

CS2100

COMPUTER ORGANISATION

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## Lecture #19

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# Sequential Logic



**NUS**  
National University  
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School of  
Computing



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(May be obscured in some slides)



## 6. Synchronous Sequential Circuits

- Building blocks: **logic gates** and **flip-flops**.
- Flip-flops make up the **memory** while the gates form one or more combinational sub-circuits.
- We have discussed *S-R* flip-flop, *J-K* flip-flop, *D* flip-flop and *T* flip-flop.



# 6.1 Flip-flop Characteristic Tables

- Each type of flip-flop has its own behaviour, shown by its **characteristic table**.

$J$	$K$	$Q(t+1)$	Comments
0	0	$Q(t)$	No change
0	1	0	Reset
1	0	1	Set
1	1	$Q(t)'$	Toggle

$S$	$R$	$Q(t+1)$	Comments
0	0	$Q(t)$	No change
0	1	0	Reset
1	0	1	Set
1	1	?	Unpredictable

$D$	$Q(t+1)$
0	0      Reset
1	1      Set

$T$	$Q(t+1)$
0	$Q(t)$ No change
1	$Q(t)'$ Toggle



## 6.2 Sequential Circuits: Analysis (1/7)

- Given a sequential circuit diagram, we can analyze its behaviour by deriving its *state table* and hence its *state diagram*.
- Requires *state equations* to be derived for the flip-flop inputs, as well as *output functions* for the circuit outputs other than the flip-flops (if any).
- We use  $\mathbf{A}(t)$  and  $\mathbf{A}(t+1)$  (or simply  $\mathbf{A}$  and  $\mathbf{A}^+$ ) to represent the present state and next state, respectively, of a flip-flop represented by  $A$ .



## 6.2 Sequential Circuits: Analysis (2/7)

- Example using  $D$  flip-flops

State equations:

$$A^+ = A \cdot x + B \cdot x$$

$$B^+ = A' \cdot x$$

Output function:

$$y = (A + B) \cdot x'$$

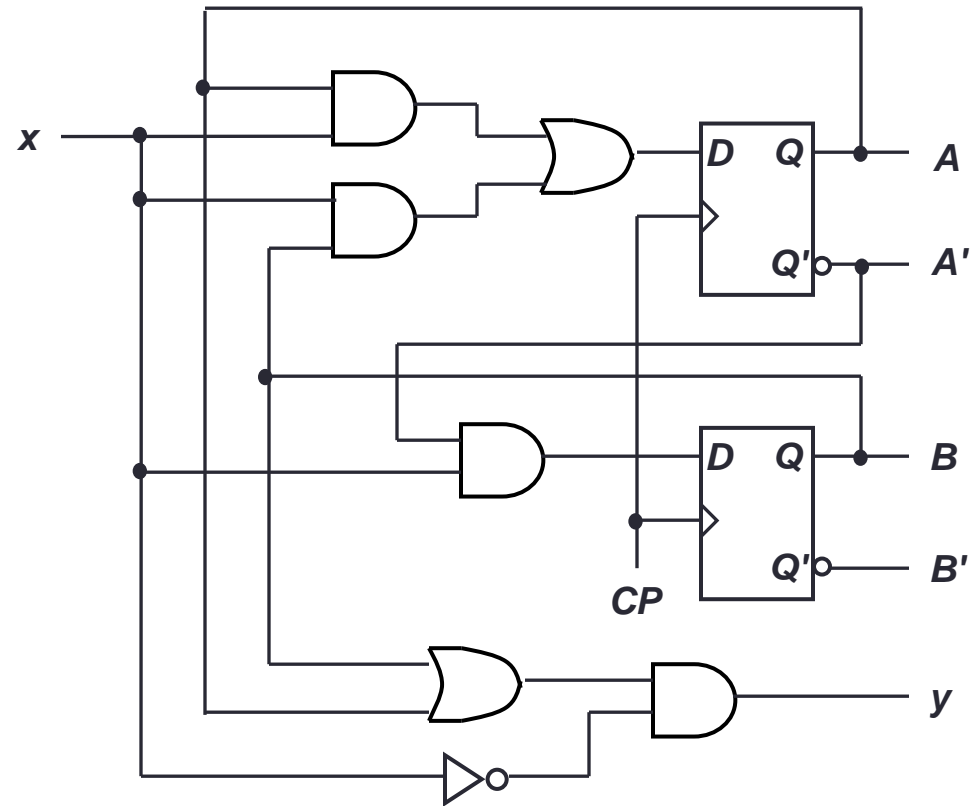


Figure 1



## 6.2 Sequential Circuits: Analysis (3/7)

- From the *state equations* and *output function*, we derive the **state table**, consisting of all possible binary combinations of present states and inputs.
- State table
  - Similar to truth table.
  - Inputs and present state on the left side.
  - Outputs and next state on the right side.
- $m$  flip-flops and  $n$  inputs  $\rightarrow 2^{m+n}$  rows.



## 6.2 Sequential Circuits: Analysis (4/7)

- **State table** for circuit of Figure 1:

State equations:

$$A^+ = A \cdot x + B \cdot x$$

$$B^+ = A' \cdot x$$

Output function:

$$y = (A + B) \cdot x'$$

Present State		Input	Next State		Output
A	B		A <sup>+</sup>	B <sup>+</sup>	
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0





## 6.2 Sequential Circuits: Analysis (5/7)

- Alternative form of state table:

Full table

Present State		Input	Next State		Output
A	B		A <sup>+</sup>	B <sup>+</sup>	
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

Compact table

Present State	Next State		Output	
	x=0	x=1	x=0	x=1
AB	A <sup>+</sup> B <sup>+</sup>	A <sup>+</sup> B <sup>+</sup>	y	y
00	00	01	0	0
01	00	11	1	0
10	00	10	1	0
11	00	10	1	0



## 6.2 Sequential Circuits: Analysis (6/7)

- From the *state table*, we can draw the *state diagram*.
- State diagram
  - Each state is denoted by a circle.
  - Each arrow (between two circles) denotes a transition of the sequential circuit (a row in state table).
  - A label of the form  $a/b$  is attached to each arrow where  $a$  (if there is one) denotes the inputs while  $b$  (if there is one) denotes the outputs of the circuit in that transition.
- Each combination of the flip-flop values represents a state. Hence,  $m$  flip-flops  $\rightarrow$  up to  $2^m$  states.

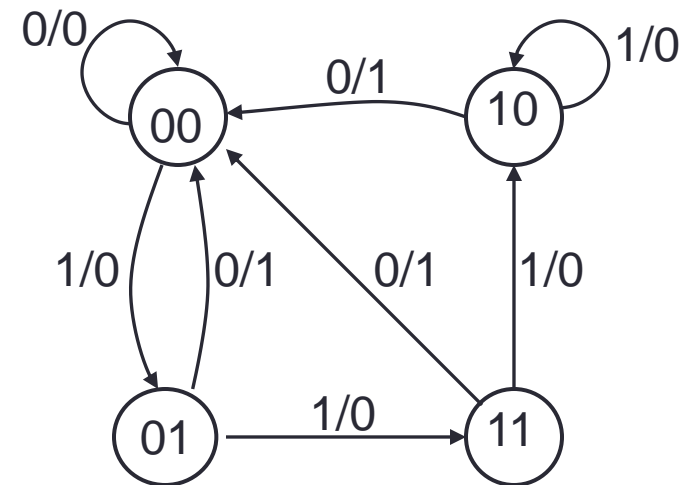


## 6.2 Sequential Circuits: Analysis (7/7)

- **State diagram** of the circuit of Figure 1:

Present State	Next State		Output	
	$x=0$	$x=1$	$x=0$	$x=1$
$AB$	$A^+B^+$	$A^+B^+$	$y$	$y$
00	00	01	0	0
01	00	11	1	0
10	00	10	1	0
11	00	10	1	0

**DONE!**



## 6.2 Flip-flop Input Functions (1/3)

- The outputs of a sequential circuit are functions of the present states of the flip-flops and the inputs. These are described algebraically by the *circuit output functions*.
  - In Figure 1:  $y = (A + B) \cdot x'$
- The part of the circuit that generates inputs to the flip-flops are described algebraically by the *flip-flop input functions* (or *flip-flop input equations*).
- The flip-flop input functions determine the next state generation.
- From the flip-flop input functions and the characteristic tables of the flip-flops, we obtain the next states of the flip-flops.

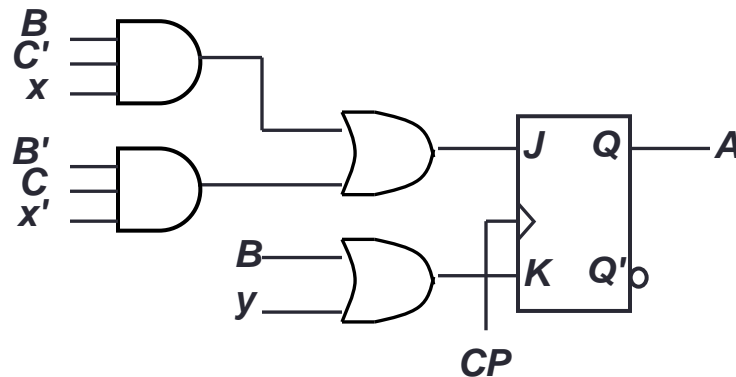


## 6.2 Flip-flop Input Functions (2/3)

- Example: circuit with a  $JK$  flip-flop.
- We use 2 letters to denote each flip-flop input: the first letter denotes the input of the flip-flop ( $J$  or  $K$  for  $J$ - $K$  flip-flop,  $S$  or  $R$  for  $S$ - $R$  flip-flop,  $D$  for  $D$  flip-flop,  $T$  for  $T$  flip-flop) and the second letter denotes the name of the flip-flop.

$$JA = B \cdot C' \cdot x + B' \cdot C \cdot x'$$

$$KA = B + y$$



## 6.2 Flip-flop Input Functions (3/3)

- In Figure 1, we obtain the following **state equations** by observing that  $Q^+ = DQ$  for a  $D$  flip-flop:

$$A^+ = A \cdot x + B \cdot x \quad (\text{since } DA = A \cdot x + B \cdot x)$$

$$B^+ = A' \cdot x \quad (\text{since } DB = A' \cdot x)$$

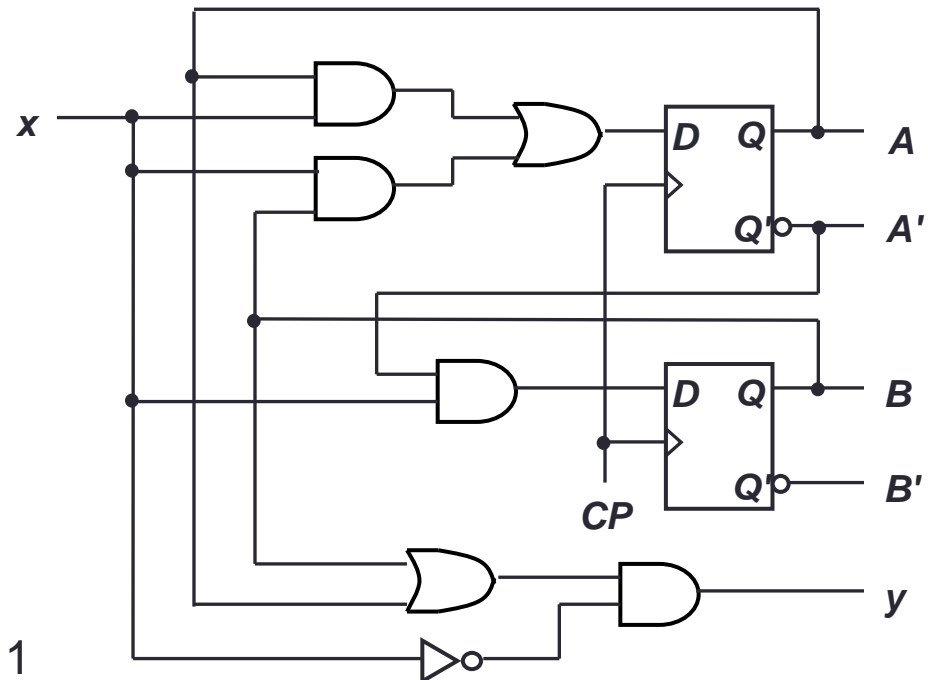
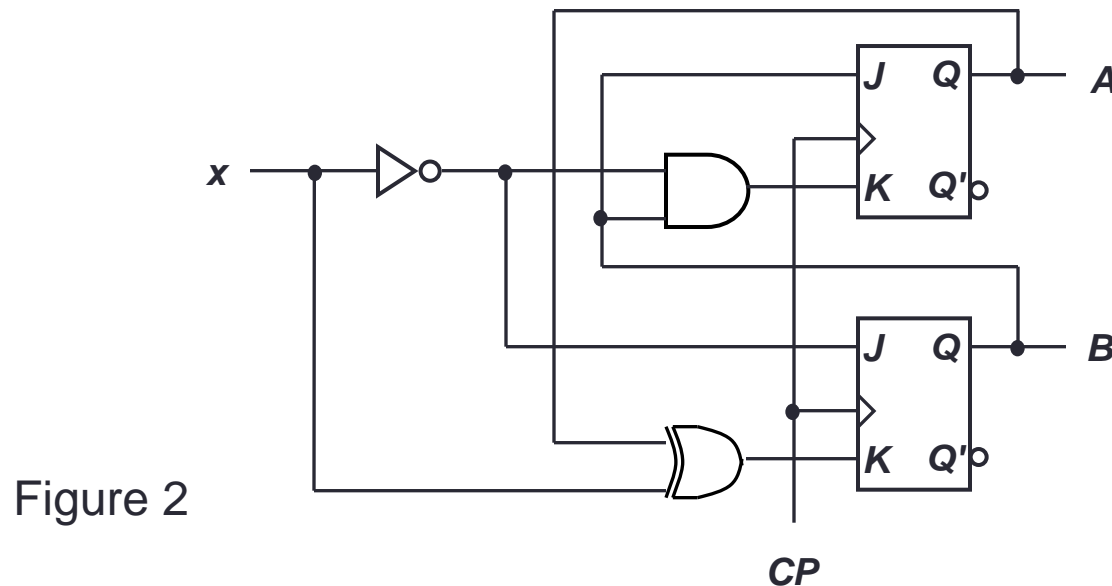


Figure 1



## 6.2 Analysis: Example #2 (1/3)

- Given Figure 2, a sequential circuit with two  $J$ - $K$  flip-flops  $A$  and  $B$ , and one input  $x$ .



- Obtain the **flip-flop input functions** from the circuit:

$$JA = B$$

$$JB = x'$$

$$KA = B \cdot x'$$

$$KB = A' \cdot x + A \cdot x' = A \oplus x$$



## 6.2 Analysis: Example #2 (2/3)

$$JA = B$$

$$KA = B \cdot x'$$

$$JB = x'$$

$$KB = A' \cdot x + A \cdot x' = A \oplus x$$

- Fill the **state table** using the above functions, knowing the characteristics of the flip-flops used.

<i>J</i>	<i>K</i>	<i>Q(t+1)</i>	Comments
0	0	<i>Q(t)</i>	No change
0	1	0	Reset
1	0	1	Set
1	1	<i>Q(t)'</i>	Toggle

Present state		Input <i>x</i>	Next state		Flip-flop inputs			
<i>A</i>	<i>B</i>		<i>A</i> <sup>+</sup>	<i>B</i> <sup>+</sup>	<i>JA</i>	<i>KA</i>	<i>JB</i>	<i>KB</i>
0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	1	1	1	1
1	1	1	1	1	1	0	0	0

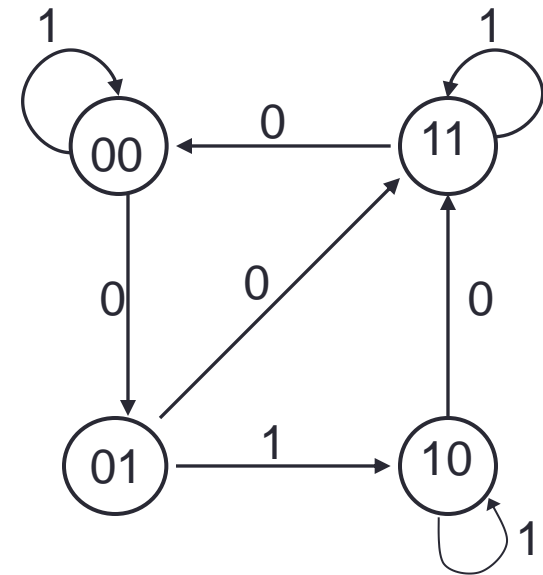




## 6.2 Analysis: Example #2 (3/3)

- Draw the **state diagram** from the state table.

Present state		Input $x$	Next state		Flip-flop inputs			
$A$	$B$		$A^+$	$B^+$	$JA$	$KA$	$JB$	$KB$
0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	1	1	1	1
1	1	1	1	1	1	0	0	0



## 6.2 Analysis: Example #3 (1/3)

- Derive the state table and state diagram of this circuit.

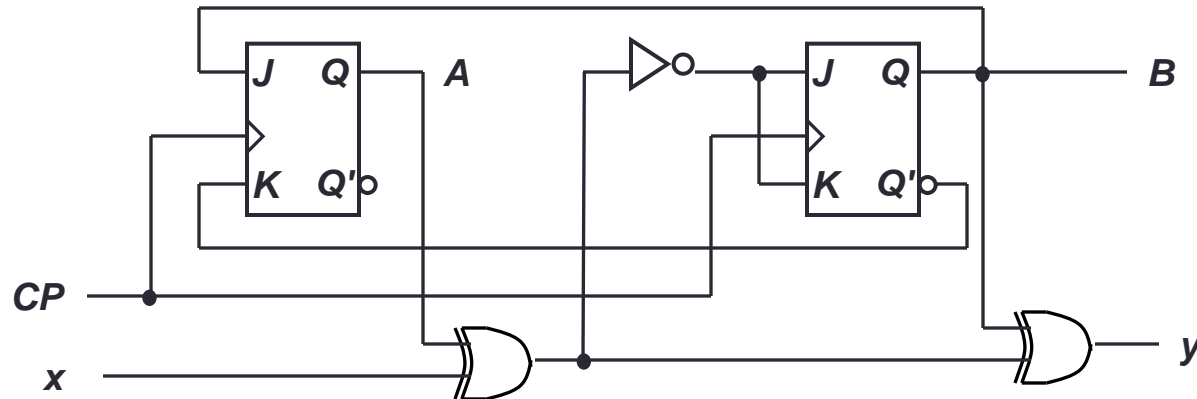


Figure 3

- Flip-flop input functions:

$$JA = B$$

$$KA = B'$$

$$JB = KB = (A \oplus x)' = A \cdot x + A' \cdot x'$$



## 6.2 Analysis: Example #3 (2/3)

- Flip-flop input functions:

$$JA = B$$

$$JB = KB = (A \oplus x)' = A \cdot x + A' \cdot x'$$

$$KA = B'$$

- State table:

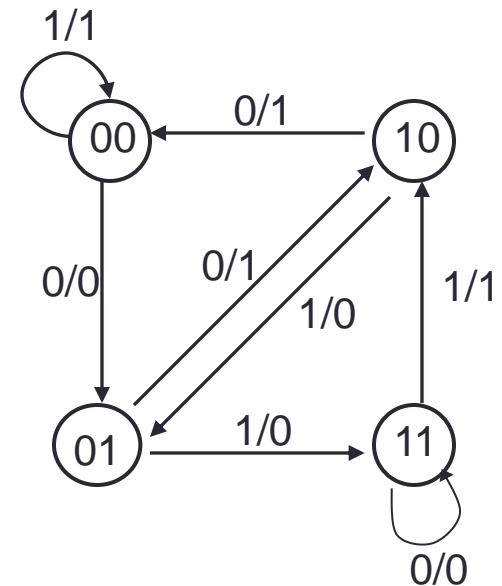
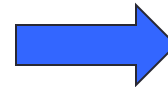
Present state		Input $x$	Next state		Output $y$	Flip-flop inputs			
$A$	$B$		$A^+$	$B^+$		$JA$	$KA$	$JB$	$KB$
0	0	0	0	1	0	0	1	1	1
0	0	1	0	0	1	0	1	0	0
0	1	0	1	0	1	1	0	1	1
0	1	1	1	1	0	1	0	0	0
1	0	0	0	0	1	0	1	0	0
1	0	1	0	1	0	0	1	1	1
1	1	0	1	1	0	1	0	0	0
1	1	1	1	0	1	1	0	1	1



## 6.2 Analysis: Example #3 (3/3)

### ■ State diagram:

Present state		Input $x$	Next state		Output $y$	Flip-flop inputs			
$A$	$B$		$A^+$	$B^+$		$JA$	$KA$	$JB$	$KB$
0	0	0	0	1	0	0	1	1	1
0	0	1	0	0	1	0	1	0	0
0	1	0	1	0	1	1	0	1	1
0	1	1	1	1	0	1	0	0	0
1	0	0	0	0	1	0	1	0	0
1	0	1	0	1	0	0	1	1	1
1	1	0	1	1	0	1	0	0	0
1	1	1	1	0	1	1	0	1	1



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