Safe Asset Shortages and Aggregate Demand in a Global Economy*

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Abstract

This paper investigates the consequences of global safe asset shortages for aggregate economic activity. We build a model with two countries (Home and Foreign) and emphasize two heterogeneities: (i) Home has more developed financial markets and (ii) a smaller number of risk-averse agents compared to Foreign. We find that safe asset demand by Foreign causes a "safety trap" (liquidity trap in the market for safe assets) with depressed output in both countries. Under imperfect fiscal policy (taxes are distortionary and imperfectly adjustable over time), there is a trade-off for safe public debt provision: it expands output everywhere but generates wasteful distortions in the issuing country. Thus, an externality arises that results in the underprovision of safe public debt from a global point of view. Finally, we explore whether Home has an incentive to close down its capital account. This has the upside of avoiding a safety trap, yet the downsides of paying higher debt servicing costs and foregoing net interest income on its asset position.

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1 Introduction

The motivation for this paper originates from two stylized macroeconomic facts: First, the large demand for safe stores of value in the global economy compared to a relatively scarce supply. Second, the fact that a large fraction of safe assets in the global economy is supplied by the US, whereas most of the demand for safe assets comes from China, oil producing economies and Japan. These facts have been identified by the literature concerned with "global imbalances" yet this literature remains silent on if (and how) these phenomena impact aggregate economic activity. Recently, Caballero and Farhi (2015) – referred to CF (2015) below – have taken a step into this direction: They analyze how excess demand for safe assets can drag the economy into a liquidity trap in the market for safe assets ("safety trap"), causing a drop in output. However, their paper focuses exclusively on a closed economy environment. This is the gap our work aims to fill: To study the effects of safe asset shortages on output in a *global* economy.

CF(2015) study an OLG model with two kinds of agents: Neutrals (risk-neutral) and Knightians (risk-averse). Neutrals own Lucas trees that are subject to aggregate risk. Neutrals can (and will in equilibrium) securitize the safe part of the tree and sell it to Knightians. However, the securitization process is subject to a financial friction. Neutrals can only pledge a fraction ρ of future payouts. The key market to clear is the market for safe assets. The equilibrating price for this market is the interest rate on safe assets. By introducing price stickiness and money, the safe interest rate is subject to a zero-lower bound. If there is excess demand for safe assets at a positive interest rate, the interest rate drops, equilibrating supply and demand. However, if there is excess demand at an interest rate of zero, the adjusting margin switches from interest rates to output – since rates cannot drop any further. A drop in output brings about the equilibrating force needed to clear the market. That is, the economy ends up in a safety trap where output is depressed. A key policy implication is that the government can alleviate the safety trap (push up output) by supplying safe public debt backed by taxes.

We build an international version of the model in CF (2015) in order to study the *global* dimension of safety traps. Our model features two countries (or regions), labeled "Home" and "Foreign." It is helpful to think of Home representing the US and, broadly speaking, Foreign representing the rest of the world. We emphasize two dimensions of heterogeneity between Home and Foreign. First, we assume that Home's Neutrals can pledge a larger fraction of future payouts than Foreign's Neutrals. Second, we assume that there is a larger fraction of Knightians in Foreign compared to Home. The first dimension of heterogeneity proxies for a higher development of US financial markets compared to the rest of the world. The second dimension captures the large demand for safe savings vehicles by countries other than the US.

First, we analyze the effects of the highlighted heterogeneities between countries on "natural" safe interest rates (where "natural" refers to the safe rate that equilibrates supply and demand for safe assets without a drop in output). We find that the natural Home autarky safe interest rate lies above the natural Foreign autarky rate and when the economy integrates, the natural world safe interest rate lies in between those autarky rates. That is, opening up the economy depresses safe interest rates from Home's point of view but raises safe rates from Foreign's point of view. The intuition for this results stems from from the scarcity of safe assets. In Home, the scarcity of safe assets is less severe compared to Foreign, which results in a higher market-clearing safe interest rate. When the economy opens up, the scarcity of safe assets is essentially averaged between Home and Foreign, which implies a market-clearing world safe rate in between the autarky safe rates. This result is also mirrored in net foreign asset positions. Home builds up a positive NFA position in safe assets while Foreign has the corresponding negative position. Importantly, when safe assets are sufficiently scarce in Foreign, we find that the natural world safe interest rate is negative although the natural Home autarky rate is positive. When interest rates are constrained by the ZLB and cannot fall below zero, this implies the world economy suffers from a safety trap with depressed output. That is, Foreign's demand for safe assets pushes Home into a recession.

Second, we analyze the role of safe public debt in a global economy by allowing the government to issue safe claims backed by taxes on endowments of future agents. We impose two restrictions on the way taxation operates. Taxes are distortionary and can only be adjusted imperfectly over time. We consider these realistic assumptions capturing that lump-sum taxes are infeasible in practice and changes in tax codes are subject to a sluggish political process. In CF (2015), safe public debt has output expansionary effects without any cost. In contrast, the restrictions we impose on fiscal policy imply that besides the upside of expanding output, safe public debt has the downside of generating wasteful distortions. Under this cost-benefit trade-off for safe public debt, a number of important implications arise. We find that (i) a government may choose to not exhaust its fiscal capacity and allow the global economy to suffer from a safety trap, even though it has the ability to pull the economy out of it. Moreover, (ii) safe public debt issuance in any of the countries has expansionary effects on output in both countries, leading to an externality of public debt: As both countries gain from a public debt issuance but only one country bears the cost, there will be underprovision of safe public debt from a global perspective.

Third, we investigate if Home would find it optimal to close its capital account in order to avoid being pushed into a safety trap by Foreign. For this exercise, it is important to distinguish whether the government's object of interest is output or consumption. This is the case as in the open economy, net exports can drive a wedge between output and consumption. In the case of output, we require Home to maintain debt above a positive threshold. We find that closing down the capital account has the upside of pushing up output (by avoiding a safety trap) but the downside of paying higher debt servicing cost. When Home closes down its capital account, its safe interest rate is positive so that it foregoes a recessionary safety trap but also pays strictly positive debt servicing costs. In contrast, in the open economy, Home suffers some output losses due to the safety trap, yet avoids debt servicing costs altogether as the safe rate is at the zero lower bound. We illustrate that a higher debt threshold and higher severeness of tax distortions tilt the trade-off towards an open capital account. In addition, we show that when the debt threshold approaches zero, only the output expansionary motive remains and a closed capital account is always preferred.

When the government's object of interest is consumption, we do not require debt to be above a threshold. Thus, the debt servicing cost motive is absent. However, there is still a trade-off for the following reason. We find that the upside of closing the capital account is still present as the expansionary effect on output translates into an expansionary effect on consumption. Nonetheless, closing down the capital account also means that Home foregoes potential net interest income on its asset position. If the world economy is such that Home earns positive net interest income, then it uses this income to permanently finance consumption above net output. By closing the capital account, Home gives up these additional resources.

Finally, we show that our results on the intermediate exhaustion of fiscal capacity and global underprovision of safe assets hold irrespectively of whether the government's object of interest is output or consumption.

Related Literature. As outlined above, our paper is most closely related to CF (2015) as we directly build on their framework and extend it to an international model. Importantly, in contrast to CF (2015), we model not only the benefit but also the cost side of safe public debt provision. This leads to a number of interesting implications in the international context, e.g. an externality of safe public debt provision in the global economy.

There is also a recent paper by Caballero, Farhi and Gourinchas (2015) that uses a framework similar to CF (2015) and studies the diffusion of liquidity traps and corresponding policy options in a global economy. Their paper and our paper have been written concurrently. They focus mostly on liquidity traps in general, though also derive some implications for liquidity traps in the market for safe assets. Some of their results overlap with our findings. They also find that countries with severe asset shortages drag other countries into recessions, that public debt provision in any country expands output everywhere and that the distribution of output drops in a liquidity trap is governed by the relative degree of nominal rigidities. Their paper extensively studies the role of nominal exchange rates devaluations, a channel absent in our paper. In contrast to our work, they do not consider costs of safe public debt provision, which is why our results related to this cost side are not present in their work.

Apart from these two papers, our work is related to several strands of the literature. First, the motivation for our paper stems from the literature that identifies the shortage of safe assets in the global economy (Barclays (2012), Caballero (2006, 2010), Caballero and Krishnamurthy (2009)) and the literature rationalizing "global imbalances" by heterogeneities in financial markets (Bernanke (2005), Bernanke et al. (2011), Caballero, Farhi and Gourinchas (2008), Mendoza, Quadrini and Rıos-Rull (2009), Maggiori (2012)).

Second, there is a recent literature that focuses on how the market for safe assets affects the macroeconomy. Barro and Mollerus (2014) study a model with heterogenous degrees of risk-aversion across agents and quantitatively match safe real interest rates, equity premia and crowding-out coefficients of public bonds. Benhima and Massenot (2013) study a model where agents have decreasing relative risk-aversion and show the existence of multiple equilibria – one of these equilibria is "fear-driven" and features depressed output.

Third, there is a large literature on liquidity traps. See for example, Krugman (1998), Eggertsson and Woodford (2006), Eggertsson and Krugman (2012), Werning (2011) and references therein. A subbranch of the literature on liquidity traps focuses on this phenomenon in an open economy context. Svensson (2001, 2003) discusses policy options involving exchange rate devaluations to escape from a liquidity trap. Jeanne (2009) shows that a demand shock in one country can push the world economy into a liquidity trap and discusses how a coordinated monetary policy response can bring the economy back to first-best. Benigno and Romei (2014) analyze how global liquidity traps can arise due to sovereign deleveraging. Cook and Devereux (2013, 2014) study optimal coordinated monetary and fiscal policy responses in a global liquidity trap and evaluate whether in this context a single-currency area is superior/inferior to a regime with flexible exchange rates. Devereux and Yetman (2014) explore how capital controls can be used once monetary policy loses bite due to a worldwide liquidity trap. We are related to these papers as we study a global liquidity trap in a particular market: the market for safe assets. However, in contrast to the policy options examined in these papers, we explore the role of safe public debt as a tool for stabilization in a global economy.

Furthermore, we are related to the literature that studies the role of the government in providing additional stores of values, see Woodford (1990) and Holmstrom and Tirole (1998). In our model – as in CF (2015) – there is a role for the government to improve market outcomes as it has an advantage over the private sector in providing a particular store of value (safe assets) as it can levy taxes on agents' future income.

There is also a literature that studies the so-called "safe haven" and/or reserve currency status. See Eichengreen (1998, 2012) and Eichengreen and Flandreau (2009) for a historical discussion of this topic. More recently, He et al. (2015) identify a number of economic mechanisms that influence the determination of which country becomes the "safe haven" of the world economy. This is literature is important with respect to our work for the following reason. When there is only a single country in the world economy with the ability to provide safe assets, then our results on the externality and global underprovision of safe public debt are particularly relevant.

Finally, there is also a growing body of literature concerned with "secular stagnation," e.g. Kocherlakota (2014) and Eggertsson and Mehrotra (2014). As will become clear later on, in our model – as in CF (2015) – safety traps can be permanent so that the economy remains in a recession for an arbitrarily long period of time. Whereas the work mentioned above considers closed economy environments, there is also a recent paper by Eggertsson et al. (2015) that analyzes secular stagnation in an international context. In contrast to our paper, they explore exchange rate devaluations as a policy tool. Similar to our work, they also think about whether economies gain from closing down their capital accounts.

The remainder of this paper is structured as follows. Section 2 lays out our baseline model and derives results on interest rates and net foreign asset positions. Section 3 extends our baseline model to allow for public provision of safe assets under imperfect fiscal policy and derives the results on intermediate exhaustion of fiscal capacity, public debt externality and considers the open vs. closed capital account trade-off. Finally, Section 4 concludes. Proofs and detailed derivations are included in the Appendix.

2 Baseline model

In this section, we lay out our baseline model. This is an international version of the model studied in CF (2015). First, we present a (real) endowment economy version to illustrate the key forces at play in a simple way, and then incorporate nominal rigidities and production to explore output consequences of these forces.

2.1 The real endowment economy

We study a single-good economy that consists of two countries (or regions), which we label "Home" and "Foreign." In the following, variables and parameters corresponding to Home will be denoted without superscript, while a *-superscript is used for Foreign. Time is continuous with infinite horizon, $t \in [0, \infty)$.

Demographics. Each country is populated by a unit mass of agents. This mass is constant over time. However, individual agents are born and die at rate θ (θ^*), independent across agents. For analytical simplicity, we work with $\theta = \theta^*$.

Aggregate Risk. At each point in time, Home (Foreign) receives endowment X_t (X_t^*) of the numeraire good. The world economy is subject to aggregate risk in the following sense. There are 2 Poisson-processes that arrive with intensities (λ^+, λ^-), and affect endowment in both countries. Their stopping times are denoted by (σ^+, σ^-). We define the stopping time of the process that hits first by $\sigma \equiv \min\{\sigma^+, \sigma^-\}$. Then, endowments for the Home country are given by

$$X_t = \begin{cases} X & t < \sigma \\ \mu^- X & t \geq \sigma, \sigma = \sigma^- \\ \mu^+ X & t \geq \sigma, \sigma = \sigma^+, \end{cases}$$

where $\mu^- < 1 < \mu^+$ (analogous process for Foreign endowment). Endowment is fixed at X (X^*) before the realization of the shock and permanently jumps up/down after the shock has been realized.[1] We focus on the limiting case where $\lambda^+ \to 0, \lambda^- \to 0$.

Lucas-trees and distribution of endowments. There is one Lucas-tree in each country. A part δX_t ($\delta^* X_t^*$) of endowment accrues as dividends to the Home (Foreign) Lucas tree. The remaining part $(1-\delta)X_t$ ($(1-\delta^*)X_t^*$) is equally distributed as endowment to newborn agents. We work with $\delta = \delta^*$.

[1] Note that we have one symmetric shock for the world as a whole. We could generalize this to imperfectly correlated shocks across countries and also to shocks of heterogenous depth. It will become obvious that what matters in the end for the determination of equilibrium is simply the worst possibly outcome.

Agents. There are two kinds of agents in each country: Knightians (infinitely risk-averse), and Neutrals (risk-neutral). A fraction α (α^*) are Knightians, and their preferences are given by

$$U_t^K = \mathbb{1}\{t - dt \leq \sigma_\theta < t\}c_t + \mathbb{1}\{t \leq \sigma_\theta\} \min\{U_{t+dt}^K\}.$$

The remaining fraction $(1-\alpha)$ ($(1-\alpha^*)$) are Neutrals, and their preferences are given by

$$U_t^N = \mathbb{1}\{t - dt \leq \sigma_\theta < t\}c_t + \mathbb{1}\{t \leq \sigma_\theta\}\mathbb{E}_t\{U_{t+dt}^N\}.$$

Importantly, we will assume that agents consume exclusively at time of their death. That is, agents save the endowment they receive at birth during their life time and consume all of their wealth when they get hit by the death-shock.

Assets. We assume that the Lucas-trees are in the hands of Neutrals. Neutrals can securitize the Lucas-trees' dividends into assets that they potentially sell to Knightians.

Financial friction. We assume the securitization process is subject to a financial friction. Neutrals can only pledge a fraction ρ (ρ^*) of the Lucas-tree's returns to other agents.

We emphasize two key dimensions of heterogeneity between Home and Foreign. First of all, the pledgeability of returns is higher in Home than in Foreign, i.e. $\rho > \rho^*$. Second of all, there is a larger fraction of Knightians (risk-averse agents) in Foreign compared to Home, i.e. $\alpha^* > \alpha$. The first source of heterogeneity is supposed to proxy for a higher development of US financial markets compared to the rest of the world. The second source of heterogeneity is supposed to capture the large demand for safe savings vehicles by emerging markets in the global economy.[2]

Our focus is the equilibrium before the Poisson event is realized (ex ante equilibrium), in particular the steady state. However, as will become clear later, even when the Poisson intensities of the shocks go to zero, the equilibrium after the bad Poisson event will feed back into the ex ante equilibrium because of Knightians' preferences. In particular, it will determine the value of safe assets. In the following subsections, we first describe the equilibrium dynamics before the Poisson event, and then move to the (deterministic solution) after the bad Poisson event to retrieve the value of safe assets (and complete the ex ante equilibrium characterization). Finally, we characterize the steady state solution of the ex ante equilibrium.

[2] Apart from these two dimensions, we also allow for heterogeneity in output/size of the countries, i.e. $X \neq X^*$.

2.1.1 Equilibrium before the Poisson event

Agents' decisions. Agents receive their endowment at birth, exchange it for assets, and reinvest their wealth until death when they liquidate all of their assets and consume. The only choice is their portfolio composition. Due to their preferences, Knightians will only hold safe assets, i.e. assets whose value is invariant to the state of the world. Neutrals hold trees and will find optimal to issue safe assets to Knightians. These insights are enough to derive the aggregate dynamics.

Assets returns. The value of the Home (Foreign) tree is denote by V_t (V_t^*). This value is divided into two parts: the total value of safe assets that can be issued against the tree's dividend payments, V_t^S, and the residual, $V_t^R = V_t - V_t^S$, which we label the risky asset. The value of safe assets is the maximum value the owner of the tree can promise to deliver next instant independently of the aggregate state. Let r_t^S be the interest rate on the safe asset, and r_t the one on the risky asset. Moreover, δ_t^S (δ_t^{S*}) denotes the safe part's dividend which is to be determined endogenously.[3] The pricing equation for the safe part of Home's tree is given by

$$r_t^S V_t^S = \underbrace{\delta_t^S X_t}_{\text{dividend}} + \underbrace{\dot{V}_t^S}_{\text{capital gain}} \tag{1}$$

The intuition for this equation is standard. The return on the safe part of the tree is made up of the dividend payment plus the capital gain. Similarly, the remaining asset pricing equations are

$$r_t V_t^R = (\delta - \delta_t^S) X_t + \dot{V}_t^R \tag{2}$$

$$r_t^S V_t^{S*} = \delta_t^{S*} X_t^* + \dot{V}_t^{S*} \tag{3}$$

$$r_t V_t^{R*} = (\delta - \delta_t^{S*}) X_t^* + \dot{V}_t^{R*} \tag{4}$$

Wealth dynamics. Let W_t^K and W_t^N denote (aggregate) Home Knightian's and Neutral's wealth, respectively. Then,

$$\dot{W}_t^K = \underbrace{-\theta W_t^K}_{\text{Consumption}} + \underbrace{\alpha(1-\delta)X_t}_{\text{Newborns' wealth}} + \underbrace{r_t^S W_t^K}_{\text{return}} \tag{5}$$

[3] Once we have solved for the endogenous values of δ_t^S and δ_t^{S*}, we have to check that $\delta^S \leq \delta\rho$ and $\delta^{S*} \leq \delta^* \rho^*$. These inequalities can be verified from our solution to the equilibrium system.

This can be understood intuitively when decomposed into 3 components. The change in wealth is given by the sum of a consumption term (which equals a fraction θ of wealth as a representative subsample dies and consumes), an endowment term capturing wealth of new-born agents and finally a return term which accounts for interest payments surviving agents earn on their wealth (Knightians only hold safe assets). The remaining wealth evolution equations follow from the same intuition.

$$\dot{W}_t^N = -\theta W_t^N + (1-\alpha)(1-\delta)X_t + r_t W_t^N \tag{6}$$
$$\dot{W}_t^{K*} = -\theta W_t^{K*} + \alpha^*(1-\delta^*)X_t^* + r_t^S W_t^{K*} \tag{7}$$
$$\dot{W}_t^{N*} = -\theta W_t^{N*} + (1-\alpha^*)(1-\delta^*)X_t^* + r_t W_t^{N*} \tag{8}$$

For Neutral's wealth return, note that Neutrals always hold the risky asset, and when they also hold some of the safe asset, they must be incentivized to do so by $r_t = r_t^S$.

Market Clearing. Our economy has two markets that clear in equilibrium: The market for the consumption good and the asset market. The goods market clearing condition is given by

$$\underbrace{\theta(W_t^K + W_t^{K*} + W_t^N + W_t^{N*})}_{\text{demand for goods}} = \underbrace{X_t + X_t^*}_{\text{supply of goods}} \tag{9}$$

This says that all consumption (given by a fraction θ of wealth) equals all output. The asset market clearing conditions is

$$\underbrace{W_t^K + W_t^{K*} + W_t^N + W_t^{N*}}_{\text{aggregate asset demand}} = \underbrace{V_t^S + V_t^R + V_t^{S*} + V_t^{R*}}_{\text{aggregate asset supply}}, \tag{10}$$

which simply is that the value of all wealth (asset demand) equals the value of asset supply.

Regimes. We have to account for the fact that there are two possible regimes in this economy. First, when the supply of safe assets ($V^S + V^{S*}$) is larger than Knightians demand ($W^K + W^{K*}$). In this case, Neutrals are the marginal holders of the safe asset, which requires $r_t = r_t^S$. In the other regime, Knightian's demand exceeds safe asset supply at $r_t = r_t^S$, which causes a drop in the safe rate to equilibrate the market such that $r_t^S < r_t$. In this regime, Knightians are the marginal holder of the safe asset. We can summarize the two regimes by including the following complementary slackness conditions in our system.

$$0 = (V_t^S + V_t^{S*} - W_t^K - W_t^{K*}) \cdot (r_t - r_t^s) \tag{11a}$$

$$V_t^S + V_t^{S*} \geq W_t^K + W_t^{K*} \tag{11b}$$

$$r_t \geq r_t^s \tag{11c}$$

Value of Safe Assets. To complete the equilibrium characterization, we need to determine the value of the safe part of each tree. The Home (Foreign) safe asset value is equal to the lowest value the tree could have next instant times the fraction of this value that can be pledged, ρ (ρ^*). In this model, the mentioned lower bound is constant and corresponds to the value of the tree if the bad Poisson event hits.[4] Moreover, this value will be independent of the ex ante system we have just described.

Let V^- and V^{-*} denote the values of the Lucas-trees *after* the realization of the bad shock. Then, the value of safe assets that are available *before* the shock is given by

$$V_t^S = V^S = \rho V^- \tag{12}$$

$$V_t^{S*} = V^{S*} = \rho^* V^{-*} \tag{13}$$

We solve for the values V^- and V^{-*} from the system that is in place *after* the bad shock. This solution is presented in the next subsection.

Note that, even though we focus on the situation where the probability of an aggregate shock is vanishing, i.e. $\lambda^+, \lambda^- \to 0$, equations (12)-(13) illustrate how the bad aggregate state feeds back into the ex ante equilibrium system. Intuitively, this happens due to Knightians' min-preferences over future outcomes.

2.1.2 Equilibrium after the bad Poisson shock

Once the aggregate shock has hit, the economy is deterministic. Therefore, there are no longer risky and safe parts of the tree, so we solve for the total value of the trees.[5] Goods

[4]This result is intuitive and will be checked for the steady state solution we analyze.

[5]Moreover, we assume $\bar{\rho} > \bar{\alpha}$, so the pedgeability constraint does not influence this ex post equilibrium

and asset market clearing conditions are

$$\theta\left(W_t^{K-}+W_t^{N-}+W_t^{K-*}+W_t^{N-*}\right)=\mu^-(X+X^*)$$
$$W_t^{K-}+W_t^{N-}+W_t^{K-*}+W_t^{N-*}=V_t^-+V_t^{-*}$$

Together, they imply a constant value for the aggregate value of both trees,

$$V_t^-+V_t^{-*}=\frac{\mu^-(X+X^*)}{\theta}$$

Return equations are

$$r_t^-V_t^-=\delta\mu^-X+\dot{V}_t^-$$
$$r_t^-V_t^{-*}=\delta^*\mu^-X^*+\dot{V}_t^{-*}$$

The last three equations imply

$$r_t^-=r^-=\delta\theta \tag{14}$$
$$V_t^-=V^-=\frac{\mu^-X}{\theta} \tag{15}$$
$$V_t^{-*}=V^{-*}=\frac{\mu^-X^*}{\theta} \tag{16}$$

This pins down the values for V^S and V^{S*} and completes the the ex ante equilibrium characterization.

2.1.3 Steady state before the Poisson event

Our focus is the **steady state** equilibrium before the shock. Recall that $\lambda^+,\lambda^-\to 0$, so this ex ante steady state equilibrium will effectively be in place forever. We begin by providing a formal definition of equilibrium. Let ($\#'$) refer to the static version of equation ($\#$), i.e. the version where changes are set to zero and time subscripts are dropped.

Definition 1 (Ex Ante Equilibrium for Endowment Economy)**.** *The steady state equilibrium before the Poisson event is a wealth distribution over agents' types* $\{W^K,W^N,W^{K*},W^{N*}\}$*, asset values* $\{V^S,V^R,V^{S*},V^{R*}\}$*, returns* $\{r,r^S\}$*, and share of dividends accrued to safe assets* $\{\delta^S,\delta^{S*}\}$ *such that*

- *Asset pricing equations hold:* ($1'$)-($4'$)*.*

- *Wealth distribution over agents' types is constant:* (5′)-(8′)*.*
- *Markets clear:* (9′)-(10′)*.*
- *One of the 2 possible regimes holds:* (11′)*.*
- *Safe assets' values are constant across aggregate states and respect the financial constraint:* (12′)-(13′)*.*

where V^- *and* V^{-*} *are given by* (15) *and* (16)*, respectively.*

Note that the equilibrium system is exactly identified as we solve for 12 variables and have 12 independent equations (one of the market clearing equations in redundant by Walras' law).

Solution. First, note that safe asset supply is determined by the equilibrium after the Poisson event, and is independent of the effective regime in the ex ante equilibrium. In particular,

$$V^S = \rho\mu^-\frac{X}{\theta} \tag{17}$$

$$V^{S*} = \rho^*\mu^-\frac{X^*}{\theta} \tag{18}$$

However, wealth distributions, asset values, and interest rates will depend on the effective regime before the shock. As will be clear later, the regime selection will depend on whether the safe asset supply is enough to absorb all of Knightians' wealth. The regime where it is enough is labeled unconstrained regime, while the other is labeled constrained regime. We present both solutions and focus on the latter.

In order to characterize the solution with convenient expressions, we define for any parameter ς the weighted average of this parameter across countries as

$$\bar{\varsigma} \equiv \omega\varsigma + (1-\omega)\varsigma^*,$$

where $\omega \equiv \frac{X}{X+X^*}$ is home country's share of (potential) output .

Case 1: The unconstrained regime. In this case, Neutrals are the marginal holder of safe assets, so $r = r^S$. In this regime, we have[6]

$$
\begin{aligned}
r &= r^S = \delta\theta \\
W^K &= \alpha\frac{X}{\theta} \\
W^{K*} &= \alpha^*\frac{X^*}{\theta}
\end{aligned}
$$

This solution is valid only if aggregate Knightian's wealth is less than total safe assets value. We find that

$$
W^K + W^{K*} \leq V^S + V^{S*} \quad \Leftrightarrow \quad \bar{\rho}\mu^- \geq \bar{\alpha}
$$

We assume that the latter condition does *not* hold, and therefore, the global economy is in the constrained regime.

Assumption 1 (Safe Asset Shortage Condition)**.** *The economy is in the constrained regime where all safe assets are hold by Knightians, i.e.*

$$
\bar{\rho}\mu^- < \bar{\alpha}.
$$

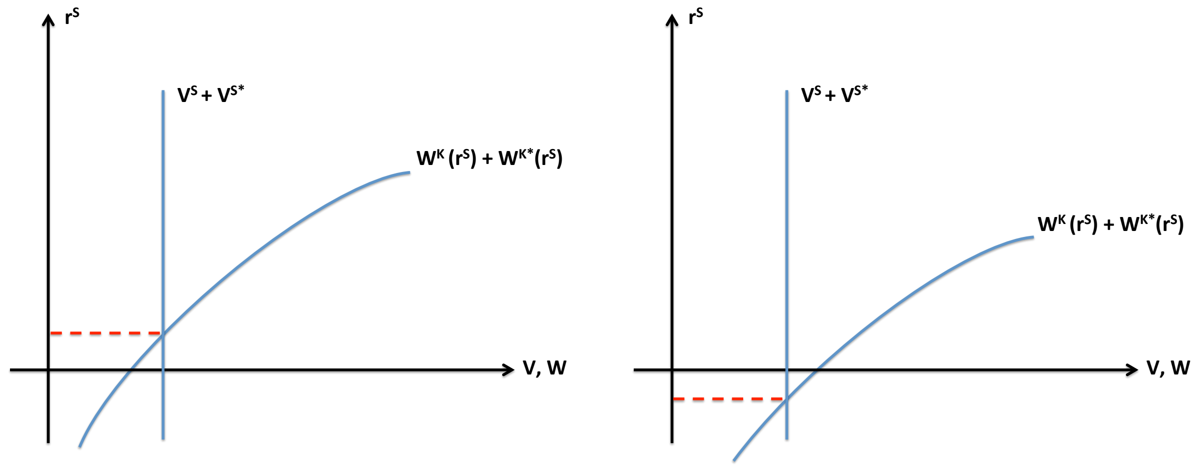
This condition is intuitive. On the one hand, both the average securitization capacity ($\bar{\rho}$) and the depth of the bad shock (μ^-) govern safe asset supply. On the other hand, the average fraction of Knightians in the economy ($\bar{\alpha}$) governs safe asset demand. If the demand factors overweigh the supply factors, safe assets are scarce and the economy ends up in the constrained regime.

Case 2: The constrained regime From now on, we focus on the analysis of the constrained regime.[7] It is important to note that in this regime a spread between risky and safe rates opens up.

$$
\begin{aligned}
r^S &= \delta\theta - (1-\delta)\theta\left(\frac{\bar{\alpha} - \bar{\rho}\mu^-}{\bar{\rho}\mu^-}\right) \\
r &= \delta\theta + (1-\delta)\theta\left(\frac{\bar{\alpha} - \bar{\rho}\mu^-}{1 - \bar{\rho}\mu^-}\right) \\
r - r^S &= (1-\delta)\theta\frac{\bar{\alpha} - \bar{\rho}\mu^-}{(1 - \bar{\rho}\mu^-)\bar{\rho}\mu^-} > 0
\end{aligned}
$$

[6] The full solution is provided in Appendix A.1.

[7] The full solution is provided in Appendix A.2.



(a) Positive equilibrium safe real interest rate (b) Negative equilibrium safe real interest rate

Figure 1: Global safe asset market equilibrium

The key element of our analysis is the market for safe assets. As we have seen, the supply of safe assets is fixed by the ex post system,

$$V^S + V^{S*} = \bar{\rho}\mu^- \frac{(X + X^*)}{\theta}$$

However, we can write the demand for safe assets as an increasing function of r^S.

$$W^K(r^S) + W^{K*}(r^S) = \frac{(1-\delta)\bar{\alpha}\,(X + X^*)}{\theta - r^S}$$

Figure 1 illustrates the market for safe assets graphically.

The safe real interest rate is the price that equilibrates the market for safe assets. If there is an excess demand for safe assets, the safe real rate drops, bringing about lower demand for safe assets such that supply and demand are equalized. Importantly, depending on the parameterization, the equilibrium safe real rate can be either positive or negative. We find that

$$r^S \leq 0 \quad \Leftrightarrow \quad \bar{\rho}\mu^- \leq (1-\delta)\bar{\alpha}. \tag{19}$$

This is intuitive. If the safe asset shortage is particularly severe, a negative safe real rate is needed to equilibrate supply and demand. So far, this does not create any issues. Since the model is entirely in real terms and agents have no alternative stores of value available, they will use safe assets to transfer wealth across time even though earning negative returns.

However, once we introduce nominal rigidities and money into the model, this changes drastically. Then, negative market-clearing safe real rates become a plague for the economy.

2.2 Nominal Rigidities and Endogenous Output Determination

In this section, we introduce nominal rigidities and money into the economy. In combination, these two factors lead to a zero-lower bound on interest rates and endogenous output determination.

We now assume that there are 2 kinds of goods: input and consumption goods. Endowment is still given by X_t (X_t^*) at every point in time, though this is in terms of input goods. Agents can transform input goods into consumption goods via a 1-1 production technology. Prices for the output good are denoted in nominal terms and are entirely rigid.[8] We assume that the price for the output good in terms of the home currency is $P = 1$ and in terms of the foreign currency is P^*. By the law-of-one price, the exchange rate is given by $E = 1/P^*$ and is constant over time. We assume that at the given prices, agents service demand until they run out of resources. Hence, output is demand determined. We define "capacity utilizations" $\xi_t, \xi_t^* \in [0, 1]$ such that output in terms of the consumption good is given by $\xi_t X_t$ and $\xi_t^* X_t^*$.

We introduce money into the economy via a cash-in-advance constraint. When the economy has money, agents have another store-of-value vehicle at hand. If the nominal interest rate is positive (note that $r = i$ due to the price rigidity), agents hold money exclusively for transaction services. However, if rates were to fall below zero, agents would substitute away from other savings vehicles and towards money, using money for both transaction services and as a store of value. Thus, we can rule out equilibria with negative interest rates and obtain a zero-lower-bound: $i_t = r_t \geq 0$. We consider a cashless limit such that we can keep the constraint on interest rates and do not have to carry around any explicit terms referring to money holdings by agent.[9]

The ex ante steady state equilibrium system of the economy with a zero-lower bound and endogenous output is given by the ex ante system from above accounting for two differences. First, endowment in terms of the output good is now given by $\xi_t X$ ($\xi_t^* X^*$). Second, there is a zero lower bound constraint. This can be written as

$$(r_t^S > 0 \ \wedge \ \xi_t = \xi_t^* = 1) \ \text{ or } \ (r_t^S = 0 \ \wedge \ \min\{\xi_t, \xi_t^*\} < 1) \tag{20a}$$

$$\xi_t, \xi_t^* \in [0, 1] \tag{20b}$$

[8] This is an extreme form of rigidity that enables us to state our insights most clearly. We relax this assumption later when we consider Philipps-curves.

[9] See CF(2015) for a closed economy equilibrium system that explicitly accounts for money holdings.

This is the key friction in our model. As in CF (2015), if there is excess demand for safe assets at a safe real rate of zero, then the equilibrating force switches from a drop in safe rates (which becomes impossible at the ZLB) to a drop in output, captured by capacity utilizations falling below unity.

Next, we formally characterize the ex ante equilibrium in this production economy. Let ($\#''$) refer to the static version of equation ($\#$) where endowment X (X^*) is replaced by output ξX ($\xi^* X^*$). Also, note that the solution after the Poisson event is the same as in the endowment model since the interest rate is strictly positive ex post. The full system and its solution are included in Appendix A.3.

Definition 2 (Ex Ante Equilibrium for Production Economy)**.** *A steady state equilibrium before the Poisson event is a wealth distribution over agents' types* $\{W^K, W^N, W^{K*}, W^{N*}\}$*, asset values* $\{V^S, V^R, V^{S*}, V^{R*}\}$*, returns* $\{r, r^S\}$*, share of dividends accrued to safe assets* $\{\delta^S, \delta^{S*}\}$*, and utilization capacities* $\{\xi, \xi^*\}$ *such that*

- *Asset pricing equations hold:* (1″)-(4″).
- *Wealth distribution over agents' types is constant:* (5″)-(8″).
- *Markets clear:* (9″)-(10″).
- *One of the 2 possible regimes holds:* (11″).
- *Safe assets' values are constant across aggregate states and respect the financial constraint:* (12″)-(13″).
- *The ZLB condition holds:* (20″).

where V^- *and* V^{-*} *are given by* (15) *and* (16)*, respectively.*

There is a crucial difference to the analysis in the closed economy. In the closed economy, the drop in output that brings about an equilibrium safe real interest rate of zero determines ξ. In the open economy, when safe interest rates are against the ZLB, a drop in world output will equilibrate world supply and world demand for safe assets. While we can determine the magnitude of the drop in world output, its distribution across countries is indeterminate.[10] This is illustrated in Figure 2. We can determine the amount by which the safe asset demand curve has to shift, though we cannot pin down how much of that shift comes through a drop

[10] Note that in Definition 2 the system has 14 variables and 13 independent equations at the ZLB.

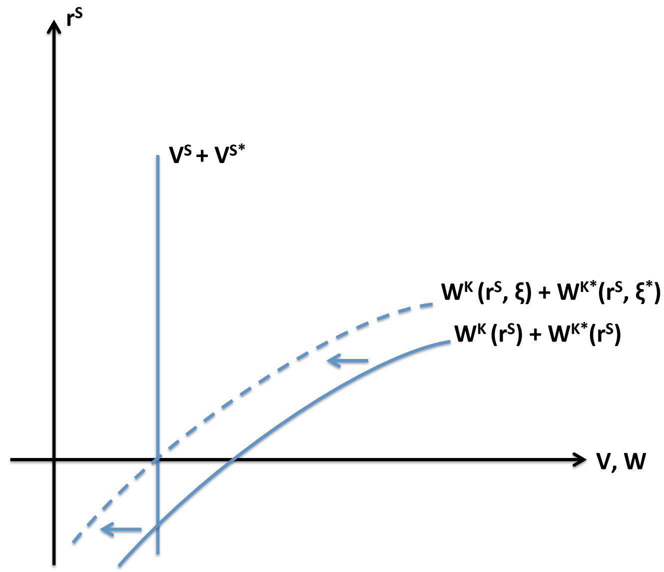


Figure 2: Output drop due to excess demand for safe assets

in Home vs. Foreign output. That is, we cannot uniquely determine the equilibrium values of ξ and ξ^* at the ZLB. In particular, we have that

$$\omega\alpha\xi + (1-\omega)\alpha^*\xi^* = \frac{\bar{\rho}\mu^-}{(1-\delta)} \tag{21}$$

This indeterminacy depends very strongly on the assumptions we have made about price rigidities. Once we move to a model that relaxes the extreme assumption of completely rigid prices, the indeterminacy goes away. In Appendix A.4, we study an extension of the model where movements in prices are governed by Philipps-curves (motivated by sticky wages) which allows us to exactly pin down the distribution of output drops across countries. In such environment, the relative degree of nominal rigidity degree is the key factor: the economy which has more rigid wages bears a larger share of the drop in world output. For the time being, we assume that output drops are equally distributed across countries (which would amount to an equal degree of nominal rigidity) and solve (21) as

$$\xi = \xi^* = \frac{\bar{\rho}\mu^-}{\bar{\alpha}(1-\delta)}. \tag{22}$$

2.3 Real Interest Rates and Net Foreign Asset Positions

In order to study our model's implications for world safe interest rates and net foreign asset positions, it is useful to define the following objects. Let $r_{aut}^{S,n}$ and $r_{aut}^{S,n*}$ be the full-capacity autarky safe real interest rates in Home and Foreign. Also, let $r^{S,n}$ be the full-capacity safe real rate of the global economy. These "natural" interest rates are potentially negative.

Recall that we emphasize two dimensions of heterogeneity between countries: Higher fraction of Knightians in Foreign compared to Home and higher securitization capacity in Home compared to Foreign, i.e. $\alpha < \alpha^*$ and $\rho > \rho^*$. We have the following

Proposition 1 (Depression of Safe Interest Rates)**.** *It holds that*

$$r_{aut}^{S,n*} < r^{S,n} < r_{aut}^{S,n} \quad \Leftrightarrow \quad \frac{\bar{\alpha}}{\bar{\rho}} > \frac{\alpha}{\rho}.$$

Under the parameterization we assume, opening up the world economy depresses safe rates from Home's point of view and increases safe rates from Foreign's point of view. Intuitively, the equilibrium safe real interest rate is determined by the severeness of the safe asset shortage. If the financially integrated economy has a more severe scarcity of safe assets compared to the Home autarky economy, the world safe real rate will be lower than the Home autarky rate (and vice versa for Foreign). This relationship of interest rates will be mirrored in net foreign asset positions. Denote Home's net foreign asset position in safe assets by NFA^S.

Proposition 2 (Safe Net Foreign Asset Positions)**.** *We have that*

$$NFA^S > 0 \quad \Leftrightarrow \quad r^{S,n} < r_{aut}^{S,n}.$$

A further illustration of these results is given by Figure 3. When the Home autarky safe rate is above the Foreign autarky safe rate, then the safe rate in a global economy will be between those two autarky rates. At this interest rate, there is an excess supply for safe assets in Home and an excess demand for safe assets in Foreign. Accordingly, Home has a positive NFA in safe assets and Foreign has a negative NFA in safe assets.

Importantly, when the safe asset shortage in Foreign is sufficiently severe, the economy can end up in the following situation.

Proposition 3 (Global Safety Traps)**.** *If* $\bar{\rho}\mu^- < (1-\delta)\bar{\alpha}$ *and* $\rho\mu^- > (1-\delta)\alpha$, *then*

$$r^{S,n} < 0 < r_{aut}^{S,n}.$$

If the safe asset shortage is mild in Home but sufficiently severe in Foreign, then the world economy features a negative natural safe rate even though the Home autarky natural safe

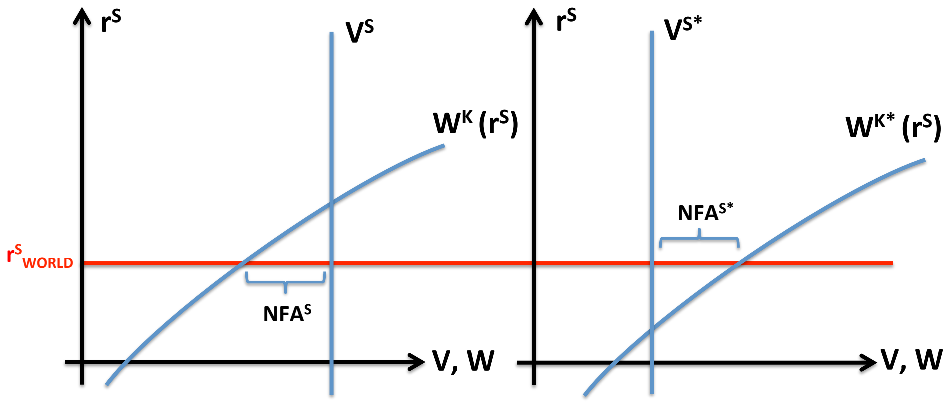


Figure 3: Safe NFA positions in a global economy

rate is above zero. Of course, in our economy with nominal rigidities and the ZLB, negative rates are not attainable. Hence, in this scenario, while the Home autarky economy is at full capacity with a safe rate above zero, the world economy is against the ZLB and experiences a recession. In short, Home gets pushed into a safety trap by Foreign's demand for safe assets.

3 Public Debt in Global Safety Traps

In this section, we analyze the role of safe public debt in global safety traps. In particular, we allow a government to issue (safe) claims, backed up by taxes on endowments of future agents. We impose two plausible restrictions on the way taxation operates. Namely, taxes are distortionary and the tax rate can only be adjusted imperfectly over time.

We analyze the situation where the government cares about (net) output, and present the following set of results. First, restricting attention to a global safety trap environment, we arrive at two main insights. (i) *Intermediate Exhaustion of Fiscal Capacity:* a government may optimally choose not to exhaust its fiscal capacity, and may allow the global economy to remain in a safety trap even when it has the ability to eliminate the safe asset shortage. (ii) *Public Debt Externality and Underprovision:* public debt issuance in any country has expansionary effects in both countries. Hence, when only the issuing country bears the costs, the debt level that maximizes domestic (net) output is always below the optimal debt level from a global perspective.

Second, we study an environment where countries are required to maintain a minimum debt level and explore whether a country that could avoid the safety trap by closing its capital account would find it optimal to do so. We show that the following trade-off arises. Compared to a financially integrated economy, closing down one's capital account has the upside of avoiding a recessionary safety trap though implies the downside of higher debt servicing cost. In accordance with this trade-off, we illustrate numerically that larger tax distortions and higher minimum requirements on the debt level tilt the government's decision towards maintaining an open capital account. Moreover, we show analytically that when the minimum debt level becomes arbitrarily small, a closed capital account is always preferred.

Then, we move to an environment where government's focus is consumption as opposed to (net) output. In this setup, we do not require a minimum debt level so that the debt servicing cost motive is absent. We illustrate an extra force at play when considering to close the capital account: net interest payments. In particular, positive net interest payments allow the country to consume beyond its production permanently. Therefore, this translates into an additional cost of closing the capital account. Finally, we show that intermediate exhaustion of fiscal capacity and public debt underprovision are also present in the environment where the government cares about consumption.

3.1 Model with costly debt provision

Public debt and taxation restrictions. We extend the baseline model by introducing public debt backed by taxes. In the following, only the home country can issue public debt as a safe asset. This asymmetry aims to capture the role of the U.S. as "safe haven" and simplifies the exposition, but results in this subsection do not rely on it. The government taxes the neutral newborns' production,[11] $(1-\alpha)(1-\delta)\xi_t X_t$. We study steady states of the model where the tax rate, τ_t , takes a constant value for each aggregate state. In particular, let τ^- (τ^+) be the tax rate after the negative (positive) realization of the Poisson event, and τ, the tax rate before it. The government issues a fixed amount of bonds at time zero,[12]

[11] Knightians are not taxed to avoid taxation to have expansionary effects through wealth reduction of agents that demand safe assets. This allows for a clearer exposition of results, but it is not necessary for our results.

[12] The government rebates lump sum the proceedings from the issuance. To whom these proceedings are rebated is not relevant because we study the steady state where there are no new issuances.

whose value we denote by D (constant over time since it is a safe asset).[13] Now, we introduce restrictions on the way taxation operates that will be key for our results.

Assumption 2 (Distortionary Taxes)**.** *A fraction ψ_t of tax collection is wasted.*

Assumption 3 (Imperfect Adjustment)**.**

$$\tau^- \leq \tau(1+\phi) \tag{23}$$

Assumption 2 captures the standard idea that there is a cost in raising taxes, and it can be thought of as a shortcut for unmodeled price distortions or administrative costs. We also assume that these distortions are only in place before the Poisson shock to simplify exposition, though this is not necessary for our results to hold. Assumption 3 captures the fact that the government cannot perfectly adjust the tax rate according to the aggregate state of the economy. In particular, the constraint asserts that the tax rate cannot increase more than ϕ percent after the bad Poisson event.[14] We consider this a plausible assumption that can be rationalized in several ways. For example, a large increase of taxes may not be politically feasible during recessions. Moreover, it might be hard for the government to exactly fine-tune the adjustment of the tax rate contingent on aggregate shocks, e.g. due to limited observability.

Also, we assume that τ^+ is set to maintain debt value after the good Poisson event, and drop this tax rate from the analysis since there is no feedback from the situation after the good shock to the ex ante equilibrium. This will be equivalent to allow the government to reduce taxes after the good shock.

Before the Poisson event, effective tax collection can be greater than the interest paid on debt, in which case surpluses (S_t) are rebated to taxed agents. After the (bad) Poisson event, the government runs a balance budget, i.e. $S_t = 0$ for $t \geq \sigma$. Then, the flow of funds equation for the government can be written as

$$S_t = \tau_t(1-\psi_t)\nu\xi_t X_t - r_t^S D \tag{24a}$$

$$S_t \geq 0 \tag{24b}$$

[13] Debt is "short term" or "instantaneous", in the sense that dividend flows, and not the asset value, adjust with market returns.

[14] We could impose the same constraint for taxes after the good Poisson event realization, but this would not influence the ex ante equilibrium.

where, for notational convenience, we denote the neutral newborn's share of output as $\nu \equiv (1-\alpha)(1-\delta)$. Accordingly, wealth dynamics for Home Neutrals are now given by

$$\dot{W}_t^N = (r_t^S - \theta)W_t^N + (1-\tau_t)\nu\xi_t X_t + S_t \tag{25}$$

Distortions are a fixed share of the tax collection before the Poisson event, $\psi_t = \psi$ for $t < \sigma$, and zero afterwards, $\psi_t = 0$ for $t \geq \sigma$. The latter is not necessary for results, but allows a simple characterization of solutions. It is helpful to think of the government as choosing tax rates $\{\tau, \tau^-\}$ subject to (23), letting the debt value and fiscal surpluses (before the Poisson event) be endogenous objects. However, as will become clear below, there will be a one-to-one map between the tax rate set after the bad Poisson event, τ^-, and the value of public debt, D, which allows the government to target the latter.

Equilibrium. Pricing equations (1″)-(4″), wealth dynamics for Home Knightians (5″), and wealth dynamics for the foreign country (7″)-(8″) are the same as in the model with production. This is also true the ZLB condition (20) and private safe asset equations (12)-(13). However, the lowest possible values of private assets (V^-, V^{-*}) in (12)-(13) change due to introduction of public debt as we describe in detail below. The regime equation (which was given by (11″) beforehand) is now given by

$$0 = (V_t^S + V_t^{S*} + D - W_t^K - W_t^{K*}) \cdot (r_t - r_t^s) \tag{26a}$$

$$V_t^S + V_t^{S*} + D \geq W_t^K + W_t^{K*} \tag{26b}$$

$$r_t \geq r_t^s \tag{26c}$$

The goods market clearing condition is altered to incorporate the waste due to distortionary taxation

$$\theta(W_t^K + W_t^{K*} + W_t^N + W_t^{N*}) = (1 - \psi_t\nu)\xi_t X_t + \xi_t^* X_t^*. \tag{27}$$

The asset market clearing accounts for the additional assets supplied by the government, i.e.

$$V_t^S + V_t^{S*} + D + V_t^R + V_t^{R*} = W_t^K + W_t^{K*} + W_t^N + W_t^{N*}. \tag{28}$$

where V_t^S is the home country's supply of *private* safe assets. To sum up, the equilibrium is characterized by the set of equation {(1″)-(5″), (7″)-(8″), (12)-(13), (24)-(28)} taking as given values (D, V^-, V^{-*}) derived below and policy instruments $\{\tau, \tau^-\}$ which must satisfy (23). As in the previous section, the equilibrium system has an indeterminacy with respect to the distribution of output drops at the ZLB. As above, we will resolve this for the time being by assuming symmetric losses in output.

Solution after bad Poisson event. Following the same steps as in subsection 2.1.2, we can derive the lowest possible value of private safe assets (V^-, V^{-*}), and the value of public debt, D. These value are given by[15]

$$D = \left(\frac{\omega\nu\tau^-}{\omega\nu\tau^- + \delta}\right)\frac{\mu^-(X+X^*)}{\theta} \tag{29}$$

$$V^- = \left(\frac{\omega\delta}{\omega\nu\tau^- + \delta}\right)\frac{\mu^-(X+X^*)}{\theta} \tag{30}$$

$$V^{-*} = \left(\frac{(1-\omega)\delta}{\omega\nu\tau^- + \delta}\right)\frac{\mu^-(X+X^*)}{\theta} \tag{31}$$

The values of private safe assets for the ex ante system are given by $V^S = \rho V^-$, and $V^{S*} = \rho^* V^{-*}$. Hence, the total safe asset supply can be written as

$$D + V^S + V^{S*} = \left(\frac{\omega\nu\tau^- + \bar{\rho}\delta}{\omega\nu\tau^- + \delta}\right)\frac{\mu^-(X+X^*)}{\theta} \tag{32}$$

Importantly, as is clear from (29), the value of debt is a strictly increasing function of government's tax rate set after the bad Poisson event.[16]

Government securitization advantage and crowding out of private safe assets. There are two types of market incompletenesses in this model: the non-pledgeability of future newborn's production, and the non-pledgeability of the $(1-\rho)$ fraction of dividends. In our framework, the government alleviates the second market incompleteness by allowing to securitize a part of future newborns' output. However, the relevant measure is *the fraction of these newly pledgeable output flows that is accrued to safe assets in the bad aggregate state* (after the bad Poisson event). Suppose the government uses a fraction φ of its taxes revenues to create safe assets (and the rest to create risky assets). Then, the total value of safe assets is given by

$$D + V^S + V^{S*} = \left(\frac{\varphi\omega\nu\tau^- + \bar{\rho}\delta}{\omega\nu\tau^- + \delta}\right)\frac{\mu^-(X+X^*)}{\theta} \tag{33}$$

and we can think of our model as the case where $\varphi = 1$. In general, the government is able to increase the supply of safe assets if and only if the fraction of output flows it securitizes into safe assets (as opposed to risky ones) is larger than private sector's pledgeability, i.e.

[15] Recall that ω is the share of potential output of the Home country.

[16] Also, the smaller the fraction of global output accrued to taxed agents, $\omega\nu \equiv \left(\frac{X}{X+X^*}\right)(1-\alpha)(1-\delta)$, the larger the tax rate increase needed to achieve a given value of debt. This implies that larger economies have an advantage in providing safe assets.

$\varphi > \bar{\rho}$.[17] The latter requirement ensures that the increase in public *safe* assets is larger than the crowding out of private *safe* assets. Even though taxation does not affect the tree dividends of the private asset, there is crowding out of private safe assets through an interest rate channel. After the bad Poisson event,[18] the presence of government policy (public debt backed by taxes) increases dividend payments of total assets (because there are now dividends on debt). At a fixed interest rate, this implies larger value of total assets (or total wealth). Since consumption is a fixed fraction of wealth, this would lead to larger consumption, however total production is fixed after the Poisson event. Therefore, interest rate must adjust. In particular, the interest rate increases to keep total asset (wealth) value constant and equilibrate the goods market. Note that there is a complete crowding out between debt, D, and private assets value after the bad shock, $(V^- + V^{-*})$. Since only a $\bar{\rho}$ fraction of latter become safe assets in the ex ante equilibrium, this fraction represents the crowding out of public debt over safe assets.

The observations discussed provide useful insights. First, as in CF (2015), it is only taxation capacity after the bad Poisson shock that can potentially increase the safe asset supply ex ante. Second, it is not enough to pledge a larger fraction of future output (after the bad shock), this newly pledgeable output flows need to be accrued (in a fraction larger than $\bar{\rho}$) to a safe asset. If a country does not have enough fiscal capacity to ensure the value of debt after the bad Poisson event and effectively issues risky claims (think of $\varphi = 0$), then it ends up decreasing the supply of safe assets (due to crowding out). Third, financial integration allows for a decrease (increase) in the crowding out of private safe assets for the more (less) financially developed country, i.e. the country with larger (smaller) ρ.

Solution before Poisson event. We begin by providing a formal definition of the steady state equilibrium.

Definition 3 (Ex Ante Equilibrium for Production Economy with Costly Debt Provision)**.** *Given government policy instruments $\{\tau, \tau^-\}$ that satisfy* (23)*, a steady state equilibrium before the Poisson event is a wealth distribution over agents' types $\{W^K, W^N, W^{K*}, W^{N*}\}$, asset values $\{V^S, V^R, V^{S*}, V^{R*}, D\}$, returns $\{r, r^S\}$, share of dividends accrued to safe assets $\{\delta^S, \delta^{S*}\}$, utilization capacities $\{\xi, \xi^*\}$, and government surplus $\{S\}$ such that*

[17] Technically, this follows from the observation that

$$\frac{d}{d\tau^-}\left[D + V^S + V^{S*}\right] > 0 \Leftrightarrow \varphi > \bar{\rho}.$$

[18] Recall that after the Poisson event the economy is deterministic so all assets have the same rate of return

- *Asset pricing equations hold:* (1″)-(4″).
- *Wealth distribution over agents' types is constant:* (5″), (25), (7″)-(8″)
- *Markets clear:* (27), (28).
- *One of the 2 possible regimes holds:* (26).
- *Safe assets' values are constant across aggregate states and respect the financial constraint:* (12″)-(13″).
- *The ZLB condition holds:* (20).
- *Government satisfies its budget flow constraint:* (24)

where D, V^-, *and* V^{-*} *are given by* (29), (30), *and* (31), *respectively. Recall that* (#″) *refers to the static version with endogenous production of equation* (#).

At the ZLB, the described system has 15 equations and 14 independent variables. As before, we keep our assumption of symmetric utilization capacities, $\xi = \xi^*$. The solution for the natural safe interest rate, i.e. the interest rate at full capacity ($\xi = 1$), is given by

$$r^{S,n}(\tau^-) = \delta\theta - (1-\delta)\theta\left[\frac{\bar{\alpha}}{\mu}\left(\frac{\omega\nu\tau^- + \delta}{\omega\nu\tau^- + \bar{\rho}\delta}\right) - 1\right] \tag{34}$$

which is increasing in τ^- as it expands the supply of total safe assets. If the shortage of safe assets is large enough, i.e. $r^{S,n} < 0$, then the global economy is in a safety trap[19] characterized by $r^S = 0$ and

$$\begin{aligned}\xi(\tau^-) &= \frac{\mu^-}{(1-\delta)\bar{\alpha}}\left[\frac{\theta(V^S + V^{S*} + D)}{X + X^*}\right] \\ &= \frac{\mu^-}{(1-\delta)\bar{\alpha}}\left[\frac{\omega\nu\tau^- + \bar{\rho}\delta}{\omega\nu\tau^- + \delta}\right]\end{aligned} \tag{35}$$

The latter expression shows that global utilization capacity is proportional to the global supply of safe assets. Therefore, in a global safety trap, the effect of public debt on global safe asset supply (discussed in detail above) translates into effects on global output (utilization capacity).

The remainder of this section considers the government's choice of tax rates $\{\tau, \tau^-\}$. First, we assume the government focuses on output net of the waste associated with distortionary

[19] See Appendix A.5 for the full steady state solution.

taxation (Subsections 3.2 and 3.3). Then, we let the government's focus change to consumption (Subsection 3.4).

3.2 Intermediate Exhaustion of Fiscal Capacity and Public Debt Underprovision

We restrict our attention to global safety traps, and illustrate the trade-offs the government faces. The analysis is restricted to (ex ante) steady state comparisons, i.e. long run effects. In particular, we assume that the government maximizes the (ex ante) home country's net output. Formally, it solves

$$\begin{aligned} \max_{\{\tau,\tau^-\}} & [1-\tau\psi\nu]\xi(\tau^-)X \\ \text{s.t. } & \tau^- \leq (1+\phi)\tau \\ & \tau,\tau^- \in [0,1] \end{aligned} \tag{36}$$

Denote (τ_D^-, τ_D) as the solution to this problem.

Intermediate Exhaustion of Fiscal Capacity. Larger τ^- increases the supply of safe assets expanding global utilization capacity. Larger τ generates more distortionary taxation, decreasing net output. Thus, if the government could perfectly decouple tax rates, it will choose $\tau^- = 1$,[20] and $\tau = 0$. However, this would violate the assumption on imperfect adjustment (23). Therefore, the latter will hold with equality. In other words, distortionary taxation and imperfect tax rate adjustments introduce an effective cost of providing safe assets, absent in CF (2015), generating a meaningful trade-off. Increasing the supply of safe assets alleviates the global safety trap, though in order to do this the government needs to increase not only taxes after the bad Poisson event (which is costless), but also before it. The latter generates tax distortions in the (ex ante) steady state, introducing an output cost of debt provision.

This trade-off implies that, when distortion considerations are large enough, the government chooses not to exhaust its fiscal capacity. In this model, the latter can be understood as $\tau^- < 1$. However, large enough distortions can prevent government from exhausting its taxation capacity independently of its limits, e.g. when the tax rate is limited by an upper bound $\bar{\tau}$ strictly lower than 1. The following proposition provides a characterization of the lower threshold on tax distortions needed for intermediate exhaustion of fiscal capacity.

[20] If some $\hat{\tau}^- < 1$ is enough to escape from the safety trap, then the government would be indifferent to set any $\tau^- \in [\hat{\tau}^-, 1]$

Proposition 4 (Intermediate Exhaustion of Fiscal Capacity)**.** *The domestically optimal (ex post) tax rate τ_D^- is smaller than 1, if and only if,*

$$\underline{\psi} \equiv \frac{\omega\delta(1-\bar{\rho})(1+\phi)}{(\nu\omega)^2 + (1+\bar{\rho})\nu\omega\delta + \delta^2\bar{\rho}} < \psi \tag{37}$$

The lower bound $\underline{\psi}$ decreases with average private pledgeability ($\bar{\rho}$), and increases with allowed tax rate adjustment (ϕ). First, larger private safe assets reduce the benefits from public debt generating less incentives to exhaust fiscal capacity, so even low tax distortions are enough to prevent the government to issue all the public debt it could. Second, when the government is able to adjust tax rates more flexibly, public debt issuance is less costly, so it requires larger distortions to justify not exhausting public debt capacity. The described intuition is present in a closed economy (set $\omega = 1$), but there are some additional insights from the open economy case. Most importantly, it is less likely for the home country to exhaust fiscal capacity when the rest of the world is less financially developed, i.e. $\rho^* < \rho$.

From now onwards, we restrict our analysis to an environment where tax distortions are high enough to imply intermediate exhaustion of fiscal capacity.

Public Debt Externality and Underprovision. The are two key components that deliver an externality of public debt provision. First, in a global safety trap, public (safe) debt issuances from a government generate output expansions in every country. Second, the cost associated with public debt (distortionary taxation) is borne only by the issuing country. The first point is a feature of financial integration, once in the safety trap, a global output drop is needed to equilibrate the safe asset market, however our model is silent about the distribution of the output drops across countries. We have assumed symmetric output drops, yet we only need that output drops are shared to some degree, i.e. $\max\{\xi, \xi^*\} < 1$, for the debt externality to appear. The second point is driven by the fact that the government needs to collect distortionary taxation to meet its debt service.

The formal result follows from comparing the solution to (36) to the solution of

$$\begin{aligned} \max_{\{\tau,\tau^-,\}} \; & [1-\tau\psi\nu]\xi(\tau^-)X + \xi^*(\tau^-)X^* \\ \text{s.t. } & \tau^- \le (1+\phi)\tau \\ & \tau, \tau^- \in [0,1] \end{aligned} \tag{38}$$

Note that the first (second) term in the objective function is the home (foreign) country's net output. Therefore, as long as $\xi^*(\tau^-)$, is an increasing function, i.e. as long as the foreign country is bearing some of the output drop due to the safety trap, there is a benefit from debt issuance not internalized from the Home's perspective. Let τ_G^- be the solution to (38) and $D(.)$ be defined by (29), then we can state the result as follows.

Proposition 5 (Public Debt Underprovision)**.** *The optimal debt provision from a global perspective $D(\tau_G^-)$ is strictly larger than the optimal debt provision $D(\tau_D^-)$ from a domestic perspective, i.e. $D(\tau_G^-) > D(\tau_D^-)$.*

It is insightful to note what kind of market failure generates this externality. It is the fixed debt price due to the ZLB that plays the key role. For example, if the foreign government were able to offer to buy home country's debt at lower interest rate (understanding the expansionary effect it would have on its economy), it would find it optimal to do so. The market would act in a similar way if public debt's price was not fixed.

3.3 Open vs. closed capital account

In this subsection, we focus on the situation where the financially integrated (global) economy experiences a safety trap, but the home country can escape from it by closing its capital account, i.e.

$$r^{S,n} < 0 < r_{aut}^{S,n}$$

We restrict attention to an environment where each country is required to maintain a certain level of public debt, and explore the cost and benefits for the home country of closing its capital account. The analysis compares the financial autarky outcome against the financial integration outcome, both under the (domestically) optimal public debt policy.

The minimum debt requirement assumption is motivated by factors that are not explicitly included in our model. For example, the government might have issued debt in the past to finance infrastructure investments or the creation of public institutions. Moreover, one can broadly think of the lower bound on debt as a proxy for required ongoing government expenditures. We formally state the assumption regarding the minimum debt level before beginning our analysis.

Assumption 4 (Minimum Debt Level)**.**

$$D \geq \underline{D} > 0 \tag{39}$$

In the following, we characterize the solutions for the home economy with an open and a closed financial account focusing on the solutions for debt levels and tax rates since we will compare the (net) output of each solution in steady state. In particular, (net) output will be given by $(1 - \psi\tau\nu)\xi X$.

Open capital account. Given our focus on a global safety trap situation, the problem the government faces is the one described by (36) with the additional restriction (39). Recall that (29) is an equilibrium condition that always holds, and τ_D^- denotes the solution to the problem when there is no minimum debt requirement. Then, the optimal (ex post) tax rate in the open economy τ_{open}^- can be characterized as[21]

$$\tau_{open}^- = \begin{cases} \tau_D^- & \text{if } D(\tau_D^-) > \underline{D} \\ D^{-1}(\underline{D}) & \text{if } D(\tau_D^-) \leq \underline{D}, \end{cases} \tag{40}$$

where $D^{-1}(\underline{D})$ is the tax rate corresponding to $\underline{D}$. Moreover, the (ex ante) rate is given by $\tau_{open} = (1 + \phi)^{-1}\tau_{open}^-$, and net output, $(1 - \psi\tau_{open}\nu)\xi(\tau_{open}^-)X$, is decreasing in $\underline{D}$ when the debt requirement is binding.

The intuition behind the latter characterization is simple. If the debt level requirement is small, it will not be binding since it is optimal to choose a debt level above the minimum requirement due to its expansionary effects on output. If this requirement is large, then it is optimal to set tax rates to match this requirement, which implies that at the margin the additional debt has a negative effect on net output because the larger taxation needed outweighs the expansionary effect of additional safe assets.

Closed capital account. We assume that under financial autarky the home country does not experience a safety trap even with no debt provision, i.e. $\rho\mu^- > \alpha(1-\delta)$. Then, output is always at potential, i.e. $\xi = 1$, and there are no output-expansionary benefits from providing public debt, yet there is still a cost due to distortionary taxation. The optimal debt provision is to maintain it as low as possible, i.e. $D = \underline{D}$. Solving the autarky ($X^* = 0$) equilibrium after the Poisson event, we recover a relation between debt value and ex post tax rate, τ_{closed}^-.

$$D_{closed}(\tau^-) = \left(\frac{\nu\tau^-}{\nu\tau^- + \delta}\right)\frac{\mu^- X}{\theta}$$

As before, this is a strictly increasing function of τ^-. The minimum debt level requirement

[21] The proof is provided in Appendix A.6

imposes two constraints on the (ex ante) tax rate τ_{closed}. First, the (ex ante) tax rate needs to ensure that the government effective tax collection is at least enough to cover interests on public debt, i.e.

$$\tau_{closed}(1-\psi)\nu X \geq r^S_{aut}(\underline{D})\underline{D} > 0, \tag{41}$$

where $r^S_{aut}(\underline{D})$ is the home country's interest rate under financial autarky and debt level $\underline{D}$, which can be characterized using (34) when $X^* = 0$ (so, $\omega = 1$, $\bar{\rho} = \rho$, and $\bar{\alpha} = \alpha$). In particular,

$$r^S_{aut}(\underline{D}) = \delta\theta - (1-\delta)\theta\left[\frac{\alpha}{\mu}\left(\frac{\nu D^{-1}_{closed}(\underline{D}) + \delta}{\nu D^{-1}_{closed}(\underline{D}) + \rho\delta}\right) - 1\right]$$

where $D^{-1}_{closed}(D)$ is the tax rate implied by debt level D. Second, the government also has to satisfy the restriction on adjustments of tax rates, i.e.

$$\tau_{closed} \geq (1+\phi)^{-1}D^{-1}_{closed}(\underline{D}) \tag{42}$$

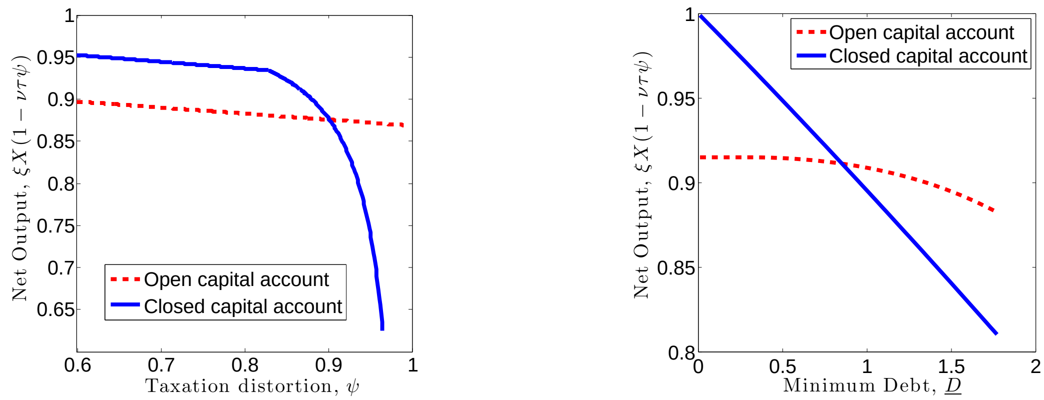
The presence of tax distortions ensures that the optimal solution for the ex ante tax rate is the minimum rate that satisfy both requirements, i.e.

$$\begin{aligned}\tau_{closed} &= \max\left\{\frac{r^S_{aut}(\underline{D})}{(1-\psi)\nu}\frac{\underline{D}}{X}, (1+\phi)^{-1}D^{-1}_{closed}(\underline{D})\right\} \\ &\equiv \max\left\{\tau^1_{closed}, \tau^2_{closed}\right\}\end{aligned} \tag{43}$$

where the last line defines the *revenue motive bound*, τ^1_{closed}, and the *adjustment motive bound*, τ^2_{closed}, of the (ex ante) tax rate. Note that the tax rate is an strictly increasing function of $\underline{D}$, which implies that net output, $(1-\psi\tau_{closed}\nu)X$, is a decreasing function of $\underline{D}$.

Costs and benefits of closing the capital account. The downside of financial integration for Home is that the excess demand for safe assets from the foreign country pushes the global economy into a safety trap that features less than full utilization capacity, i.e. $\xi(\tau^-) < 1$. The upside is that it pays no interest on its debt because the safe rate is zero. The latter implies that it has no *revenue motive* to collect taxes independently of how large its debt is. Large required debt levels only have effects on the (ex ante) tax rate through the *adjustment motive* since at some point large debt requirements force an increase in the tax rate after the bad Poisson event.[22] Closing the capital account has the advantage of

[22]Moreover, it can be proved that when the *adjustment motive* is binding in the open and closed economy, that a lower tax rate is needed in the open economy to maintain the same debt level, i.e. $D^{-1}(\underline{D}) <$



(a) Tax distortion

(b) Minimum debt requirement

Figure 4: Open vs. closed capital account

insulating the home economy from the safety trap, $\xi = 1$, but generates a *revenue motive* for tax collection since the safe interest rate is now positive.

Two parameter are key in determining how the trade-off between output losses due to a safety trap and avoiding debt servicing cost plays out. First, the parameter ψ governing the severeness of tax distortions. The larger ψ, the more attractive becomes an open capital account. This is due to the fact that larger inefficiencies imply a larger tax rate to collect any given amount of resources. We illustrate this force in Figure 4a. For a given minimum debt requirement $\underline{D}$, we vary the severeness of tax distortions. We can see that for low levels of ψ, a closed capital account is preferable but once ψ crosses a threshold, an open capital account is desirable. Second, the minimum debt requirement $\underline{D}$ is important. A low (high) debt requirement translates into a low (high) cost of closing the capital account since a positive safe interest rate will generate low (high) interest payments, and therefore tilt the trade-off towards a closed (open) capital account. This is illustrated in Figure 4b, where we hold ψ fixed and vary $\underline{D}$. While the forces described above are present in general, how they exactly play out depends on the parameterization of the model.[23]

Finally, we can show analytically that for for sufficiently small values of $\underline{D}$, a closed capital account is always preferable.

Proposition 6 (Closed Capital Account with Small Debt)**.** *$\exists \hat{D} > 0$ such that for any $\underline{D} < \hat{D}$, net output is larger with a closed capital account.*

$D^{-1}_{closed}(\underline{D})$. This follows from the fact that the (ex post) interest rate is lower in the open economy, so the same dividend tax flows generate more value.

[23]This is why we do not prove a formal proposition but rather illustrate them by means of a numerical example. The exact numerical values used to generate Figure 4 are provided in Appendix A.7.

The intuition for this stems from the fact that there is no safety trap with a closed capital account and when the debt requirement becomes arbitrarily small, the downside of debt servicing payments vanishes.

3.4 Consumption, Imports and Net Interest Income

So far, we have treated (net) output as the object of interest for the government. However, agents ultimately derive utility from consumption and in the open economy, a country's net output and consumption do not necessarily align perfectly. Thus, we turn to examine the situation where the government cares about consumption. The solution for consumption can be written as

$$C = \underbrace{\xi(\tau^{-})X(1-\nu\psi\tau)}_{\text{Net Output}} + \underbrace{\xi(\tau^{-})X\delta(1-\omega)\frac{(\nu(1-\tau\psi)-\nu^{*})}{\omega\nu(1-\tau\psi)+(1-\omega)\nu^{*}}}_{\text{Imports/Exports}} \tag{44}$$

This implies that the Home country can permanently consume beyond its (net) production, if and only if, its neutral newborns' share of output (net of taxes and rebates) is larger than the one in the Foreign country, i.e. $(1-\tau\psi)\nu > \nu^{*}$. The intuition behind this follows from interest payments and the risk premium. For Home to be able to consume above its net output permanently, it needs to finance its imports with the return on its asset position, i.e. it needs to generate a net income from interest payments.[24] The presence of the risk premium ensures larger returns for Neutrals' wealth, so a larger fraction of Neutrals is associated with greater interest payments; in turn, greater interest payments allow for financing of permanent imports.

This gap between consumption and (net) output is a key element when considering to close the capital account. In the last subsection, we analyzed the case where $r^{S,n} < 0 < r^{S,n}_{aut}$ which follows from sufficiently large differences in the heterogeneities we emphasize, $\rho > \rho^{*}$ and $\alpha < \alpha^{*}$. In such an environment, when the government's objective is consumption (not output), closing the capital account comes with an extra cost (compared to the analysis focusing on output): it prevents the Home country from enjoying a positive flow of resources due to asset returns. That is, even when there is no debt servicing motive ($\underline{D} = 0$), it might still be desirable to maintain an open capital account as this allows to enjoy additional consumption financed via returns earned on net foreign assets.

Now, we emphasize that the intermediate exhaustion of fiscal capacity and the debt underprovision results are also present when the government targets consumption. The government's

[24] Recall that NFA positions are constant in steady state.

problem is given by

$$\max_{\{\tau,\tau^-\}} C \equiv [1-\Gamma(\tau)]\xi(\tau^-)X \tag{45}$$
$$\text{s.t. } \tau^- \leq (1+\phi)\tau$$
$$\tau, \tau^- \in [0,1]$$

where

$$\Gamma(\tau) = \nu\psi\tau - \delta(1-\omega)\frac{(\nu(1-\tau\psi)-\nu^*)}{\omega\nu(1-\tau\psi)+(1-\omega)\nu^*}$$

which is strictly increasing in τ. Let (τ^-, τ_C^-) denote the solution for this problem. The following proposition illustrates the intermediate exhaustion of fiscal capacity result. The intuition is analogous to the one described for output.

Proposition 7 (Intermediate Exhaustion of Fiscal Capacity)**.** *The optimal (ex post) tax rate τ_C^- is smaller than 1, if and only if, $\psi > \underline{\psi}(\bar{\rho}, \phi)$ where the $\underline{\psi}$ is increasing in the allowed tax rate adjustment ϕ and decreasing in $\bar{\rho}$.*

Next, we turn to the debt underprovision result. Note that maximizing global consumption is equivalent to maximizing global (net) output,[25] so τ_G^- is the tax rate that maximizes global consumption. To study the presence of an externality, we need to analyze the effects of taxes on the part of global net output (or consumption) that is not accrued to the Home country. In previous subsections, this part was Foreign output, and the externality followed from the fact that ξ^* was increasing in τ^-. Now, the relevant part is foreign consumption which can be written as

$$C^* = \xi(\tau^-)(X+X^*)\frac{(1-\omega)\nu}{\omega\nu(1-\tau\psi)+(1-\omega)\nu^*} \tag{46}$$

Thus, Foreign consumption is increasing in τ^- and τ. The first effect is the output effect, larger (ex post) taxes in Home expand global safe assets and therefore output in both countries. The second effect is particular to the consumption analysis. A larger tax rate in Home reduces the Neutrals' share of wealth which decreases the overall interest payments to Home. This translates into less imports for the Home country. The effect is exactly the opposite for the Foreign country: larger interest payments, and larger imports, i.e. more consumption. The following proposition formalizes the latter intuition.

[25]By goods market clearing, it holds that $C + C^* = (1-\psi\nu)\xi X + \xi^* X^*$.

Proposition 8 (Public Debt Underprovision)**.** *The optimal debt provision from a global perspective $D(\tau_G^-)$ is strictly larger than the optimal debt provision $D(\tau_C^-)$ from a domestic perspective when targeting consumption, i.e.* $D(\tau_G^-) > D(\tau_C^-)$.

4 Conclusion

This paper has explored the output implications of safe asset shortages in a global economy. Safe asset demand from countries with low development of financial markets and large number of risk-averse agents depresses world interest rates. When interest rates are subject to a zero lower bound, then this safe asset demand pushes the world economy into a safety trap in which all countries experience output losses.

Under imperfect fiscal policy (taxes are distortionary and imperfectly adjustable over time), safe public debt has the upside of expanding output though the downside of generating wasteful distortions. Due to this trade-off, an externality arises and the global economy will suffer from underprovision of safe assets. Since safe asset demand is at the root of the problem, countries which do not suffer from safe asset shortages in autarky, have an incentive to close their capital account in order to insulate their economy from safety traps. Yet, this can imply foregoing net interest income on their asset positions.

All in all, our work indicates that safety traps are global phenomena that originate in some countries but affect the world economy as a whole. Due to the externality of safe public debt provision, it seems unlikely that the global undersupply of safe assets will be resolved by the actions of a "safe haven" alone. Moreover, if a world economy has a several countries supplying safe assets on net, these countries have an incentive to move towards a closed capital account, perhaps leading to a "capital controls war" among them. Hence, policy responses towards safety traps cry for coordination on a global scale.

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A Appendix

A.1 Solution to ex-ante system in unconstrained case

The solution to the ex-ante system in the unconstrained regime is characterized by the following expressions.

$$r = r^S = \delta\theta$$

$$\begin{aligned}
\delta^S &= \mu^-\rho\delta & \delta^{S*} &= \mu^-\rho^*\delta^* \\
V^S &= \rho\mu^-\frac{X}{\theta} & V^{S*} &= \rho^*\mu^-\frac{X^*}{\theta} \\
V^R &= (1-\rho\mu^-)\frac{X}{\theta} & V^{R*} &= (1-\rho^*\mu^-)\frac{X^*}{\theta} \\
W^K &= \frac{\alpha X}{\theta} & W^{K*} &= \frac{\alpha^* X^*}{\theta} \\
W^N &= \frac{(1-\alpha)X}{\theta} & W^{N*} &= \frac{(1-\alpha)X^*}{\theta}
\end{aligned}$$

A.2 Solution to ex-ante system in constrained case

The solution to the ex-ante system in the constrained regime is characterized by the following expressions.

$$\begin{aligned}
r^S &= \delta\theta - \theta(1-\delta)\frac{\tilde{\alpha}-\tilde{\rho}\mu^-}{\tilde{\rho}\mu^-} \\
r &= \delta\theta + \theta(1-\delta)\frac{\tilde{\alpha}-\tilde{\rho}\mu^-}{1-\tilde{\rho}\mu^-}
\end{aligned}$$

$$\begin{aligned}
\delta^S &= \mu^-\delta\rho - \frac{\rho}{\bar{\rho}}(1-\delta)(\bar{\alpha}-\bar{\rho}\mu^-) & \delta^{S*} &= \mu^-\delta\rho^* - \frac{\rho^*}{\bar{\rho}}(1-\delta)(\bar{\alpha}-\bar{\rho}\mu^-) \\
V^S &= \rho\mu^-\frac{X}{\theta} & V^{S*} &= \rho^*\mu^-\frac{X*}{\theta} \\
V^R &= \frac{(\delta-\delta^S)X}{r} & V^{R*} &= \frac{(\delta-\delta^{S*})X^*}{r} \\
W^K &= \frac{\alpha}{\bar{\alpha}}\frac{X}{\theta}\bar{\rho}\mu^- & W^{K*} &= \frac{\alpha^*}{\bar{\alpha}}\frac{X^*}{\theta}\bar{\rho}\mu^- \\
W^N &= \frac{(1-\alpha)(1-\delta)X}{\theta-r} & W^{N*} &= \frac{(1-\alpha^*)(1-\delta)X^*}{\theta-r}
\end{aligned}$$

A.3 Ex-ante system and solution with nominal rigidities and ZLB

The ex-ante steady state equilibrium system of the economy with a zero-lower bound and endogenous output can be written as follows.

Wealth dynamics

$$0 = -\theta W^K + r^S W^K + \alpha(1-\delta)\xi X \tag{47}$$
$$0 = -\theta W^N + r W^N + (1-\alpha)(1-\delta)\xi X \tag{48}$$
$$0 = -\theta W^{K*} + r^S W^{K*} + \alpha^*(1-\delta^*)\xi^* X^* \tag{49}$$
$$0 = -\theta W^{N*} + r W^{N*} + (1-\alpha^*)(1-\delta^*)\xi^* X^* \tag{50}$$

Pricing equations

$$r^S V^S = \delta^S \xi X \tag{51}$$
$$r V^R = (\delta - \delta^S)\xi X \tag{52}$$
$$r^S V^{S,*} = \delta^{S*} \xi^* X^* \tag{53}$$
$$r V^{R,*} = (\delta^* - \delta^{S*})\xi^* X^* \tag{54}$$

Market clearing

$$\theta(W^K + W^{K*} + W^N + W^{N*}) = \xi X + \xi^* X^* \tag{55}$$
$$W^K + W^{K*} + W^N + W^{N*} = V^S + V^R + V^{S*} + V^{R*} \tag{56}$$

Regime

$$0 = (V^S + V^{S*} - W^K - W^{K*}) \cdot (r - r^S) \tag{57}$$
$$V^S + V^{S*} \geq W^K + W^{K*}$$
$$r \geq r^S$$

Safe assets

$$V^S = \rho V^- \tag{58}$$
$$V^{S*} = \rho^* V^{-*} \tag{59}$$

ZLB

$$(r^S > 0 \ \wedge \ \xi = \xi^* = 1) \ \text{ or } \ (r^S = 0 \ \wedge \ \min\{\xi, \xi^*\} < 1) \tag{60}$$

To express the solution of this system, define

$$\tilde{\alpha}(\xi, \xi^*) \equiv \frac{\alpha \xi X + \alpha^* \xi^* X^*}{\xi X + \xi^* X^*}, \qquad \tilde{\rho}(\xi, \xi^*) \equiv \frac{\rho X + \rho^* X^*}{\xi X + \xi^* X^*}$$

Then, the solution for the unconstrained regime can be characterized as

$$r^S = \delta\theta - \theta(1-\delta)\frac{\tilde{\alpha} - \tilde{\rho}\mu^-}{\tilde{\rho}\mu^-}$$
$$r = \delta\theta + \theta(1-\delta)\frac{\tilde{\alpha} - \tilde{\rho}\mu^-}{1 - \tilde{\rho}\mu^-}$$

$$\begin{aligned}
\delta^S &= \mu^- \delta \rho - \frac{\rho}{\tilde{\rho}}(1-\delta)(\tilde{\alpha} - \tilde{\rho}\mu^-) & \delta^{S*} &= \mu^- \delta \rho^* - \frac{\rho^*}{\tilde{\rho}}(1-\delta)(\tilde{\alpha} - \tilde{\rho}\mu^-) \\
V^S &= \rho \mu^- \frac{X}{\theta} & V^{S*} &= \rho^* \mu^- \frac{X*}{\theta} \\
V^R &= \frac{(\delta - \delta^S)X}{r} & V^{R*} &= \frac{(\delta - \delta^{S*})X^*}{r} \\
W^K &= \frac{\alpha}{\tilde{\alpha}} \frac{X}{\theta} \tilde{\rho}\mu^- & W^{K*} &= \frac{\alpha^*}{\tilde{\alpha}} \frac{X^*}{\theta} \tilde{\rho}\mu^- \\
W^N &= \frac{(1-\alpha)(1-\delta)X}{\theta - r} & W^{N*} &= \frac{(1-\alpha^*)(1-\delta)X^*}{\theta - r}
\end{aligned}$$

We have basically solve the system treating ξ, ξ^* as parameters. Having obtained the solution laid out above, we check whether $r^S \geq 0$ for $\xi = \xi^* = 1$. If this is the case, the solution is given by the expressions above with $\xi = \xi^* = 1$. If $r^S < 0$, we have $r^S = 0$ and obtain a relationship between $\xi = \xi^*$ that we display in the text.

A.4 Relaxing fully rigid prices

This section lays out a version of our international model where prices are sticky, though not completely rigid. This is interesting for the following reason. When we study the 2-country model without inflation, we can only arrive at a linear relationship between ξ and ξ^* – we cannot pin them down separately. However, in the version with *some* price stickiness, we are able to do that. This extension is an international version of the model with price adjustments in CF (2015).

We assume that prices are sticky downward (not upward) and adjustments are governed by Philipps-curves. In particular, we specify

$$\begin{aligned}\pi &\geq -\left(\kappa_0 + \kappa_1(1-\xi)\right)\\ \pi^* &\geq -\left(\kappa_0^* + \kappa_1^*(1-\xi^*)\right),\end{aligned}$$

where the κs are parameters. We assume that when the economy is below potential, i.e. $\xi < 1$ ($\xi^* < 1$), prices fall as fast as possible. These restrictions will on the prices in Home and Foreign will be captured by two complementary slackness conditions in the equilibrium system. We assume monetary authorities set nominal interest rates according to Taylor-rules that are specified below.

The steady-state equilibrium is characterized by wealth dynamics (47) - (50), asset pricing equations (51)-(54), market clearing conditions (55) - (56), and the following.

Safe asset regime condition

$$W^K + W^{K*} = V^S + V^{S*} \tag{61}$$

Philipps-curves

$$\left[\pi + \left(\kappa_0 + \kappa_1(1-\xi)\right)\right](1-\xi) = 0 \tag{62}$$

$$\left[\pi^* + \left(\kappa_0^* + \kappa_1^*(1-\xi^*)\right)\right](1-\xi^*) = 0 \tag{63}$$

Taylor-rules and ZLB

$$i = \max\{0, r^{S,nat} + \hat{\pi} + \phi(\pi - \hat{\pi})\} \tag{64}$$

$$i^* = \max\{0, r^{S,nat} + \hat{\pi}^* + \phi^*(\pi^* - \hat{\pi}^*)\} \tag{65}$$

Fisher equations

$$i = r^S + \pi \tag{66}$$

$$i^* = r^S + \pi^* \tag{67}$$

Safe assets

$$V^S = \bar{V}^S \tag{68}$$

$$V^{S*} = \bar{V}^{S*} \tag{69}$$

Law-of-one-price

$$\frac{\dot{E}}{E} = \pi - \pi^*. \tag{70}$$

In full rigor, we would use a backward-inductive step to solve for the value of safe assets as functions of parameters (this would include solving the system after the aggregate shock has been realized). This will include judgement about ex-post equilibrium selection (potentially there are multiple ex-post equlibria). We abstract from these issues here by assigning *some* fixed number to the value of safe assets.

We have a set of 19 variables to solve for: real returns $\{r, r^S\}$, wealth distribution over agents' types $\{W^K, W^N, W^{K*}, W^{N*}\}$, asset values $\{V^S, V^R, V^{S*}, V^{R*}\}$, share of dividends accrued to safe assets $\{\delta^S, \delta^{S*}\}$, utilization capacities $\{\xi, \xi^*\}$, nominal returns $\{i, i^*\}$, inflation rates $\{\pi, \pi^*\}$, and exchange rate depreciation $\frac{\dot{E}}{E}$. And we have 19 independent equations from the system above. We can solve this but the solution is possibly non-unique. Thus, we attempt a solution by means of an AS-AD approach.

For Home, while $\xi < 1$, the AS-curve is given by

$$\pi = -\kappa_0 - \kappa_1 + \kappa_1 \xi.$$

At $\xi = 1$, the AS-curve becomes vertical. The AD-curve has two segments. The first is when $i = 0$, then

$$\pi = \frac{\alpha(1-\delta)\xi X + \alpha^*(1-\delta)\xi^* X^*}{\bar{V}^S + \bar{V}^{S*}} - \theta.$$

The second is when $i > 0$, which gives

$$\pi = \hat{\pi} + \frac{1}{\phi - 1} \frac{\alpha(1-\delta)(1-\xi)X + \alpha^*(1-\delta)(1-\xi^*)X^*}{\bar{V}^S + \bar{V}^{S*}}$$

The kink connecting the two segments is characterized by

$$\pi = \frac{\phi - 1}{\phi}\hat{\pi} - \frac{r^{S,nat}}{\phi}$$

Completely analogous, we have for Foreign that as long as $\xi^* < 1$, the AS-curve is given by

$$\pi^* = -\kappa_0^* - \kappa_1^* + \kappa_1^* \xi^*$$

and turns vertical once $\xi^* = 1$. The upward-sloping part of the AD-curve ($i^* = 0$) is given by

$$\pi^* = \frac{\alpha(1-\delta)\xi X + \alpha^*(1-\delta)\xi^* X^*}{\bar{V}^S + \bar{V}^{S*}} - \theta.$$

The downward-sloping part ($i^* = 0$) is given by

$$\pi^* = \hat{\pi}^* + \frac{1}{\phi^* - 1} \frac{\alpha(1-\delta)(1-\xi)X + \alpha^*(1-\delta)(1-\xi^*)X^*}{\bar{V}^S + \bar{V}^{S*}}$$

The kink for Foreign is characterized by

$$\pi^* = \frac{\phi^* - 1}{\phi^*}\hat{\pi}^* - \frac{r^{S,nat}}{\phi^*}$$

In the following, we characterize the equilibrium where both countries are in a recession. Equalizing the AS- and AD-curve in Home and Foreign respectively yields

$$\begin{aligned} -\kappa_0 - \kappa_1 + \kappa_1 \xi &= \frac{\alpha(1-\delta)X}{\bar{V}^S + \bar{V}^{S*}}\xi + \frac{\alpha^*(1-\delta)X^*}{\bar{V}^S + \bar{V}^{S*}}\xi^* - \theta \\ -\kappa_0^* - \kappa_1^* + \kappa_1^* \xi^* &= \frac{\alpha(1-\delta)X}{\bar{V}^S + \bar{V}^{S*}}\xi + \frac{\alpha^*(1-\delta)X^*}{\bar{V}^S + \bar{V}^{S*}}\xi^* - \theta \end{aligned}$$

This gives 2 equations in ξ and ξ^* that we can solve. Some algebra yields

$$\xi^* = \frac{\frac{\alpha(1-\delta)X}{\bar{V}^S + \bar{V}^{S*}} \frac{\theta - \kappa_0^* - \kappa_1^*}{\frac{\alpha(1-\delta)X}{\bar{V}^S + \bar{V}^{S*}} - \kappa_1} + \kappa_1^* + \kappa_0^* - \theta}{\kappa_1^* - \frac{\alpha^*(1-\delta^*)X^*}{\bar{V}^S + \bar{V}^{S*}} + \frac{\alpha(1-\delta)X}{\bar{V}^S + \bar{V}^{S*}} \frac{\frac{\alpha^*(1-\delta^*)X^*}{\bar{V}^S + \bar{V}^{S*}}}{\frac{\alpha(1-\delta)X}{\bar{V}^S + \bar{V}^{S*}} - \kappa_1}}$$

We can exploit the relationship between the two AS-curves to find a simple expression for ξ as a function of ξ^*.

$$\xi = \frac{(\kappa_0 - \kappa_0^*) + (\kappa_1 - \kappa_1^*)}{\kappa_1} + \frac{\kappa_1^*}{\kappa_1}\xi^*$$

Finally, note that in the recessionary equilibrium, we have $i = i^* = 0$ which implies $\pi = \pi^* = -r^{S,nat}$. Thus, exchange rate will be constant, i.e. $\frac{\dot{E}}{E} = 0$.

A.5 Solution for Model with Costly Debt Provision

We study the ex-ante steady state equilibrium in the constrained regime where safe rates are against the ZLB. This is characterized by the following set of equations.

Asset pricing equations

$$r^S V^S = \delta^S \xi X \tag{71}$$
$$r^S V^{S*} = \delta^{S*} \xi X^* \tag{72}$$
$$r V^R = (\delta - \delta^S) \xi X \tag{73}$$
$$r V^{R*} = (\delta - \delta^{S*}) \xi X^* \tag{74}$$

Wealth dynamics

$$(\theta - r^S) W^K = \alpha (1 - \delta) \xi X \tag{75}$$
$$(\theta - r^S) W^{K*} = \alpha^* (1 - \delta) \xi X^* \tag{76}$$
$$(\theta - r) W^N = (1 - \tau) \nu \xi X + S \tag{77}$$
$$(\theta - r) W^{N*} = \nu^* \xi X^* \tag{78}$$

Market Clearing

$$W^K + W^{K*} + W^N + W^{N*} = V^S + V^R + V^{S*} + V^{R*} + D \tag{79}$$
$$\theta \left(V^S + V^{S*} + V^R + V^{R*} + D \right) = (1 - \psi\nu) \xi X + \xi X^* \tag{80}$$

Constrained regime

$$V^S + V^{S*} + D = W^K + W^{K*} \tag{81}$$

Safe assets

$$V^S = \rho V^- \tag{82}$$
$$V^{S*} = \rho^* V^{*-} \tag{83}$$

ZLB

$$r^S = 0 \tag{84}$$

Government flow

$$S = \tau (1 - \psi) \nu \xi X - r^S D \tag{85}$$

These blocks correspond to asset pricing, wealth dynamics, market clearing, the constrained regime equation, the equations for safe assets, the (binding) ZLB constraint and the government flow budget constraint. The full solution to this system is given by

$$\xi = \frac{\mu^-}{\bar{\alpha}(1-\delta)} \frac{\omega\nu\tau^- + \bar{\rho}\delta}{\omega\nu\tau^- + \delta}$$
$$r = \theta - \theta \frac{(1-\delta)(1-\bar{\alpha}) - \psi\tau\nu\omega}{(1-\delta)(1-\bar{\alpha}) - \psi\tau\nu\omega + \delta}$$

$$\delta^S = 0 \qquad \delta^{S*} = 0$$
$$V^S = \rho \frac{\omega\delta}{\omega\nu\tau^- + \delta} \frac{\mu^-(X + X^*)}{\theta} \qquad V^{S*} = \rho^* \frac{(1-\omega)\delta}{\omega\nu\tau^- + \delta} \frac{\mu^-(X + X^*)}{\theta}$$
$$W^K = \frac{\alpha}{\bar{\alpha}} \frac{\mu^-}{\theta} \frac{\omega\nu\tau^- + \bar{\rho}\delta}{\omega\nu\tau^- + \delta} X \qquad W^{K*} = \frac{\alpha^*}{\bar{\alpha}} \frac{\mu^-}{\theta} \frac{\omega\nu\tau^- + \bar{\rho}\delta}{\omega\nu\tau^- + \delta} X^*$$

and with these expressions at hand, we can calculate

$$W^N = \frac{(1-\psi\tau)\nu X}{\theta - r} \frac{\mu^-}{\bar{\alpha}(1-\delta)} \frac{\omega\nu\tau^- + \bar{\rho}\delta}{\omega\nu\tau^- + \delta} \qquad W^{N*} = \frac{\nu^* X^*}{\theta - r} \frac{\mu^-}{\bar{\alpha}(1-\delta)} \frac{\omega\nu\tau^- + \bar{\rho}\delta}{\omega\nu\tau^- + \delta}$$
$$V^R = \frac{\delta\xi X}{r} \qquad V^{R*} = \frac{\delta\xi X^*}{r},$$

which completes the full solution.

A.6 Solution with minimum debt requirement

We consider the same problem as in (38) with the additional constraint $D > \underline{D}$. Since (29) delivers an strictly increasing relationship between D and τ^-, the constraint is effectively a lower bound on τ^-, denote it as $D^{-1}(\underline{D})$. If the solution to (38) satisfies the constraint, i.e. $\tau_G^- < D^{-1}(\underline{D})$, then the extra constraint is not binding and the solution is unchanged. If τ_G^- does not satisfies debt constraint, the solution is given by $D^{-1}(\underline{D})$ because the function is globally concave, i.e. $Y_G'(\tau^-) > 0$ and $Y_G''(\tau^-) < 0$.

A.7 Numerical example

The parametrization used for the numerical examples is reported in the following table. The examples aim to illustrate the forces present in the model, and not to provide quantitative statements. When illustrating the effects of debt requirement $\underline{D}$ (tax distortions ψ), we fix $\psi = 0.8$ ($\underline{D} = 0.6$).

Parameter	Value
δ	0.5
θ	0.05
μ^-	0.2
(ρ, ρ^*)	(0.8, 0.5)
(α, α^*)	(0.2, 0.8)
(X, X^*)	(1,0.33)

To present the effects neatly, we use different values of ϕ in each example. For the debt requirement (tax distortions) example, we use $\phi = 1.5$ ($\phi = 0.11$).

A.8 Proof of Proposition 1

Note that the natural safe real interest rate in the world economy, $r^{S,n}$, is given by

$$r^{S,n} = \delta\theta - (1-\delta)\theta \left(\frac{\bar{\alpha} - \bar{\rho}\mu^-}{\bar{\rho}\mu^-} \right).$$

We obtain the natural autarky safe real rate in Home, $r^{S,n}_{aut}$, as the limiting case of the above expression when $X^* \to 0$. Analogously, we obtain $r^{S,n*}_{aut}$ from the above expression when $X \to 0$. We have

$$r^{S,n}_{aut} = \delta\theta - (1-\delta)\theta \left(\frac{\alpha - \rho\mu^-}{\rho\mu^-} \right)$$
$$r^{S,n*}_{aut} = \delta\theta - (1-\delta)\theta \left(\frac{\alpha^* - \rho^*\mu^-}{\rho^*\mu^-} \right).$$

Now, it follows straightforwardly that the respective inequalities hold if and only if the condition given in the Proposition is satisfied.

A.9 Proof of Proposition 2

We calculate Home's NFA position in safe assets as

$$NFA^S \equiv V^S - W^K = \rho\mu^- \frac{X}{\theta} - \frac{\alpha}{\bar{\alpha}} \frac{X}{\theta} \bar{\rho}\mu^-.$$

Hence,

$$NFA^S > 0 \Leftrightarrow \frac{\bar{\alpha}}{\bar{\rho}} > \frac{\alpha}{\rho},$$

which completes the proof.

A.10 Proof of Proposition 3

We have that

$$r^{S,n} = \delta\theta - (1-\delta)\theta\left(\frac{\bar{\alpha} - \bar{\rho}\mu^{-}}{\bar{\rho}\mu^{-}}\right) < 0 \Leftrightarrow \bar{\rho}\mu^{-} < (1-\delta)\bar{\alpha}$$

and

$$r^{S,n}_{aut} = \delta\theta - (1-\delta)\theta\left(\frac{\alpha - \rho\mu^{-}}{\rho\mu^{-}}\right) > 0 \Leftrightarrow \rho\mu^{-} > (1-\delta)\alpha.$$

A.11 Proof proposition 4

We start from problem (36). Recall that function $\xi(\tau^{-})$ is given by (35), and τ_D^{-} denotes the solution for τ^{-}. It is straightforward to proof that the objective function is increasing in τ^{-}, and decreasing in τ, so constraint $\tau^{-} \leq (1+\phi)\tau$ will be binding. Then, home net output (the objective function) can be written as

$$Y(\tau^{-}) = (1 - (1+\phi)^{-1}\tau^{-}\psi\nu)\xi(\tau^{-})X$$

It is straightforward to verify that $Y'(\tau^{-}) > 0$ and $Y''(\tau^{-}) < 0$. Let $Y'(1,\psi)$ denote $Y'(1)$ as a function of tax inefficiencies ψ. It is straightforward to show $\frac{dY'(1,\psi)}{d\psi} < 0$. Define $\underline{\psi}$ as the value of ψ such that $Y'(1,\underline{\psi}) = 0$. Then, for any $\psi > \underline{\psi}$, we have that $Y'(1) < 0$ holds, i.e. $\tau_D^{-} < 1$. Also, for any $\psi < \underline{\psi}$, we have $Y'(1) > 0$ and $\tau_D^{-} = 1$. The boundary shown in the proposition is equivalent to $Y'(1,\underline{\psi}) = 0$.

A.12 Proof proposition 5

Recall that we focus on the situation where there is intermediate exhaustion of fiscal capacity, i.e. $\tau_D^{-} < 1$. Problem (38) is symmetric to problem (36) except for an extra term in the objective function, $\xi(\tau^{-})X$. It is straightforward to prove that the objective is increasing in τ^{-} and decreasing in τ which implies that the adjustment constraint will be binding (as in the domestic problem). Let $Y_G(\tau^{-})$ denote global net output as function of τ^{-} imposing the adjustment constraint at equality.

$$Y_G(\tau^{-}) = Y(\tau^{)} + \xi(\tau^{-})X^{*}$$

where $Y(\tau^{)}$ the Home net output. It can be shown that $Y_G'(\tau^{-}) > 0$ and $Y_G''(\tau^{-}) < 0$. Therefore, for any $a \in [0,1)$, we have that $\tau_G^{-} > a$, if and only if, $Y'(a) > 0$. Let $a = \tau_D^{-}$, then we have that

$$Y_G'(\tau_D^-) = \underbrace{Y'(\tau_D^-)}_{=0} + \xi'(\tau_D^-)X^* > 0$$

where the first term is zero because of the FOC of problem (36) and the inequality follows from the fact that $xi(\tau^-)$ is an strictly increasing function. This implies $\tau_G^- < \tau_D^-$.

A.13 Proof proposition 6

With no minimum debt requirement,$\underline{D} = 0$, net output in the Home country with a closed capital account is given by X, while the corresponding value with an open capital account is given by $\xi(\tau^-)(1-\nu\psi\tau^-)X$. Recall that we are studying the situation where the financially integrated economy is in a safety trap while the Home country is not in financial autarky. It is clear that $\xi(\tau^-)(1-\nu\psi\tau^-)X > X$, so the proposition follows from the continuity the solutions on $\underline{D}$.

A.14 Proof proposition 7

We start from problem (45). Recall that function $\xi(\tau^-)$ is given by (35), and τ_C^- denotes the solution for τ^-. It is straightforward to proof that the objective function is increasing in τ^-, and decreasing in τ, so constraint $\tau^- \leq (1+\phi)\tau$ will be binding. Then, home net output (the objective function) can be written as

$$C(\tau^-) = (1 - \Gamma((1+\phi)^{-1}\tau^-))\xi(\tau^-)X$$

It is straightforward to verify that $C'(\tau^-) > 0$ and $C''(\tau^-) < 0$. Therefore, for any $a \in (0,1]$, we have that $\tau_C^- < a$, if and only if, $C'(a) < 0$. Let $C'(1,\psi,\phi,\bar{\rho})$ denote $C'(1)$ as a function of ψ, ϕ, and $\bar{\rho}$. It can be shown that $\frac{C'(1,\psi,\phi,\bar{\rho})}{d\psi} < 0$, $\frac{C'(1,\psi,\phi,\bar{\rho})}{d\phi} > 0$, and $\frac{C'(1,\psi,\phi,\bar{\rho})}{d\bar{\rho}} < 0$. Define $\underline{\psi}^c(\phi,\bar{\rho})$ as the value of ψ such that $C'(1,\underline{\psi}^c,\phi,\bar{\rho}) = 0$. It follows from the derivatives' signs that $\underline{\psi}^c_\phi > 0$, and $\underline{\psi}^c_{\bar{\rho}} < 0$. Moreover, for any $\psi > \underline{\psi}^c$, we have that $C'(1) < 0$ holds, i.e. $\tau_C^- < 1$. Also, for any $\psi < \underline{\psi}^c$, we have $C'(1) > 0$ and $\tau_C^- = 1$.

A.15 Proof proposition 8

By an argument completely symmetric to the one in proof of proposition 5, it is enough to prove that Foreign consumption C^* is increasing in in τ^- when imposing the adjustment constraint at equality. We can write foreign consumption as in equation (46), so the proposition follows.