}$$

Average weaning weight, w_t, is determined by forage availability, λ_t, and the maximum weaning weight, $\tilde{w}$.

$$w_t = \begin{cases} \tilde{w}\left(1 - \frac{(1-\lambda_t)}{3}\right) & \text{if } \lambda_t < 1 \\ \tilde{w} & \text{if } \lambda_t >= 1 \end{cases} \tag{14}$$

Forage availability, λ_t, is further described in section 1.2.4.

1.2.3 Cow Revenues

Cow revenues are not dependent on weight as culled cows are assumed to be sold for a standard, per-cow price.

$$R_{cows,t} = p_{\theta,t} * \theta_t * y_t \tag{15}$$

The price of cows in the simulation is held constant at \$850/cow.

1.2.4 Insurance Revenues

1.2.5 Forage

Forage production per pair, F_t, is the dot product of forage potential, α_t, and the monthly rainfall, ρ_t, divided by the ratio of acres per cow and carrying capacity. α_t and ρ_t are vectors with a length of twelve representing each month of the year.

In an average rainfall year with undegraded forage potential, if the herd size is equal to the carrying capacity, then $F_t = 1$. As rainfall or forage potential increases (decreases), F_t increases (decreases). As herd size increases (decreases), F_t decreases (increases).

$$F_t = \frac{\alpha_t \cdot \rho_t}{\%K} \tag{16}$$

$$\%K = \frac{\frac{l}{\theta_t}}{K} \tag{17}$$

where l is the number of acres grazed, θ_t is the size of the herd (head of cows, not including calves or yearlings), and K is defined as the sustainable carrying capacity of the ranch in an average year (acres/cow).

When the herd is at its carrying capacity ($l/\theta_t = K$), $\%K = 1$. When the carrying capacity is exceeded, $\%K > 1$, leading to a reduction of forage per pair. In other words, at any given level of forage potential and rainfall, the forage per pair is smaller when the herd increases. Likewise, when $l/\theta_t < K$, the forage per pair increases (holding forage and rainfall constant).

Perfect adaptation, $\tilde{a}_t$, is defined as the gap between full forage production per pair ($F_t = 1$) and actual forage production per pair. It is a unitless measure that is best interpreted as a percentage.

$$\tilde{a}_t \equiv 1 - F_t \tag{18}$$

Actual adaptation, a_t, is defined as the portion of the gap between full forage production per pair and actual forage production per pair that is filled with drought adaptation measures, such as the purchase of additional feed. We define this as the ratio of expenditures on adaptation and cost of perfect adaptation scaled by the perfect adaptation, $\tilde{a}_t$. We assume that the costs of adaptation are linear.

$$a_t \equiv \frac{\text{Expenditures on adaptation}}{C(\tilde{a}_t)} \tilde{a}_t \tag{19}$$

Grazing pressure, G_t, depends on the forage production, the herd size, and the level of investment in adaptation. At a sustainable equilibrium, $G_t = 0$. We define this equilibrium as follows: when forage production per pair plus drought adaptation measures are equal to 1, then there is zero grazing pressure. As forage production per pair plus drought adaptation measures fall below 1, then there is positive grazing pressure leading to a degradation of forage potential. As forage production per pair plus drought adaptation measures rise above 1, then there is negative grazing pressure leading to a recovery of forage potential if it is not already at full health ($\sum \alpha_t = 1$).

$$G_t \equiv 1 - (F_t + a_t) \tag{20}$$

Forage potential increases or decreases based on the grazing pressure on the land. If $G_t < 0$, then forage potential increases (subject to a maximum, $\bar{\alpha}$). If $G_t > 0$, then forage potential decreases. For each monthly value in the vector α_t, the previous years' forage value is multiplied by a percentage that is determined by the grazing pressure and a scaling factor, x. The parameter x will be determined by testing the equation for a variety of values. The end goal is to decrease the forage potential by 1-2% per year when grazing pressure is high, as recommended by the team of ranching experts at the University of Wyoming.

When there is negative grazing pressure, the forage potential may increase to a maximum of $\bar{\alpha}$ which is determined by the MLRA plant growth curves. The sum of the elements of the $\bar{\alpha}$ vector must equal to 1.

$$\alpha_t = \alpha_{t-1}\left(1 - \frac{G_{t-1}}{x}\right) \tag{21}$$

$$\max \sum_{i=1}^{12} \alpha_i = \sum_{i=1}^{12} \bar{\alpha}_i = 1 \tag{22}$$

where i indexes the twelve elements of the vector that represent the forage growth potential for each month.

2 Scripts

2.1 global.R

1. Sources other scripts
2. Javascript coding
3. Populate a new environment with rainfall gauge info: `getStationGauge()`
4. Populate a new environment with constant (user) variables: `getConstantVars()`
5. Setting additional variables: acres, start years, simulation lengths
6. Create state variables for practice and full runs: `getSimVars()`
7. Create lists of variables for practice and full runs: `practiceRuns`, `simRuns`
8. Establish additional settings

2.2 load.R

Loads necessary packages

2.3 shinySupportFunctions.R

1. `getJulyInfo` function: Calculates available and predicted forage in July, creates a UI to display info and allows user to select adaptation level.
   - Called in `simUI.R`
2. `getCowSell` function: Creates a UI for the user to select how many cows and calves to sell. Called in `simUI.R`.
3. `shinyInsurance` function: Calculates premium and indemnification for a specific year and grid cell. Currently, returns are summed but this could be done on a index interval basis instead.

2.4 forageFunctions.R

- `getForagePotential` function: Returns an index representing annual forage production for a given gridcell or station gauge's annual precipitation record. Called in `calfCowFunctions.R`.
- `whatIfForage` function: calculates expected forage for a given scenario. Called in `shinySupportFunctions.R` and `simUI.R`.

- `getMLRAWeights` function: Computes forage potential weights using the mean of plant growth curves by MRLA for a specified state. Called in `initialFunctions.R`.
- `COOP_in_MLRA` function: Returns the MLRA in which a specified coop site is located. Called in `initialFunctions.R`.

2.5 adaptationFunctions.R

- `calculateAdaptationIntensity` function: Takes forage potential and an adaptation intensity factor to provide a scalar of drought action. If forage potential is above 1 (no drought), then this variable goes to 0 (no adaptation). Called in `shinySupportFunctions.R` and `simUI.R`.

2.6 costRevenueFunctions.R

- `calculateExpSales` function: Calculates expected calf revenues for non-drought year.
- `calculateFeedCost` function: Calculates the costs of purchasing additional feed. Called in `getAdaptCost` in `costRevenueFunctions.R`.
- `CalculateRentPastCost` function: Calculates the costs of renting pasture and trucking pairs. Called in `getAdaptCost` in `costRevenueFunctions.R`.
- `getAdaptCost` function: Calculates the cost of adaptation based on strategy, intensity needed, days, and herd size. Called in `shinySupportFunctions.R` and `simUI.R`.

2.7 initialFunctions.R

- `getConstantVars` function: Reads in constant variables into a `constvars` environment using the file `data/constant_vars.csv`. Called in `global.R`.
- `getSimVars` function: Creates list of simulation variables. Called in `global.R`.
- `getStationGauge` function: Returns precipitation record and locational attributes for the target location. Default is Central Plains Experimental Range (CPER) but alternative locations at COOP sites across Colorado may be specified. Called in `global.R`.
- `createResultsFrame` function: This function creates a theoretical previous result from the year before the simulation begins right now this assumes that there was no drought the year before the simulation and revenues were 0. These assumptions are likely unrealistic and can be adjusted to accomodate different scenarios. Called in `shinySupportFunctions.R` and `server.R`.

2.8 calfCowFunctions.R

- `AdjWeanSuccess` function: Adusts weaning success downward for the year of the drought and the following year. Called in `simUI.R`.
- `calfDroughtWeight` function: If forage potential is less than 1, then the calf weight is less than the optimal weight. Called in `shinySupportFunctions.R` and `simUI.R`.
- `calfWeanWeight` function: Computes calf weights based on station/grid cell forage potential for a n-year period. Called in `initialFunctions.R`.
- `shinyHerd` function: calculates the size of herd for the shiny app. Called in `simUI.R`.

2.9 assetFunctions.R

- `CalcCowAssets` function: Calculates the cow assets for each year. Called in `initialFunctions`.

3 Function Details

Key functions in the model are listed and described in alphabetical order.

3.1 AdjWeanSuccess

- Function: `AdjWeanSuccess`
  - Description: Adjusts weaning success downward for the year of the drought and the following year. If forage production falls below 1 in year $t = 1$, then weaning percentage falls slightly in year $t = 1$ and more drastically in year $t = 2$. If forage production falls below 1 in a year $t = 1$ where weaning percentage was already decremented because of previous forage production deficits or insufficient culling, then weaning percentage falls further in years $t = 1, 2$ than it would have if the starting point was at the maximum weaning percentage.
  - Inputs: `totalForage`, `totalForage1ya`, `normal.wn.succ`
  - Output: `wn.succ`
  - Assumptions: This equation is based on what I consider to be "reasonable" estimates of weaning success based on forage potential. These fall roughly in line with body condition scores from the *Nutrient Requirements of Beef Cattle*, but are only ballpark estimates.

The weaning percentage maximum is 88%, represented by $\tilde{\omega}$. Weaning percentage in year t, ω_t, is given by:

$$\omega_t = \begin{cases} \tilde{\omega} & \text{if } F_t, F_{t-1} \geq 1 \\ \tilde{\omega} * F_t^{(1/4)} & \text{if } F_t < 1 \wedge F_{t-1} \geq 1 \\ \tilde{\omega} * F_{t-1} & \text{if } F_t \geq 1 \wedge F_{t-1} < 1 \\ \tilde{\omega} * F_t^{(1/4)} * F_{t-1} & \text{if } F_t, F_{t-1} < 1 \end{cases} \tag{23}$$

Code:

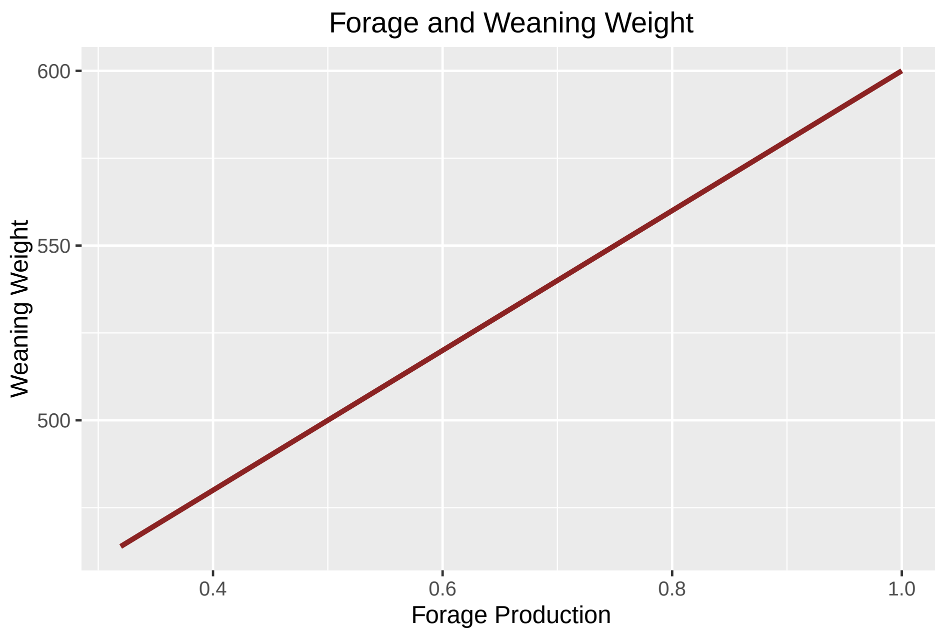
```
wn.succ <- NULL
if(totalForage < 1 & totalForage1ya >= 1){
  wn.succ <- normal.wn.succ * totalForage^.25
}else if(totalForage >= 1 & totalForage1ya < 1){
  wn.succ <- normal.wn.succ * totalForage1ya
}else if(totalForage < 1 & totalForage1ya < 1){
  wn.succ <- normal.wn.succ * totalForage1ya * totalForage^.25
}else if(totalForage >= 1 & totalForage1ya >= 1){
  wn.succ <- normal.wn.succ
}
```

3.2 calfWeanWeight

Function: `calfDroughtWeight`

- Description:
- Inputs:
  - $\bar{w}$ = `normal.wn.wt` (average calf weight at weaning under normal conditions (pounds)),
  - λ = `totalForage` (percentage of normal forage from both rangeland production and supplemental feed)
- Output: w_t = `wn.wt` (calf weight at weaning in year t)

$$w_t = \begin{cases} \bar{w}\left(1 - \frac{(1-\lambda)}{3}\right) & \text{if } \lambda < 1 \\ \bar{w} & \text{if } \lambda >= 1 \end{cases} \tag{24}$$



3.3 getStationGauge

The model is currently set for a default to the Central Plans Experimental Range (CPER), but alternative locations for COOP sites in Colorado may also be used.

- Function: `getStationGauge`
  - Description: Returns precipitation record and locational attributes for the target location. Default is Central Plains Experimental Range (CPER).
  - Inputs: `target.loc` (target location. default set to `CPER`)
  - Outputs: a list called `station.gauge` that contains `monthlyPrecipWeights` (the weight given to precipitation in each month in terms of how much it contributes to annual forage), `precip` (station or gauge precipitation data from 1948 to 2015), `gridCell` (Target Grid Cell for reading in PRF index values at a given point in time).

If CPER (default):

1. Monthly precipitation weights are based on the plant growth curves for rangeland with loamy soils in major land resource area 67B (see Figure 1, an area that encompasses a large portion of Eastern Colorado including the CPER site (Site ID: R067BY002CO). We use the plant growth curve for Western Wheatgrass, Blue Grama, Green Needlegrass, Fourwing Saltbush Plant Community (Growth curve number

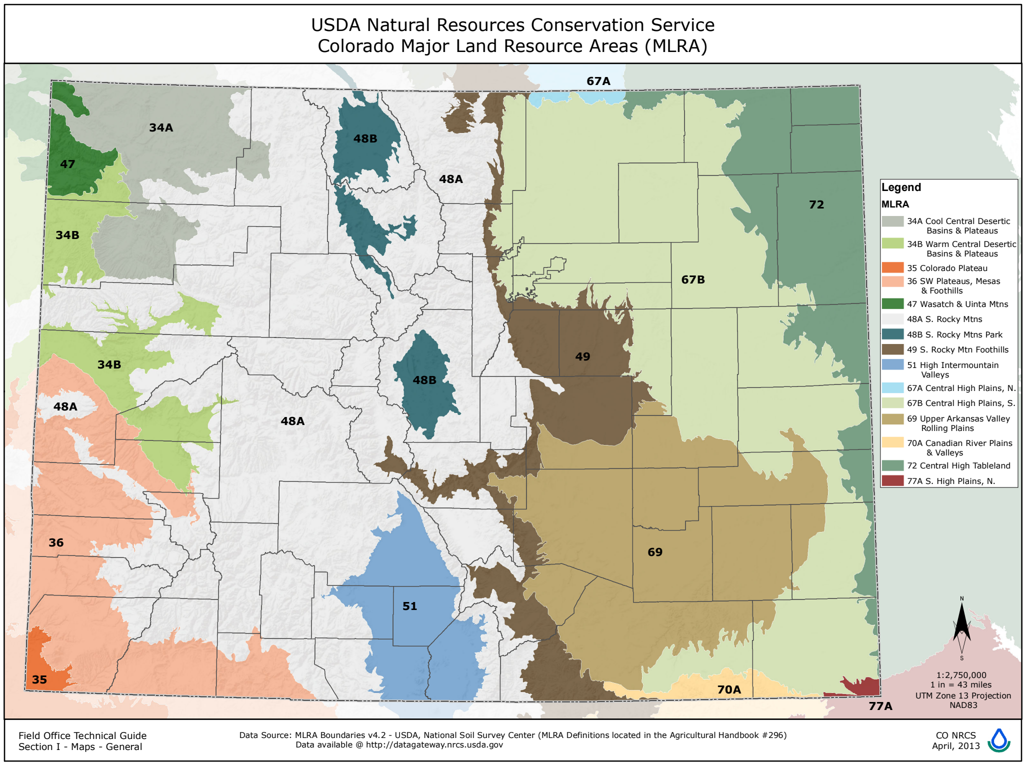


Figure 1: Major Land Resource Areas for Colorado

CO6701), which is well suited for grazing and is maintained with properly stocked prescribed grazing.

These monthly weights are the percentage of plant growth that is expected in each month during a normal year. Under normal conditions, total annual production averages 1300 pounds per acre. These weights add up to 1.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
0	0	0.02	0.08	0.2	0.28	0.15	0.12	0.1	0.05	0	0

2. Station Gauge, historical precipitation totals dating back to 1948, which are also read in from the Excel model. Precip totals are collected at CPER itself and do not rely on precip data from COOP sites.

3. The target grid cell 25002 is assigned to the PRF grid cell (assuming this is the correct grid cell)

4 Default Model Values

- Starting herd size: 600 cows
- Ranch size: 3000 acres
- Carrying capacity: 5 acres/cow
- Maximum calf weaning weight: 600 lbs
- Default calf sale rate: 75%
- Maximum weaning percentage: 88%
- Cow price: $850/cow
- Calf price at weaning: $1.40/pound
- Cow mortality rate: 4%
- Practice round years: 1951-1955
- Game round years: 1999-2008
- Household expenses: $60,000
- Starting cash assets: $90,000
- Operating costs: $500/cow
- Insurance Coverage Level: 90%
- Interest rate: 5%