CS2040S Data Structures and Algorithms

Hashing! (Part 2)

Today: More Hashing!

Java hashing

Collision resolution: open addressing

Table (re)sizing

Review: Symbol Table Abstract Data Type Which of the following is *not* typically a symbol table operation?

- 1. insert(key, data)
- 2. delete(key)
- 3. successor(key)
- 4. search(key)
- 5. None of the above.

Review: Symbol Table Abstract Data Type

Which of the following is *not* typically a symbol table operation?

- 1. insert(key, data)
- 2. delete(key)
- 3. successor(key)
- 4. search(key)
- 5. None of the above.

Abstract Data Types

Symbol Table

public interface	SymbolTable	
void	insert(Key k, Value v)	insert (k,v) into table
Value	search(Key k)	get value paired with k
void	delete(Key k)	remove key k (and value)
boolean	contains(Key k)	is there a value for k?
int	size()	number of (k,v) pairs

Direct Access Tables

Attempt #1: Use a table, indexed by keys.

0	null
1	null
2	item1
3	null
4	null
5	item3
	null
7	null
8	item2
9	null

Universe $U=\{0..9\}$ of size m=10.

(key, value)

(2, item1)

(8, item2)

(5, item3)

Assume keys are distinct.

Direct Access Tables

Problems:

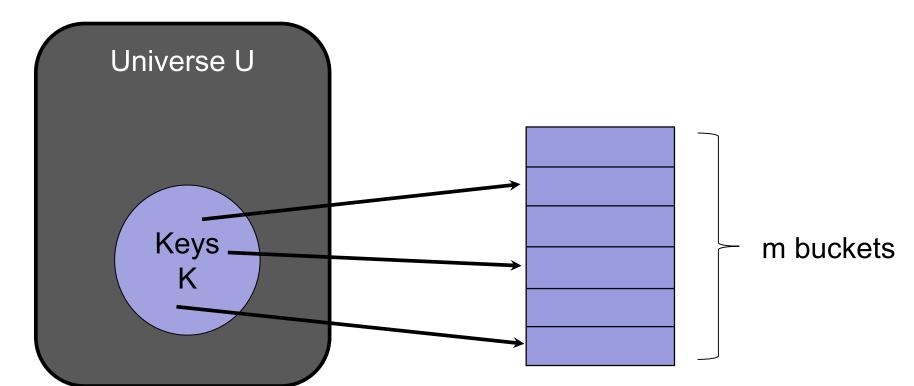
- Too much space
 - If keys are integers, then table-size > 4 billion

- What if keys are not integers?
 - Where do you put the key/value "(hippopotamus, bob)"?
 - Where do you put 3.14159...?

Hash Functions

Problem:

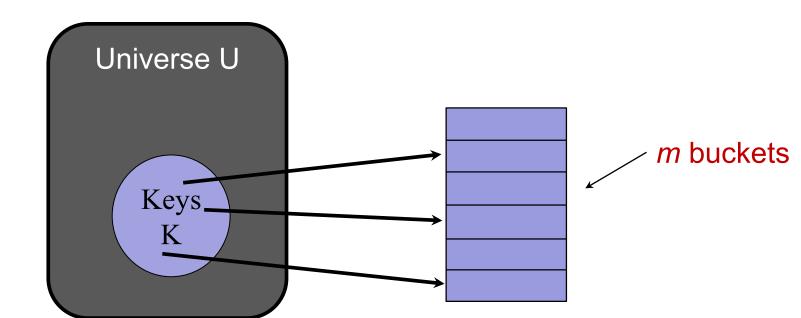
- Huge universe *U* of possible keys.
- Smaller number *n* of actual keys.
- How to map *n* keys to $m \approx n$ buckets?



Hash Functions

Define hash function $h : U \square \{1..m\}$

- Store key k in bucket h(k).



Hash Functions

Collisions:

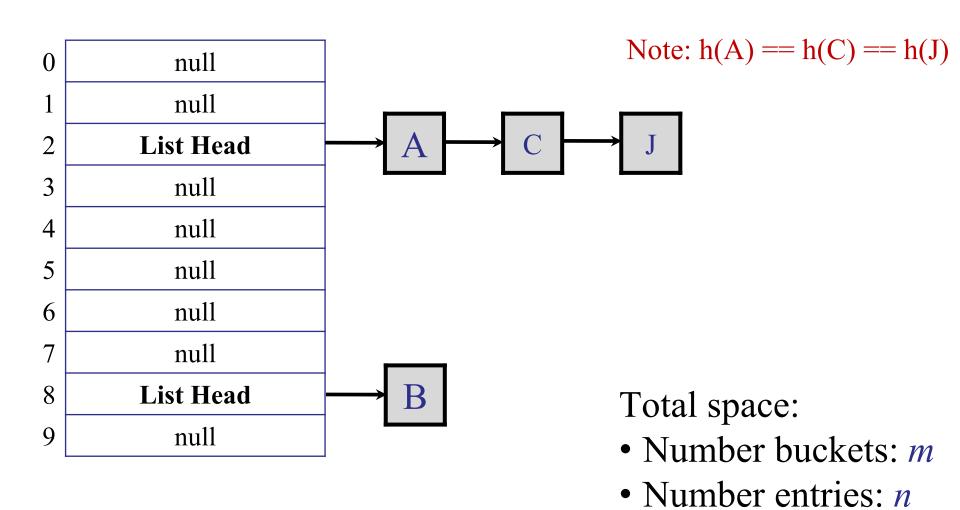
- We say that two <u>distinct</u> keys k_1 and k_2 collide if:

$$h(k_1) = h(k_2)$$

- Unavoidable!
 - The table size is smaller than the universe size.
 - The pigeonhole principle says:
 - There must exist two keys that map to the same bucket.
 - Some keys must collide!

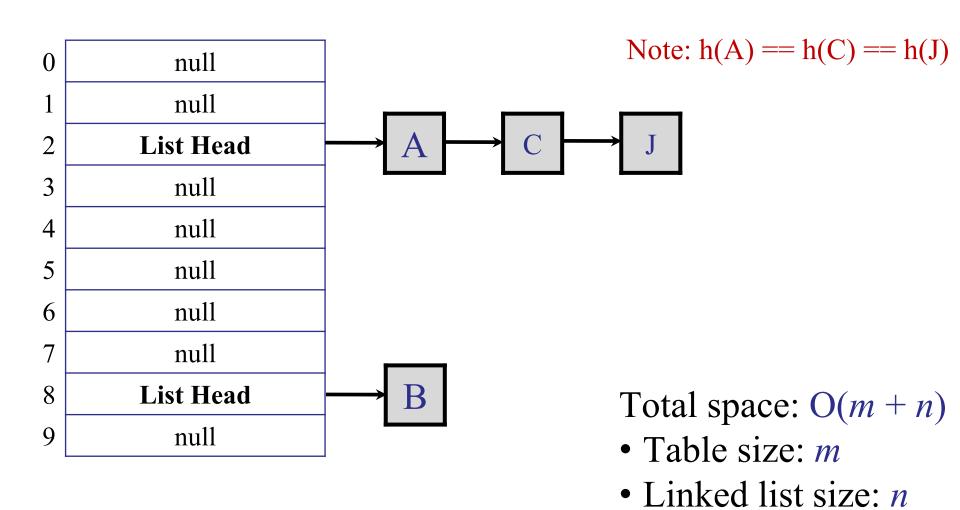
Chaining

Each bucket contains a linked list of items.



Chaining

Each bucket contains a linked list of items.



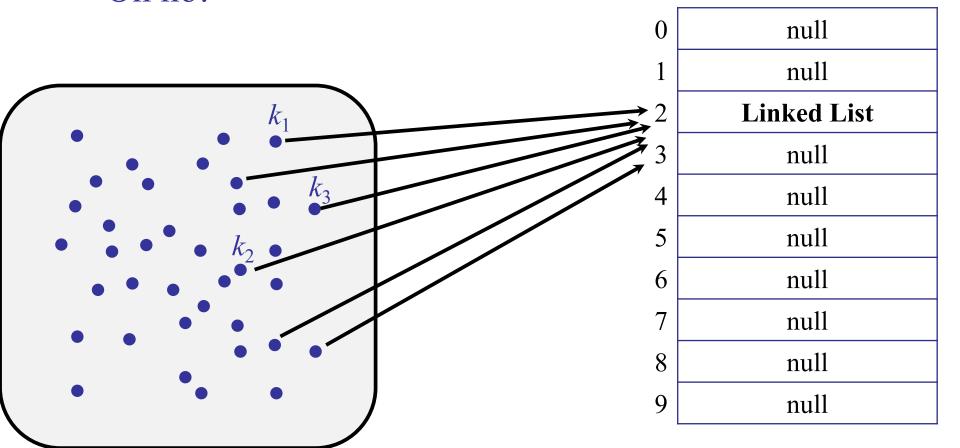
Operations:

- insert(key, value)
 - Calculate h(key)
 - Lookup h(key) and add (key, value) to the linked list.

- search(key)
 - Calculate h(key)
 - Search for (key, value) in the linked list.

What if all keys hash to the same bucket!

- Worst-case search costs O(n)
- Oh no!



The Simple Uniform Hashing Assumption

- Every key is equally likely to map to every bucket.
- Keys are mapped independently.

Assume hash function has this property, even if it may not!

Intuition:

- Each key is put in a random bucket.
- Then, as long as there are enough buckets, we won't get too many keys in any one bucket.

The Simple Uniform Hashing Assumption

- Assume:
 - *n* items
 - *m* buckets
- Define: load(hash table) = n/m= average # items / bucket.

Expected search time = 1 + expected # items per bucket

hash function + array access

The Simple Uniform Hashing Assumption

- Assume:
 - *n* items
 - *m* buckets
- Define: load(hash table) = n/m= average # items / buckets.

- Expected search time = 1 + n/mlinked list traversal hash function + array access

The Simple Uniform Hashing Assumption

- Assume:
 - *n* items
 - $m = \Omega(n)$ buckets, e.g., m = 2n

- Expected search time = 1 + n/m= O(1)

Searching:

- Expected search time = 1 + n/m = O(1)
- Worst-case search time = O(n)

Inserting:

- Worst-case insertion time = O(1)

** In this case, inserting allows duplicates...

Preventing duplicates requires searching.

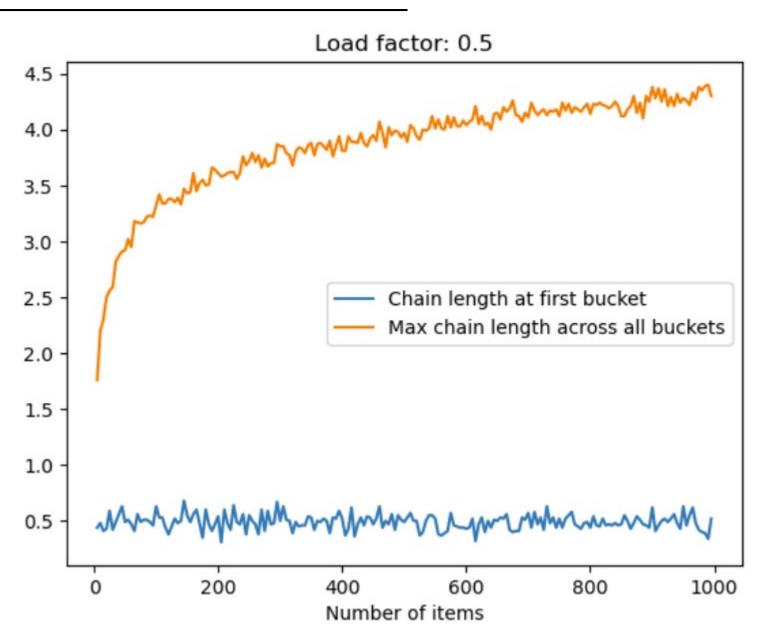
What if you insert n elements in your hash table?

What is the expected *maximum* cost?

- Analogy:
 - Throw n balls in m = n bins.
 - What is the maximum number of balls in a bin?

Cost: $\Theta(\log n / \log \log n)$

(See CS5330 for a proof.)



Some Remark

How to reduce maximum chain length?

• Power of two choices!: Use two hash functions h_1 and h_2 . For a key k, store in bucket $h_1(k)$ if it has the shorter chain, otherwise store in bucket $h_2(k)$. The maximum chain length becomes $O(\log \log n)$ instead of $O(\log n)$ in expectation!

• IMO, one of the most stunning facts in algorithm design.

Hashing: Recap

Problem: coping with large universe of keys

- Number of possible keys is very, very large.
- Direct Access Table takes too much space

Hash functions

- Use hash function to map keys to buckets.
- Sometimes, keys collide (inevitably!)
- Use linked list to store multiple keys in one bucket.

Analyze performance with simple uniform hashing.

- Expected number of keys / bucket is O(n/m) = O(1).

Today

• Java hashing

java.util.Map

```
public interface java.util.Map<Key, Value>
           void clear()
                                         removes all entries
        boolean contains Key (Object k) is k in the map?
        boolean contains Value (Object v) is v in the map?
          Value get(Object k)
                                         get value for k
          Value put (Key k, Value v)
                                        adds (k,v) to table
          Value remove(Object k)
                                        remove mapping for k
            int size()
                                         number of entries
```

java.util.Map

Parameterized by key and value. Not necessarily comparable

public interface java.util.Map<Key, Value> void clear() removes all entries boolean contains Key (Object k) is k in the map? boolean contains Value (Object v) is v in the map? Value get (Object k) get value for k Value put (Key k, Value v) adds (k,v) to table Value remove (Object k) remove mapping for k int size() number of entries

nublic interface java util Manckey

java.util.Map

Search by key.

772 1 110 >

public interface	java.utii.Map <key, th="" vaiue<=""><th></th></key,>	
void	clear()	removes all entries
boolean	containsKey(Object k)	is k in the map?
boolean	containsValue(Object v)	is v in the map?
Value	get(Object k)	get value for k
Value	put(Key k, Value v)	adds (k,v) to table
Value	remove(Object k)	remove mapping for k
int	size()	number of entries

java.util.Map

Search by value. (May not be efficient.)

public interface	java.util.Map <key, th="" value<=""><th>e></th></key,>	e>
void	clear()	removes all entries
boolean	containsKey(Object k)	is k in the map?
boolean	containsValue(Object v)	is v in the map?
Value	get(Object k)	get value for k
Value	put(Key k, Value v)	adds (k,v) to table
Value	remove(Object k)	remove mapping for k
int	size()	number of entries

java.util.Map

Can use any Object as key?

```
public interface java.util.Map<Key, Value>
                                         removes all entries
           void clear()
        boolean contains Key (Object k) is k in the map?
        boolean contains Value (Object v) is v in the map?
          Value get (Object k)
                                         get value for k
          Value
                put(Key k, Value v)
                                         adds (k,v) to table
          Value remove (Object k)
                                         remove mapping for k
            int size()
                                         number of entries
```

java.util.Map

Put new (key, value) in table.

```
public interface java.util.Map<Key, Value>
                                           removes all entries
            void
                clear()
         boolean contains Key (Object k) is k in the map?
         boolean contains Value (Object v) is v in the map?
           Value get (Object k)
                                           get value for k
           Value put (Key k, Value v)
                                           adds (k,v) to table
           Value remove (Object k)
                                           remove mapping for k
             int size()
                                           number of entries
```

Map Interface in Java

java.util.Map<Key, Value>

- No duplicate keys allowed.
- No mutable keys

If you use an *object* as a key, then you can't modify that object later.

Symbol Table

Key Mutability

```
SymbolTable<Time, Plane> t =
           new SymbolTable<Time, Plane>();
Time t1 = new Time(9:00);
Time t2 = new Time(9:15);
t.insert(t1, "SQ0001");
t.insert(t2, "SQ0002");
t1.setTime(10:00);
x = \text{new Time}(9:00);
t.search(x);
```

What time does this plane depart at?

Symbol Table Moral: Keys should be immutable.

Key Mutability

Examples: Integer, String

```
SymbolTable<Time, Plane> t =
            new SymbolTable<Time, Plane>();
Time t1 = new Time(9:00);
Time t2 = \text{new Time (9:15)};
t.insert(t1, "SQ0001");
t.insert(t2, "SQ0002");
t1.setTime(10:00);
x = \text{new Time}(9:00);
t.search(x);
```

Design Decisions

Allow duplicate keys?

- No: need to search on insertion
- Yes: faster insertion

What to do if user inserts duplicate key?

- Replace existing key.
- Add new value (i.e., key has two values).
- Error.

Insert empty/null value?

- Deletes existing (key, value) pair.
- Creates a null value.
- Error.

java.util.Map

```
public interface java.util.Map<Key, Value>
Set<Map.Entry<Key, Value> entrySet() set of all mappings

Set<Key> keySet() set of all keys

Collection<Value> values() collection of all values
```

Note: not sorted

not necessarily efficient to work with these sets/collections.

What is wrong here?

Example:

There is a bug here!

```
Map<String, Integer> ageMap = new Map<String, Integer>();
ageMap.put("Alice", 32);
ageMap.put("Bernice", 84);
ageMap.put("Charlie", 7);

Integer age = ageMap.get("Alice")
```

- Key-type: String
- Value-type: Integer

What is wrong here?

Example:

Map is an interface!

Cannot instantiate an interface.

```
Map<String, Integer> ageMap = new Map<String, Integer>();

ageMap.put("Alice", 32);

ageMap.put("Bernice", 84);

ageMap.put("Charlie", 7);

Integer age = ageMap.get("Alice")
```

- Key-type: String
- Value-type: Integer

Map Class in Java

Example: HashMap

```
Map<String, Integer> ageMap = new HashMap<String, Integer>();
ageMap.put("Alice", 32);
ageMap.put("Bernice", 84);
ageMap.put("Charlie", 7);

Integer age = ageMap.get("Alice");
System.out.println("Alice's age is: " + age + ".");
```

- Key-type: String
- Value-type: Integer

Map Class in Java

Example: HashMap

```
Map<String, Integer> ageMap = new HashMap<String, Integer>();
ageMap.put("Alice", 32);
ageMap.put("Bernice", null);
ageMap.put("Charlie", 7);

Integer age = ageMap.get("Bob");
if (age==null){
    System.out.println("Bob's age is unknown.");
}
```

- Returns "null" when key is not in map.
- Returns "null" when value is null.

Map Classes in Java

HashMap

Symbol Table

- containsKey
- contains Value
- entrySet
- get
- isEmpty
- keySet
- put
- putAll
- remove
- values

TreeMap

Dictionary

- containsKey
- contains Value
- entrySet
- get
- isEmpty
- keySet
- put
- putAll
- remove
- values

Map Classes in Java

HashMap

Symbol Table

TreeMap

Dictionary

- ceilingEntry
- ceilingKey
- descendingKeySet
- firstEntry
- firstKey
- floorEntry
- floorKey
- headMap
- higherEntry
- higherKey
- ... (and more)

Hashing in Java

How does your program know which hash function to use?

```
HashMap<MyFoo, Integer> hmap = new ...
MyFoo foo = new MyFoo();
hmap.put(foo, 8);
```

Every object supports the method:

```
int hashCode()
```

Java Object

Every class implicitly extends Object

public class	Object	
Object	clone()	creates a copy
boolean	equals(Object obj)	is obj equal to this?
void	finalize()	used by garbage collector
Class	getClass()	returns class
int	hashCode()	calculates hash code
void	notify()	wakes up a waiting thread
void	notifyAll()	wakes up all waiting threads
String	toString()	returns string representation
void	wait(…)	wait until notified

Hashing in Java

How does your program know which hash function to use?

```
HashMap<MyFoo, Integer> hmap = new ...
MyFoo foo = new MyFoo();
int hash = foo.hashCode();
hmap.put(foo, 8);
```

Every object supports the method:

```
int hashCode()
```

Rules:

- Always returns the same value, if the object hasn't changed.
- If two objects are equal, then they return the same hashCode.

Is it legal for every object to return 32?

Every object supports the method:

```
int hashCode()
```

Rules:

- Always returns the same value, if the object hasn't changed.
- If two objects are equal, then they return the same hashCode.

Is it *legal* for every object to return 32? (YES)

Every object supports the method:

```
int hashCode()
```

Default Java implementation:

- hashCode returns the memory location of the object
- Every object hashes to a different location

```
Must implement/override hashCode () for your class.
```

• CAVEAT: hashCode() returns an int, so using it directly to specify the bucket would require a table of size 2³²...not good!

```
/**
 * Returns index for hash code h.
 */
static int indexFor(int h, int length) {
   return h & (length-1);
}
```

• In HashMap, the hashCode is truncated to fit the table size. Usually, table size is a power of 2, so above code extracts suffix of appropriate length from hashCode.

e.g. table size of 16

16 = 10000 (in binary/base 2)

15 - 1 = 01111 (in binary/base 2)

e.g. table size of 16

16 = 10000 (in binary/base 2)

15 - 1 = 01111 (in binary/base 2)

e.g. table size of 16

example hash code = 45

45 = 101101 (in binary)

e.g. table size of 16

16 = 10000 (in binary/base 2)

15 - 1 = 01111 (in binary/base 2)

e.g. table size of 16

example hash code = 45

45 = 101101 (in binary) use index = 1101 (in binary) i.e. index = 13

Java Library Classes

Integer

Long

String

Integer

```
public int hashCode() {
                                                 Rules:
    return value;
                                                  Always returns the same value, if the object hasn't
                                               - If two objects are equal, then they return the same
```

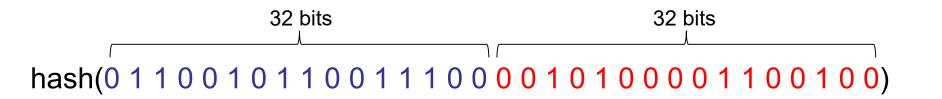
Note: hashcode is always a 32-bit integer.

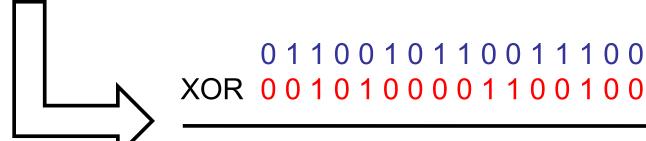
Note: every 32-bit integer gets a unique hashcode.

What do you do for smaller hash tables? Can there be collisions?

```
Collision can happen!
```

```
public int hashCode() {
  return (int) (value ^ (value >>> 32));
```





100110111111000

String

```
public int hashCode() {
  int h = hash; // only calculate hash once
  if (h == 0 \&\& count > 0) \{ // empty = 0 \}
       int off = offset;
       char val[] = value;
       int len = count;
       for (int i = 0; i < len; i++) {
            h = 31*h + val[off++];
       hash = h;
  return h;
```

String

HashCode calculation:

hash =
$$s[0]*31^{(n-1)} + s[1]*31^{(n-2)} + s[2]*31^{(n-3)} + ... + s[n-2]*31 + s[n-1]$$

Why did they choose 31?

String

HashCode calculation:

```
hash = s[0]*31^{(n-1)} + s[1]*31^{(n-2)} + s[2]*31^{(n-3)} + ... + s[n-2]*31 + s[n-1]
```

Why did they choose 31? Prime, 2^5-1

```
public class Pair {
    private int first;
    private int second;
    Pair(int a, int b) {
         first = a;
         second = b;
```

```
public void testPair() {
    HashMap<Pair, Integer> htable =
             new HashMap<Pair,
Integer>();
    Pair one = new Pair (20, 40);
    htable.put(one, 7);
    Pair two = new Pair (20, 40);
    int question = htable.get(two);
```

htable.get(new Pair(20, 40)) == ?

- 1. 1
- 2. 7
- 3. 11
- √4. null

```
Pair one = new Pair(20, 40);
Pair two = new Pair(20, 40);
one.hashCode() != two.hashCode()
```

```
Pair one = new Pair (20, 40);
Pair two = new Pair (20, 40);
htable.put(one, "first item");
htable.get(one) → "first item"
htable.get(two) _ null
```

```
public class Pair {
    private int first;
    private int second;
    Pair(int a, int b) {
         first = a;
         second = b;
    int hashCode() {
         return (first ^ second);
```

```
Pair one = new Pair (20, 40);
Pair two = new Pair (20, 40);
htable.put(one, "first item");
htable.get(one) → "first item"
htable.get(two) → null
one.equals(two) - false
```

Every object supports the method:

```
int hashCode()
```

Rules:

- Always returns the same value, if the object hasn't changed.
- If two objects are equal, then they return the same hashCode.
- Must redefine .equals to be consistent with hashCode.

```
Pair one = new Pair (20, 20);
Pair two = new Pair (20, 20);
htable.put(one, "first item");
htable.get(one) → "first item"
htable.get(two) → null
```

Every object supports the method:

```
boolean equals (Object o)
```

Rules:

- Reflexive: x.equals(x) == true
- Symmetric: x.equals(y) == y.equals(x)
- **Transitive**: x.equals(y), y.equals(z) \rightarrow x.equals(z)
- Consistent: always returns the same answer
- Null is null: x.equals(null) → false

Every object supports the method:

boolean equals (Object o)

```
boolean equals (Object p) {
     if (p == null) return false;
     if (p == this) return true;
     if (!(p instanceOf Pair)) return false;
     Pair pair = (Pair)p;
     if (pair.first != first) return false;
     if (pair.second != second) return
false:
     return true;
```

```
public V get(Object key) {
  if (key == null) return getForNullKey();
  int hash = hash(key.hashCode());
  for (Entry<K, V> e = table[indexFor(hash, table.length)];
        e != null;
       e = e.next)
         Object k;
         if (e.hash==hash
  &&((k=e.key)==key)||key.equals(k)))
               return e.value;
  return null;
```

```
public V get(Object key) {
  if (key == null) return getForNullKey();
  int hash = hash(key.hashCode());
  for (Entry<K, V> e = table[indexFor(hash, table.length)];
       e != null;
       e = e.next)
         Object k;
         if (e.hash==hash
  &&((k=e.key)==key)||key.equals(k)))
               return e.value;
  return null;
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public V get(Object key) {
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       e = e.next)
         Object k;
         if (e.hash==hash
  &&((k=e.key)==key)||key.equals(k)))
               return e.value;
  return null;
```

```
public V get(Object key) {
   if (key == null) return getForNullKey();
   int hash = hash(key.hashCode());
  for (Entry<K, V> e = table[indexFor(hash, table.length)];
        e != null;
        e = e.next)
         Object k;
         if (e.hash==hash
  & ((k=e.key) == key) \mid | key.equals(k))
               return e.vaiue,
  return null;
```

```
public V get(Object key) {
  if (key == null) return getForNullKey();
  int hash = hash(key.hashCode());
  for (Entry<K, V> e = table[indexFor(hash, table.length)];
       e != null;
       e = e.next)
         Object k;
         if (e.hash==hash
  &&((k=e.key)==key)||key.equals(k)))
               return e.value;
  return null;
```

```
public V get(Object key) {
  if (key == null) return getForNullKey();
  int hash = hash(key.hashCode()); ????
  for (Entry<K, V> e = table[indexFor(hash, table.length)];
       e != null;
       e = e.next)
         Object k;
         if (e.hash==hash
  &&((k=e.key)==key)||key.equals(k)))
               return e.value;
  return null;
```

```
// This function ensures that hashCodes that differ only
// by constant multiples at each bit position have a
// bounded number of collisions (approximately 8 at
// default load factor).
```

```
static int hash(int h) {
   h ^= (h >>> 20) ^ (h >>> 12);
   return h ^ (h >>> 7) ^ (h >>> 4);
}
```

```
// This function ensures that hashCodes that differ only
// by constant multiples at each bit position have a
// bounded number of collisions (approximately 8 at
// default load factor).
```

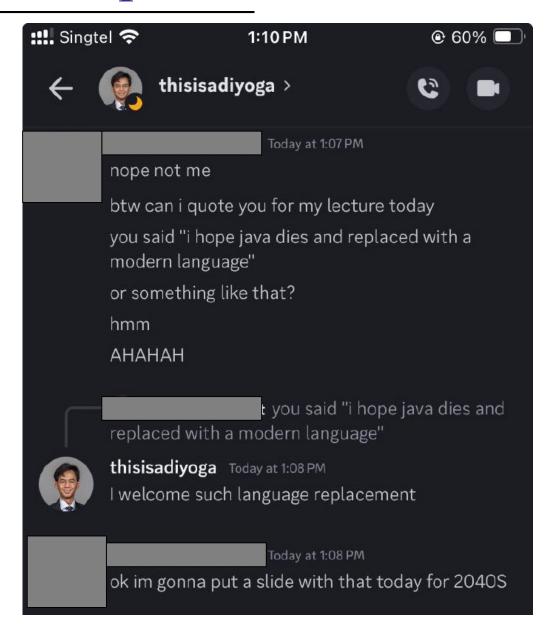
```
static int hash(int h) {
  h ^= (h >>> 20) ^ (h >>> 12);
  return h ^ (h >>> 7) ^ (h >>> 4);
}
```

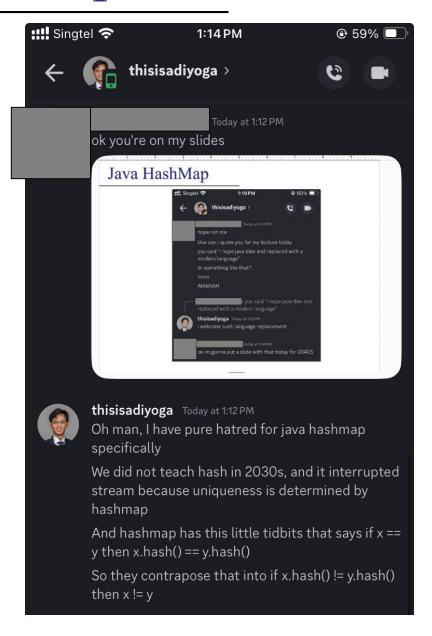
Eldon! Show them the documentation!

- CS2040S Slides Reminder

```
public V get(Object key) {
  if (key == null) return getForNullKey();
   int hash = hash(key.hashCode());
  for (Entry<K, V> e = table[indexFor(hash, table.length)];
        e != null;
        e = e.next)
         Object k;
         if (e.hash==hash
  &&((k=e.key)==key)||key.equals(k)))
               return e.value;
  return null;
                        Java checks if the key is equal to the
```

item in the hash table before returning it!







thisisadiyoga Today at 1:12 PM

Oh man, I have pure hatred for java hashmap specifically

We did not teach hash in 2030s, and it interrupted stream because uniqueness is determined by hashmap

And hashmap has this little tidbits that says if x == y then x.hash() == y.hash()

So they contrapose that into if x.hash() != y.hash() then x != y

thisisadiyoga Today at 1:14 PM

Then stream.distinct no longer works on your class unless you override the hash function

Today & Next Week

Java hashing

• Collision resolution: open addressing

Table (re)sizing