Monetary Policy and the Cross-Section of Currency Risk-Premia*

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Abstract

Countries whose currencies appreciate during crises pay low interest rates on their debt but forego exchange rate depreciations as a tool for macroeconomic stabilization. This paper studies how these considerations shape optimal monetary policy of a small open economy (Home) that issues uncontingent Home-currency denominated external debt and is subject to nominal rigidities in price-setting. Aggregate demand management considerations call for a risk-premium on the Home bond that is decreasing (increasing) in the covariation of the global business cycle with Home productivity shocks (demand shocks for Home-produced goods). Insurance considerations call for a risk-premium on the Home bond that is increasing in the covariation of Home and Foreign output and decreasing in Foreign's risk-aversion. For countries with a larger fraction of Home-produced goods in consumption, optimal policy tilts towards aggregate demand management and for countries with a larger amount of Home-currency external debt, optimal policy tilts towards insurance. Empirically, I find support for the model's predictions that countries' currency risk-premia are increasing in the procyclicality of demand shocks and decreasing in the procyclicality of productivity shocks.

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1 Introduction

Exchange rate policies are a key instrument used by central banks around the world in order to cope with macroeconomic shocks. In particular, exchange rate depreciations can be employed to move international relative prices, thereby stimulating aggregate demand for domestically produced goods.[1] Simultaneously, exchange rate movements have important implications for financial market outcomes: Bonds denominated in currencies which appreciate during crises pay lower real interest rates.[2] This gives rise to a trade-off: Using exchange rate depreciations as a tool for stabilization during crises comes at the downside of higher ex-ante interest rates on Home-currency denominated debt.

In fact, this trade-off has surfaced prominently during the Great Recession. Countries like the US, Japan and Switzerland, whose currencies are prime examples of "safe" currencies, have long been enjoying the upside of extraordinarily low interest rates on their debt. As a direct flip side of this, the US dollar, Japanese Yen, and Swiss franc have appreciated massively after the onset of the financial crisis in 2008. This has caused concerns among policymakers over aggregate demand shortfalls due to declines in international competitiveness.[3]

In this paper, I analyze monetary policy for an economy that issues Home-currency denominated external debt and is subject to nominal rigidities in price-setting. To that end, I develop and test a theory of monetary policy where an exchange rate depreciation (appreciation) stimulates (dampens) aggregate demand for Home-produced goods and simultaneously raises (lowers) ex-ante interest rates on Home-currency denominated bonds as it makes these assets "riskier" ("safer") for international investors.

As risk-premia are absent in any linearized solution, I characterize the *full non-linear solution* of an open-economy New-Keynesian model which allows me to study the interactions of monetary policy and currency risk-premia (risk-premia on Home-currency denominated bonds). The model features a small open economy ("Home"), a large rest of the world ("Foreign"), and

[1] This is effective as long as prices for home-produced goods are, at least partially, sticky in domestic currency. Based on these grounds, Friedman (1953) famously makes the case for flexible exchange rates.

[2] This is simply the risk-based view of the uncovered-interest-parity puzzle: Bonds denominated in currencies that appreciate during "bad" times have negative currency risk-premia as they provide a good hedge against risk.

[3] Eichengreen (2010) provides a summary of the debate.

there are two kinds of goods: tradables and non-tradables. Non-tradable goods are produced using labor and production is hampered by nominal rigidities: Producers of non-tradable goods have to set prices in Home-currency one period in advance. Prices for tradable goods are exogenous and denominated in Foreign currency.[4] Asset markets are incomplete in the sense that only uncontingent Home- and Foreign-currency denominated bonds can be traded and Home holds a portfolio of external Home-currency denominated debt and external Foreign-currency denominated assets.[5] Any asset is priced by a global stochastic discount factor (SDF), which is the Foreign consumer's marginal utility of consumption. Hence, Home-currency denominated bonds have a positive (negative) risk-premium if their real payout covaries negatively (positively) with the global SDF. Home has control over its monetary policy and sets its nominal exchange rate vis-a-vis Foreign.

I analyze the problem of a Home social planner choosing monetary policy under full commitment in order to maximize welfare over attainable competitive equilibria. On the one hand, the presence of nominal rigidities gives rise to the well-known aggregate demand management motive for monetary policy. By depreciating its exchange rate, Home can depress the price of Home-produced relative to Foreign goods, thereby reallocating demand and stimulating production of Home goods. On the other hand, due to the combination of incomplete markets and Home-currency denominated external debt, exchange rate movements influence resource transfers from Home to Foreign *in real terms*: If Home appreciates its exchange rate in a state of the world where the global SDF is high, then the Home bond's payout in real terms rises that state, providing insurance to Foreign in "bad" times. The corresponding increase in covariation of the Home bond's real payout with the global SDF entails a benefit in form of a lower risk-premium on the Home bond – Foreign investors are willing to hold the Home bond at a lower interest rate. In order to understand what shapes the optimal policy, it is helpful to focus on two benchmark cases.

First, consider the exchange rate behavior if policy was only concerned with aggregate demand

[4] This can be interpreted as a world where prices for tradable goods are set in a dominant currency, e.g. the US dollar, which is arguably a good approximation of the data (see Gopinath (2016)).

[5] A portfolio structure of this kind is a good approximation for developed economies, which is what both my model and the empirical analysis in my paper focus on. Understanding monetary policy in the context of an economy with nominal rigidities, incomplete markets, and Foreign-currency denominated debt – as is oftentimes the case for emerging economies – would require a different kind of theory, as provided by Wang (2019).

management (i.e. closing the output gap in every state of the world). I show that the associated risk-premium on the Home bond is determined by the covariance of the global SDF with shocks to productivity of and demand for Home-produced goods.

When a positive shock to the productivity of domestic producers is realized, it is efficient to increase production. However, due to nominal rigidities, producers are unable to cut prices which results in an aggregate demand shortfall for Home-produced goods. Hence, it is optimal to depreciate the exchange rate, depressing the international relative price of Home-produced goods in order to stimulate aggregate demand and raise production up to the efficient level. Accordingly, the more positively Home productivity shocks are correlated with the global SDF, the higher the induced risk-premium on the Home bond. For example, if a country's total factor productivity is highly positively correlated with the global business cycle (i.e. highly negatively correlated with the global SDF), it is optimal to depreciate in booms and appreciate in downturns – inducing a negative risk-premium on the Home bond.

When a positive shock to aggregate demand for Home-produced goods hits, it would be efficient for producers to raise prices as long as the optimal level of production does not increase one-for-one. As price resetting by producers is constrained when nominal rigidities are present, such a shock results in excess aggregate demand for Home-produced goods. Increasing the international relative price of Home-produced goods is brought about by an exchange rate appreciation, reducing aggregate demand for Home goods down to the efficient level. The more positively correlated Home demand shocks are with the global SDF, the lower the induced risk-premium on the Home bond. For example, if a country is specialized in exporting a product with highly procyclical demand (e.g. a commodity such as crude oil), it is optimal to appreciate in booms and depreciate in downturns – inducing a positive risk-premium on the Home bond.

Second, consider the exchange rate behavior and induced risk-premium on the Home bond if policy was only concerned with using exchange rate movements to influence the amount of insurance provided to Foreign (i.e. implementing perfect risk-sharing). I show that the associated risk-premium on the Home bond is determined by the covariance of Home and Foreign tradables and by Foreign's risk aversion (relative to Home's).

Due to the presence of incomplete markets, there generically exist states of the world where Home's and Foreign's SDFs diverge, i.e. there is a gap between Home's and Foreign's intertemporal marginal rate of substitution. For any state of the world where the Foreign SDF exceeds the Home SDF, Home can reap gains from intertemporal trade of resources: By appreciating its exchange rate in that particular state, Home induces a transfer of resources to Foreign and is being compensated in form of a lower risk-premium on its bonds.

I find that two factors are key for the shape of the exchange rate which implements perfect risk-sharing and the corresponding risk-premium it induces on the Home bond: (i) The risk-premium is increasing in the covariation of Home and Foreign output of tradable goods. The larger the covariation of Home and Foreign tradables, the more costly it is for Home to appreciate (i.e. transfer resources to Foreign) in states of the world where Foreign's SDF is high. (ii) The risk-premium is decreasing in Foreign's risk aversion relative to Home's. Holding constant any covariation of Home and Foreign tradables, the compensation (in form of a lower risk-premium) Home receives for an appreciation in a state of the world where Foreign's SDF is high, increases in Foreign's risk aversion.

Under nominal rigidities and incomplete markets, the optimal policy trades-off the above described motives and induces a risk-premium that is a weighted average of the risk-premia induced by the aggregate demand management and insurance considerations. This trade-off is shaped by two key parameters. First, as the amount of external Home-currency denominated debt grows, it is optimal to tilt towards insurance considerations. Particularly for countries with large stocks of Home-currency external debt, it is suboptimal to let monetary policy only be guided by aggregate demand management considerations – for example, it can be beneficial to appreciate in crises in order to reap the benefits of low interest rates on Home bonds. Second, the degree of openness. As Home becomes more closed (fraction of Home-produced goods in consumption basket rises), Home finds it optimal to tilt towards aggregate demand management.

My model yields a set of testable implications with respect to the cross-section of currency risk-premia. I analyze a panel data set of OECD countries spanning the time period 1983 − 2010 and find support for the model's predictions with respect to the aggregate demand management motive's implications for currency risk-premia. First, I find that the more positively correlated a country's real labor productivity is with global output, the lower a country's average annualized currency risk-premium. Second, I examine if a country's currency risk-premium is increasing in its export-to-GDP-share (as a proxy for exposure to procyclical demand for exports). I do not find supporting evidence for the latter hypothesis. However, a resolution for this anomaly is to control for the composition of a a country's exports, e.g. whether it mainly produces commodities (whose demand is highly procyclical) or final goods (whose demand tends to be less procyclical). In the data, commodity exporters have higher currency risk-premia than final goods exporters.

Related Literature. My paper is related to several strands of the literature. First, there is a vast literature on monetary policy in open-economy models. A comprehensive survey is provided by Corsetti, Dedola and Leduc (2010). My paper is part of a subset of this literature which focuses on the case of incomplete markets so that country portfolios play a crucial role. My main contribution with respect to the existing literature is to characterize the interaction of monetary policy and currency risk-premia. Fanelli (2019) studies a framework similar to mine but focuses on a first-order approximation to optimal policy and allows for portfolio choice. In contrast to his work, I study the existence and characteristics of risk-premia and restrict attention to exogenous portfolios. Also, Benigno (2009*a*), Benigno (2009*b*), Senay and Sutherland (2019), Devereux and Sutherland (2008) and Devereux and Sutherland (2011) develop approximation methods to characterize the equilibrium with portfolio choice up to the first-order. They do not study the full optimal monetary policy and endogenous risk-premia are absent in their solutions. An exception to the common practice of using linearized solutions is the work by Tille and van Wincoop (2010) who develop an approximation technique that does allow for endogenous risk-premia, but they study a real model without a role for monetary policy. Finally, Hassan, Mertens and Zhang (2019) also study the incentive to use monetary policy to influence one's risk-premium, but focus on a different form of market incompleteness (market segmentation) and abstract from aggregate demand management considerations.

Second, there is also a vast literature on currency risk-premia, pioneered by Bilson (1978) and Fama (1984). Subsequently, the literature has studied deviations from uncovered interest parity either via portfolio-based methods ("carry-trade-puzzle") or regression-based methods ("forward-premium-puzzle").[6] In recent work, Hassan and Mano (2019) provide an overarching framework that makes commonalitites and differences of these approaches transparent. Hassan (2013), Richmond (2019), and Ready, Roussanov and Ward (2017) build and test models that identify heterogeneity in country size, trade network centrality, and mode of production (commodities vs finished consumption goods) as potential sources of persistent cross-country differences that can account for the existence of unconditional currency risk-premia. My paper contributes to this literature by proposing cross-country differences in monetary policy as an explanation for the cross-section of currency risk-premia. In addition, I show how cross-country differences in monetary policy are driven by underlying cross-sectional variation in country fundamentals. Closest to my idea of linking the cross-section of currency risk-premia to cross-sectional heterogeneity in monetary policy is the work by Backus et al. (2013) who present evidence that Australia (a country with persistently high interest rates) follows a more expansionary monetary policy relative to the US (a country with persistently low interest rates). However, they do no apply this approach to the entire cross-section of currency risk-premia nor do they offer a model-based perspective on why cross-country differences in monetary policy arise.

Third, there is a literature documenting and exploring the consequences of so-called "financial globalization." Lane and Milesi-Ferretti (2007) document the large increases in cross-border holding of financial assets over the course of the last decades. Gourinchas and Rey (2007) focus on the evolution of external assets and liabilities of the US and quantify both the "exorbitant privilege" (gains from low borrowing cost) and the "exorbitant duty" (cost from large wealth transfers during crises). Caballero, Farhi and Gourinchas (2008) link "global imbalances" to heterogeneity in financial development and Caballero, Farhi and Gourinchas (2016) explore the consequences of this phenomenon for output. I contribute to this literature by exploring the consequences of large country portfolios for the conduct of monetary policy and its implications for the cross-section of currency risk-premia.[7]

[6] Important contributions include Engel (1996), Bekaert, Hodrick and Marshall (1996), Bansal (1997), Bansal and Dahlquist (2000), Backus, Foresi and Telmer (2002), Lustig and Verdelhan (2007), Brunnermeier, Nagel and Pedersen (2008), Alvarez, Atkeson and Kehoe (2009), Verdelhan (2010), Lustig, Verdelhan and Roussanov (2011), Bansal and Shaliastovich (2013), and Engel (2016).

[7] Gabaix and Maggiori (2015) also link currency risk-premia to cross-country portfolios but rely on frictions in financial intermediation, absent in my model.

Finally, there is a literature exploring the causes and consequences of safe assets and currencies. Eichengreen and Flandreau (2012) explore the rise of the US dollar as an international reserve currency. He, Krishnamurthy and Milbradt (2019) build a model that links the safety of an asset to coordination equilibria. Farhi and Maggiori (2018) study the problem of a reserve issuer exerting monopolistic supply power while being subject to confidence crises. Brunnermeier et al. (2016) propose a concept for a "Eurozone"-safe-asset and Brunnermeier and Huang (2019) develop a concept for a safe asset by and for emerging economies. My contribution relative to this literature is to show how monetary policy (through its effects on exchange rates) makes certain classes of assets "safer" for international investors and to explore how much optimal policy will be driven by this concern.

Outline. The remainder of the paper is structured as follows. In Section 2, I set up the model. The optimal policy is characterized in Section 3. Solutions for several benchmark cases are presented in Section 4. In Section 5, I characterize the solution to the full problem. Empirical analysis is presented in Section 6. Finally, Section 7 concludes.

2 Model

The world consists of a small open economy (SOE) called "Home" and a large rest of the world, "Foreign." There are two time periods, $t \in \{0, 1\}$. In period $t = 1$, a state of the world $s \in \mathcal{S}$ is realized.[8] A representative agent inhabits Home and has preferences over consumption of non-tradable goods, tradable goods, and labor given by

$$U(C_{T,0}, C_{NT,0}, N_0) + \beta \cdot \mathbb{E}_0 \left[U(C_{T,1}, C_{NT,1}, N_1) \right],$$

where the consumption of non-tradables is a Dixit-Stiglitz aggregator over varieties

[8] In my companion paper, Vutz (2019), I show how the implications of optimal monetary policy for currency risk-premia generalize to an infinite horizon setting with Calvo-pricing.

$$C_{NT} = \left(\int_0^1 C_{NT}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}},$$

and ϵ denotes the elasticity of substitution between different varieties of the non-tradable good. The Home agent receives an endowment of tradables, wages for hours worked, profits from firm ownership and transfers from the government. I assume that the Home agent does *not* have access to financial markets. As will become clear below, all asset positions are assumed by the government directly.[9] Hence, the agent's budget constraints (expressed in terms of Home currency) are

$$P_{T,0}C_{T,0} + P_{NT,0}C_{NT,0} = P_{T,0}Y_{T,0} + W_0 N_0 + \Pi_0 + T_0, \tag{1}$$

where T_0 is a transfer from the government. In period $t = 1$, state s

$$P_{NT}(s)C_{NT}(s)dj + P_T(s)C_T(s) \leq W(s)N(s) + P_T(s)Y_T(s) + \Pi(s) + T(s). \tag{2}$$

In each period, the Home agent maximizes period utility over choices of hours worked and consumption of tradables and non-tradables. The corresponding first-order conditions are given by

$$\frac{U_{C_{T,t=0}}}{P_{T,t=0}} = \frac{U_{C_{NT,t=0}}}{P_{NT,t=0}} \tag{3}$$

$$-\frac{U_{N,t=0}}{W_{t=0}} = \frac{U_{C_{T,t=0}}}{P_{T,t=0}} \tag{4}$$

[9] The assumption that positions in asset markets are assumed by the government, instead of private agents, is not crucial. I could also allow private agents to access financial markets and the government could still implement a particular portfolio using a set of state-contingent portfolio taxes/subsidies.

and in period $t = 1$, for each state $s \in \mathcal{S}$

$$\frac{U_{C_T}(s)}{P_T(s)} = \frac{U_{C_{NT}}(s)}{P_{NT}(s)} \tag{5}$$

$$-\frac{U_N(s)}{W(s)} = \frac{U_{C_T}(s)}{P_T(s)}. \tag{6}$$

These conditions characterize the agent's optimal choice of labor vs leisure and tradables consumption vs non-tradables consumption. The agent equates the marginal utility of tradable and non-tradable goods (normalized by the respective prices) and works the number of hours such that the marginal disutility of labor equals the marginal utility of consumption adjusted for the real wage.

2.1 Home producers

There is a continuum of Home producers $j \in [0, 1]$. Each producer j has access to a production technology which enables him to produce a unique differentiated variety of the non-tradable consumption good,

$$Y_{NT}^j(s) = A(s) N^j(s),$$

where $N^j(s)$ denotes the labor hired by producer j and total factor productivity productivity, $A(s)$, is homogenous across Home producers. Each Home producer sets the price for his good, facing a downward sloping demand curve given by

$$C_{NT}^j(s) = \left(\frac{P_{NT}^j(s)}{P_{NT}(s)} \right)^{-\epsilon} C_{NT}(s).$$

In period $t = 0$, prices are flexible and each producer $j \in [0, 1]$ solves

$$\max_{P^j_{NT,t=0}} \left[P^j_{NT,t=0} - (1+\tau_{L,0}) \frac{W_{t=0}}{A_{t=0}} \right] \cdot \left(\frac{P^j_{NT,t=0}}{P_{NT,t=0}} \right)^{-\epsilon} C_{NT,t=0}$$

where $\tau_{L,0}$ is a labor tax levied by the Home government (described in more detail below). The resulting price is given by

$$P^j_{NT,t=0} = (1+\tau_{L,0}) \frac{\epsilon}{\epsilon - 1} \frac{W_{t=0}}{A_{t=0}} \tag{7}$$

and is identical across producers. The corresponding profits (per producer and in the aggregate) are

$$\Pi_0 = \left[P_{NT,0} - (1+\tau_{L,0}) \frac{W_0}{A_0} \right] C_{NT,0} = \left[\frac{\epsilon}{\epsilon - 1} - 1 \right] \cdot (1+\tau_{L,0}) \cdot \frac{W_0}{A_0} \cdot C_{NT,0} \tag{8}$$

I assume producers have to set prices for period $t = 1$ in advance. Each producer j solves

$$\max_{P^j_{NT}} \int \pi(s) SDF(s) \left[P^j_{NT} - (1+\tau_L) \frac{W(s)}{A(s)} \right] \cdot \left(\frac{P^j_{NT}}{P_{NT}(s)} \right)^{-\epsilon} C_{NT}(s) ds$$

and the FOC can be written as

$$P^j_{NT} = (1+\tau_L) \frac{\epsilon}{\epsilon - 1} \frac{\int \pi(s) SDF(s) C_{NT}(s) (1+\tau_L(s)) \frac{W(s)}{A(s)} ds}{\int \pi(s) SDF(s) C_{NT}(s) ds} \tag{9}$$

Corresponding profits in period $t = 1$ and state s are given by

$$\Pi(s) = \left[P^j_{NT} - (1+\tau_L)\frac{W(s)}{A(s)}\right] \cdot C_{NT}(s) \tag{10}$$

2.2 Home Government

The Home government imposes labor taxes $(\tau_{L,0}, \tau_L)$ on Home producers and makes lump-sum transfers $(T_0, T(s))$ to Home agents. There is also a Home central bank in control of monetary policy which is described by a schedule for the nominal exchange rates between Home and Foreign $(E(s))$, where

$$E \equiv \frac{\text{units of Home currency}}{\text{unit of Foreign currency}},$$

i.e. a higher value of E means the Home ER depreciates . For period $t = 0$, the exchange is normalized to 1. Finally, Home issues portfolio of Home-currency- and Foreign-currency-denominated debt. I treat this portfolio as exogenous.

The initial period government budget constraint is

$$qB + q^F B^F + \tau_{L,0} W_0 N_0 = T_0, \tag{11}$$

where

$$q^F = \int \pi(s) SDF^F(s) \frac{P^F_{T,0}}{P^F_T(s)} ds, \tag{12}$$

is the price of a Foreign-currency denominated uncontingent bond and

$$q = \int \pi(s) SDF^F(s) \frac{P^F_{T,0}}{E(s) P^F_T(s)} ds \tag{13}$$

is the bond price of the uncontingent Home bond. The Foreign stochastic discount factor, $SDF^F(s)$, applies to payouts in tradable goods. The prices for the tradable good in Foreign currency are given by $(P^F_{T,0}, P^F_T(s))$ are prices for the tradable good in Foreign currency. Home takes both the Foreign stochastic discount factor and the Foreign-currency-denominated prices of tradable goods as given. In period $t = 1$, the government's budget constraint is given by

$$B + E(s)B^F + T(s) = \tau_L(s)W(s)N(s). \tag{14}$$

The government's objective is to maximize Home household utility using choices of policy instruments $\tau_{L,0}, (\tau_L(s)), (E(s)), (T_0, T(s))$. Note that the choice of an exchange rate schedule $(E(s))$ maps into an *endogenous* risk-premium which is defined as

$$\text{risk premium} \equiv \frac{1}{q} \cdot \mathbb{E}\left[\frac{1}{E}\right] - \frac{1}{q^F},$$

which is the expected excess return of going long in the Home and short the Foreign bond.

2.3 Market Clearing

Note that the conditions describing in the markets for labor and non-traded consumption goods are given by

$$C_{NT,0} = A_0 N_0 \tag{15}$$

$$C_{NT}(s) = A(s)N(s) \tag{16}$$

while market clearing conditions for the tradable consumption good are not required due Home economy being of negligible size with respect to the rest of the world.

2.4 Definition of equilibrium

Before I provide the definition of an equilibrium, note that the economy is subject to shocks to the Home endowment of tradables, Foreign-currency prices of tradables, the Foreign stochastic discount factor, and Home productivity. Hence, the following variables are exogenously given.

$$Y_{T,0}, (Y_T(s))_s, P^F_{T,0}, (P^F_T(s))_s, (SDF^F(s))_s, A_0, (A(s))_s.$$

Moreover, by definition, the prices of tradable goods in terms of Home currency are given by

$$P_{T,0} = P^F_{T,0} \tag{17}$$

$$P_T(s) = E(s)P^F_T(s). \tag{18}$$

Then, the definition of equilibrium is as follows.

Definition 1. *A competitive equilibrium for the small open economy consists of*

- *an allocation* $C_{T,0}, (C_T(s))_s, C_{NT,0}, (C_{NT}(s))_s, N_0, (N(s))_s$
- *prices* $P_{T,0}, (P_T(s))_s, P_{NT,0}, P_{NT}, W_0, (W(s))_s$

- *firms' profits* $\Pi_0, (\Pi(s))_s$
- *policy tools* $\tau_{L,0}, \tau_L, (E(s))_s, T_0, (T(s))_s, B, B^F$

such that

- *agents maximize, i.e. (1)-(6) hold*
- *firms maximize, i.e. (7)-(10) hold*
- *the government BCs are satisfied, i.e. (11)-(14) hold*
- *markets clear, i.e. (15)-(16) hold.*

Given this definition, I proceed by analyzing optimal policy.

3 Optimal Policy

In this section, I lay out and characterize the solution to the problem of a social planner maximizing Home agent's utility over implementable competitive equilibria. In order to keep the formulation tractable, I follow along the lines of Farhi and Werning (2017) and assume that (i) preferences between consumption and labor are separable and (ii) preferences over consumption are homothetic. Then, agent's intratemporal first-order condition with respect to consumption gives that the ratio of non-tradable to tradable consumption can be expressed as a monotonic function of relative prices. That is,

$$C_{NT}(s) = f\left(\frac{P_T(s)}{P_{NT}}\right) \cdot C_T(s),$$

where $f(\cdot)$ is an increasing function. Using the fact that $P_T(s) = E(s)P_T^F(s)$, this can be written as

$$C_{NT}(s) = f\left(\frac{E(s)P_T^F(s)}{P_{NT}}\right) \cdot C_T(s). \tag{19}$$

This condition makes transparent how the planner can employ monetary policy to affect the equilibrium allocation. As prices of non-tradables are sticky in Home currency and Foreign currency prices of tradables are exogenous, an exchange rate depreciation ($E(s) \uparrow$), raises the effective relative price of tradables relative to non-tradables. According to (19), the agent's response is to shift consumption away from tradables and towards non-tradables – which stimulates domestic production. Moreover, using (19), the agent's period utility can be expressed in terms of an indirect utility function depending on only on tradables consumption and relative prices

$$V\left(C_T(s), \frac{P_T(s)}{P_{NT}}\right) \equiv U\left(C_T(s), f\left(\frac{P_T(s)}{P_{NT}}\right) \cdot C_T(s), \frac{f\left(\frac{P_T(s)}{P_{NT}}\right) \cdot C_T(s)}{A(s)}\right)$$

Then, the planning can be formulated such that the planner chooses consumption of tradables, non-tradables prices, and an exchange rate schedule directly and uses the remaining policy tools (taxes on labor and lump-sum transfers) to implement his allocation of choice as a competitive equilibrium. A detailed description of the decentralization is given in Apendix A.1. The planning problem is given by

$$\begin{aligned}
\max_{(E(s))_s, C_{T,0}, (C_T(s))_s, P_{NT,0}, P_{NT}} \quad & V\left(C_{T,0}, \frac{P^F_{T,0}}{P_{NT,0}}\right) + \beta \cdot \int \pi(s) V\left(C_T(s), \frac{E(s)P^F_T(s)}{P_{NT}}\right) ds \\
\text{s.t.} \quad C_{T,0} &= Y_{T,0} + \frac{qB}{P^F_{T,0}} + \frac{q^F B^F}{P^F_{T,0}} \\
C_T(s) &= Y_T(s) - \frac{B}{E(s)P^F_T(s)} - \frac{B^F}{P^F_T(s)} \\
q &= \int SDF^F(s) \frac{P^F_{T,0}}{E(s)P^F_T(s)} \pi(s) ds \\
q^F &= \int SDF^F(s) \frac{P^F_{T,0}}{P^F_T(s)} \pi(s) ds,
\end{aligned}$$

where the first 2 constraints are the consolidated Home agent and Home government budget constraints and the latter two constraints are the pricing equations for the Home and Foreign bond.

3.1 Characterization of optimal policy

Denoting the Lagrange-multipliers on the initial-period and state-s budget constraints by μ and $\lambda(s)$ respectively, the optimal policy is characterized as follows.

$$(C_{T,0}) \qquad \mu = V_{C_T,0} \tag{20}$$

$$(C_T(s)) \qquad \lambda(s) = V_{C_T,s} \tag{21}$$

$$(P_{NT,0}) \qquad 0 = V_{p,0} \tag{22}$$

$$(P_{NT}) \qquad 0 = \int \pi(s) V_{p,s} \frac{-E(s)P_T^F(s)}{P_{NT}^2} ds \tag{23}$$

$$(E(s)) \qquad 0 = \beta\pi(s)V_{p,s}\frac{P_T^F(s)}{P_{NT}} + \mu \cdot \frac{\partial q}{\partial E(s)} \cdot \frac{B}{P_{T,0}^F} + \beta\pi(s)\lambda(s)\frac{B}{E(s)^2 P_T^F(s)} \tag{24}$$

Equations (20) and (21) simply state that the shadow value of relaxing the initial period budget constraint is given by $V_{C_T,0}$, whereas the shadow value of relaxing state s budget constraint is given by $\lambda(s)$.

The FOC with respect to the non-tradables price in the initial period says that the planner adjusts this price until the welfare gain of doing so is 0. This stems from the fact that there are no nominal rigidities in the initial period so that production efficiency is guaranteed.

In contrast, the FOC with respect to the non-tradables price for period 1 shows that the welfare effect of moving this price is set to 0 on average, not state-by-state. This is due to to the presence of nominal rigidities which require this price to be set before the state of the world is realized.

Finally, the FOC with respect to $E(s)$, the exchange rate in state s, has 3 components. First, marginally depreciating the exchange moves relative prices in state s, hence impacting welfare. Second, since any exchange rate movement is anticipated by Foreign investors, the depreciation feeds back into the bond price q. Third, depreciating the exchange rate reduces the real value of any outstanding Home-currency denominated debt. The planner will set the pick the exchange rate such that the combined effect of these 3 factors is equal to 0.

3.2 Sharper characterization of exchange rate FOC

It is insightful to solve out the first-order condition with respect to the exchange rate further. As seen above, the FOC for a marginal depreciation is given by

$$0 = \beta\pi(s)V_p(s)\frac{P_T^F(s)}{P_{NT}} + V_{C_T,0} \cdot \frac{\partial q}{\partial E(s)} \cdot \frac{B}{P_{T,0}^F} + \beta\pi(s)V_{C_T}(s)\frac{B}{E(s)^2 P_T^F(s)}, \tag{25}$$

where I have replaced the Lagrange multipliers by the partials of the value function with respect to consumption. Recall that the first summand stems from the fact that an exchange rate depreciation raises the relative price of tradables relative to non-tradables. If this has a positive or negative effect on welfare depends on the existence and sign of the output gap. Formally, define the output gap as

$$\tau(s) \equiv 1 + \frac{1}{A(s)}\frac{U_N(s)}{U_{C_{NT}}(s)}. \tag{26}$$

Then,

$$\begin{aligned}
\tau(s) = 0 &\Leftrightarrow U_{C_{NT}}(s) = -\frac{U_N(s)}{A(s)} \Leftrightarrow \text{labor is efficient} \\
\tau(s) > 0 &\Leftrightarrow U_{C_{NT}}(s) > -\frac{U_N(s)}{A(s)} \Leftrightarrow \text{labor is inefficiently low} \\
\tau(s) < 0 &\Leftrightarrow U_{C_{NT}}(s) < -\frac{U_N(s)}{A(s)} \Leftrightarrow \text{labor is inefficiently high}
\end{aligned}$$

and using this definition of $\tau(s)$, it holds that

$$V_p(s) = \frac{f'(p(s))}{p(s)} C_T(s) U_{C_T}(s) \tau(s). \tag{27}$$

Using this expression for the partial derivative of the value function with respect to relative prices, (24) can be expressed as

$$0 = \beta\pi(s) \frac{f'(p(s))}{p(s)} C_T(s) U_{C_T}(s) \tau(s) \frac{P_T^F(s)}{P_{NT}} + V_{C_T,0} \cdot \frac{\partial q}{\partial E(s)} \cdot \frac{B}{P_{T,0}^F} + \beta\pi(s) V_{C_T}(s) \frac{B}{E(s)^2 P_T^F(s)} \tag{28}$$

This allows for an intuitive interpretation of the first summand. An exchange rate depreciation in some state s raises the relative prices of tradables to non-tradables by $P_T^F(s)/P_{NT}$ at the margin, stimulating spending on non-tradables. If and only if the output gap in state s is positive (labor is inefficiently low), the resulting effect on welfare is positive. Next, I tackle the latter terms. Recall that by definition, the Home bond price q is given by

$$q = \int \pi(s) SDF^F(s) \frac{P_{T,0}^F}{E(s) P_T^F(s)} ds.$$

Hence, when the exchange rate marginally depreciates in state s, the change in q is

$$\frac{\partial q}{\partial E(s)} = -\pi(s) SDF^F(s) \frac{P_{T,0}^F}{P_T^F(s)} \frac{1}{E(s)^2} < 0$$

Intuitively, as the value of the exchange rate in state s becomes more depressed, the uncontingent claim to one unit of Home currency is getting less valuable in real terms, as evaluated by the Foreign stochastic discount factor. Using this result, I can rewrite (28) and state the following

Proposition 1. *The optimal choice for the exchange rate in state s is characterized by*

$$0 = \beta\pi(s)\frac{f'(p(s))}{p(s)}C_T(s)U_{C_T}(s)\tau(s)\frac{P_T^F(s)}{P_{NT}} + \pi(s)\cdot V_{C_T,0}\cdot\left[\beta\frac{V_{C_T}(s)}{V_{C_T,0}} - SDF^F(s)\right]\cdot\frac{B}{E(s)^2P_T^F(s)}.$$

That is, for any state s, the planner targets a weighted average of the output gap and the gap between the Home and Foreign stochastic discount factor. The intuition for the terms involving the gap in stochastic discount factors is as follows. Moving the exchange rate in state s allows Home to transfer between today and state s tomorrow – despite the fact that Home's portfolio consists exclusively of uncontingent assets. By depreciating the exchange rate in state s, Home obtains more resources which it values from an ex-ante perspective by $\beta V_{C_T}(s)/V_{C_{T,0}}$. However, as this depreciation is priced in, Foreign requires to be compensated at its valuations $SDF^F(s)$. Hence, as long as $\beta V_{C_T}(s)/V_{C_{T,0}} > SDF^F(s)$, Home reaps welfare gains by depreciating its exchange rate. However, due to its effect on relative prices, any movement in the exchange rate simultaneously moves production of non-tradables, which is why the planner balances these two motives. It is instructive to specialize the above condition to two special case. First, consider the case where prices are flexible so relative prices automatically adjust to close all output gaps.

Proposition 2. *If prices are flexible, the optimal choice for the exchange rate in state s is characterized by*

$$\frac{\beta V_{C_T}(s)}{V_{C_{T,0}}} = SDF^F(s).$$

As output gaps are closed irrespectively of the policy choice, the planner uses the exchange rate exclusively to implement transfers between today and state s tomorrow. The polar opposite case obtains when prices are rigid and the planner has access to a complete set of state-contingent claims. Then, the planner can achieve perfect risk-sharing by buying and selling state-contingent claims and uses the exchange rate to close the output gap state-by-state.

Proposition 3. *If prices are rigid and the planner can trade a full set of state-contingent claims, the optimal choice of the exchange rate in state s will be such that*

$$\tau(s) = 0.$$

To build a better understanding of how the planner chooses to trade-off between these two motive and how this will map into endogenous risk-premia, I lay out several benchmark cases in the following section.

4 Building intuition: Solution for benchmark cases

In order to develop a better understanding of the forces shaping the solution to the planner's problem, I specialize the utility function to

$$U(C_T, C_{NT}, N) = \alpha \cdot \log(C_T) + (1-\alpha) \cdot \log(C_{NT}) - \kappa \cdot \frac{N^{1+\psi}}{1+\psi} \tag{29}$$

and assume that the Foreign stochastic discount factor is given by

$$SDF^F(s) = \beta^F \left(\frac{Y_T^F(s)}{Y_{T,0}^F} \right)^{-\gamma_F}, \tag{30}$$

which allows me to provide a sharp characterization of a number of insightful benchmark cases. Note that under this specification, the condition characterizing efficient production, i.e.

$$U_{C_{NT}}(s) = -\frac{U_N(s)}{A(s)} \tag{31}$$

becomes

$$\frac{1-\alpha}{C_{NT}(s)} = \frac{\kappa \cdot N(s)^{\psi}}{A(s)} \Leftrightarrow N(s) = \left(\frac{1-\alpha}{\kappa}\right)^{\frac{1}{1+\psi}} \tag{32}$$

The efficient amount of labor is constant as income and substitution effects cancel out. First, I focus on the case where prices are rigid and the planner can trade a full set of state-contingent claims. In this case, the planner uses the exchange rate solely for the purpose of closing output gaps. Second, I study the case where prices are rigid, markets are incomplete and the planner closes the output gap in all states while neglecting risk-sharing concerns. Third, I study the case where prices are flexible and markets are incomplete. Then, the planner uses the exchange rate solely for the purpose of implementing perfect risk-sharing. Fourth, I study the case where prices are rigid, markets are incomplete and the planner implements perfect risk-sharing while neglecting output gaps. I obtain closed form solutions for cases (i) and (ii) and provide intuition for cases (iii) and (iv) which I verify numerically. A natural guess is that the solution to the planner's problem will be a weighted average of the solution under cases (ii) and (iv). I confirm this intuition by means of numerical explorations below.

For the remainder of the analysis, I fix the exogenous Foreign-currency prices of tradables goods at $P_{T,0}^{F} = P_{T}^{F}(s) = 1$.

4.1 Benchmark case 1: Access to complete markets and rigid prices

When the Home planner has access to a full-set of state-contingent claims, he implements perfect-risk sharing via asset trades so that

$$C_T(s) = \frac{\beta}{\beta^F}\left(\frac{Y_T^F(s)}{Y_{T,0}^F}\right)^{\gamma_F} C_{T,0}, \tag{33}$$

where consumption in the initial period is given by

$$C_{T,0} = \frac{1}{1+\beta}\left[Y_{T,0} + \int \pi(s)\beta^F \left(\frac{Y_T^F(s)}{Y_{T,0}^F}\right)^{-\gamma_F} Y_T(s)ds\right] \tag{34}$$

The exchange rate is then solely used to close all output gaps, i.e. implement

$$N(s) = \left(\frac{1-\alpha}{\kappa}\right)^{\frac{1}{1+\psi}}.$$

Using the implementability condition (19), I can solve for the exchange rate as

$$E(s) = \frac{\alpha}{1-\alpha}\frac{P_{NT}}{P_T^F}\frac{A(s)\left(\frac{1-\alpha}{\kappa}\right)^{\frac{1}{1+\psi}}}{\frac{\beta}{\beta_F}\left(\frac{Y_T^F(s)}{Y_{T,0}^F}\right)^{\gamma_F} C_{T,0}} \tag{35}$$

Next, I analyze the exchange rate's response to shocks and summarize my findings in the following

Proposition 4. *Suppose the planner has access to complete markets and prices are rigid. Then, all else equal, the optimal exchange rate*

- *depreciates in response to positive shocks to Home productivity* ($A(s)$),
- *does not respond to shocks to Home endowment* ($Y_T(s)$),
- *appreciates in response to positive Foreign endowment shocks* ($Y_T^F(s)$).

Economic Intuition. When a positive Home productivity shock hits, it is efficient to produce more non-tradables while the efficient consumption of tradables is unchanged. Accordingly, the planner depreciates the exchange rate to make non-tradables cheaper relative to tradables. When a positive shock to Foreign prices hits, the efficient amounts of tradables and non-tradables consumption are unaffected so the planner will appreciate the exchange rate in order to keep the effective relative price constant. When a higher value for the Foreign endowment is realized, it is efficient to receive a larger transfer from Foreign due to the complete markets risk-sharing arrangement. Facing an increase in wealth, the Home agent is inclined to expand spending on both tradables and non-tradables. However, the efficient production and hence consumption of non-tradables is unchanged. By depreciating the exchange rate, tradables become relatively cheaper so that a higher consumption of tradables can be implemented while the consumption of non-tradables remains at its pre-shock level. When a higher value for the Home endowment is realized, the planner will not move the exchange rate as the optimal consumption of tradables and non-tradables and the effective relative price are unaffected.

4.2 Benchmark case 2: ER that closes output gap under rigid prices and incomplete markets

Now, I turn to the setup where prices are rigid and the planner does not have access to complete markets – instead, he holds an exogenous portfolio of uncontingent Home-currency and Foreign-currency denominated debt denoted by (B, B^F). The policy of closing the output gap state-by-state under rigid prices and incomplete markets is not optimal as it neglects any risk-sharing concerns. Yet, it is helpful to study this as a benchmark which will be informative with respect to the full solution of the planner's problem. Note that by the budget constraint, it holds that

$$C_T(s) = Y_T(s) - \frac{B}{E(s)P_T^F} - \frac{B^F}{P_T^F} \tag{36}$$

Since prices are rigid, the implementability condition (19) holds, which is given by

$$C_{NT}(s) = \frac{1-\alpha}{\alpha}\frac{E(s)P_T^F}{P_{NT}}C_T(s) \tag{37}$$

Solving for the exchange rate that implements optimal amount of labor state-by-state results in

$$E(s) = \frac{\frac{\alpha}{1-\alpha}\left(\frac{1-\alpha}{\kappa}\right)^{\frac{1}{1+\psi}} A(s)P_{NT} + B}{P_T^F Y_T(s) - B^F}, \tag{38}$$

Again, it is helpful to analyze how the exchange rate responds to shocks while keeping in mind that that I study the case $B > 0$ (Home-currency denominated debt). The following proposition summarizes my results.

Proposition 5. *Suppose markets are incomplete and prices are rigid. Then, all else equal, the exchange rate which closes the output gap state-by-state*

- *depreciates in response to positive shocks to Home productivity* $(A(s))$,
- *appreciate in response to positive shocks to Home endowment* $(Y_T(s))$,
- *does not respond to shocks to Foreign endowment* $(Y_T^F(s))$.

Economic Intuition. There are a number of differences with respect to benchmark case 1. First, note that an exchange rate depreciation still makes tradables more expensive relative to non-tradables and hence reallocates spending towards non-tradables. Second, note that there is now an additional effect stemming from the existence of positive uncontingent Home-currency debt. As the exchange rate depreciates, the real-value of these liabilities is depressed, implying a wealth transfer from Foreign to Home. This positive wealth effect will trigger an increase in spending on both tradables and non-tradables. With respect to the different shocks, these forces play out as follows. When a positive Home productivity shock is realized, it is efficient to produce more. In order to shift spending towards non-tradables, the planner depreciates the exchange rate. As described above, this triggers a wealth transfer from Foreign to Home, further stimulating spending on both tradables and non-tradables. Thus, the exchange rate response with respect to Home productivity shocks will be qualitatively in line with benchmark case 1 but the depreciation required to close the output gap for a given produvitivty shock is decreasing in the amount of outstanding Home-currency debt.[10] When a positive shock to Home endowment of tradables hits, the efficient amount of production is unaffected. However, the increase in wealth leads the agent to increase spending on both tradables and non-tradables. As the

[10] A formal proof for this statement is provided in Appendix A.5.

planner is only concerned with closing the output gap, it is optimal to appreciate the exchange rate to maintain a constant level of non-tradables consumption (and production). This is in contrast to benchmark case 1 where it was optimal to not responds to Home endowment shocks. When a Foreign endowment shock hits, the efficient amount of production is unaffected, as are relative prices and Home's wealth. Hence, it is optimal not to respond with an exchange rate movement. Finally, a positive shock to Foreign prices is buffered via an exchange rate appreciation, as in the complete markets case.

Implications for risk-premia. What does the exchange rate behavior imply for the risk-premium on the Home bond? Note that non-zero risk premia can obtain in equilibrium only if there is variation in the Foreign stochastic discount factor, i.e. there are shocks to Foreign endowment $Y_T^F(s)$. First, suppose shocks to the Foreign stochastic discount factor are the only source of uncertainty in the economy. As seen above, it is optimal to not let the exchange rate respond to these shocks, so the resulting risk-premium are nil. However, if there are additional shocks to either Home productivity or Home endowment of tradables and these shocks are correlated with the Foreign stochastic discount factor, non-zero risk-premia obtain. I summarize my findings in the following

Proposition 6. *For the exchange rate which closes the output gap state-by-state under rigid prices and incompletes markets, the following holds. If there are shocks to Foreign endowment* $Y_T^F(s)$

- *but no shocks to Home productivity* $(A(s))$ *or Home endowment of tradables* $(Y_T(s))$, *then the risk-premium is 0.*
- *and shocks to Home productivity* $A(s)$ *such that* $cov(A, Y_T^F) > 0$, *then the risk-premium is positive. If* $cov(A, Y_T^F) < 0$, *then the risk-premium is negative.*
- *and shocks to Home tradables* $Y_T(s)$ *such that* $cov(Y_T^F, Y_T) > 0$, *then the risk-premium is positive. If* $cov(Y_T^F, Y_T) < 0$, *then the risk-premium is negative.*

This concludes my analysis of how the motive to stabilize the economy shapes the exchange rate and hence the existence and behavior of endogenous risk-premia.

4.3 Benchmark case 3: ER that implements perfect risk-sharing under flexible prices and incomplete markets

Now, I switch gears and assume that the planner is only concerned with implementing perfect risk-sharing. As a starting point, I assume that prices are flexible. The latter assumption is relaxed in the following section. As before, markets are incomplete and the planner holds and exogenous portfolio of uncontingent Home- and Foreign-currency denominated debt denoted by (B, B^F). Due to flexible prices, it automatically holds that $\tau(s) = 0$ and perfect risk-sharing is characterized by

$$C_T(s) = \frac{\beta}{\beta^F} \left(\frac{Y_T^F(s)}{Y_{T,0}^F} \right)^{\gamma_F} C_{T,0} \tag{39}$$

where $C_{T,0} = Y_{T,0} + qB + q^F B^F$. Combining (39) with the budget constraint under incomplete markets, which is given by

$$C_T(s) = Y_T(s) - \frac{B}{E(s)P_T^F} - \frac{B^F}{P_T^F}, \tag{40}$$

yields

$$E(s) = \frac{B}{P_T^F \left[Y_T(s) - \frac{\beta}{\beta^F} \left(\frac{Y_T^F(s)}{Y_{T,0}^F} \right)^{\gamma_F} C_{T,0} \right] - B^F} \tag{41}$$

This expression allows me to analytically characterize how the exchange rate responds to shocks.

Proposition 7. *Suppose markets are incomplete and prices are flexible. Then, all else equal, the exchange rate which implements perfect risk-sharing*

- *does not respond to shocks to Home productivity* $(A(s))$,

- *appreciates in response to positive shocks to Home endowment* ($Y_T(s)$),
- *depreciates in response to shocks to Foreign endowment* ($Y_T^F(s)$).

Economic Intuition. When a positive Home productivity shock is realized, this does not change Home's wealth of tradables good nor the desired consumption of tradables. Hence, it is optimal to not respond with an exchange rate movement. In response to a positive Home endowment shock, Home's wealth in tradables rises, but the tradables consumption consistent with perfect risk-sharing calls for no response. Hence, increase the transfer to Foreign by appreciate the exchange rate. When a Foreign endowment shock hits, perfect risk-sharing calls for a higher consumption of tradables, hence depreciate the exchange rate in order to reduce the transfer received from Foreign. When a positive shock to Foreign prices is realized, the the Foreign currency value of the transfer to/from Home goes up. If this transfer is to Foreign, appreciate your ER to keep the effective transfer constant. Vice versa, if this transfer is to Home, depreciate your ER.

Perfect risk-sharing and covariation with Foreign output. Proposition 7 immediately implies that if Home and Foreign endowments are negatively correlated, it is optimal to appreciate the Home exchange rate during states of the world where the global stochastic discount factor is high (i.e. states where $Y_T^F(s)$ is low) and the Home bond will have a negative risk-premium. However, as we see in the data that output across countries is fairly positively correlated,[11] I focus on the case where $cov(Y_T, Y_T^F) > 0$. This case is more subtle as shocks to Home and Foreign endowment of tradables call for movements of the exchange rate in opposite directions. In order to analyze the behavior of the exchange rate and the implied risk-premium on the Home bond, define the transfer *Home makes to Foreign in state s* as

$$TI(s) \equiv Y_T(s) - C_T(s) = Y_T(s) - \frac{\beta}{\beta^F} \left(\frac{Y_T^F(s)}{Y_{T,0}^F} \right)^{\gamma_F} C_{T,0}. \tag{42}$$

Keep in mind that due to the presence of Home-currency denominated debt, increasing the transfer to Foreign is brought about via an appreciation (and vice versa for decreasing the

[11] See Stockman and Tesar (1995).

transfer). The key statistic determining the sign of the risk-premium on the Home bond is given by

$$
\begin{aligned}
cov(TI, Y_T^F) &= cov(Y_T - C_T, Y_T^F) && (43)\\
&= cov(Y_T, Y_T^F) - cov(C_T, Y_T^F) && (44)\\
&= cov(Y_T, Y_T^F) - \underbrace{cov\left(\frac{\beta}{\beta^F}\left(\frac{Y_T^F(s)}{Y_{T,0}^F}\right)^{\gamma_F} C_{T,0}, Y_T^F\right)}_{>0}. && (45)
\end{aligned}
$$

As described above, a sufficient condition for $cov(TI, Y_T^F) < 0$ (appreciate during crises) is $cov(Y_T, Y^F) < 0$. However, more interestingly, (45) demonstrates that Home can find it optimal to appreciate during crises even when it is procyclical, i.e. $cov(Y_T, Y_T^F) > 0$. For a given positive correlation of Home and Foreign endowments of tradables, this case can be obtained for a sufficiently high value γ_F. The intuition for this is that when Foreign becomes more and more risk-averse, Home reaps greater compensation of providing insurance to Foreign which overcompensates for the fact that this entails higher transfer to Foreign in states where Home endowment of tradables is depressed.

4.4 Benchmark case 4: ER that implements perfect risk-sharing under rigid prices and incomplete markets

Finally, I study the case where markets are incomplete, prices are rigid and the planner implements perfect risk-sharing. Note that this is *not* equivalent to benchmark case 3 for the following reason. When prices are rigid and the planner uses the exchange rate to implement perfect risk-sharing, this will lead to the existence of non-zero output gaps. In turn, the presence of these output gaps will influence the insurance desired by the planner. This is an application of the result of Farhi and Werning (2016.) who show that the laissez-faire complete markets risk-sharing arrangement is inefficient when the unconstrained efficient allocation is unattainable. In my framework, this shows up as

$$V_{C_T}(s) = U_{C_T}(s) \cdot \left[1 + \frac{1-\alpha}{\alpha}\tau(s)\right]. \tag{46}$$

The planner's marginal valuation of one additional unit of the tradable good is given by the agent's marginal utility adjusted for a term involving the output gap. If the output gap is positive ($\tau(s) > 0$), insurance becomes more valuable as higher wealth will lead agents to expand spending on both tradables and non-tradables which helps stimulate the inefficiently low production. Hence, the system of equations characterizing the outcome for this benchmark case are

$$C_T(s) = \frac{\beta}{\beta^F}\left(\frac{Y_T^F(s)}{Y_{T,0}^F}\right)^{\gamma_F}\left[1 + \frac{1-\alpha}{\alpha}\tau(s)\right] C_{T,0} \tag{47}$$

$$C_{T,0} = Y_{T,0} + qB + q^F B^F \tag{48}$$

$$\tau(s) \equiv 1 + \frac{1}{A(s)}\frac{U_N(s)}{U_{C_{NT}}(s)} = 1 - \left(\frac{N(s)}{N^*}\right)^{1+\psi} \tag{49}$$

$$C_T(s) = Y_T(s) - \frac{B}{E(s)P_T^F} - \frac{B^F}{P_T^F} \tag{50}$$

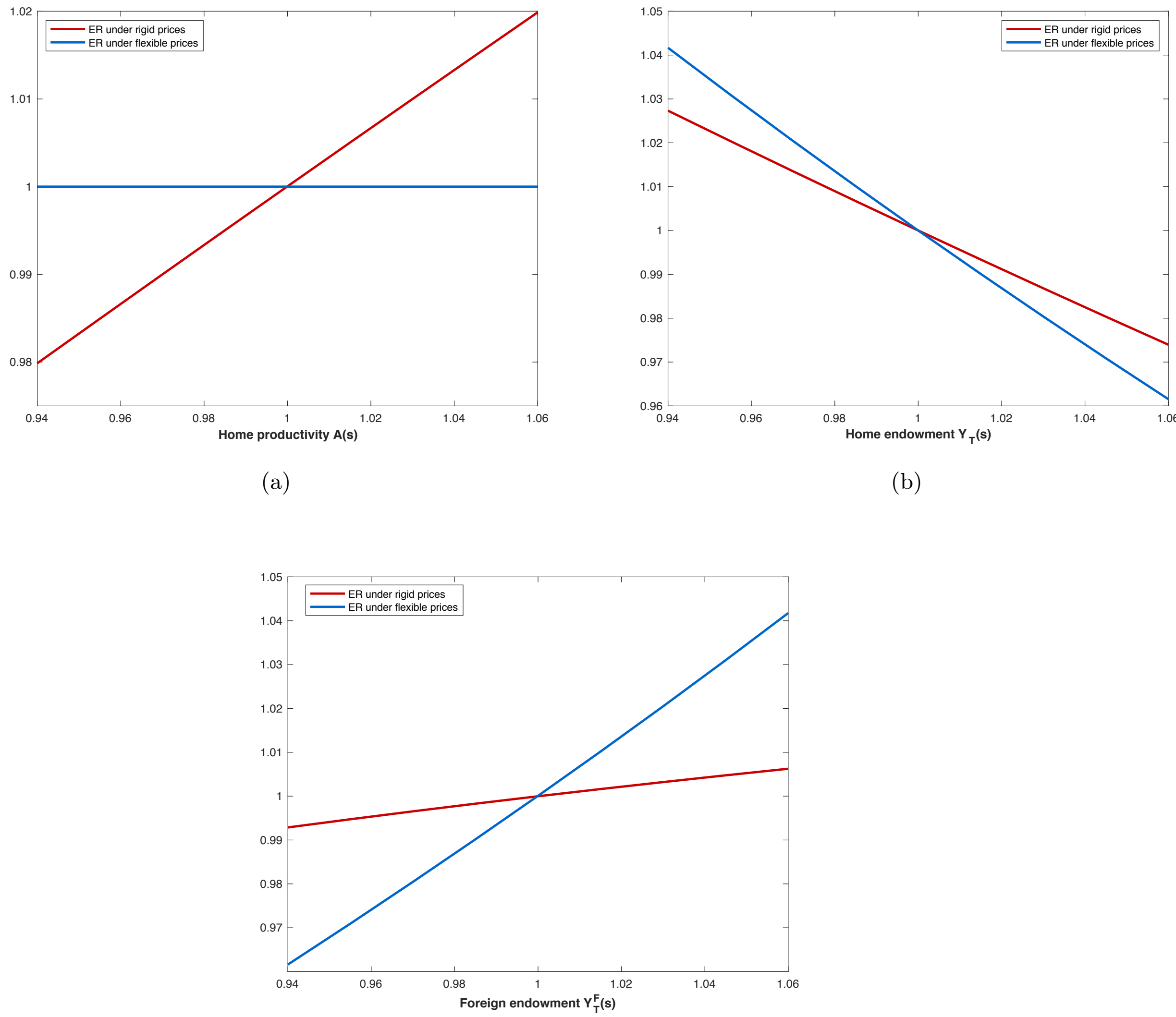
$$\frac{A(s)N(s)}{C_T(s)} = \frac{1-\alpha}{\alpha}\frac{E(s)P_T^F}{P_{NT}}, \tag{51}$$

where $N^* = ((1-\alpha)/\kappa)^{1/(1+\psi)}$ denotes the amount of labor that corresponds to the efficient level of production. As this system does not allow for an easily interpretable analytic solution, I explore the exchange rate's behavior in response to shocks numerically for a benchmark calibration shown in Table 1. I explore shocks for $A(s), Y_T(s), Y_T^F(s)$ separately which each follow a truncated normal distribution (cut off 3 standard deviation above and below the mean) with standard deviation of 2%.

Economic Intuition. First, I analyze what happens when shocks to Home productivity are present. Under flexible prices, it was optimal to not react to these shocks. However, under rigid prices, keeping the exchange rate flat after a positive shock to Home productivity opens up a positive labor wedge, raising the effective marginal utility from tradables. Hence, the planner

Baseline calibration	
Home discount factor	$\beta = 0.99$
Foreign discount factor	$\beta^F = 0.99$
Foreign CRRA	$\gamma_F = 1$
Openness	$\alpha = 0.5$
Inverse Frisch-elasticity	$\psi = 2$
Home-currency debt	$B = 1.5$
Foreign-currency debt	$B^F = -1.5$

Table 1: Baseline calibration for Figure 1



(a) (b) (c)

Figure 1: Exchange rate responses to shocks under rigid prices and incomplete markets

lets the exchange rate depreciate to receive a greater transfer. This behavior is depicted in Figure 1a.

Second, suppose there is a shock to Home endowment. Under flexible prices, it was optimal to appreciate the exchange rate in response to positive $Y_T(s)-$shock in order to engineer a larger transfer from Home to Foreign. Under rigid prices, such an appreciation implies that Home agent shift consumption away from tradables so that a positive labor wedge opens up. This increases the marginal utility of tradables. In response, the planner finds it optimal to dampen the initial appreciation to limit the increase of transfers to the rest of the world. In sum, the planner still responds to positive Home endowment shocks with an appreciation, although less so than under flexible prices. Figure 1b illustrates this.

Finally, in response to a positive shock to Foreign tradables, it was optimal to depreciate under flexible prices, in order to receive larger transfer from Foreign. However, under rigid prices, a depreciation makes the Home agent shift resources towards non-tradable goods, opening up a negative labor wedge. This, in turn, depresses the marginal valuation of tradables, which leads the planner to dampen the initial depreciation. In sum, it is still optimal to depreciate in response to a shock to Foreign tradables, yet this depreciation is less pronounced than under flexible prices. For the benchmark calibration, this is shown in Figure 1c.

Implications for risk-premia. When the behavior of the exchange rate is as under the baseline calibration, the following holds.[12]

- If there are only shocks to Foreign endowment ($Y_T^F(s)$), the risk-premium is negative.
- If there are shocks to Foreign endowment $Y_T^F(s)$ and shocks to Home productivity ($A(s)$) such that $cov(Y_T^F, A) > 0$, then the risk premium is positive. If $cov(Y_T^F, A) < 0$, the risk-premium can take either sign.
- If there are shocks to Foreign endowment ($Y_T^F(s)$) and shocks to Home endowment ($Y_T(s)$) such that $cov(Y_T^F, Y_T) < 0$, then the risk premium is negative. If $cov(Y_T^F, Y_T) > 0$, the risk-premium can take either sign.

[12] A reasonable question to ask is whether the existence of non-zero output gaps under rigid prices could not just dampen the response of the exchange rate to shocks relative to the flexible price case but actually reverse it. In Appendix A.11, I show that this is cannot be the case.

The case where $cov(Y_T^F, Y_T) > 0$, is akin to what happens when implementing perfect risk-sharing under flexible prices. Positive shocks to the Foreign endowment of tradables call for an appreciation (to induce a greater transfer to Foreign), but they tend to realize when Home endowment for tradables is low, which calls for a depreciation (to induce a greater transfer from Foreign to Home). Under rigid prices, there are – in addition to these opposing forces – effects through non-zero output gaps. As $Y_T(s)$ drops, the negative wealth effect induces the Home agent to reduce spending on both tradable and non-tradabale goods, despite the fact that the efficient amount of non-tradable consumption is unchanged. Thus, this will open a positive output gap which *reduces* the incentive to *provide* insurance to Foreign. So the more positive the output gaps that open up in response to shocks to tradable goods, the greater is the compensation needed to induce Home to provide insurance to Foreign.

5 Solution to the planning problem

Having worked through the solution of the benchmark cases (i)-(iv) gives a firm intuition for what characteristics to expect from the solution to the planning problem under rigid prices and incomplete markets where the planner trades-off manipulating the risk-premium on Home bonds against closing output gaps. Namely, I expect the optimal exchange rate to be a weighted average of the exchange rate of benchmark case (ii) (closing the output gap under rigid prices and incomplete markets) and benchmark case (iv) (implementing perfect risk-sharing under rigid prices and incomplete markets and). In the following, I characterize the optimal exchange rate behavior numerical for the cases when (a) only shocks to Foreign endowment ($Y_T^F(s)$) are present and (b) shocks to Foreign endowment ($Y_T^F(s)$) and Home endowment ($Y_T(s)$) are present.

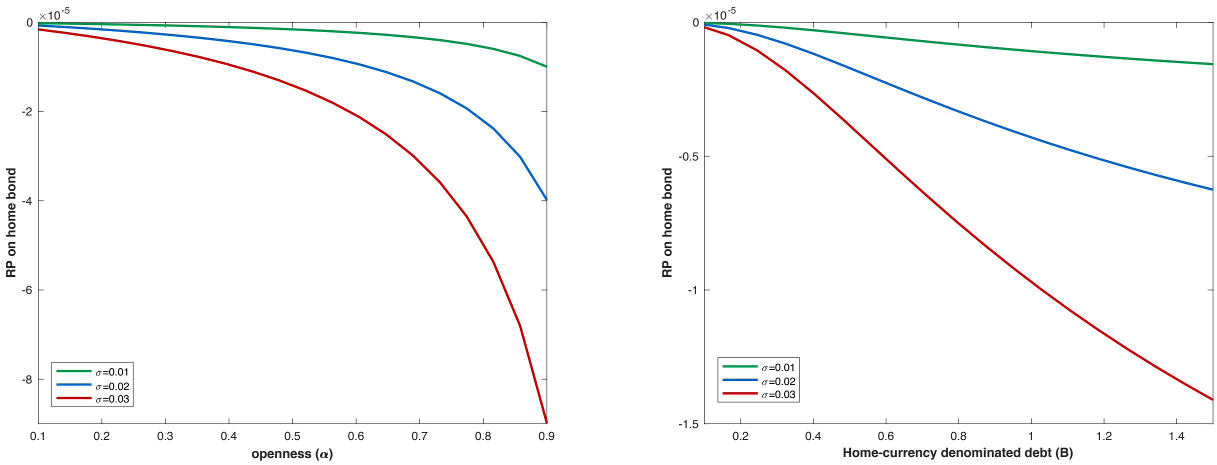
I analyze comparative statics with respect to 2 key parameters: first, α, which simultaneously parameterizes Home's degree of openness and the fraction of Home's production susceptible to changes in international relative prices (which is given by $(1-\alpha)$). Second, Home's amount of Home-currency denominated external debt B. When the economy's openness α increases, this has 3 effects. First, the weight Home places on tradable increases whereas the weight on non-tradable goods decreases. Second, closing any given output gap requires a larger movement of the exchange rate (as can be seen from (19)). Third, the existence of non-zero output gaps feeds less into the risk-sharing condition (see (47)). As can be seen below, I find numerically that the first effect dominates. As B increases, any given gap in stochastic discount factors becomes more costly (as can be seen from the expression in Proposition 29) which is why the planner puts a larger weight on implementing perfect risk-sharing.

5.1 Only shocks to Foreign endowment $Y_T^F(s)$

When shocks to Foreign endowment ($Y_T^F(s)$) are the only source of uncertainty, the exchange rate behavior is as follows. As I have discussed in Section 4.2, macroeconomic stabilization does not call for exchange rate movements in response to $Y_T^F(s)-$shocks. Accordingly, the exchange rate closing the output gap state-by-state is flat and implies that the risk-premium on the Home bond is 0. However, the risk-sharing motive calls for a participation in the fluctuations of Foreign output. Hence, it is optimal to appreciate the exchange rate in states where $Y_T^F(s)$ is low, and, vice versa, depreciate the exchange rate in states of the world where $Y_T^F(s)$ is high. Thus, the exchange rate implementing the perfect risk-sharing arrangement induces a negative risk-premium on the Home bond. In Figure 5a, I display the resulting risk-premium for different levels of opennness, α. As $\alpha \to 0$, the planner cares only about stabilization, which is why the risk-premium converges to 0. As α increases, the risk-sharing concerns become more and more important, resulting in a more negative risk-premium. In Figure 5b, I display the risk-premium for different levels of Home-currency denominated debt B. As B increases, any given deviation from perfect risk-sharing becomes more costly from the planner's point of view, even though he does care about stabilization (the level of openness is fixed at some $\alpha > 0$). Again, the more the planner tilts towards risk-sharing, the more negative becomes the risk-premium.

5.2 Positively correlated shocks to Foreign endowment $Y_T^F(s)$ and Home endowment $Y_T(s)$

Studying the case where only shocks to Foreign endowment are present provides a useful starting point to understand the solution to the full problem. Yet, the fact that the Home

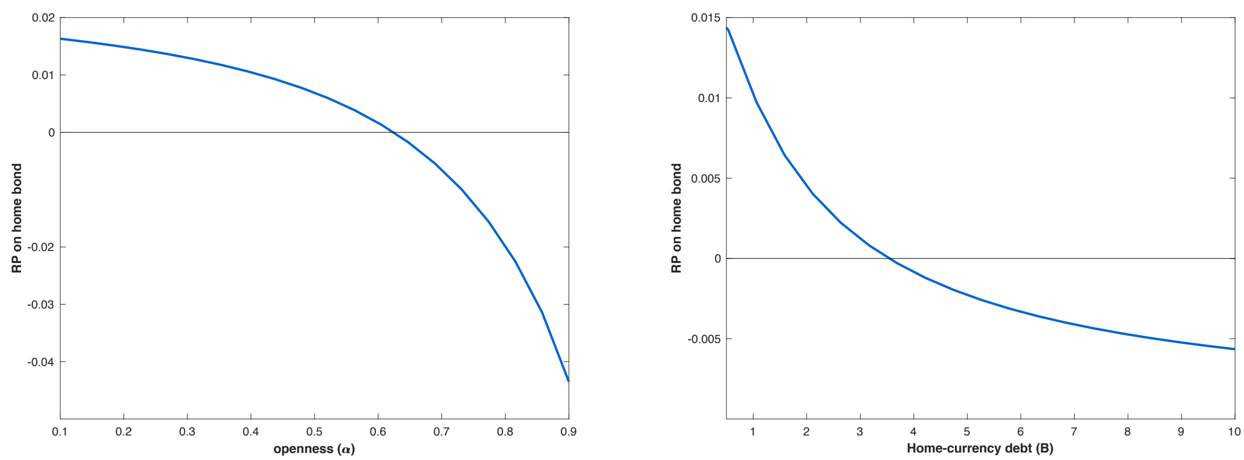


(a) RP on Home bond as a function of openness (b) RP on Home bond as a function of Home-currency debt

Figure 2: Risk-premia on Home bond under Foreign endowment shocks

currency appreciates in bad states of the world (i.e. Home provides insurance to Foreign) follows immediately from the fact that Home's endowment is acyclical – which is obviously an extreme assumption. Thus, I now analyze the case when there are shocks to both Foreign and Home endowment. To keep matters interesting, I focus on the case where the shocks are *positively* correlated. I show that the stabilization motive calls for a positive risk-premium but the insurance motive can still call for a negative risk-premium, despite the fact that shocks are perfectly correlated.

I start out by describing how the exchange rate that closes the output gap reacts to these shocks. As seen above, shocks to Foreign endowment do not call for any response. On the contrary, when Home endowment suffers a negative shock, agent's are less wealthy and contract spending on both tradable and non-tradable goods. The bust propagates from one sector of Home's economy to another. At the same time, the efficient production of non-tradables is unchanged. Hence, it is optimal to depreciate the exchange rate in order to reallocate spending towards non-tradabales, providing stabilization. As result, the the exchange rate closing the output gap state-by-state induces a positive risk-premium on the Home bond. With respect to the risk-sharing motive, there are now two opposing forces. On the one hand, a higher realization of $Y_T(s)$ pushes the optimal response towards an appreciation in order to share resources with Foreign. However, due to the positive correlation of shocks, these are states where also high values for $Y_T^F(s)$ are realized, pushing the optimal responds towards a depreciation to participte in Foreign's resources. I explore this trade-off numerically for perfectly positively correlated shocks and find that for sufficiently large degrees of Foreign risk-aversion, the perfect risk-sharing arrangement calls for higher transfer from Home to Foreign during crises. I illustrate how these stabilization versus insurance motive play out for a degree of Foreign risk-aversion $\gamma_F = 8$ in Figure 3. When the economy is very closes or debt is small, the planner tilts towards the stabilization motive, implying a positive risk-premium on the Home bond. When the economy becomes more open (non-tradables production less important) or debt increases (more Home-currency denominated debt), the risk-sharing motive becomes more important, which leads to a decrease in the risk-premium. For sufficiently large values of openness or debt, the risk-premium becomes negative.



(a) RP on Home bond as a function of openness

(b) RP on Home bond as a function of Home-currency debt

Figure 3: Risk-premia on Home bond under Home and Foreign endowment shocks

6 Empirical Analysis

In the following section, I evaluate whether the model's main predictions are supported by the data.

6.1 Testable implications

As described in detail above, the optimal policy weighs managing aggregate demand against manipulating the risk-premium on the Home bond. In turn, both the risk-premium induced by aggregate demand management and the risk-premium induced are functions of the underlying shocks and structural parameters of the economy. In particular, the risk-premium associated with aggregate demand management is

- decreasing in $cov(A, Y_T^F)$ and
- increasing in $cov(Y_T, Y_T^F)$.

The risk-premium implied by optimal risk-premium manipulation is

- increasing $cov(Y_T, Y_T^F)$ and
- decreasing in γ_F.

Hence, the underlying correlations of shocks give robust predictions with respect to the equilibrium risk-premium implied by the model. Also note that as the optimal policy will tilt towards aggregate demand management for a large fraction of Home produced goods in the consumption basket (α small) and tilt towards risk-sharing for large amounts of Home-currency external debt (B large), yet those parameters are not informative about the sign of the model implied risk-premium in an unconditional sense. I focus on testing whether the cross-sectional variation in currency risk-premia can be explained by cross-sectional variation in the underlying shocks $cov(A, Y_T^F)$ and $cov(Y_T, Y_T^F)$.

6.2 Data description

I study a panel data set of OECD countries over the time span $1976-2010$. A number of OECD countries are excluded either due to poor data availability or because they hold a significant amount of Foreign-currency denominated external debt so that the model's predictions do not necessarily apply. Importantly, the sample is necessarily unbalanced due to the introduction of the Euro in 1999. Thus, all Euro member countries are contained as separate entities prior to 1999, and dropped thereafter when the Euro is introduced. The full sample consists of

19 countries and is displayed in Appendix A.12. I report results both for the full sample and for a subset of so-called "G10" currencies in which the bulk of Foreign exchange trading takes place.[13] The latter sample consists of data on Australia, Canada, the Euro (after 1999), Germany (before 1999), Great Britain, Japan, Norway, New Zealand, and Sweden.

For each country in my sample, I obtain forward premia and realized currency excess returns from the dataset made available by Hassan and Mano (2019). As is standard in the literature, forward premia and currency excess returns over any time horizon n and any currency k are quoted against the US dollar. That is, going long in the currency of country k at time t over a horizon of length n entails an excess return of

$$rx^k_{t,t+n} = i^k_{t,t+n} - i^{US}_{t,t+n} - \Delta s^k_{t,t+n} = (f^k_{t,t+n} - s^k_t) - \Delta s^k_{t,t+n}$$

where s^k_t is the exchange rate of currency k against the US dollar, $f^k_{t,t+n}$ is the forward rate of currency k against the US dollar at time t and horizon n, and the last equality assumes covered interest rate parity which implies forward premia and interest rate differentials are equal to each other.

In addition, I collect data from the OECD database on labor productivity (real GDP per hours worked; which I use as a proxy for Home productivity shocks), exports-to-GDP-ratio (which I use as exposure to procyclical shocks for Home-produced goods) and real GDP growth of the G7 (which I use as a proxy for the global business cycle),[14] all at annual frequencies. In order to focus on business cycle fluctuations, I detrend these time-series via HP-filtering.

6.3 Covariation of productivity with the global business cycle

First, I evaluate whether there is support for the model's prediction that currency risk-premia are decreasing in a country's covariation of productivity with the global business cycle. For each country k, I run a regression of the growth rate of real labor productivity (real GDP per

[13] See Ready, Roussanov and Ward (2017).

[14] The G7-members are Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States. Hence, G7 GDP accounts for the bulk of global output.

hours worked) on a measure of the global business cycle, which here is the real GDP per capita growth rate of the G7. In mathematical terms,

$$g^k_{\text{laborprod},t} = \alpha_k + \beta_k \cdot g^{G7}_{y,t} + \delta_k \cdot x^k_t + \varepsilon^k_t, \tag{52}$$

where x^k_t is a vector of controls. Having obtained a cross-section of coefficients $\{\beta_k\}_k$, I then run a cross-sectional regression of average realized currency excess returns on the β_k, i.e.

$$\overline{rx}_k = \lambda_0 + \lambda_1 \cdot \beta_k + \lambda_2 \cdot c_k + u_k, \tag{53}$$

where $\overline{rx}_k$ is the sample average of currency excess returns earned by going long (investing) in the currency of country k and going short (borrowing) in US dollar, i.e.

$$\overline{rx}_k \equiv \frac{1}{T_k} \sum_t rx_{k,t}$$

and c_k denotes a vector of controls.

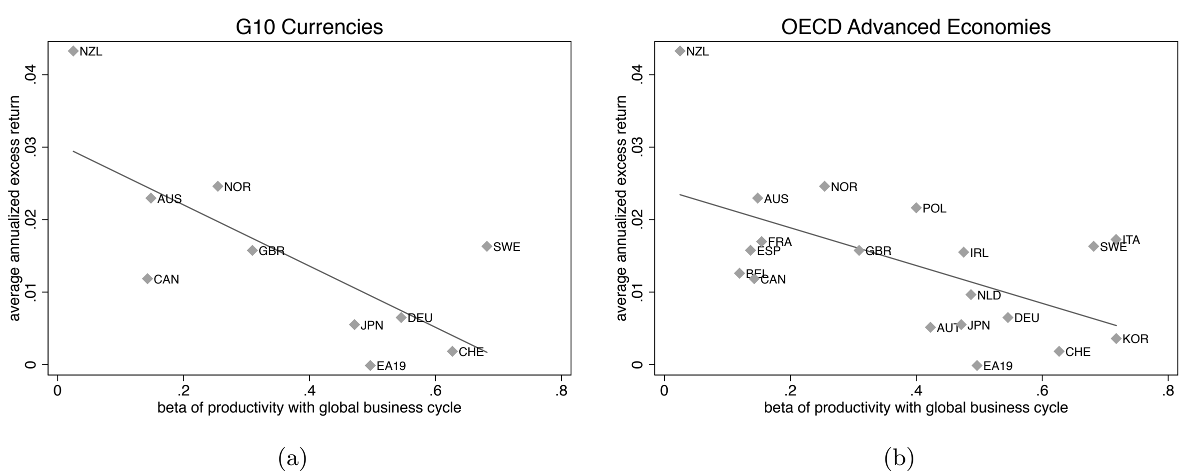


Figure 4: Average realized currency excess returns decrease in procyclicality of productivity

For both the full sample and the "G10" sample, I find that higher covariation of of productivity with the global business cycle is associated with a lower currency risk-premium and the effect is statistically significant (at the 5% level). See Appendix A.13 for a detailed regression table. This is consistent with the model's prediction. The scatterplots and linear fit for both samples are displayed in Figure 4. In the appendix, I show that these findings are robust to using different measures of the global business cycle (e.g. US real GDP per capita or consumption) and to including various controls.

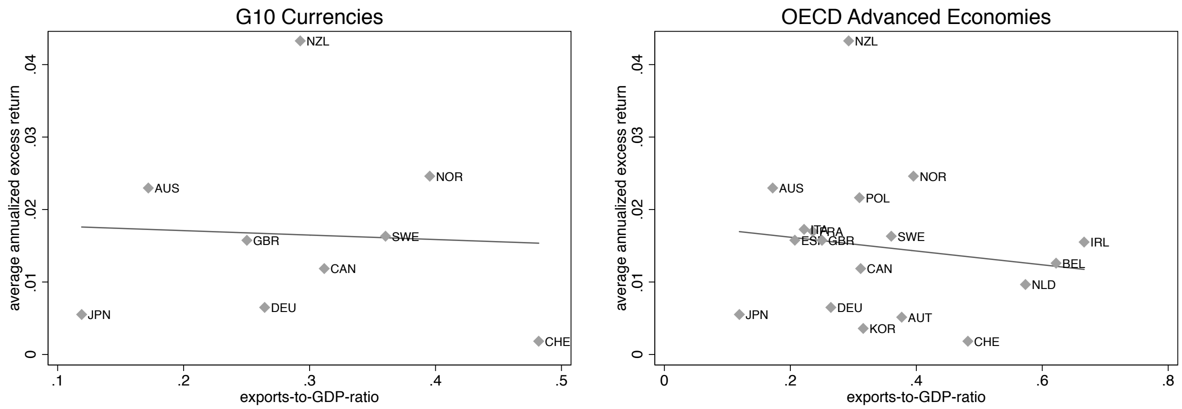
6.4 Exposure to export-demand shocks

Second, I evaluate the model's predictions that a country's currency risk-premium is increasing in its covariation of endowment of the tradable good with the global business cycle. As discussed above, the "endowment of tradables" in the model can be interpreted as the country's resources generated from exporting goods to the rest of the world. Building on this interpretation, I use a country's exports-GDP-share as a first-pass proxy for exposure to export-demand shocks. If this is a valid proxy, then the model predicts that currency risk-premia are increasing in this metric. For each country in the sample, I calculate the time-series average of this share (which exhibits a high degree of persistence) and run the following specification.

$$\overline{rx}_k = \lambda_0 + \lambda_1 \cdot \text{expgdp}_k + \lambda_2 \cdot c_k + u_k, \tag{54}$$

where expgdp_k denotes country k's export-to-GDP-share (averaged over time) and c_k is a vector of controls. For both samples, I do not find a positive value for the coefficient on expgdp^k nor any statistical significance. Results are displayed in Figure 5.

A potential reason for not being able to find support for the prediction that currency risk-premia are increasing in the exposure to export-demand shocks can be that the exports-GDP-ratio is too simple of a metric to proxy for exposure. In particular, besides the relative size of the export sector, it might be useful to control for the *kind* of goods a country exports. For example, while countries such as Australia, New Zealand, and Norway (countries with high currency



(a) (b)

Figure 5: Average realized currency excess returns do not increase in export-GDP-share

risk-premia) primarily export basic commodity goods, countries such as Germany and Switzerland (countries with low currency risk-premia) specialize in exporting finished consumption goods. If demand for and/or prices of commodities are more procyclical than for finished goods, this provides a possible reconciliation of the above empirical finding with the model's prediction.

7 Conclusion

I have analyzed monetary policy in a small open economy that issues Home-currency denominated external debt and is subject to nominal rigidities in price-setting. An exchange rate depreciation (appreciation) simultaneously stimulates (dampens) aggregate demand for Home-produced goods and raises (lowers) ex-ante interest rates on Home-currency denominated bonds as it makes these assets "riskier" ("safer").

Aggregate demand management considerations call for a risk-premium on the Home bond that is decreasing (increasing) in the covariation of the global business cycle with Home productivity shocks (demand shocks for Home-produced goods). Insurance considerations call for a risk-premium on the Home bond that is increasing in the covariation of Home and Foreign output and decreasing in Foreign's risk-aversion. Importantly, the larger a country's stock of Home-currency denominated external debt, the more optimal policy tilts towards insurance.

My model yields a number of testable implications with respect to the cross-section of currency risk-premia. Empirically, I find that – in line with aggregate demand management considerations – countries' currency risk-premia are increasing in the procyclicality of demand shocks and decreasing in the procyclicality of productivity shocks.

These findings leave several promising avenues for future research. First, incorporating elements such imperfect exchange rate pass-through (for example due to the existence of intermediate inputs) would allow for a more granular analysis of the aggregate demand management motive and its implications for currency risk-premia. Second, augmenting the model by elements such as disaster risk or habit-persistence in consumption would allow to assess the *quantitative* impact of cross-sectional differences in monetary policy on the cross-section currency risk-premia. Third, studying the model under a central bank with *limited* commitment will yield additional insights with respect to the countries that emerge as natural insurance providers for the rest of the world.

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A Appendix

A.1 Decentralization of solution to planning problem

Solving the planning problem results in optimal quantities of tradables consumption $C_{T,0}$ and $(C_T(s))$, desired prices for non-tradables $P_{NT,0}$ and P_{NT}, and an exchange rate schedule $(E(s))$. The remaining variables are backed out as follows.

- prices for tradables denominated in domestic currency from the consistency conditions

$$P_{T,0} = P^F_{T,0}$$
$$P_T(s) = E(s)P^F_T(s)$$

- consumption of non-tradables is given by

$$C_{NT,0} = f\left(\frac{P^F_{T,0}}{P_{NT,0}}\right) C_{T,0}$$
$$C_{NT}(s) = f\left(\frac{E(s)P^F_T(s)}{P_{NT}}\right) C_T(s)$$

- obtain the corresponding amounts of labor from market clearing conditions

$$N_0 = \frac{C_{NT,0}}{A_0}$$
$$N(s) = \frac{C_{NT}(s)}{A(s)}$$

- obtain wages from labor-leisure condition

$$W_0 = -\frac{U_{N,0}}{\frac{U_{C_T,0}}{P_{T,0}}}$$
$$W(s) = -\frac{U_N(s)}{\frac{U_{C_T}(s)}{P_T(s)}}$$

- pick labor taxes $\tau_{L,0}$ and τ_L such that the firm FOCs hold

$$P_{NT,0} = (1+\tau_{L,0})\frac{\epsilon}{\epsilon-1}\frac{W_0}{A_0} \quad \text{for all}$$
$$P_{NT} = (1+\tau_L)\frac{\epsilon}{\epsilon-1}\frac{\int \pi(s)SDF(s)P_{NT}^{\epsilon}(s)C_{NT}(s)\frac{W(s)}{A(s)}ds}{\int \pi(s)SDF(s)P_{NT}^{\epsilon}(s)C_{NT}(s)ds} \quad \text{for}$$

- obtain government transfers T_0 and $T(s)_s$ from the GBCs

$$T_0 = qB + q^F B^F + \tau_{L,0}W_0N_0$$
$$T(s) = \tau_L W(s)N(s) - B - E(s)B^F$$

A.2 Lagrangian for planning problem

- start by writing the corresponding Lagrangian

$$\begin{aligned}\mathcal{L} =& V\left(C_{T,0}, \frac{P_{T,0}^F}{P_{NT,0}}\right) + \beta \cdot \int \pi(s)V\left(C_T(s), \frac{E(s)P_T^F(s)}{P_{NT}(s)}\right)ds \\ &+ \mu \cdot \left[Y_{T,0} + \frac{qB}{P_{T,0}^F} + \frac{q^F B^F}{P_{T,0}^F} - C_{T,0}\right] \\ &+ \int \beta\pi(s)\lambda(s)\left[Y_T(s) - \frac{B}{E(s)P_T^F(s)} - \frac{B^F}{P_T^F(s)} - C_T(s)\right]ds\end{aligned}$$

A.3 FOCs for endogenous portfolio

- finally, the FOCs for Home (B) and Foreign currency (B^F) debt

$$\mu \cdot \frac{q}{P_{T,0}^F} = \beta \cdot \int \pi(s)\lambda(s)\frac{1}{E(s)P_T^F(s)}ds$$
$$\mu \cdot \frac{q^F}{P_{T,0}^F} = \beta \cdot \int \pi(s)\lambda(s)\frac{1}{P_T^F(s)}ds$$

- *note: if I study the version of the model without portfolio choice, I simply drop the last 2 FOCs and treat B and B^F as exogenous in the remaining FOCs and constraints*

A.4 Solution system for full planning problem with endogenous portfolio

- a solution of the planning problem consists of $2 \cdot S + 5$ variables
  - values for tradables consumption: $C_{T,0}, (C_T(s))$
  - values for prices of non-tradables: $P_{NT,0}, P_{NT}$
  - values for the nominal exchange rate: $(E(s))$
  - values for debt: B, B^F
- the solution to the following system of $2 \cdot S + 5$ equations gives us the values for these variables

$$
\begin{aligned}
0 &= V_{p,0} \\
0 &= \int \pi(s) V_{p,s} \frac{-E(s)P_T^F(s)}{P_{NT}^2} ds \\
0 &= \beta\pi(s)V_{p,s}\frac{P_T^F(s)}{P_{NT}} + \pi(s)\cdot V_{C_T,0}\cdot\left[\beta\frac{V_{C_T}(s)}{V_{C_T,0}} - SDF^F(s)\right]\cdot\frac{B}{E(s)^2P_T^F(s)} \\
C_{T,0} &= Y_{T,0} + \frac{qB}{P_{T,0}^F} + \frac{q^FB^F}{P_{T,0}^F} \\
C_T(s) &= Y_T(s) - \frac{B}{E(s)P_T^F(s)} - \frac{B^F}{P_T^F(s)} \\
V_{C_T,0}\cdot\frac{q}{P_{T,0}^F} &= \beta\cdot\int \pi(s)V_{C_T}(s)\frac{1}{E(s)P_T^F(s)}ds \\
V_{C_T,0}\cdot\frac{q^F}{P_{T,0}^F} &= \beta\cdot\int \pi(s)V_{C_T}(s)\frac{1}{P_T^F(s)}ds
\end{aligned}
$$

A.5 Optimal Policy under complete markets and flexible prices

Hence, the planner's problem is now given by

$$
\begin{aligned}
\max_{(E(s))_s, C_{T,0}, (C_T(s))_s, P_{NT,0}, P_{NT}, (B(s))_s} \quad & V\left(C_{T,0}, \frac{P_{T,0}^F}{P_{NT,0}}\right) + \beta\cdot\int \pi(s)V\left(C_T(s), \frac{E(s)P_T^F(s)}{P_{NT}}\right)ds \\
\text{s.t.}\quad C_{T,0} &= Y_{T,0} + \frac{\int q(s)B(s)ds}{P_{T,0}^F} \\
C_T(s) &= Y_T(s) - \frac{B(s)}{E(s)P_T^F(s)} \\
q(s) &= \pi(s)SDF^F(s)\frac{P_{T,0}^F}{E(s)P_T^F(s)}
\end{aligned}
$$

The first-order conditions for $B(s)$ and $E(s)$ are given by

$$
\begin{aligned}
V_{C_T,0} \cdot \frac{q(s)}{P^F_{T,0}} &= \beta\pi(s)\lambda(s)\frac{1}{E(s)P^F_T(s)} \\
0 &= \beta\pi(s)V_p(s)\frac{P^F_T(s)}{P_{NT}} + \mu\frac{\partial q(s)}{\partial E(s)}B(s) + \beta\pi(s)\lambda(s)\frac{B}{E(s)P^F_T(s)}
\end{aligned}
$$

These imply

$$
\begin{aligned}
\frac{\beta V_{C_T}(s)}{V_{C_T,0}} &= SDF^F(s) \\
V_p(s) &= 0
\end{aligned}
$$

That is, there is perfect risk-sharing and the output gap is closed in all states of the world. The intuition is simple. Under complete markets, transactions in state-contingent securities suffice to implement perfect risk-sharing so that the exchange rate can be used to close the output gap state-by-state.

A.6 Value function partials in terms of output gap

$$
V_{C_T}(s) = U_{C_T}(s) \cdot \left[1 + \frac{f(p(s))}{p(s)}\tau(s)\right] \tag{55}
$$

$$
V_p(s) = \frac{f'(p(s))}{p(s)} C_T(s) U_{C_T}(s)\tau(s) \tag{56}
$$

A.7 First-order conditions in terms of output gap

Using the definition of the output gap provided in the next, the first-order condition with respect to the nominal exchange rate in state $s \in \mathcal{S}$ can be expressed as

$$
0 = \pi(s)\beta\frac{f'(p(s))}{E(s)}U_{C_T}(s)C_T(s)\tau(s) + U_{C_{T,0}}\pi(s)\left[\frac{\beta V_{C_T}(s)}{V_{C_T,0}} - SDF^F(s)\right]\frac{B}{E(s)^2 P^F_T(s)}.
$$

This illustrates transparently the planner's trade-off. Using his set of available instruments (the exchange rate schedule $(E(s))_s$), he wants to close the output gap $\tau(s)$ but simultaneously equalize Home and Foreign stochastic discount factors. Thus, the planner will trade-off these two motives.

A.8 Proof of Proposition 4

The exchange rate is given by

$$E(s) = \frac{\alpha}{1-\alpha} \frac{P_{NT}}{P_T^F(s)} \frac{A(s)\left(\frac{1-\alpha}{\kappa}\right)^{\frac{1}{1+\psi}}}{\frac{\beta}{\beta_F}\left(\frac{Y_T^F(s)}{Y_{T,0}^F}\right)^{\gamma_F} C_{T,0}}.$$

Take two states $s, s' \in \mathcal{S}$. First, suppose $A(s') > A(s)$ and $Y_T(s') = Y_T(s), Y_T^F(s) = Y_T^F(s'), P_T^F(s') = P_T^F(s)$. Then,

$$\frac{E(s')}{E(s)} = \frac{A(s')}{A(s)} > 1.$$

Second, suppose $Y_T(s') > Y_T(s)$ and $A(s') = A(s), Y_T^F(s) = Y_T^F(s'), P_T^F(s') = P_T^F(s)$. Then,

$$\frac{E(s')}{E(s)} = 1.$$

Third, suppose $Y_T^F(s') > Y_T^F(s)$ and $A(s') = A(s), Y_T(s) = Y_T(s'), P_T^F(s') = P_T^F(s)$. Then,

$$\frac{E(s')}{E(s)} = \left(\frac{Y_T^F(s)}{Y_T^F(s')}\right)^{\gamma_F} < 1.$$

Finally, suppose $P_T^F(s') > P_T^F(s)$ and $A(s') = A(s), Y_T(s) = Y_T(s'), Y_T^F(s') = Y_T^F(s)$. Then,

$$\frac{E(s')}{E(s)} = \frac{P_T^F(s)}{P_T^F(s')} < 1.$$

A.9 Proof of Proposition 5

The exchange rate is given by

$$E(s) = \frac{\frac{\alpha}{1-\alpha}\left(\frac{1-\alpha}{\kappa}\right)^{\frac{1}{1+\psi}} A(s)P_{NT} + B}{P_T^F(s)Y_T(s) - B^F}$$

Take two states $s, s' \in \mathcal{S}$. First, suppose $A(s') > A(s)$ and $Y_T(s') = Y_T(s), Y_T^F(s) = Y_T^F(s'), P_T^F(s') = P_T^F(s)$. Then,

$$\frac{E(s')}{E(s)} = \frac{\frac{\alpha}{1-\alpha}\left(\frac{1-\alpha}{\kappa}\right)^{\frac{1}{1+\psi}} A(s')P_{NT} + B}{\frac{\alpha}{1-\alpha}\left(\frac{1-\alpha}{\kappa}\right)^{\frac{1}{1+\psi}} A(s)P_{NT} + B} > 1.$$

Second, suppose $Y_T(s') > Y_T(s)$ and $A(s') = A(s), Y_T^F(s) = Y_T^F(s'), P_T^F(s') = P_T^F(s)$. Then,

$$\frac{E(s')}{E(s)} = \frac{P_T^F Y_T(s) - B^F}{P_T^F Y_T(s') - B^F} < 1.$$

Third, suppose $Y_T^F(s') > Y_T^F(s)$ and $A(s') = A(s), Y_T(s) = Y_T(s'), P_T^F(s') = P_T^F(s)$. Then,

$$\frac{E(s')}{E(s)} = 1.$$

Finally, suppose $P_T^F(s') > P_T^F(s)$ and $A(s') = A(s), Y_T(s) = Y_T(s'), Y_T^F(s') = Y_T^F(s)$. Then,

$$\frac{E(s')}{E(s)} = \frac{P_T^F(s)Y_T - B^F}{P_T^F(s')Y_T - B^F} < 1.$$

A.10 Proof of Proposition 7

The exchange rate is given by

$$E(s) = \frac{B}{P_T^F(s)\left[Y_T(s) - \frac{\beta}{\beta^F}\left(\frac{Y_T^F(s)}{Y_{T,0}^F}\right)^{\gamma_F} C_{T,0}\right]}$$

Take two states $s, s' \in \mathcal{S}$. First, suppose $A(s') > A(s)$ and $Y_T(s') = Y_T(s), Y_T^F(s) = Y_T^F(s'), P_T^F(s') = P_T^F(s)$. Then,

$$\frac{E(s')}{E(s)} = 1.$$

Second, suppose $Y_T(s') > Y_T(s)$ and $A(s') = A(s), Y_T^F(s) = Y_T^F(s'), P_T^F(s') = P_T^F(s)$. Then,

$$\frac{E(s')}{E(s)} = \frac{Y_T(s) - \frac{\beta}{\beta^F}\left(\frac{Y_T^F}{Y_{T,0}^F}\right)^{\gamma_F} C_{T,0} - B^F}{Y_T(s') - \frac{\beta}{\beta^F}\left(\frac{Y_T^F}{Y_{T,0}^F}\right)^{\gamma_F} C_{T,0} - B^F} < 1.$$

Third, suppose $Y_T^F(s') > Y_T^F(s)$ and $A(s') = A(s), Y_T(s) = Y_T(s'), P_T^F(s') = P_T^F(s)$. Then,

$$\frac{E(s')}{E(s)} = \frac{P_T^F(s)\left[Y_T(s) - \frac{\beta}{\beta^F}\left(\frac{Y_T^F(s)}{Y_{T,0}^F}\right)^{\gamma_F} C_{T,0}\right] - B^F}{P_T^F(s)\left[Y_T(s) - \frac{\beta}{\beta^F}\left(\frac{Y_T^F(s')}{Y_{T,0}^F}\right)^{\gamma_F} C_{T,0}\right] - B^F} > 1.$$

Finally, suppose $P_T^F(s') > P_T^F(s)$ and $A(s') = A(s), Y_T(s) = Y_T(s'), Y_T^F(s') = Y_T^F(s)$. Then,

$$\frac{E(s')}{E(s)} = \frac{P_T^F(s')\left[Y_T - C_T\right] - B^F}{P_T^F(s)\left[Y_T - C_T\right] - B^F},$$

which is greater than 1 for $(Y_T - C_T) > 0$, smaller than 1 for $(Y_T - C_T) < 0$ and equal to 0 for $Y_T = C_T$.

A.11 Proof of non-reversal of exchange rate response to shocks under rigid prices relative to the response under flexible prices

Consider two states s and s' so that realization of all shocks are identical except $Y_T^F(s') < Y_T^F(s)$. Under flexible prices, it is optimal to appreciate the exchange ratea in state s' relative to its value in s. Suppose that under rigid prices, the opposite is the case. That is, suppose $E(s') > E(s)$. This implies $\tau(s') < \tau(s)$ which, in turn, implies that

$$\left[1 + \frac{1-\alpha}{\alpha}\tau(s')\right] < \left[1 + \frac{1-\alpha}{\alpha}\tau(s)\right].$$

Then,

$$\left(\frac{Y_T^F(s')}{Y_{T,0}^F}\right)^{\gamma_F}\left[1 + \frac{1-\alpha}{\alpha}\tau(s')\right] < \left(\frac{Y_T^F(s)}{Y_{T,0}^F}\right)^{\gamma_F}\left[1 + \frac{1-\alpha}{\alpha}\tau(s)\right]$$

which means that the transfer to the rest of the world is smaller in state s than in state s'. This implies $E(s) > E(s')$ which is a contradiction.

A.12 Overview of countries in sample and respective sample length

Country / Currency	Sample length
Australia	1985 - 2010
Austria	1986 - 1998
Belgium	1983 - 1998
Canada	1985 - 2010
Switzerland	1983 - 2010
Germany	1983 - 1998
Euro	1999 - 2010
Spain	1986 - 1998
France	1983 - 1998
United Kingdom	1983 - 1998
Ireland	1986 - 1998
Italy	1984 - 1998
Japan	1983 - 2010
South Korea	2002 - 2010
Netherlands	1983 - 1998
Norway	1985 - 2010
New Zealand	1985 - 2010
Poland	1996 - 2010
Sweden	1985 - 2010

A.13 Regression table for productivity shocks

Note that specifications (1) and (2) are for the full sample of OECD advanced economies whereas specifications (3) and (4) are for the sample of “G10” currencies.

	(1)	(2)	(3)	(4)
	Excess return	Excess return	Excess return	Excess return
beta	-0.0261*	-0.0241*	-0.0422*	-0.0418*
	(-2.89)	(-2.65)	(-3.02)	(-2.71)
exports-GDP-share		-0.00583		0.0195
		(-0.43)		(0.60)
constant	0.0241***	0.0260***	0.0305***	0.0257*
	(6.03)	(4.46)	(5.09)	(2.54)
N	19	18	10	9

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

A.14 Regression table for export-demand shocks

Note that specifications (1) and (2) are for the full sample of OECD advanced economies whereas specifications (3) and (4) are for the sample of "G10" currencies.

	(1)	(2)	(3)	(4)
	Excess return	Excess return	Excess return	Excess return
exports-GDP-share	-0.00952	-0.00583	-0.00614	0.0195
	(-0.61)	(-0.43)	(-0.14)	(0.60)
beta		-0.0241*		-0.0418*
		(-2.65)		(-2.71)
constant	0.0181**	0.0260***	0.0183	0.0257*
	(3.08)	(4.46)	(1.36)	(2.54)
N	18	18	9	9

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$