

CS2100

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COMPUTER ORGANISATION

Lecture #23

Cache

Part II: Set/Fully Associative Cache



NUS
National University
of Singapore

School of
Computing



Questions?

Ask at

<https://sets.netlify.app/module/676ca3a07d7f5ffc1741dc65>

OR

Scan and ask your questions here!
(May be obscured in some slides)



Lecture #23:

Cache II: Set/Fully Associative Cache

1. Types of Cache Misses
2. Block Size Trade-off
3. Set Associative Cache
4. Fully Associative Cache
5. Block Replacement Policy
6. Additional Examples
7. Summary
8. Exploration



1. (Recall) Types of Cache Misses

- **Compulsory misses**
 - On the first access to a block; the block must be brought into the cache
 - Also called **cold start misses** or **first reference misses**
- **Conflict misses**
 - Occur in the case of direct mapped cache or set associative cache, when several blocks are mapped to the same block/set
 - Also called **collision misses** or **interference misses**
- **Capacity misses**
 - Occur when blocks are discarded from cache as cache cannot contain all blocks needed

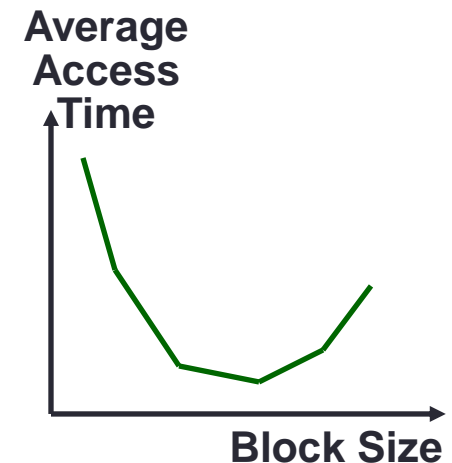
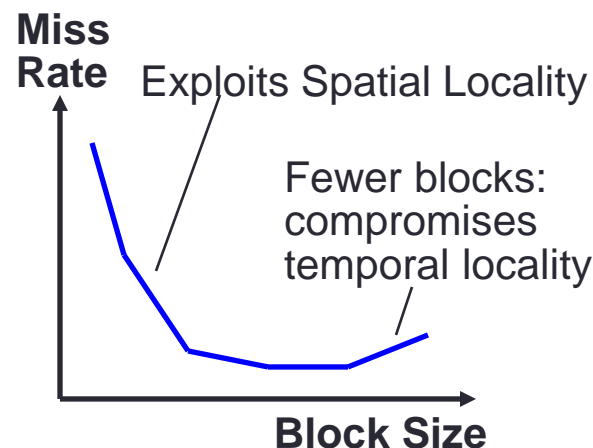
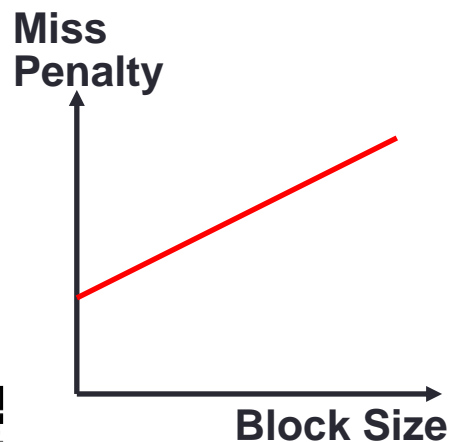


2. Block Size Trade-off (1/2)

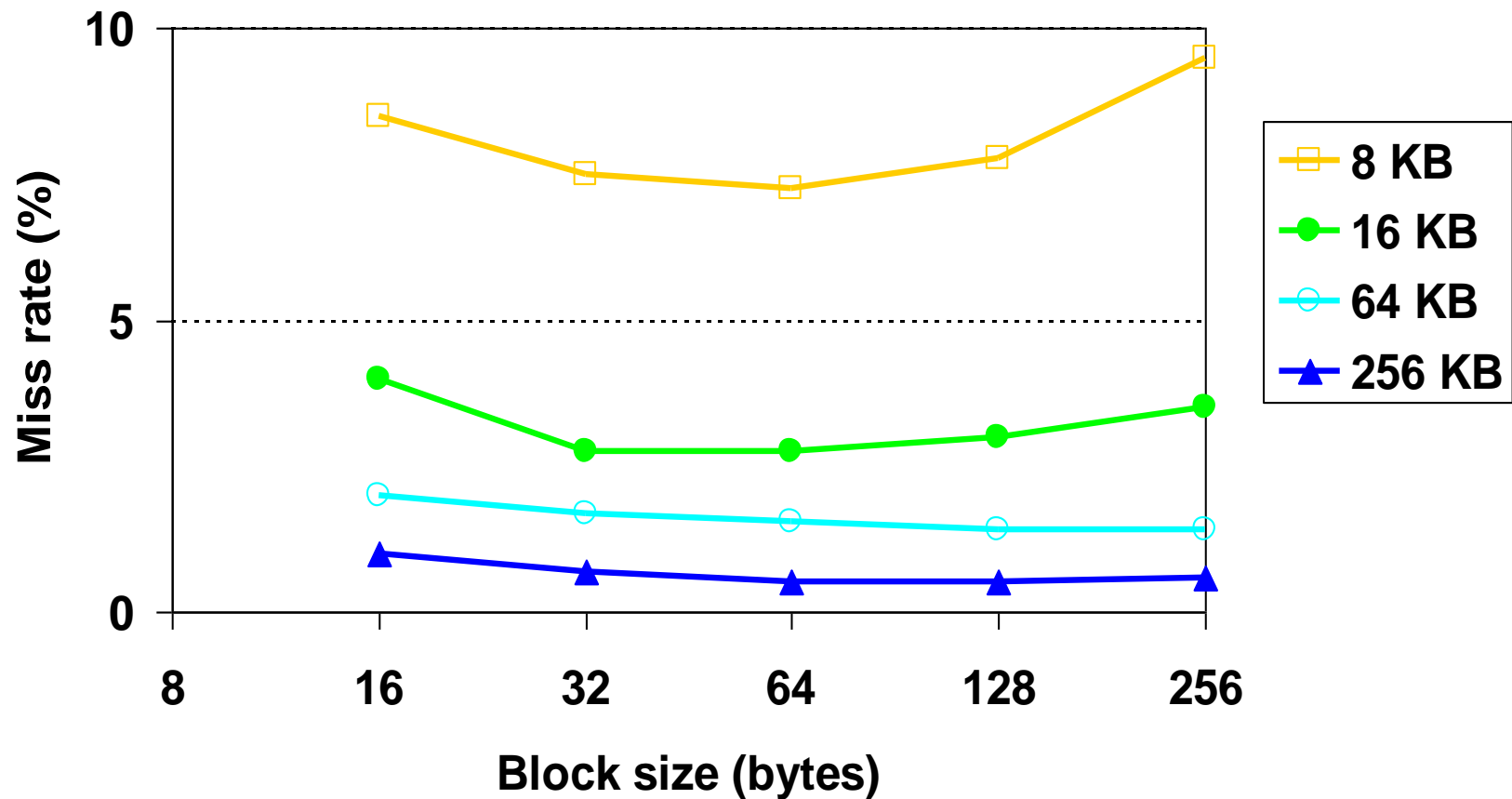
Average Access Time

$$= \text{Hit rate} \times \text{Hit Time} + (1 - \text{Hit rate}) \times \text{Miss penalty}$$

- Larger block size:
 - + Takes advantage of spatial locality
 - Larger miss penalty: Takes longer time to fill up the block
 - If block size is too big relative to cache size
 - Too few cache blocks → miss rate will go up



2. Block Size Trade-off (2/2)



3. Set Associative (SA) Cache

- Compulsory misses

- On the first access to a block, it must be brought into the cache
- Also called cold start misses

Solution:
Set Associative Cache

- Conflict misses

- Occur in the case of direct mapped cache or set associative cache, when several blocks are mapped to the same block/set
- Also called collision misses or interference misses

- Capacity misses

- Occur when blocks are discarded from cache as cache cannot contain all blocks needed



3. Set Associative Cache: Analogy



Many book titles start with “T”

→ Too many conflicts!

Hmm... how about we give more slots per letter, 2 books start with “A”, 2 books start with “B”, etc?



3. Set Associative (SA) Cache

- **N-way Set Associative Cache**

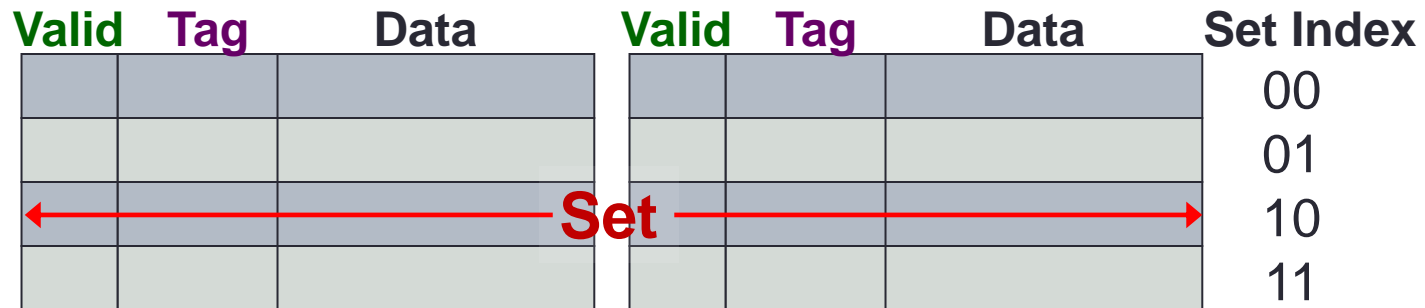
- A memory block can be placed in a fixed number (**N**) of locations in the cache, where **N** > 1

- **Key Idea:**

- Cache consists of a number of sets:
 - Each **set contains N cache blocks**
- Each memory block maps to a unique cache set
- Within the set, a memory block can be placed in **any** of the **N** cache blocks in the set



3. Set Associative Cache: Structure

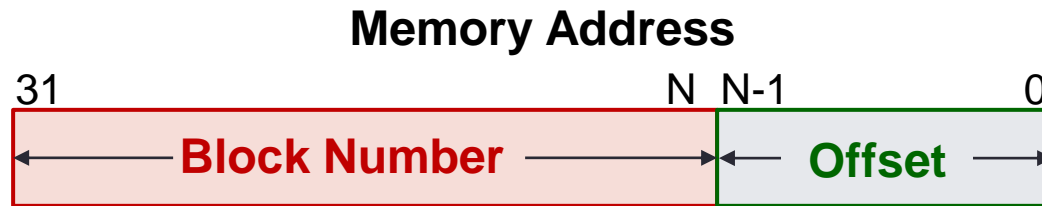


2-way Set Associative Cache

- An example of 2-way set associative cache
 - Each set has two cache blocks
 - A memory block maps to a **unique set**
 - In the set, the memory block can be placed in **either of the cache blocks**
- ➔ Need to search both to look for the memory block



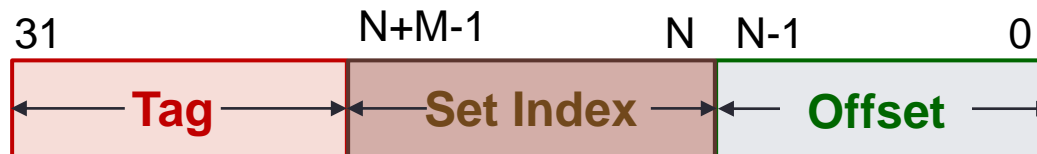
3. Set Associative Cache: Mapping



Cache Block size = 2^N bytes

Cache Set Index

= (BlockNumber) modulo (NumberOfCacheSets)



Cache Block size = 2^N bytes

Number of cache sets = 2^M

Offset = N bits

Set Index = M bits

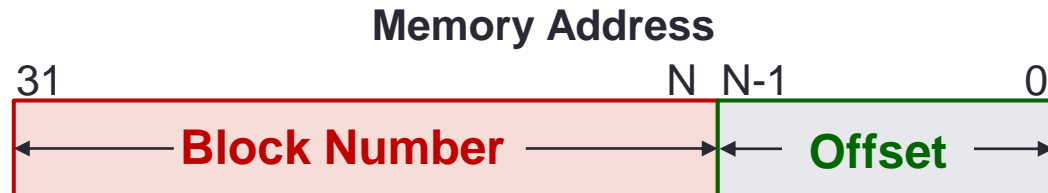
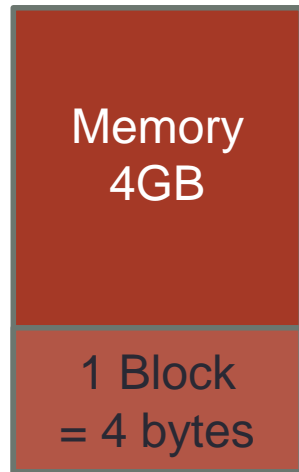
Tag = $32 - (N + M)$ bits

Observation:

It is essentially unchanged from the direct-mapping formula



3. Set Associative Cache: Example



Offset, $N = 2$ bits

Block Number = $32 - 2 = 30$ bits

Check: Number of Blocks = 2^{30}



Number of Cache Blocks

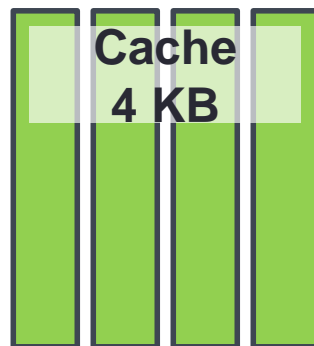
= $4\text{KB} / 4\text{bytes} = 1024 = 2^{10}$

4-way associative, number of sets

= $1024 / 4 = 256 = 2^8$

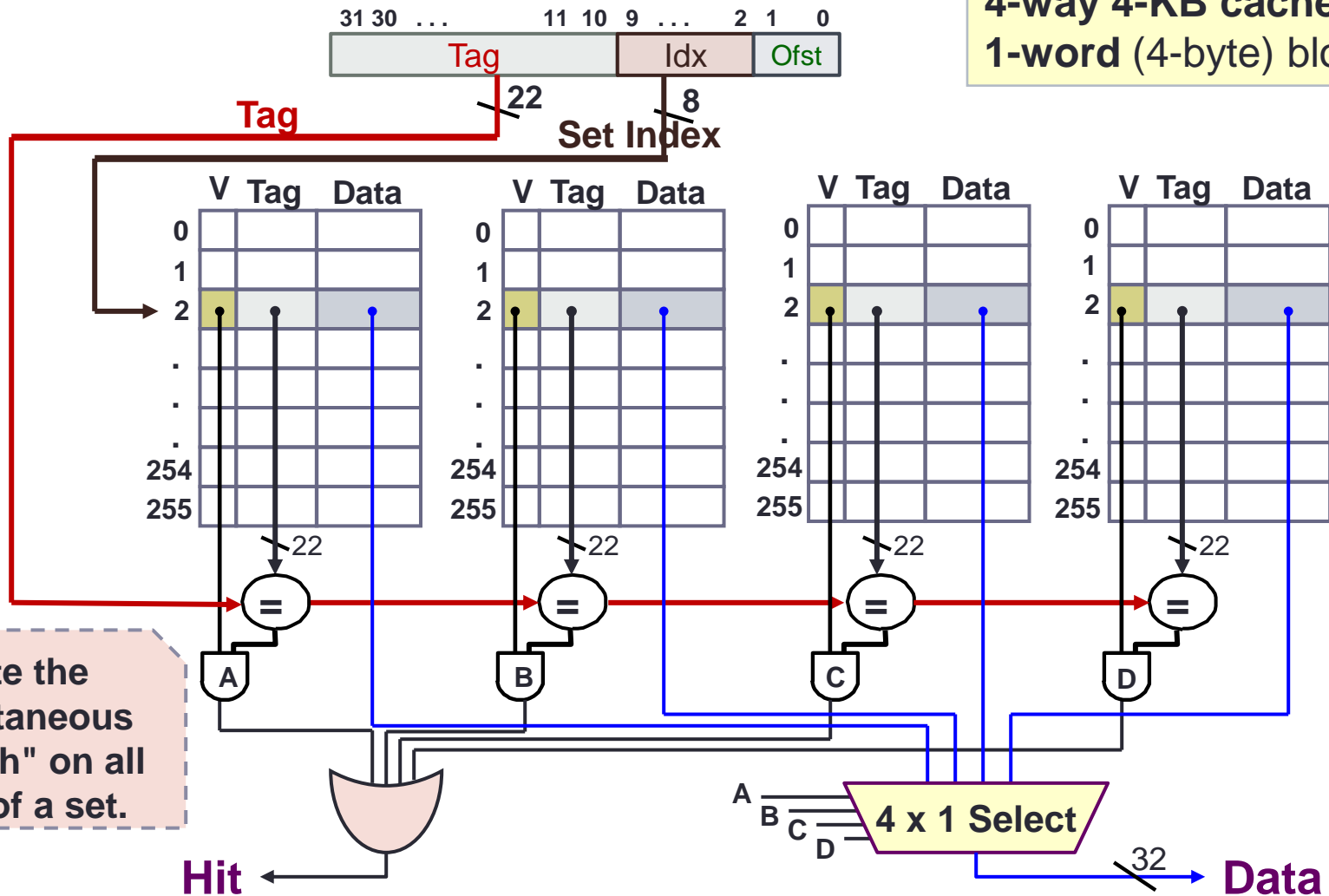
Set Index, $M = 8$ bits

Cache Tag = $32 - 8 - 2 = 22$ bits

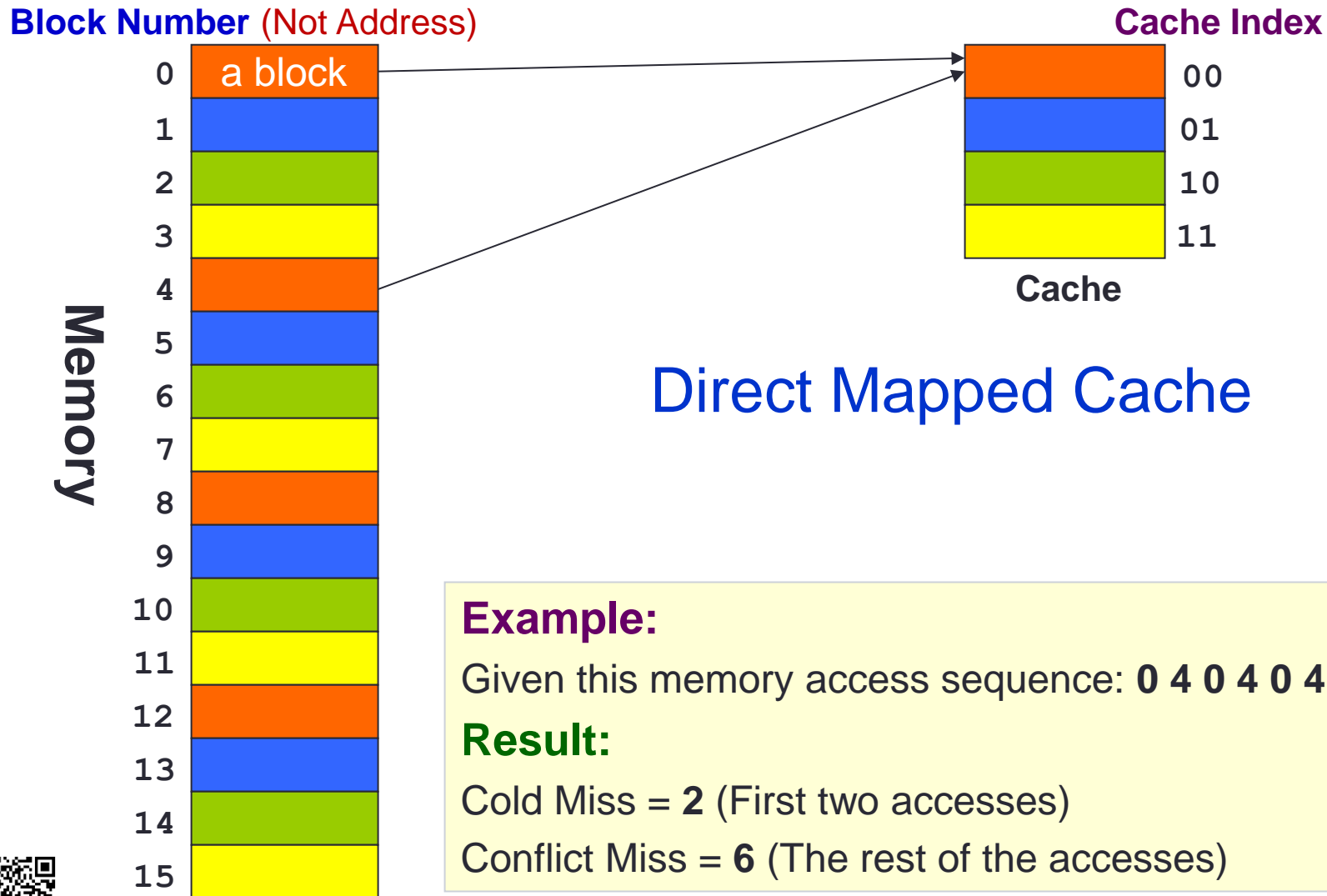


3. Set Associative Cache: Circuitry

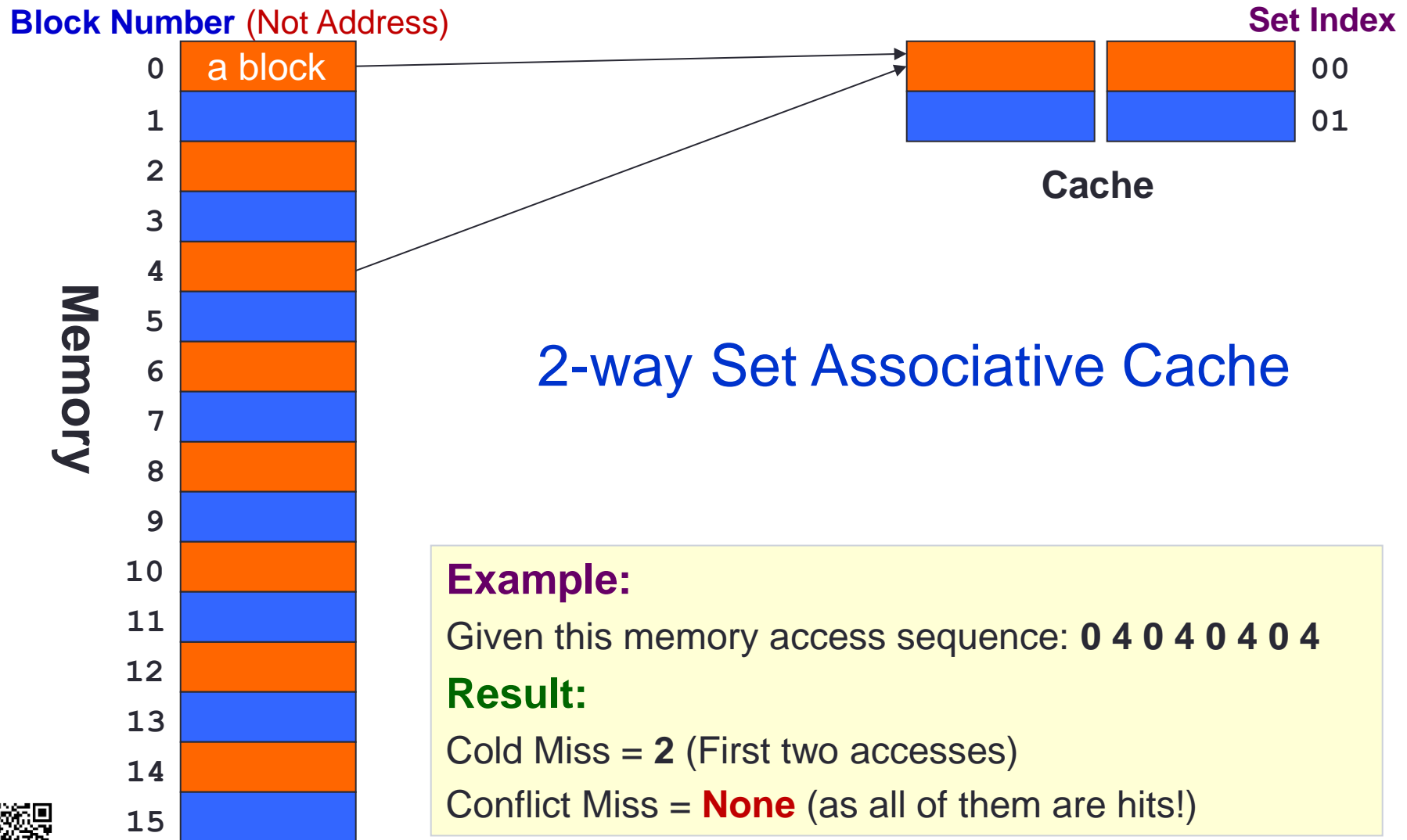
**4-way 4-KB cache:
1-word (4-byte) blocks**



3. Advantage of Associativity (1/3)



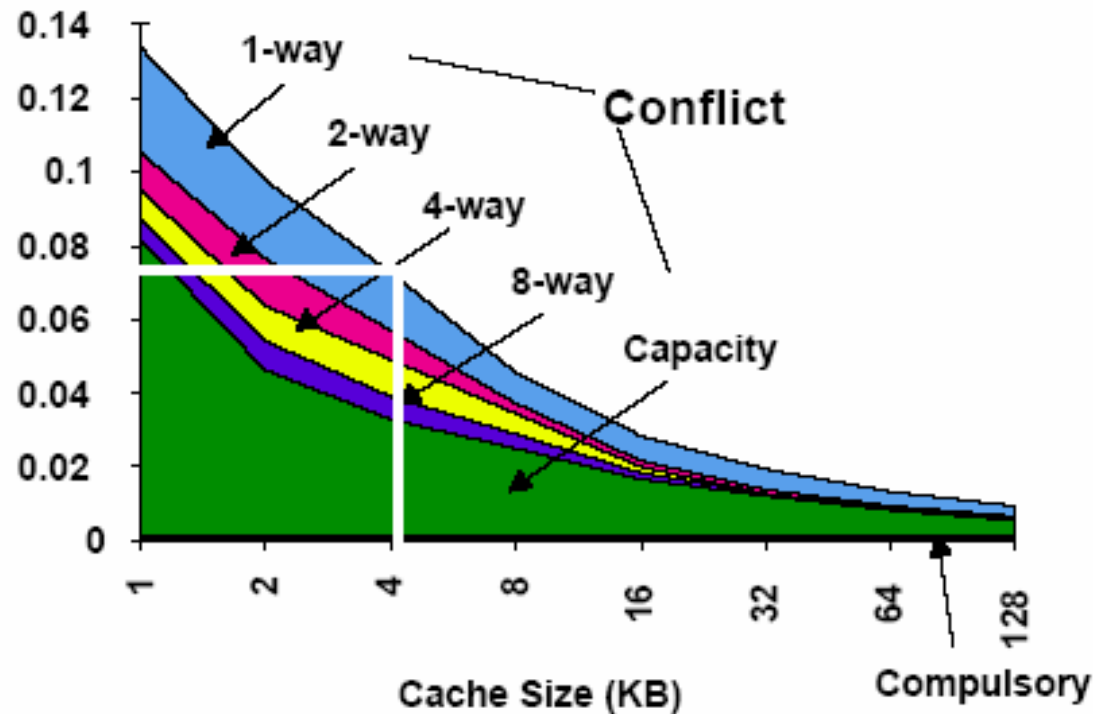
3. Advantage of Associativity (2/3)



3. Advantage of Associativity (3/3)

Rule of Thumb:

A direct-mapped cache of size N has about the same miss rate as a 2-way set associative cache of size $N/2$



3. SA Cache Example: Setup

- Given:
 - Memory access sequence: **4, 0, 8, 36, 0**
 - 2-way set-associative cache with a total of four 8-byte blocks → **total of 2 sets**
 - Indicate hit/miss for each access



Offset, **N** = 3 bits

Block Number = $32 - 3 = 29$ bits

2-way associative, number of sets = $2 = 2^1$

Set Index, **M** = 1 bits

Cache Tag = $32 - 3 - 1 = 28$ bits



3. SA Cache Example: Load #1

Miss
4, 0, 8, 36, 0

Tag

Index Offset

- Load from 4 →

00000000000000000000000000000000 0 100

Check: Both blocks in **Set 0** are invalid [**Cold Miss**]

Result: Load from memory and place in **Set 0 - Block 0**

Set Index	Block 0				Block 1			
	Valid	Tag	W0	W1	Valid	Tag	W0	W1
0	0 1	0	M[0]	M[4]	0			
1	0				0			



3. SA Cache Example: Load #2

Miss Hit
4, **0**, 8, 36, 0

Tag

Index Offset

■ Load from **0** →

00000000000000000000000000000000 0 000

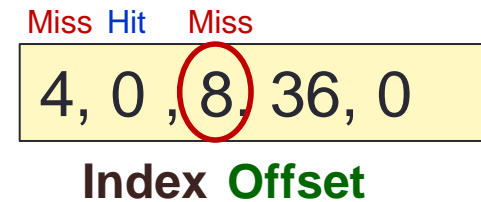
Result:

[Valid and Tags match] in **Set 0-Block 0** [**Spatial Locality**]

Set Index	Block 0				Block 1			
	Valid	Tag	W0	W1	Valid	Tag	W0	W1
0	1	0	M[0]	M[4]	0			
1	0				0			



3. SA Cache Example: Load #3



- [illegible]

Check: Both blocks in **Set 1** are invalid [**Cold Miss**]

Result: Load from memory and place in **Set 1 - Block 0**

Set Index	Block 0				Block 1			
	Valid	Tag	W0	W1	Valid	Tag	W0	W1
0	1	0	M[0]	M[4]	0			
1	0 1	0	M[8]	M[12]	0			

3. SA Cache Example: Load #4

Miss	Hit	Miss	Miss	
4	0	8	36	0
Index		Offset		

■ Load from 36 → 0000000000000000000000000000000010 0 100

Check: [Valid but tag mismatch] **Set 0 - Block 0**
 [Invalid] **Set 0 - Block1 [Cold Miss]**

Result: Load from memory and place in **Set 0 - Block 1**

Set Index	Block 0				Block 1			
	Valid	Tag	W0	W1	Valid	Tag	W0	W1
0	1	0	M[0]	M[4]	0 1	2	M[32]	M[36]
1	1	0	M[8]	M[12]	0			



3. SA Cache Example: Load #5

Miss	Hit	Miss	Miss	Hit
4	0	8	36	0
Index		Offset		

- Load from 0 ➡

0000000000000000000000000000	0	<u>000</u>
------------------------------	---	------------

Check: [Valid and tags match] **Set 0-Block 0**

[Valid but tags mismatch] Set 0-Block1

[Temporal Locality]

Set Idx	Block 0				Block 1			
	Valid	Tag	W0	W1	Valid	Tag	W0	W1
0	1	0	M[0]	M[4]	1	2	M[32]	M[36]
1	1	0	M[8]	M[12]	0			



4. Fully Associative (FA) Cache

- **Compulsory misses**

- On the first access to a block; the block must be brought into the cache
- Also called **cold start misses** or **first reference misses**

- **Conflict misses**

- Occur in the case of direct mapped cache or set associative cache, when several blocks are mapped to the same block/set
- Also called **collision misses**

Occurs in
Fully Associative Cache

- **Capacity misses**

- Occur when blocks are discarded from cache as cache cannot contain all blocks needed



4. Fully Associative (FA) Cache: Analogy



Let's not restrict the book by title any more. A book can go into **any** location on the desk!



4. Fully Associative (FA) Cache

■ Fully Associative Cache

- A memory block can be placed in any location in the cache

■ Key Idea:

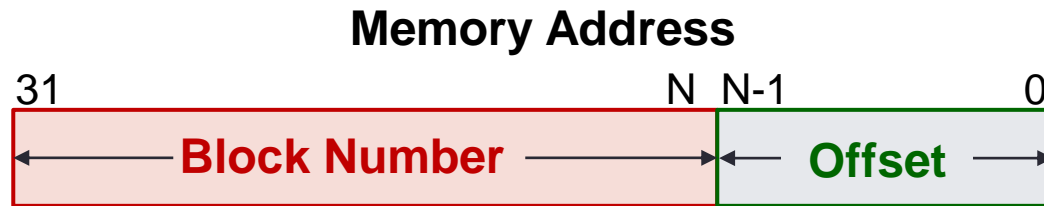
- Memory block placement is no longer restricted by cache index or cache set index

++ Can be placed in any location, **BUT**

--- Need to search all cache blocks for memory access



4. Fully Associative Cache: Mapping



Cache Block size = 2^N bytes



Cache Block size = 2^N bytes

Number of cache blocks = 2^M

Offset = N bits

Tag = 32 - N bits

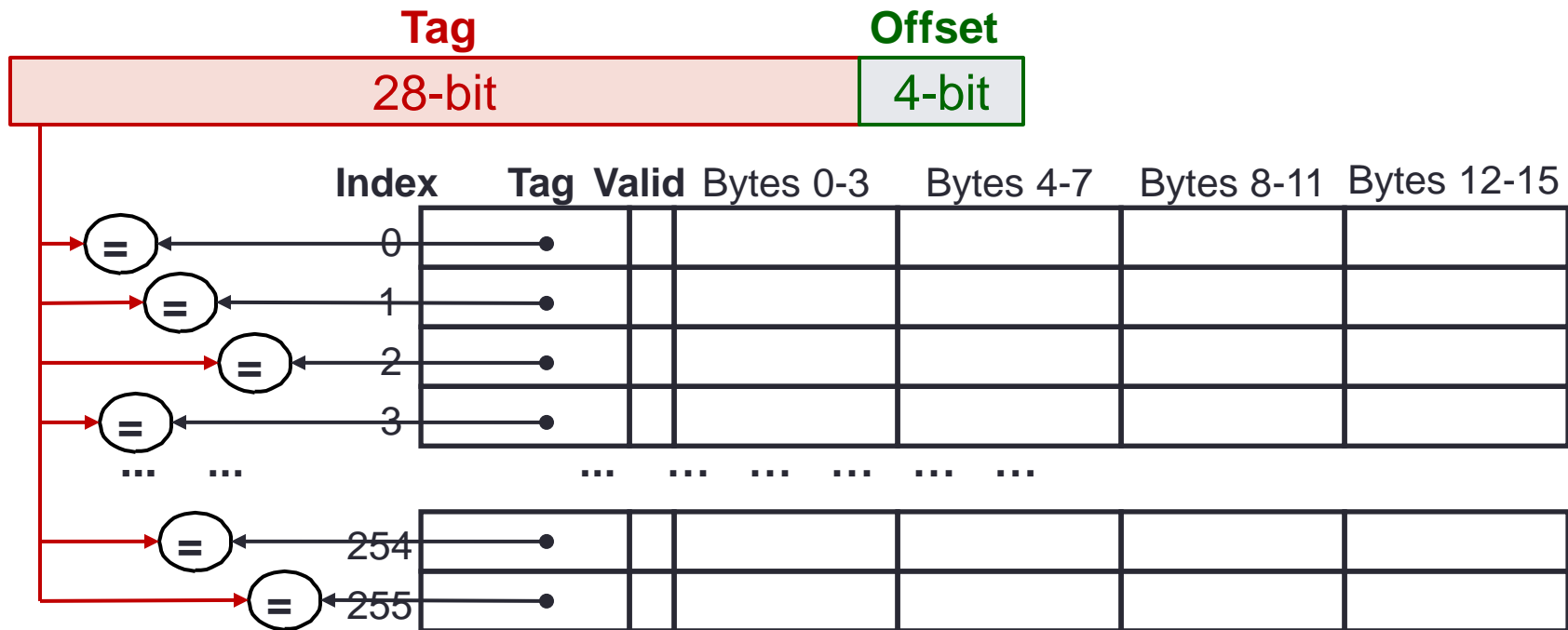
Observation:

The block number serves as the tag in FA cache.



4. Fully Associative Cache: Circuitry

- Example:
 - 4KB cache size and 16-Byte block size
 - Compare tags and valid bit in parallel



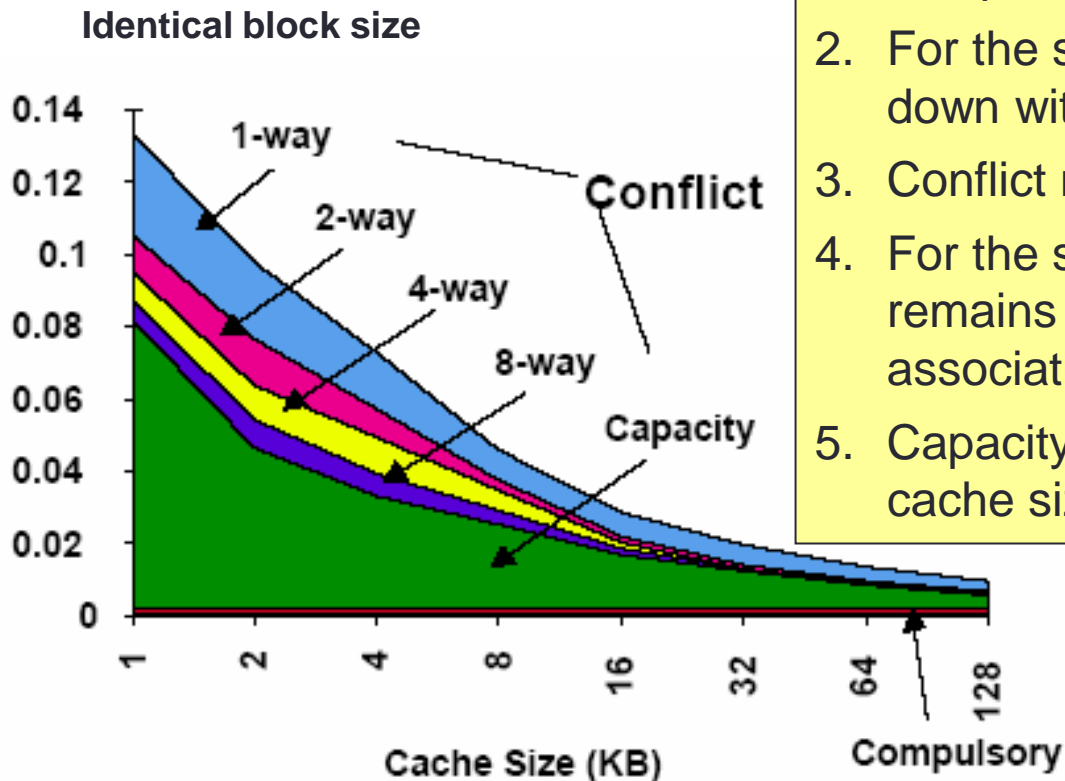
No Conflict Miss (since data can go anywhere)



4. Cache Performance

Observations:

1. Cold/compulsory miss remains the same irrespective of cache size/associativity.
2. For the same cache size, conflict miss goes down with increasing associativity.
3. Conflict miss is 0 for FA caches.
4. For the same cache size, capacity miss remains the same irrespective of associativity.
5. Capacity miss decreases with increasing cache size.



Total Miss = Cold miss + Conflict miss + Capacity miss

Capacity miss (FA) = Total miss (FA) – Cold miss (FA), when Conflict Miss $\rightarrow 0$

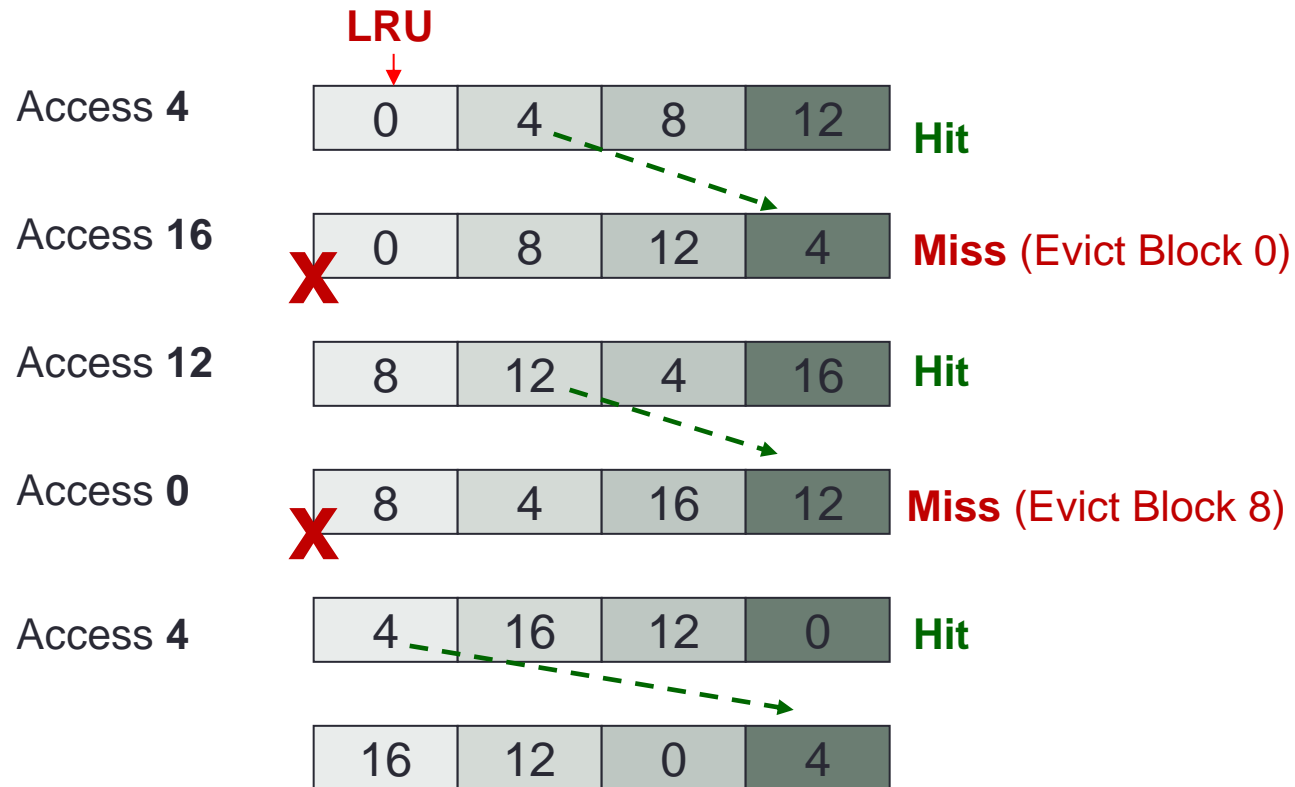
5. Block Replacement Policy (1/3)

- **Set Associative or Fully Associative Cache:**
 - Can choose where to place a memory block
 - Potentially replacing another cache block if full
 - Need **block replacement policy**
- **Least Recently Used (LRU)**
 - **How:** For cache hit, record the cache block that was accessed
 - When replacing a block, choose one which has not been accessed for the longest time
 - **Why:** Temporal locality



5. Block Replacement Policy (2/3)

- Least Recently Used policy in action:
 - 4-way SA cache
 - Memory accesses: **0 4 8 12 4 16 12 0 4**



5. Block Replacement Policy (3/3)

- Drawback for LRU
 - Hard to keep track if there are many choices
- Other replacement policies:
 - First in first out (FIFO)
 - Random replacement (RR)
 - Least frequently used (LFU)



6. Additional Examples #1

■ **Direct-Mapped** Cache:

- Four 8-byte blocks

■ Memory accesses:

4, 8, 36, 48, 68, 0, 32

Addr:	Tag	Index	Offset
4:	00...000	00	100
8:	00...000	01	000
36:	00...001	00	100
48:	00...001	10	000
68:	00...010	00	100
0:	00...000	00	000
32:	00...001	00	000

Index	Valid	Tag	Word0	Word1
0	0 1	0 1 1 1 2 0	M[0] M[32] M[32] M[64] M[0] M[4]	M[4] M[36] M[36] M[68] M[4] M[12]
1	0 1	0	M[8]	M[12]
2	0 1	1	M[48]	M[52]
3	0			



6. Additional Examples #2

■ Fully-Associative Cache:

- Four 8-byte blocks
- LRU Replacement Policy

■ Memory accesses:

4, 8, 36, 48, 68, 0, 32

Addr:	Tag	Offset
4:	00...00000	100
8:	00...00001	000
36:	00...00100	100
48:	00...00110	000
68:	00...01000	100
0:	00...00000	000
32:	00...00100	000

Index	Valid	Tag	Word0	Word1
0	0 1	0 8	M[0] M[64]	M[4] M[68]
1	0 1	1 0	M[8] M[0]	M[12] M[4]
2	0 1	4	M[32]	M[36]
3	0 1	6	M[48]	M[52]



6. Additional Examples #3

■ 2-way Set-Associative Cache:

- Four 8-byte blocks
- LRU Replacement Policy

■ Memory accesses:

4, 8, 36, 48, 68, 0, 32

Addr:	Tag	Index	Offset
4:	00...0000	0	100
8:	00...0000	1	000
36:	00...0010	0	100
48:	00...0011	0	000
68:	00...0100	0	100
0:	00...0000	0	000
32:	00...0010	0	000

Set Index	Block 0				Block 1			
	Valid	Tag	Word0	Word1	Valid	Tag	Word0	Word1
0	0 1	0	M[0]	M[4]	0 1	2	M[32]	M[36]
		3	M[48]	M[52]		4	M[64]	M[68]
		0	M[0]	M[4]		2	M[32]	M[36]
1	0 1	0	M[8]	M[12]	0			



7. Summary: Cache Organizations

One-way set associative (direct mapped)

Block	Tag	Data
0		
1		
2		
3		
4		
5		
6		
7		

Two-way set associative

Set	Tag	Data	Tag	Data
0				
1				
2				
3				

Four-way set associative

Set	Tag	Data	Tag	Data	Tag	Data	Tag	Data
0								
1								

Eight-way set associative (fully associative)

Tag	Data	Tag	Data	Tag	Data	Tag	Data	Tag	Data	Tag	Data	Tag	Data	Tag	Data



7. Summary: Cache Framework (1/2)

Block Placement: Where can a block be placed in cache?

Direct Mapped:

- Only one block defined by index

N-way Set-Associative:

- Any one of the **N** blocks within the set defined by index

Fully Associative:

- Any cache block

Block Identification: How is a block found if it is in the cache?

Direct Mapped:

- Tag match with only one block

N-way Set Associative:

- Tag match for all the blocks within the set

Fully Associative:

- Tag match for all the blocks within the cache



7. Summary: Cache Framework (2/2)

Block Replacement: Which block should be replaced on a cache miss?

Direct Mapped:

- No Choice

N-way Set-Associative:

- Based on replacement policy

Fully Associative:

- Based on replacement policy

Write Strategy: What happens on a write?

Write Policy: Write-through vs write-back

Write Miss Policy: Write allocate vs write no allocate



8. Exploration: Improving Cache Penalty

Average Access Time

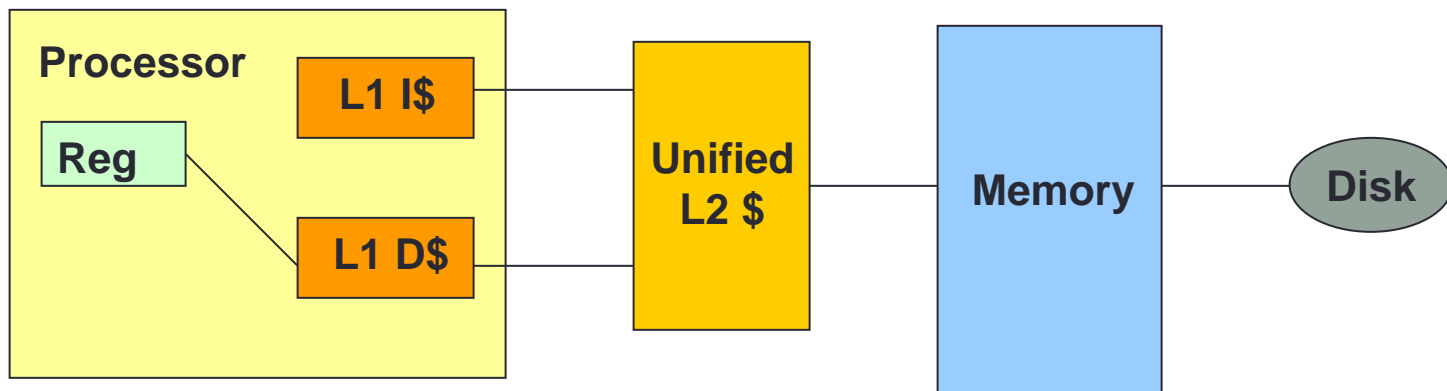
$$= \text{Hit rate} \times \text{Hit Time} + (1 - \text{Hit rate}) \times \text{Miss penalty}$$

- So far, we tried to improve Miss Rate:
 - Larger block size
 - Larger Cache
 - Higher Associativity
- What about **Miss Penalty**?

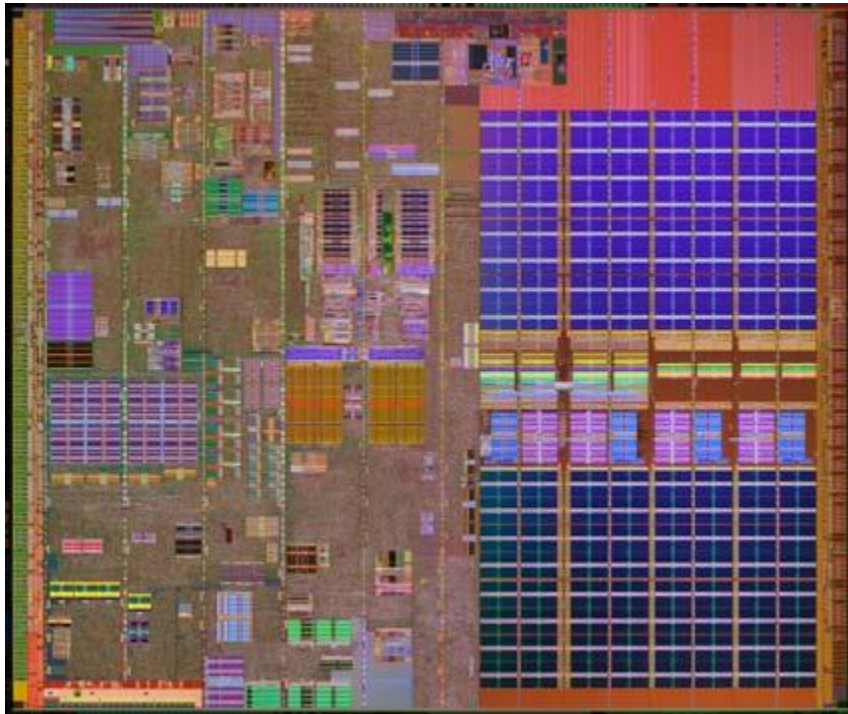


8. Exploration: Multilevel Cache

- Options:
 - Separate data and instruction caches, or a unified cache
- Sample sizes:
 - **L1**: 32KB, 32-byte block, 4-way set associative
 - **L2**: 256KB, 128-byte block, 8-way associative
 - **L3**: 4MB, 256-byte block, Direct mapped



8. Exploration: Intel Processors

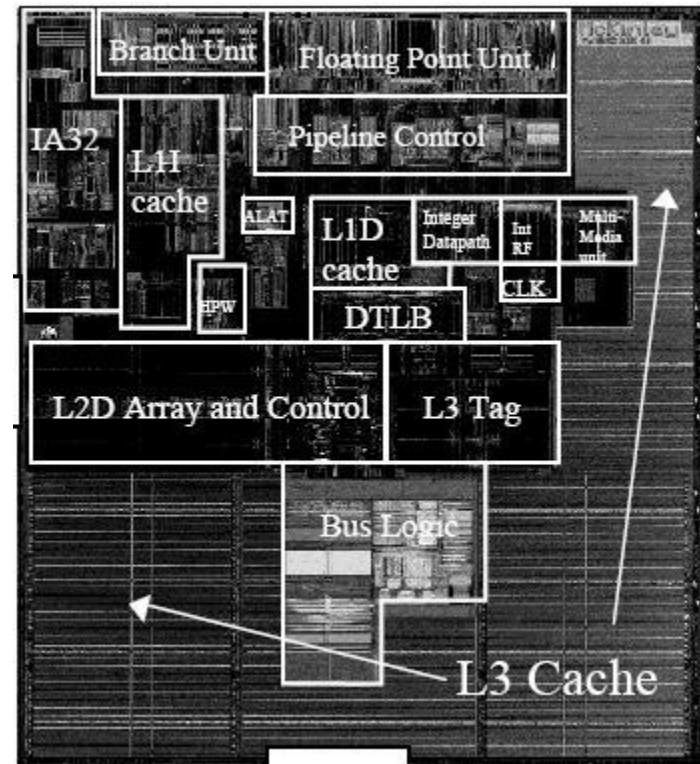


Pentium 4 Extreme Edition

L1: 12KB I\$ + 8KB D\$

L2: 256KB

L3: 2MB



Itanium 2 McKinley

L1: 16KB I\$ + 16KB D\$

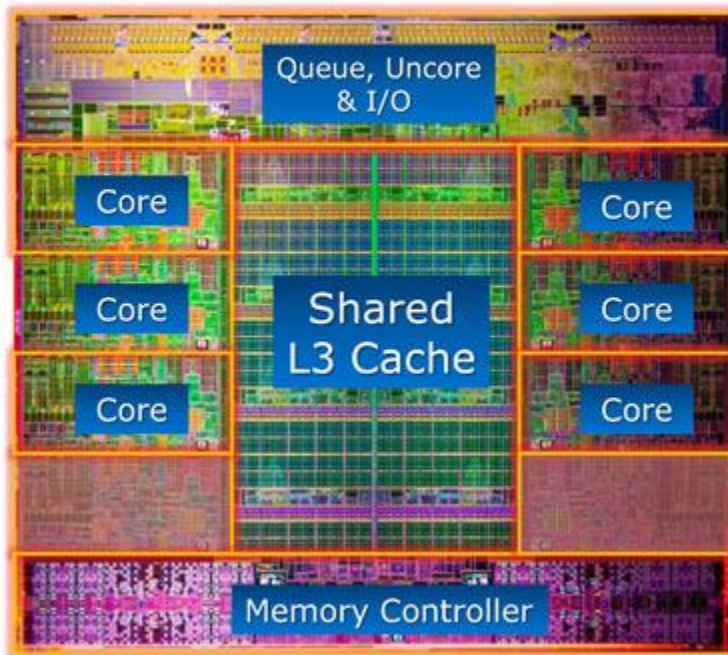
L2: 256KB

L3: 1.5MB – 9MB



8. Exploration: Trend: Intel Core i7-3960K

Intel® Core™ i7-3960X Processor Die Detail



Intel Core i7-3960K

per die:

- 2.27 billion transistors
- 15MB shared Inst/Data Cache (LLC)

per Core:

- 32KB L1 Inst Cache
- 32KB L1 Data Cache
- 256KB L2 Inst/Data Cache
- up to 2.5MB LLC



Reading

- **Large and Fast: Exploiting Memory Hierarchy**
 - Chapter 7 sections 7.1 – 7.2 (3rd edition)
 - Chapter 5 sections 5.1 – 5.2 (4th edition)



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