

CS2040S

# Data Structures and Algorithms

The End

Thanks to the tutors

for all their hard work!

CS2040S

# Data Structures and Algorithms

What comes next?

# Today's Plan

- ❑ **Announcements**
- ❑ **What comes next?**
- ❑ **Top 8 Algorithm Tips**
- ❑ **Wrap**

# Announcements

# Final Exam:

- Date: Saturday April 26
- Time: 13:00
- Location: MPSH

*(check specific hall assignment)*

(This is not official)

**Please double check  
time and location!**

# Final Exam:

- Topics: *everything*
- Cheat sheet: Bring 1 (double-sided, A4) sheet of notes.
- Calculator/phone: NO

# Final Exam:

Format: *Similar to Midterm Paper (Sample to be released)*

## Short answer advice:

1. Keep your answer short.
2. You think, “if I write more, I may write what the professor is looking for.”
3. I think, “if they wrote a lot, they probably don’t really understand the point.”  
(Getting to exactly the point is important)
4. The more you write, the more chance you write something that is false.
5. Draw pictures to explain what you mean.
6. Part of the test is communication. If I don’t understand your answer, you will not get credit, even if it is the “right idea” or your algorithm “works.”



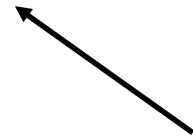
# Announcements

# Teaching Feedback:

- Please fill in departmental teaching feedback.
- Please fill in Coursemology feedback survey.

(One last chance at a few XP!)

Open tomorrow...



# Problem Sets:

Last chance for submissions: Friday, 11:59pm.

# Summer plans:

Overall, this was a great class!

(Most of you did really well...)

A few of you may need to shore up some basics.

(Experience and comfort with coding, basic programming skills are important.)

(In more advanced classes like CS2109S, assignments will expect that you can easily implement BFS, for example.)

(We may contact you after exams are over with some suggestions.)

Good idea to practice for technical interview:

<https://github.com/yangshun/tech-interview-handbook>

<https://leetcode.com/>

<https://nus.kattis.com/courses/CS2040>

# Today's Plan

- ❑ **Announcements**
- ❑ **What comes next?**
- ❑ **Top 8 Algorithm Tips**
- ❑ **Wrap**

# Algorithms Everywhere

Web browser:  
Parsing  
Substring manipulation  
XML trees

Internet

Internet routing:  
TCP (congestion control)  
IP routing  
BGP (Bellman-Ford)  
Content caching  
DNS

Google:  
PageRank  
String matching

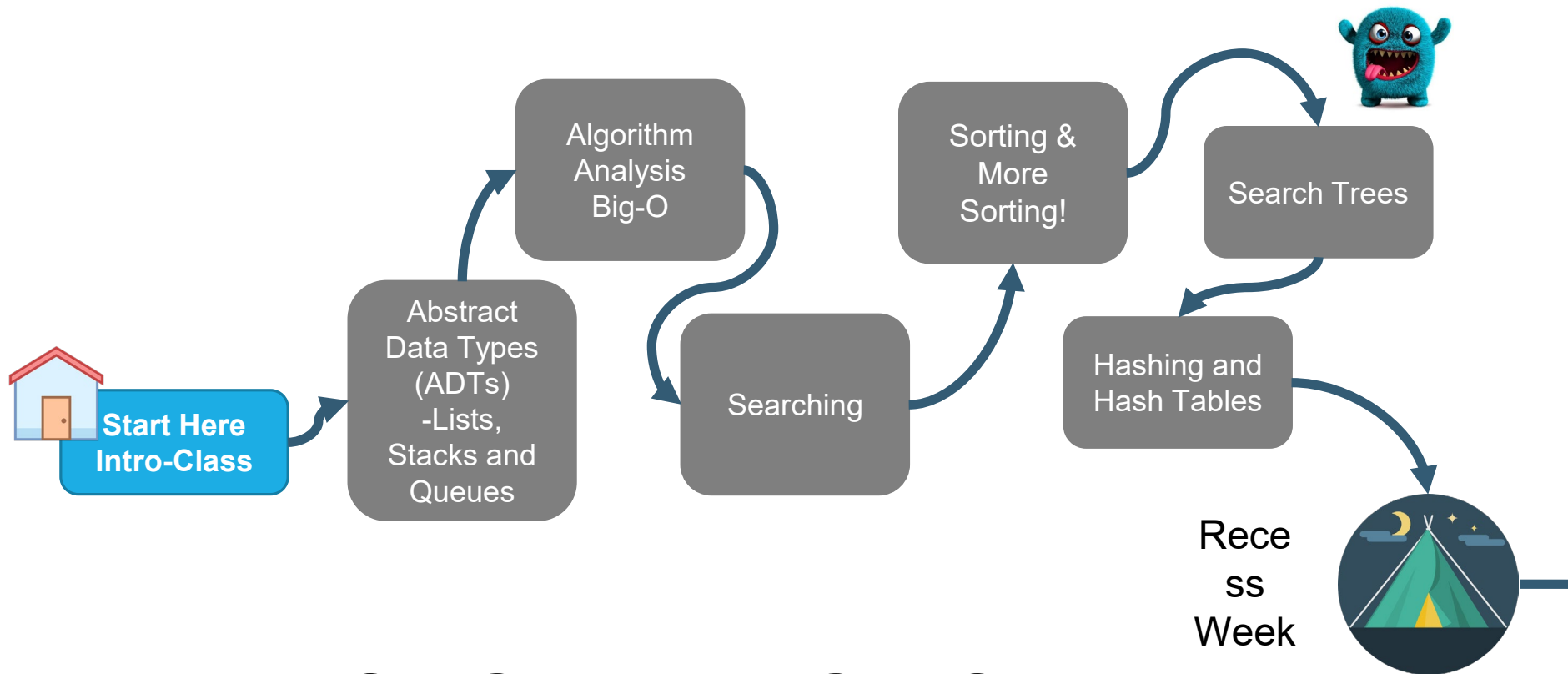
Web servers:  
Load balancing  
Scheduling  
Memory allocation

Database:  
B-trees  
Search  
Sorting



What is CS2040S about?

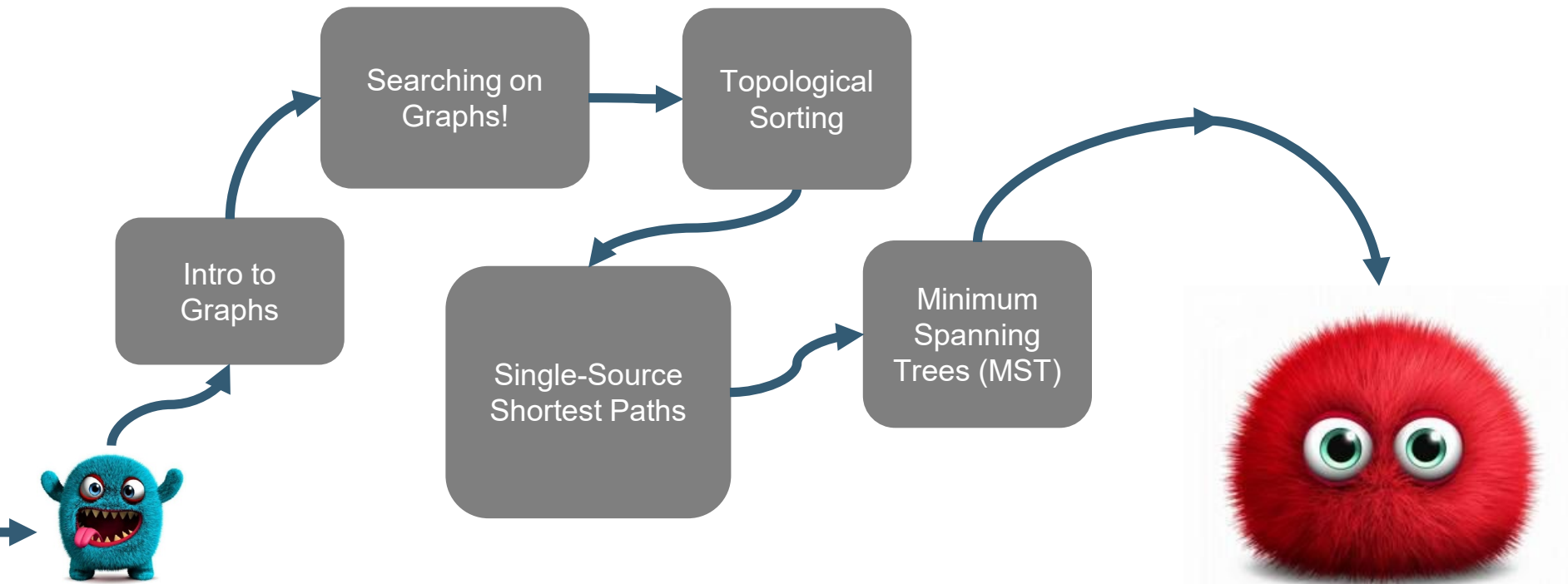
# COURSE STRUCTURE



## PART I: ORGANIZING YOUR DATA



# COURSE STRUCTURE



## PART II: MODELLING AND SOLVING PROBLEMS

# Data Structures

## Problem Solving

## Algorithms

# Desirable features of your algorithm:



How do you choose the right algorithm for the right problem?

How do you design new algorithms for new problems?

# FRAMEWORK: ALGORITHM & DATA STRUCTURE

What problem does it solve?

How does it work?

How to implement it?

What is its asymptotic performance?

What is its real world performance?

What are the trade-offs?



# GOALS

By the end of this course, you should be able to:

- **Apply algorithmic thinking and techniques** for solving computational problems.
- **Describe the structure and operation** of different **data structures and algorithms** under the standard computational model.
- **Assess the suitability** of different data-structures and algorithms for a specific computational problem.
- **Adapt existing data-structures and algorithms** to solve specific computational problems.

What comes next?

# What's next?

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Topic 1: Searching and Sorting

Topic 2: Trees

Topic 3: Hashing

Topic 4: Shortest Paths

Topic 5: Minimum Spanning Trees

# Topic 1: Searching and Sorting

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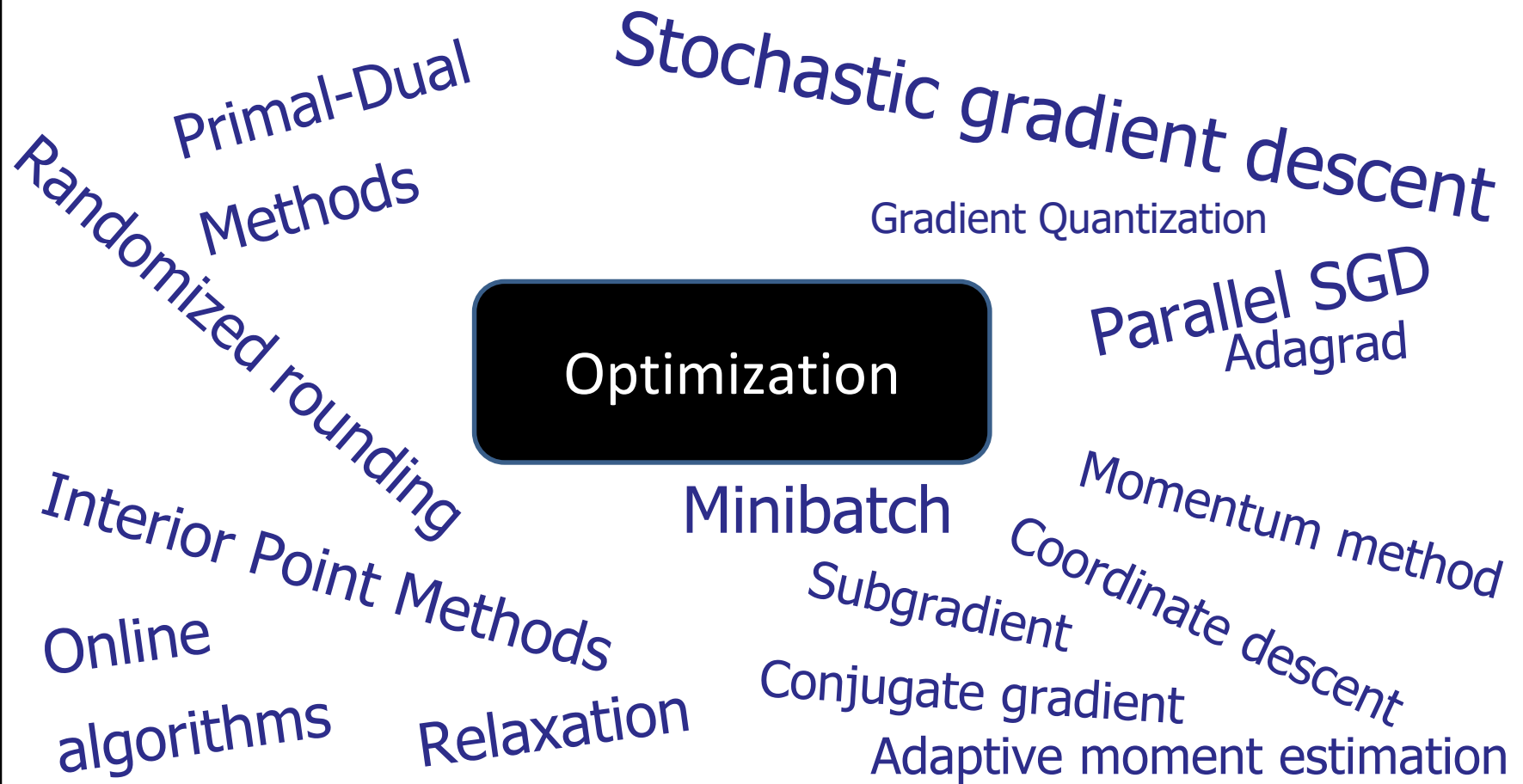
## Binary Search:

- Fast way to search monotonic data.
- Fast way to find max/min for convex data.

□ Optimization



Huge area of research and development:



Optimization algorithms:  
General techniques.

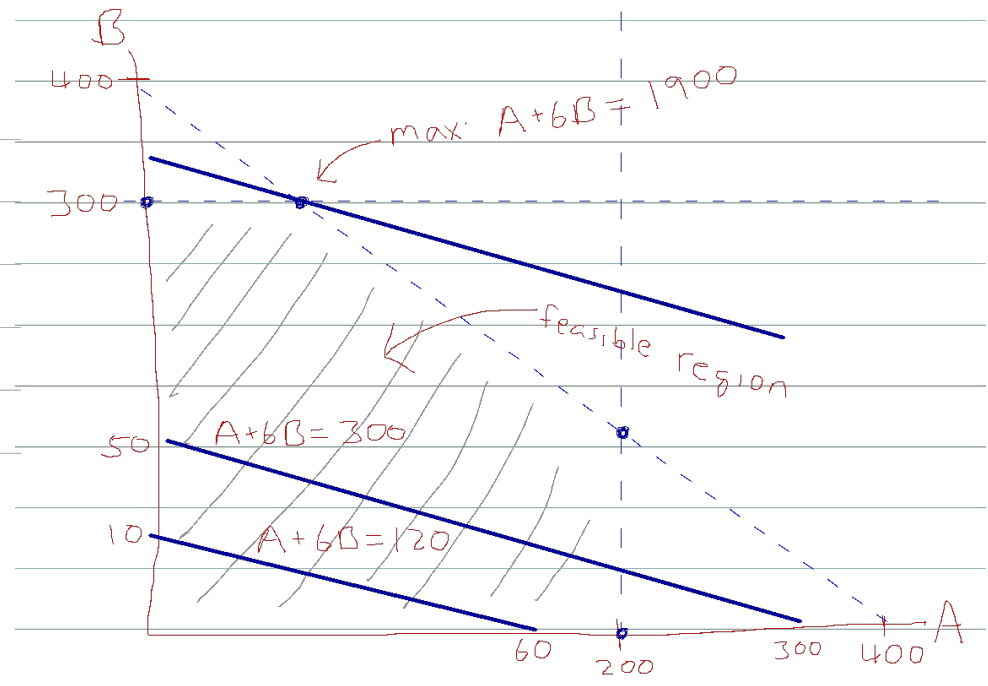
Machine learning:  
How to train a model?

# Optimization Algorithms

## Linear Programming:

- How to optimize a linear function subject to linear constraints.
- E.g., simplex method

$$\begin{aligned} &\max (A+6B) \\ &\text{where: } A \leq 200 \\ &\quad B \leq 300 \\ &\quad A+B \leq 400 \\ &\quad A \geq 0 \\ &\quad B \geq 0 \end{aligned}$$



# Optimization Algorithms

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## Linear Programming:

- How to optimize a linear function subject to linear constraints.
- E.g., simplex method

## Applications:

- Graph algorithms (e.g., max-flow)
- Approximation algorithms (e.g., weighted vertex cover, weighted set cover, multicommodity flow)
- And many, many real-world problems.

# Optimization Algorithms

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## Linear Programming:

- How to optimize a linear function subject to linear constraints.
- E.g., simplex method

## And more...

- Semidefinite programming
- Integer linear programming (NP-hard)
- Quadratic programming (NP-hard)
- Constraint satisfaction (NP-hard)

# Topic 1: Searching & Sorting

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## Fast Sorting Algorithms:

- QuickSort
- MergeSort
- HeapSort

## Key properties:

- Running time
- Space usage
- In-place

# Sorting Faster!

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Multi-pivot QuickSort...

# Sorting Faster!

---

## Yahoo TeraSort:

- Each node has:
    - 8 cores: 2GHz
    - 8 GB RAM
    - 4 disks: 4TB each
  - 40 nodes / rack (interconnect: 1GB/s switch)
  - 25-100 racks (interconnect: 8GB/s switch)
- ~ 16,000 cores

# Sorting Faster!

---

## Yahoo TeraSort:

- Each node has:
    - 8 cores: 2GHz
    - 8 GB RAM
    - 4 disks: 4TB each
  - 40 nodes / rack (interconnect: 1GB/s switch)
  - more racks (interconnect: 8GB/s switch)
- 50,400 cores

2013:

Yahoo (Hadoop) sorts  
100TB of data in 72  
minutes.



# Sorting Faster!

---

## DataBricks TeraSort:

- 206 nodes
- 6,592 cores

### Record (2014):

DataBricks (Spark) sorts  
100TB of data in 23  
minutes.

## DataBricks PetaSort:

- 190 nodes
- 6,080 cores

### Record (2014):

DataBricks (Spark) sorts  
1PB of data in 234  
minutes.

# Sorting Faster!

---

Tencent Sort:

- 512 nodes
- 5,024 cores

Record (2016):  
98.8 seconds

# Sorting Faster!

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It's a race:

- High performance clusters.
- Hardware interconnect.
- Parallel performance.

## Topic 2: Search Trees

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Many types of search trees:

- **AVL trees** and Red-Black trees
- **B-trees**
- **Skip Lists**
- **Tries**
- **Interval Trees, Order Statistics Trees**
- **Range Trees, kd-Trees**
- Splay trees

# Key tree-related questions:

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## High dimensional data:

- How should we store higher dimensional data?
- How should we index higher dimensional data?
- How should we search higher dimensional data?

## Examples:

R-trees, spatial indices, quadtrees, Z-order curve, ...

# Key tree-related questions:

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Performance?

# Predicting Performance

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Example: 100 TB of data

1) Store data sorted in an array

- ⇒ Scan all the data:  $O(n)$
- ⇒ (Binary) search:  $O(\log n)$

2) Store data in a linked list

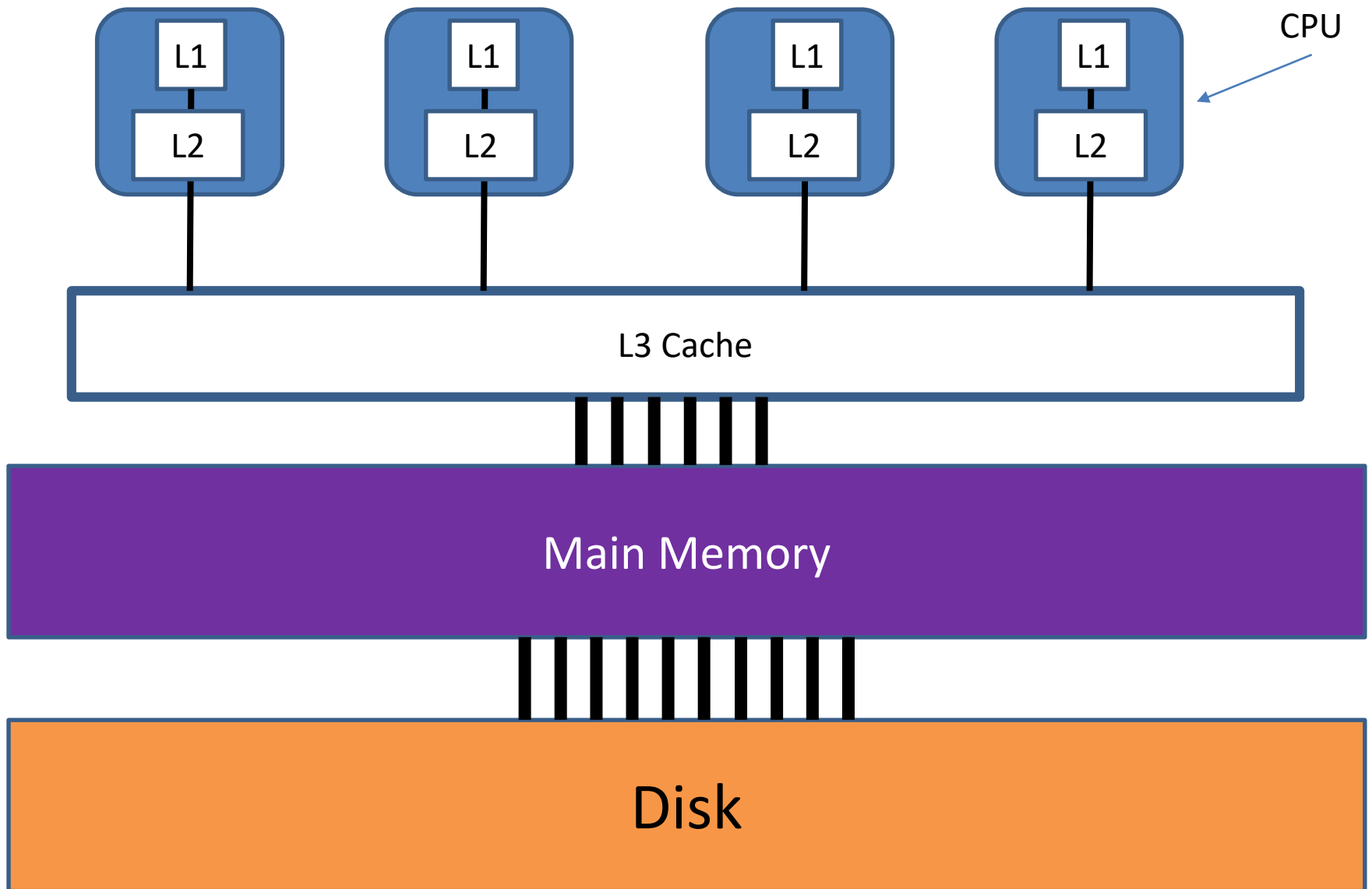
- ⇒ Scan all the data:  $O(n)$
- ⇒ Search:  $O(n)$

3) Store data in a red-black tree

- ⇒ Scan all the data:  $O(n)$
- ⇒ Search:  $O(\log n)$

Analysis is not predicting performance very well!

# A Real Computer (?)





# Disks

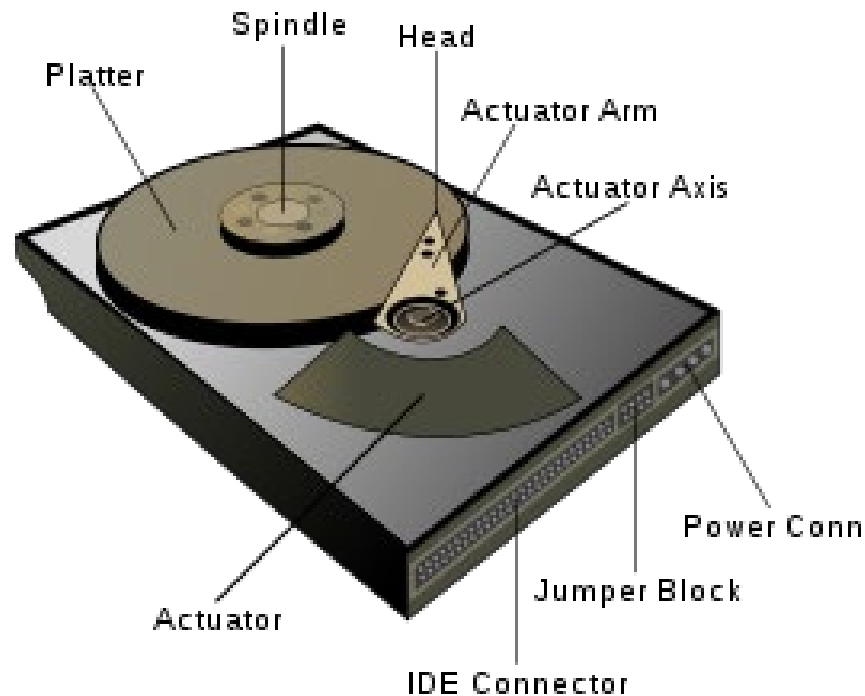
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Where is most data stored? **Hard disk!**

- Magnetic
- Mechanical
- Slow (6000rpm = 10ms)

Two step access:

1. *seek (find right track)*
2. *read track*

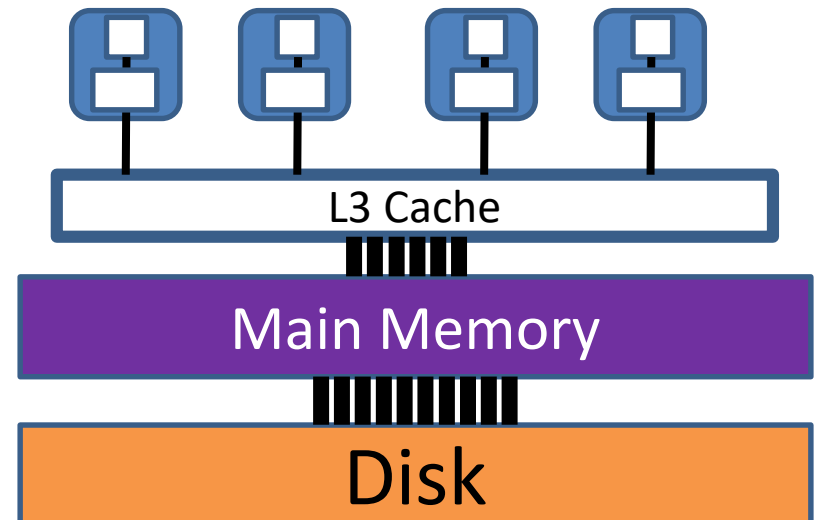


# Haswell Architecture (2-18 cores)

Memory Type	size	line size	clock cycles
L1 cache	64 KB	64 B	~4
L2 cache	256 KB	64 B	~10
L3 cache	2-40 MB	64 B	40-74
L4 (optional)	128 MB		
Main Memory	< 128 GB	16 KB	~200-350
SSD Disk	BIG	Variable (e.g., 16KB)	~20,000
Disk	BIGGER	Variable (e.g., 16KB)	~20,000,000

## Notes:

- Several other "caches" e.g., TLB, micro-op cache, instruction cache, etc.
- L1/L2 caches are per core.
- L3/L4 cache are shared per socket.
- Main memory shared cross socket.



# Haswell Architecture

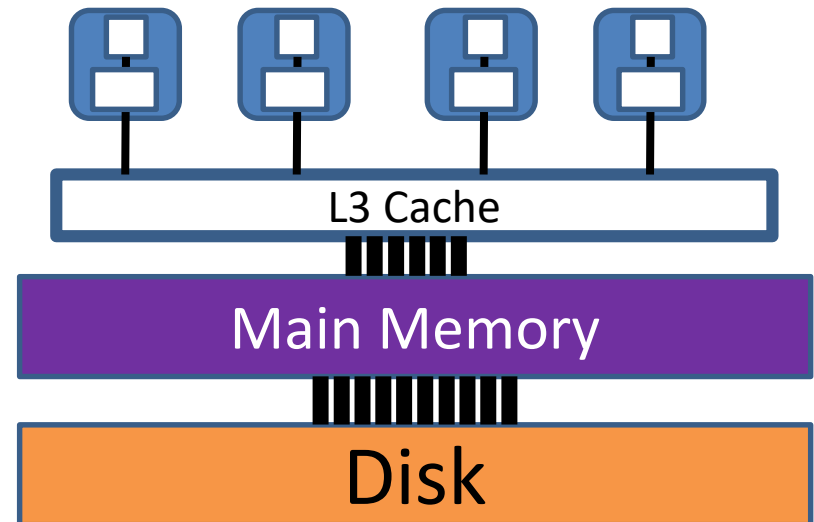
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A simple example calculation:

What fraction of operations "hit" each cache?

- ⇒ 90% L1 hit rate (4 cycles)
- ⇒ 8% L2 hit rate (10 cycles)
- ⇒ 2% main memory (300 cycles)

← *Just an example..*



# Haswell Architecture

## A simple example calculation:

What fraction of operations "hit" each cache?

- ⇒ 90% L1 hit rate (4 cycles)
- ⇒ 8% L2 hit rate (10 cycles)
- ⇒ 2% main memory (300 cycles)

← Just an example..

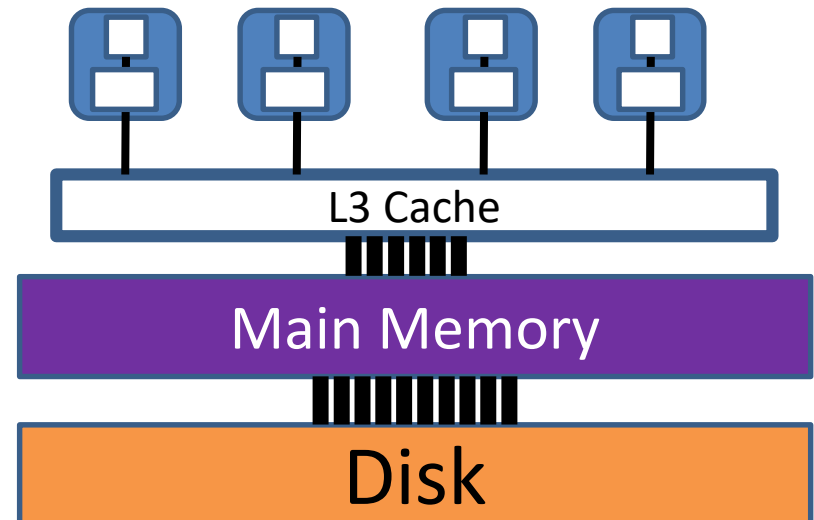
What fraction of time for each cache?

- ⇒ 35% waiting for L1
- ⇒ 8% waiting for L2
- ⇒ 57% waiting for main memory

Conclusion:

98% cache hit ?

57% waiting on main memory



# Haswell Architecture

## A simple example calculation:

What fraction of operations "hit" each cache?

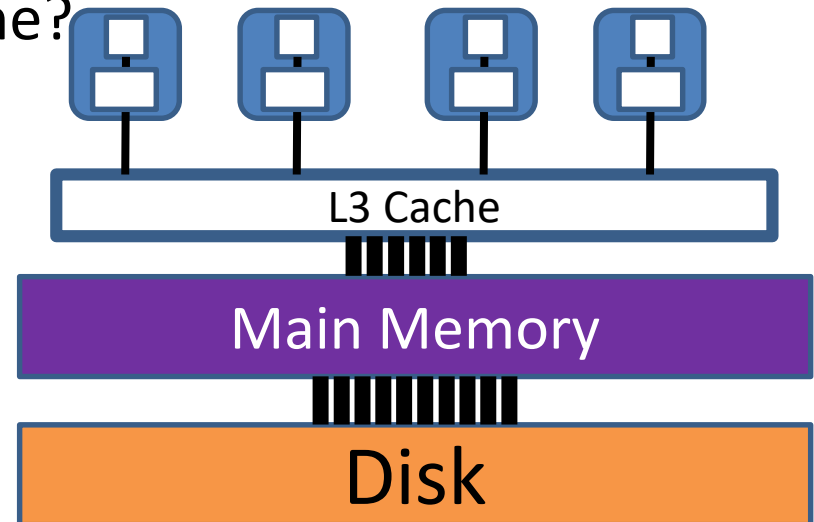
- ⇒ 90% L1 hit rate (4 cycles)
- ⇒ 8% L2 hit rate (10 cycles)
- ⇒ 1.8% main memory (300 cycles)
- ⇒ 0.2% disk (20,000,000 cycles)

← Just an example..

What fraction of time for each cache?

- ⇒ 99.98% waiting for disk

Disk is much, much worse!



# Where is the bottleneck?

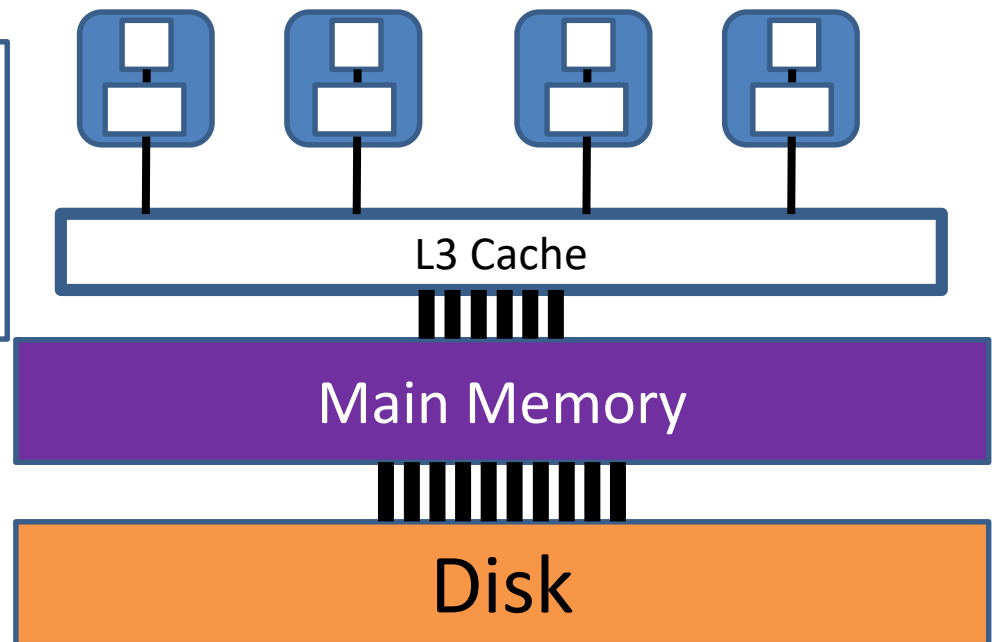
The bottleneck depends on the application:

- Small working set data lives in L1/L2 cache ☐ **fast**.
- Medium working set data lives in main memory ☐ **bottleneck is memory latency**.
- Big data lives on disk ☐ **bottleneck is disk latency / bandwidth**.

For most applications,  
one level dominates the  
cost.

*(Costs grow fast!*

*Largest level dominates.)*



# B-trees

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## Basic facts

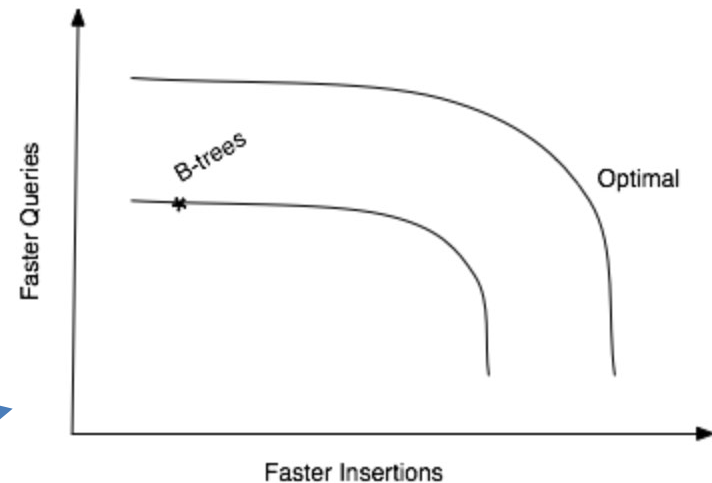
- One of the most important data structures out there today. (Variants used in all major databases.)
- Very fast. (Not just asymptotic analysis, but in practice nearly impossible to beat a well-implemented B-tree.)
- Benefit comes both from good cache performance, low overhead, good parallelization, etc.

# Faster Trees?

Goal:

A external memory data structure with fast searches, and super-fast insertions/deletions.

“Write-optimized data structure.”



Percona advertisement graph  
(not technical)

*B-trees are not on the optimal insert/query tradeoff curve*



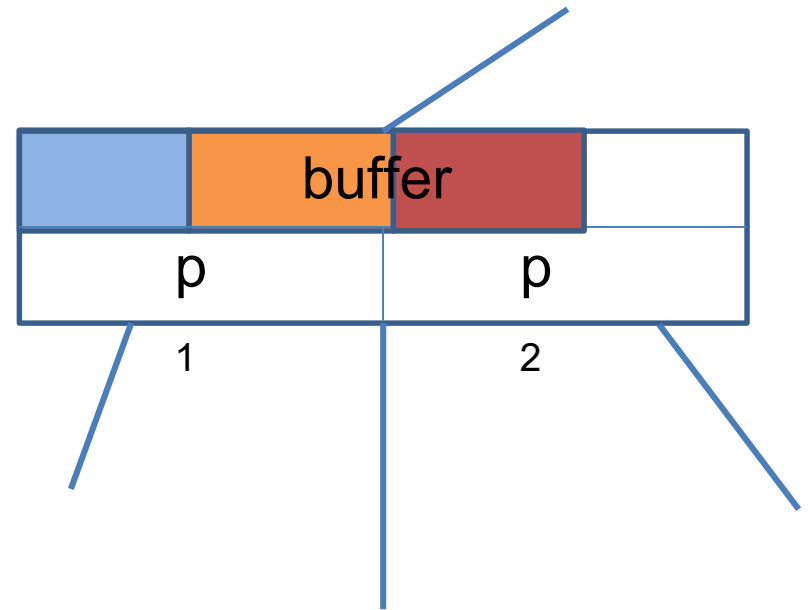
# Buffer Tree

## Summary

Cost of operations:

insert/delete:  $O\left(\frac{1}{B} \log n\right)$

search:  $O(\log n)$



# Faster Trees?

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Goal:

A external memory data structure with fast searches, and super-fast insertions/deletions.

“Write-optimized data structure.”

Examples:

- LSM: log-structured merge trees
- COLA: cost-oblivious lookahead array

Hot area of DBS research today...

See:  
**BetRFs**

# Topic 3: Hash Tables

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Two key types of hash tables:

- Chaining
- Open Addressing

# Better Collision Resolution

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## Chaining:

- $O(1)$  *expected* search
- $O(1)$  *worst-case* insertion

## Cuckoo Hashing:

← Neat, newer hashing method!

- $O(1)$  *worst-case* search
- $O(1)$  *expected* insertion

# More realistic hash functions

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How well does linear probing really work?

Does open addressing work with realistic hash functions?

- Example: limited independence hash functions
- Example: tabular hashing

YES: it works really well!

# Better hash functions

---

## Faster hash functions:

- Example: xxHash, etc.
- Example: tabular hashing

*Fastest today??  
For GPUs??*

## Cryptographic hash functions:

- Example: MD6,
- Example: SHA-512

*Are these secure?*

# Topic 3: Hash Tables

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Use case of a hash table: maintain a set of items

Hash Sets / Filters:

- Fingerprint Hash Tables
- Bloom Filters

# Better filters

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## Quotient Filters:

- Optimal trade-off error vs. space.
- Practical and easy to implement

## Cuckoo Filters:

- Interesting alternative to a Bloom Filter

Optimizations: Bloomier filters, compact approximators...

Key question: minimize space, minimize error



# Better filters

## Learned Bloom Filters:

- Can we use machine learning to train better Bloom Filters?

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### A Model for Learned Bloom Filters, and Optimizing by Sandwiching

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Michael Mitzenmacher  
School of Engineering and Applied Sciences  
Harvard University  
michaelm@eecs.harvard.edu

#### Abstract

Recent work has suggested enhancing Bloom filters by using a pre-filter, based on applying machine learning to determine a function that models the data set the Bloom filter is meant to represent. Here we model such *learned Bloom filters*, with the following outcomes: (1) we clarify what guarantees can and cannot be associated with such a structure; (2) we show how to estimate what size the learning function must obtain in order to obtain improved performance; (3) we provide a simple method, sandwiching, for optimizing learned Bloom filters; and (4) we propose a design and analysis approach for a learned Bloomier filter, based on our modeling approach.

- Can we get a better trade-off between common-case and worst-case performance?

# Applications of Filters

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## Caches:

- What is in your cache?
- Check bloom filters before accessing cache.

## Learned Bloom Filters:

- Use machine learning to choose most useful items to store in filter.
- Then we need less space!

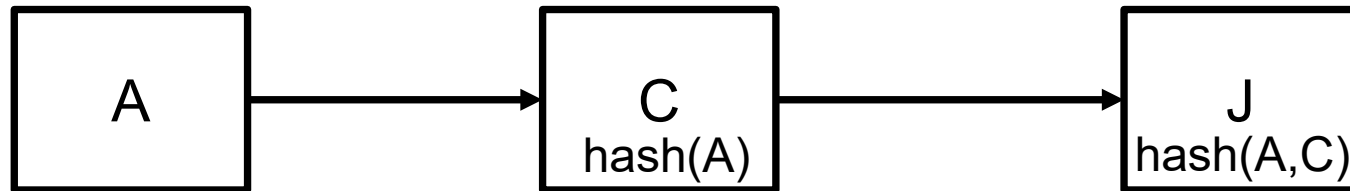
See: <https://papers.nips.cc/paper/7328-a-model-for-learned-bloom-filters-and-optimizing-by-sandwiching.pdf>

# Blockchains

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Blockchain = Hashchain

- proof-of-work == inverting hash function
- hash summarizes chain (see: Merkle trees!)



# Topic 4: Shortest Paths

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Several situations:

- Unweighted graphs
- Directed acyclic graphs
- Graphs with negative weights
- Graph with positive weights
- All-pairs shortest paths

# Topic 4: Shortest Paths

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Key algorithms:

- BFS
- Topological sort
- Bellman-Ford
- Dijkstra
- Floyd-Warshall

# Topic 5: Minimum Spanning Trees

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Key algorithms:

- Prim's
- Kruskal's

Sub-components:

- Union-Find
- Priority Queues

# Key Questions

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Big Data

Parallelism

Distributed Network Applications

# Parallel Algorithms

## Moore's Law

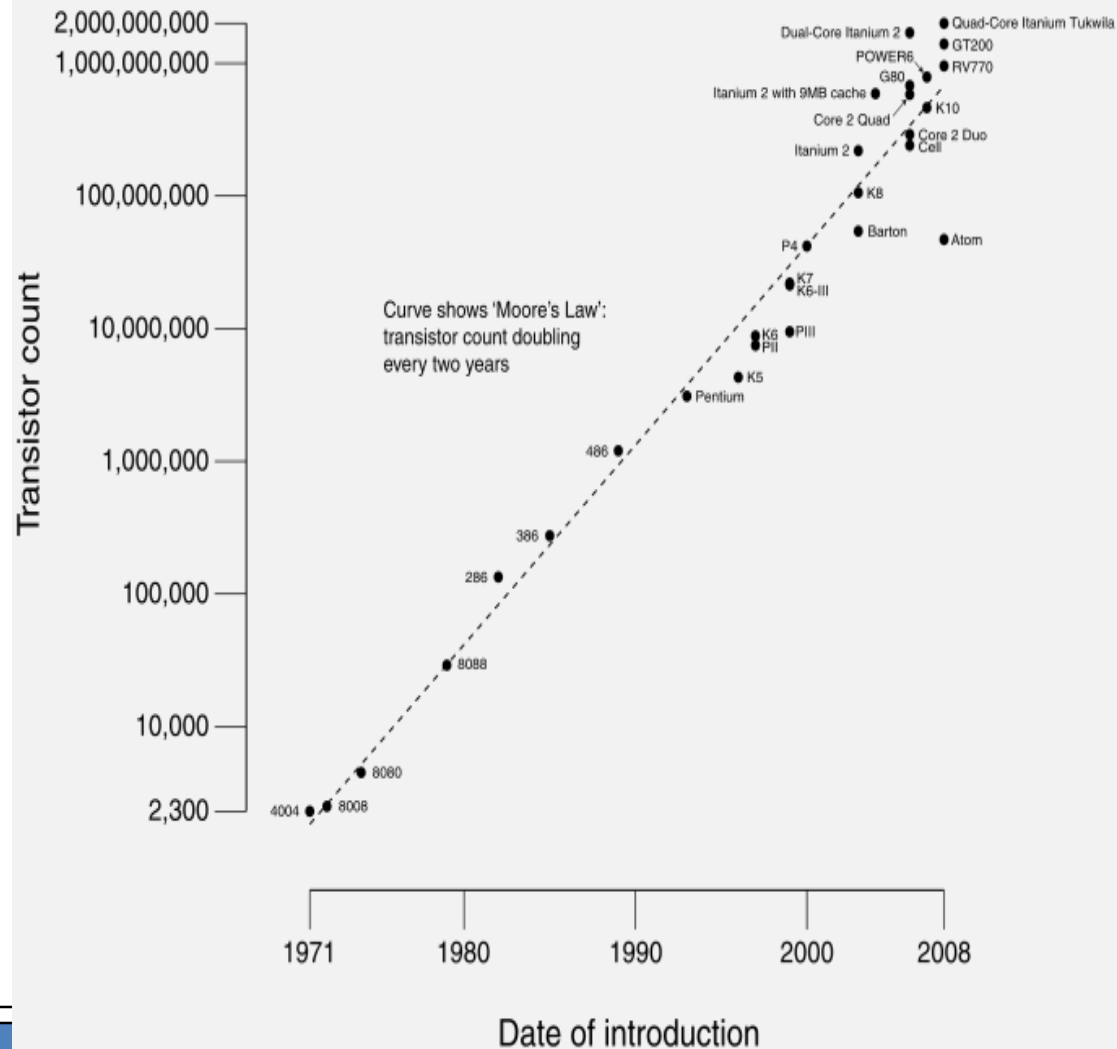
Number of transistors  
doubles every 2 years!

“The complexity for minimum component costs has increased at a rate of roughly a factor of two per year... Certainly over the short term this rate can be expected to continue, if not to increase.” Gordon Moore, 1965

Limits will be reached  
in 10-20 years...maybe.

Source: Wikipedia

CPU Transistor Counts 1971-2008 & Moore's Law





# Parallel Algorithms

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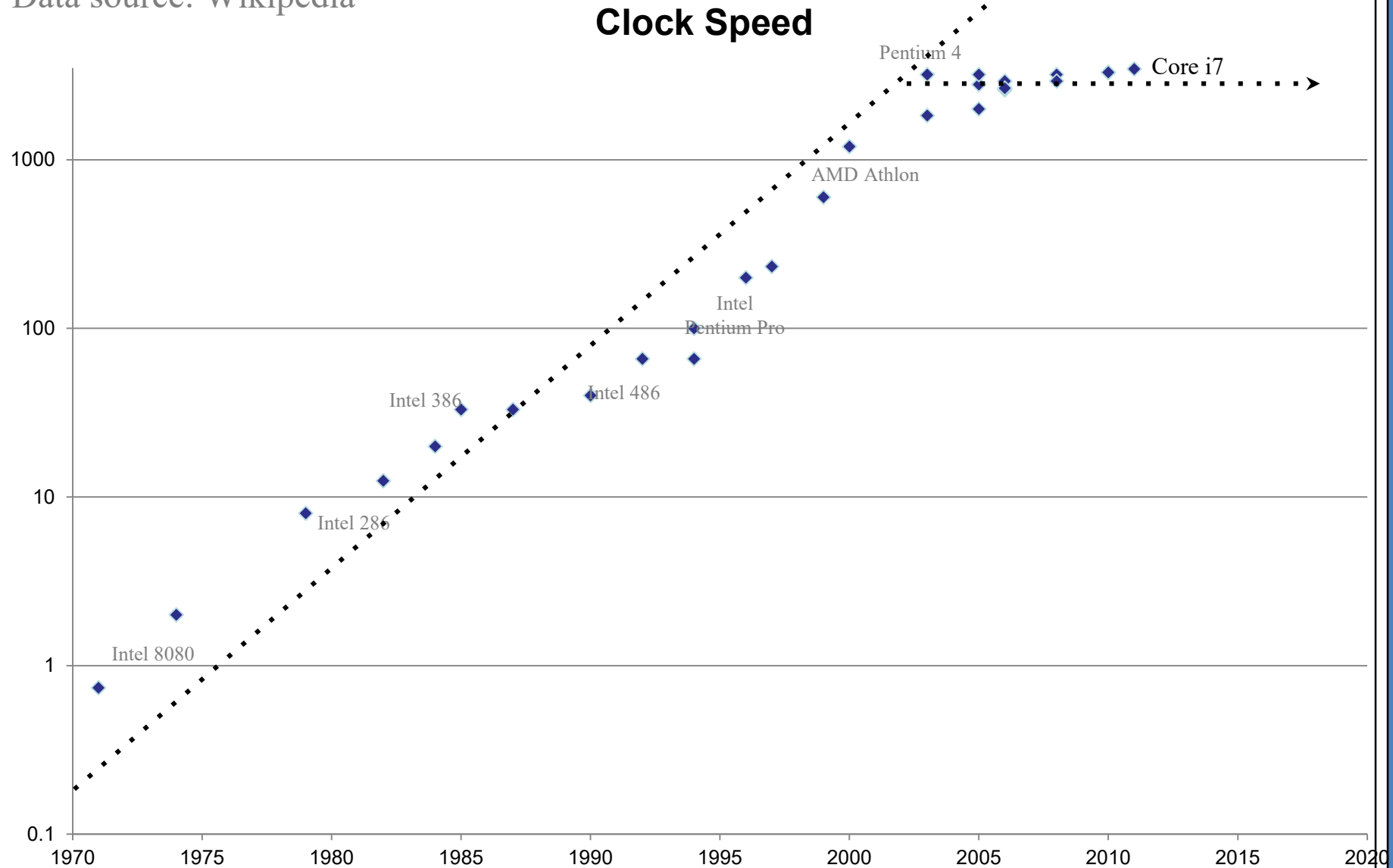
## More transistors == faster computers?

- More transistors per chip  $\square$  smaller transistors.
- Smaller transistors  $\square$  faster
- Conclusion:

Clock speed doubles every two years, also.

# Parallel Algorithms

Data source: Wikipedia



# Parallel Algorithms

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What to do with more transistors?

- More functionality
  - GPUs, FPUs, specialized crypto hardware, etc.
- Deeper pipelines
- More clever instruction issue (out-of-order issue, scoreboarding, etc.)
- More on chip memory (cache)

Limits for making faster processors?

# Parallel Algorithms

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## Problems with faster clock speeds:

- Heat
  - Faster switching creates more heat.
- Wires
  - Adding more components takes more wires to connect.
  - Wires don't scale well!
- Clock synchronization
  - How do you keep the entire chip synchronized?
  - If the clock is too fast, then the time it takes to propagate a clock signal from one edge to the other matters!

# Parallel Algorithms

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## Conclusion:

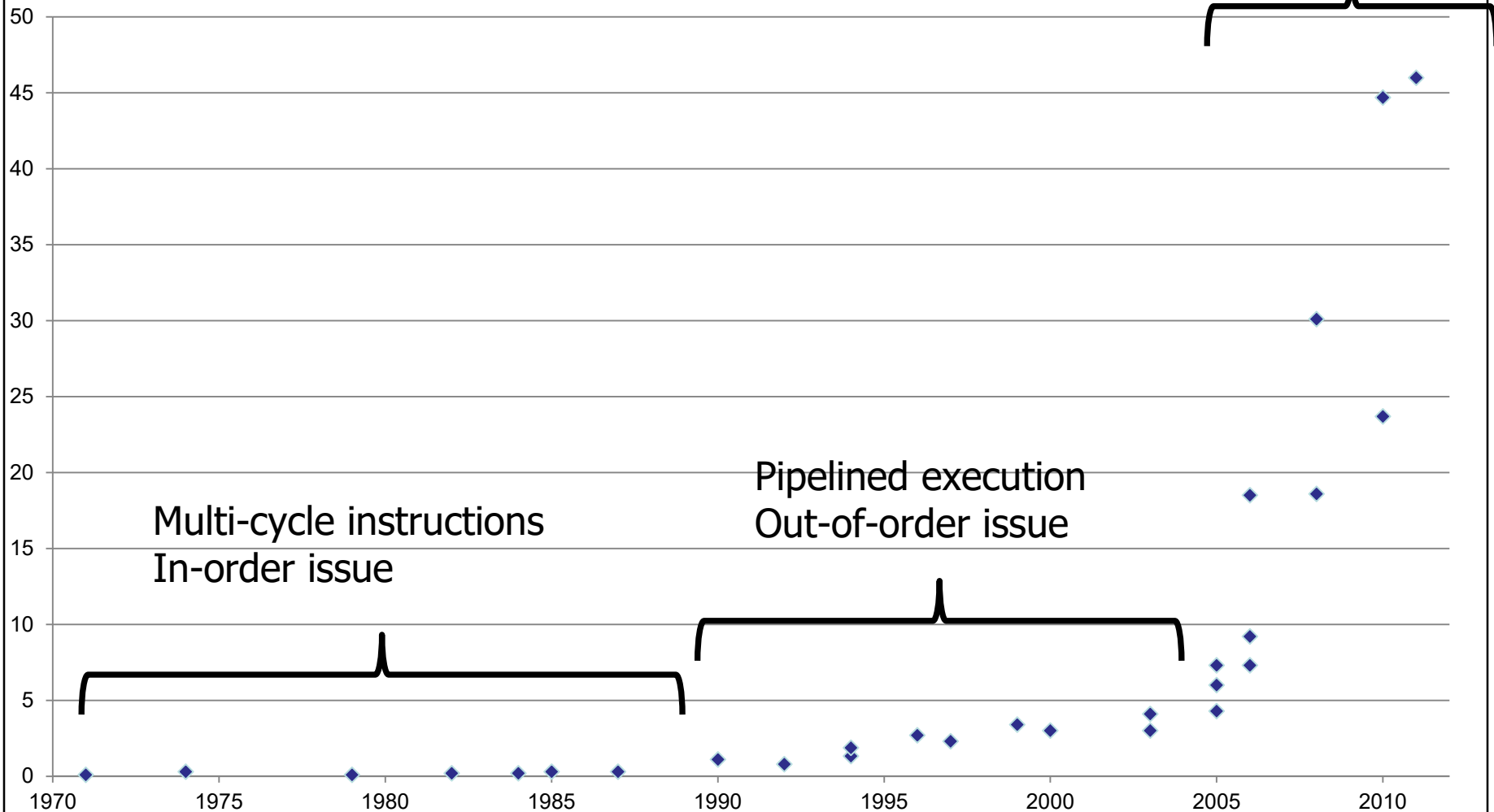
- We have lots of new transistors to use.
- We can't use them to make the CPU faster.

What do we do?

# Parallel Algorithms

Data source: Wikipedia

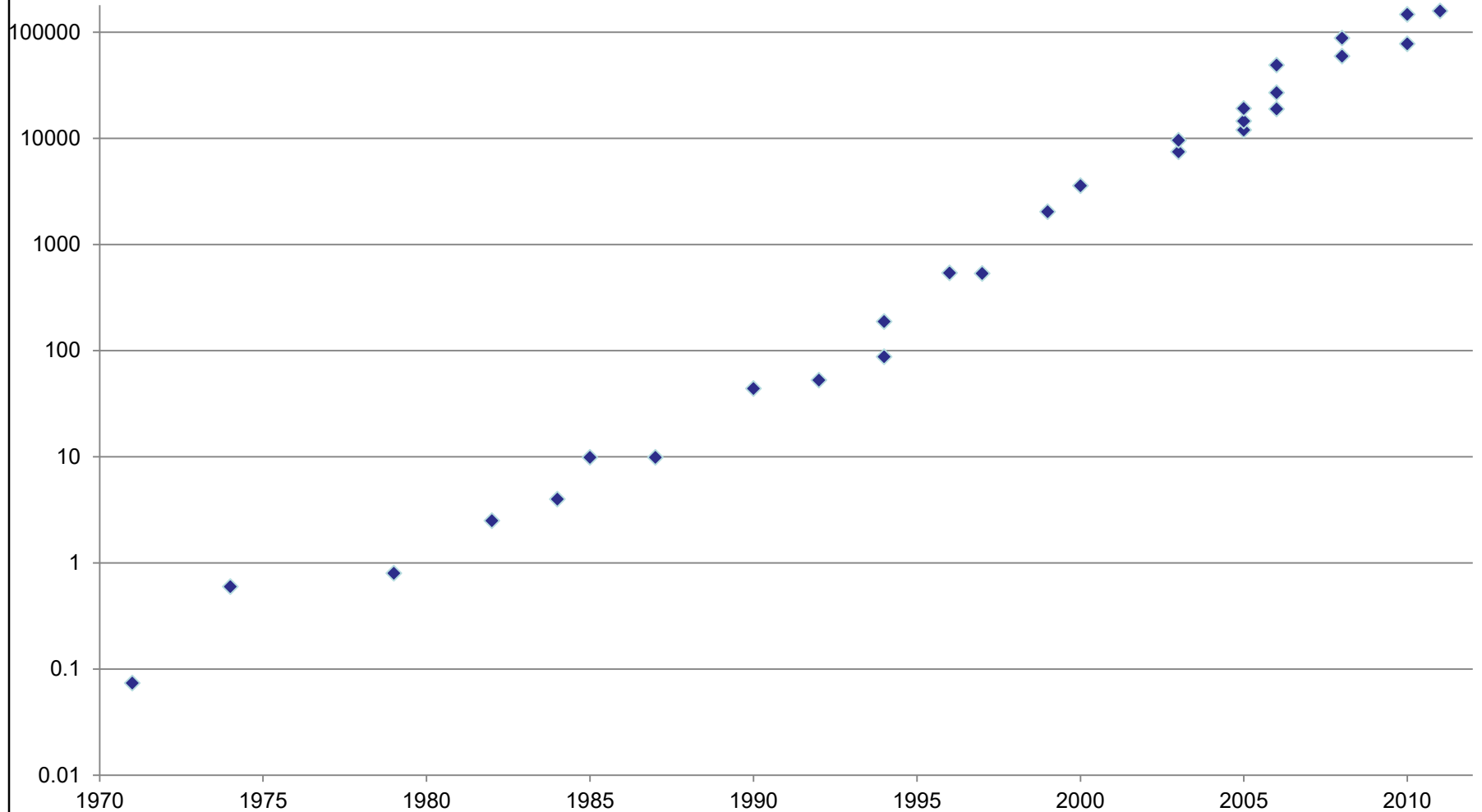
## Instructions per Clock Cycle



# Parallel Algorithms

Data source: Wikipedia

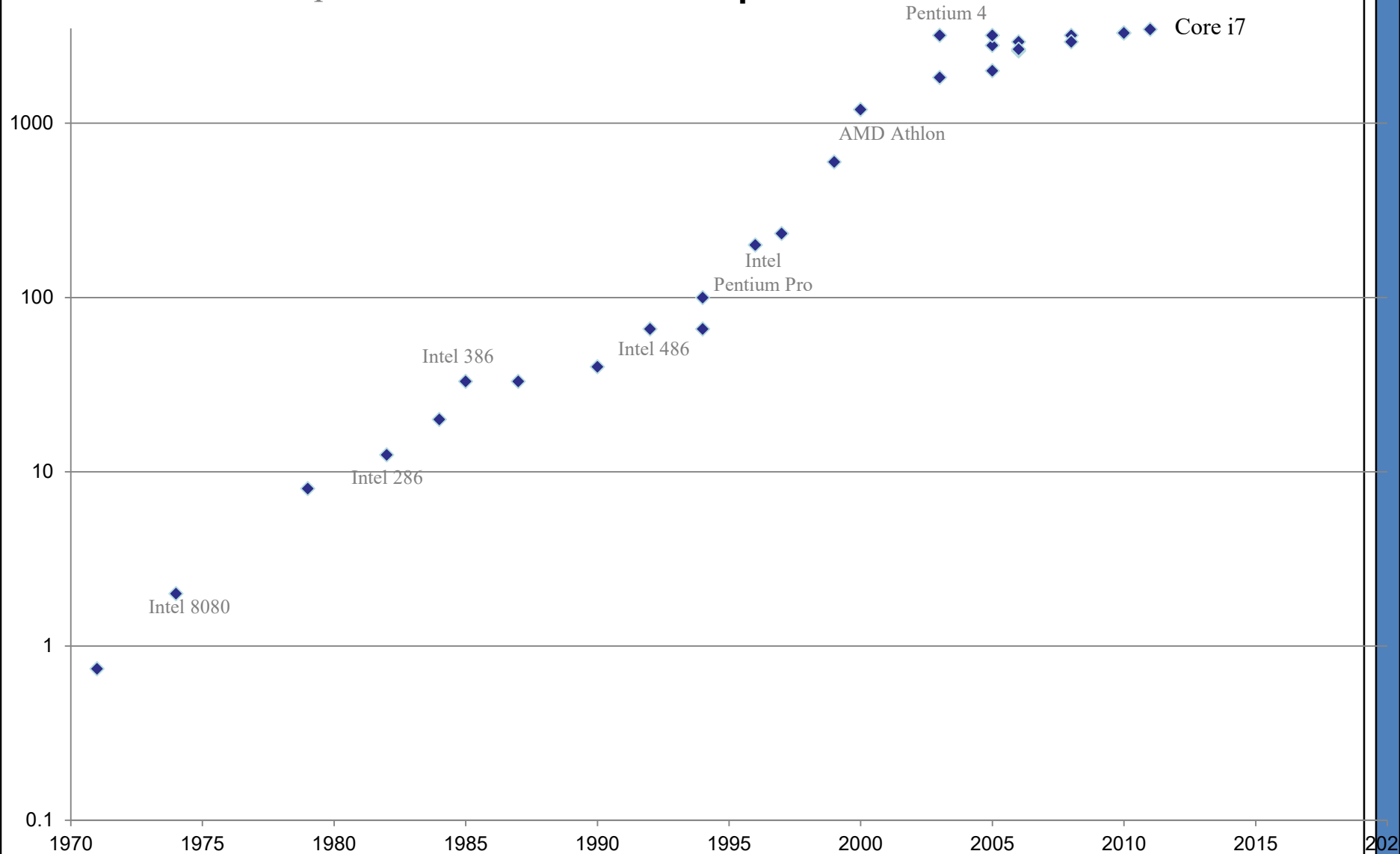
## Instructions per Second



# Parallel Algorithms

Data source: Wikipedia

## Clock Speed





# Parallel Algorithms

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To make an algorithm run faster:

- Must take advantage of multiple cores.
- Many steps executed at the same time!

# Parallel Algorithms

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## Different types of parallelism:

- multicore
  - on-chip parallelism: synchronized, shared caches, etc.
- multisocket
  - closely coupled, highly synchronized, shared caches
- cluster / data center
  - connected by a high-performance interconnect
- distributed networks
  - slower interconnect, less tightly synchronized

# Parallel Algorithms

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## Map-Reduce:

- Target: high-performance clusters
- Focus: data (not computation)

## Inventor: Google

- processing web data

Today: ubiquitous (Amazon, Yahoo, Facebook, etc,.)

- Hadoop, etc.

# Map-Reduce Model

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## Basic round:

1. **Map**: process each (key, value) pair
2. **Shuffle**: group items by key
3. **Reduce**: process items with same key together

## Plan:

Load data from disk.

Execute several rounds.

Save (key, value) pairs, sorted by key.

# Parallel Algorithms

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A huge amount of ongoing research on how to process big graphs fast in parallel...

A huge amount of ongoing research on how to efficiently find shortest paths, spanners, MSTs, and more in a network...

# What's next?

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Topic 1: Searching and Sorting

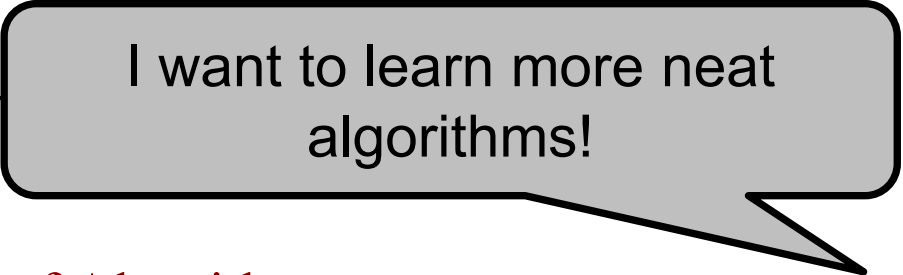
Topic 2: Trees

Topic 3: Hashing

Topic 4: Shortest Paths

Topic 5: Minimum Spanning Trees

# What next?



I want to learn more neat algorithms!

Algorithms modules:

- CS3230: Design and Analysis of Algorithms
- CS3233: Competitive Programming
- CS4231: Parallel and Distributed Algorithms
- CS4234: Optimization Algorithms
- CS4261 Algorithmic Mechanism Design
- CS4268 Quantum Computing
- CS5234: Algorithms at Scale
- CS5330: Randomized Algorithms

# What next?

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## Theory:

- CS4232: Theory of Computation
- CS3234: Logic for Proofs and Programs
- CS4269 Fundamentals of Logic in Computer Science
- CS5230: Computational Complexity

Can we rank how hard different types of problems are?

What does it really mean when we say a problem is **NP-hard**?

How do you show that a problem is hard?



# What next?



I want to build cool systems!

Software engineering modules:

- CS2103: Software engineering
- CS4211: Formal methods for software engineering
- CS4218: Software testing and debugging

System design and programming modules:

- CS3216: Software development on evolving platforms
- CS3217: Software engineering on modern application platforms

# What next?

I want to build an AGI that will take over the world and subjugate the human race!

## Machine Learning and AI:

- **CS2109: Intro to Machine Learning and AI**
- CS4243 Computer Vision and Pattern Recognition
- CS4244 Knowledge Representation and Reasoning
- CS4246 AI Planning and Decision Making
- CS4248 Natural Language Processing

# What next?



Wait, there's more?

Specialized modules:

- Distributed Systems
- Computer Security
- Game Design
- Computer Graphics
- Computational Biology
- Wireless computing and sensor networks
- Etc...

# What next?

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## Research:

- UROP (Undergraduate Research Opportunities)
- Turing Programme
- FYP

# Algorithm Design Tips

8

## Small examples

- At least 5 items/nodes.
- Avoid special cases.
- Understand why problem is hard.
- Look for useful structure.

7

## Naïve solutions:

- Simple algorithms are useful.
- Baseline: complicated algorithm should be better than the simple one!
- De-optimize your algorithm.

6

## Special cases:

- Try easier versions of the problem.
- Trees? Planar graphs?
- Do those cases generalize?

5

## Best possible solution?

- What is your target.
- Look for natural “lower bounds.”
- How good is good enough?

4

## Metrics:

- How to measure goodness of your solution?
- What do you want?
- Can you get better performance by looking at a different metric?

3

## Invariants

- What is always true?
- What does the problem require?
- What does the algorithm guarantee?

2

## Binary search:

- Many, many algorithms can be optimized by using binary search.

1

## Divide-and-conquer:

- Look for subproblems.
- Look for structure.
- Wishful thinking: if only we could solve X, then we could...

# The End

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## Goals:

- ☐ Apply algorithmic thinking and techniques.
- ☐ Describe the structure and operation of different data structures.
- ☐ Assess the suitability of different data-structures and algorithms.
- ☐ Adapt existing data-structures and algorithms.

# The End

---

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- ☐ Apply algorithmic thinking and techniques.
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I hope everyone has had fun...

# The End

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I hope everyone has had fun...

If you ever want to chat about algorithms, just drop by our offices...



## A little bit of my personal opinion

- Yes, grades are important, but remember:  
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- You might take longer to learn, so long  
as you keep doing so even after the  
semester has ended.

Some of your TAs are still learning new things about 2040S this semester.

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- “After this how do I get better on algorithms on my own?”
    - Solve problems! Actively Google for puzzles. Solve them yourself.
- E.g. sources: Kattis, Codeforces, Leetcode, MIT/CMU/Stanford tutorial pdfs or exam papers.

# The End

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## Goals:

- ☐ Apply algorithmic thinking and techniques.
- ☐ Describe the structure and operation of different data structures.
- ☐ Assess the suitability of different data-structures and algorithms.
- ☐ Adapt existing data-structures and algorithms.

I hope everyone has had fun...

If you ever want to chat about algorithms, just drop by our offices...

And it's over... congratulations!