Efficient Attributed Scattering Center Extraction Using Gradient-Based Optimization for SAR Image

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***Abstract*—The attributed scattering center (ASC) model provides a concise representation of high-frequency electromagnetic scattering of targets, making it widely applicable to synthetic aperture radar target characteristic analysis and recognition. However, ASC extraction, as a non-convex parameter estimation problem, is computationally time-consuming in global optimization due to the non-convexity of loss function. While gradient-based optimization methods in deep learning frameworks can significantly improve efficiency, applying them directly to non-convex optimization remains challenging. To solve this issue, this letter proposes an efficient gradient-based ASC extraction method. First, A more convex-like loss function is designed using windowing techniques to broaden the convergence region and reduce the difficulty of non-convex optimization. Then, an image-frequency domain joint parameter initialization and estimation strategy are developed, achieving fully gradient-based parameter estimation. Validation on simulated and measured data confirms the effectiveness of the proposed method. While maintaining accuracy, it improves ASC extraction efficiency for tank targets from a minute-level to a second-level. The codes related to this work are available at https://github.com/jw-07/ASC_extraction/.**

***Index Terms*—synthetic aperture radar (SAR), attributed scattering center (ASC), gradient-based optimization**

I. Introduction

SYNTHETIC aperture radar (SAR), with its high-resolution imaging capability, can precisely characterize the electromagnetic scattering properties of typical targets. The parametrized attributed scattering center (ASC) model [1] provides a concise and physically interpretable representation of scattering mechanisms by inversely modeling the target radar echoes. Based on this model, the backscattering characteristics of targets can be analysed, enabling both scattering property inversion [2], [3], [12]and key feature extraction for recognition [4]–[7]. However, the effective characteristic inversion relies on accurate parameter estimation, i.e., extracting reliable scattering center (SC) parameters from echo data. Therefore, studying efficient and accurate ASC extraction methods is crucial for target characteristic analysis and recognition tasks.

ASC extraction is essentially a nonlinear and non-convex parameter estimation problem, where target echoes are reconstructed from SC parameters. Existing methods are mainly divided into image-domain and frequency-domain methods. Image-domain methods estimate ASC parameters by segmenting SAR images and initializing parameters accordingly, but their performance heavily depends on segmentation accuracy, which degrades significantly under low signal-to-noise ratio (SNR) conditions [8], [9]. As a result, frequency-domain methods have become the preferred approach for achieving more precise results. Some scholars employed the orthogonal matching pursuit (OMP) method, constructing a large parameter dictionary for parameter estimation. Although these approaches achieve good performance, they suffer from high spatial complexity [10], [11]. Jing et al. [12] introduced heuristic optimization algorithms into frequency-domain ASC extraction to address this issue, leveraging the global search capability of the genetic algorithm (GA) for parameter estimation. Although GA achieved promising estimation results, the slow convergence of global optimization makes the estimation process highly time-consuming.

This letter proposes an efficient and accurate ASC extraction method based on a gradient optimization in deep learning frameworks. Since ASC extraction is a non-linear and non-convex problem, gradient-based optimization methods are not directly applicable. First, we analyse the problem and determine that a gradient-based method should be designed from two aspects: designing a more convex-like loss function and developing a more accurate parameter initialization strategy to enhance the applicability of gradient optimization. Then, by visualizing the loss function and adopting the windowing operation from signal processing to suppress side lobes (local minima), we design a more convex-like loss function. Furthermore, based on the characteristics of the ASC position and distributed parameters, we propose a joint image-frequency domain initialization strategy. Finally, the ASC extraction is implemented in the frequency domain using fully gradient-based optimization, achieving both accuracy and efficiency.

II. Challenge of ASC extraction

In the high-frequency domain, the radar echo of complex targets can be modeled as a superposition of multiple ASCs. The echo of complex target can be expressed as:

$$E(f,\phi)=\sum_{k=1}^{N}[A_k\left(j\frac{f}{f_c}\right)^{\alpha_k}\operatorname{sinc}\left(\frac{2\pi f}{c}L_k\sin\left(\phi-\bar{\phi}_k\right)\right)$$
$$\cdot\exp\left(\frac{-j4\pi f}{c}\left(x_k\cos\phi+y_k\sin\phi\right)\right)\exp\left(-2\pi f\gamma_k\sin\phi\right)]+n \tag{1}$$

where, f, ϕ are the frequency and azimuth axes, respectively. N denotes ASCs' number, which depends on target complexity and is either set to a large value or determined adaptively by a stopping criterion from Eq. 11, the echo of k-th ASC is represented by the parameter set $\theta_k=\left[A_k,\alpha_k,x_k,y_k,L_k,\bar{\phi}_k,\gamma_k\right]$, which correspond to the amplitude, frequency-dependent factor, azimuth/distance coordinates, length, azimuth angle, and

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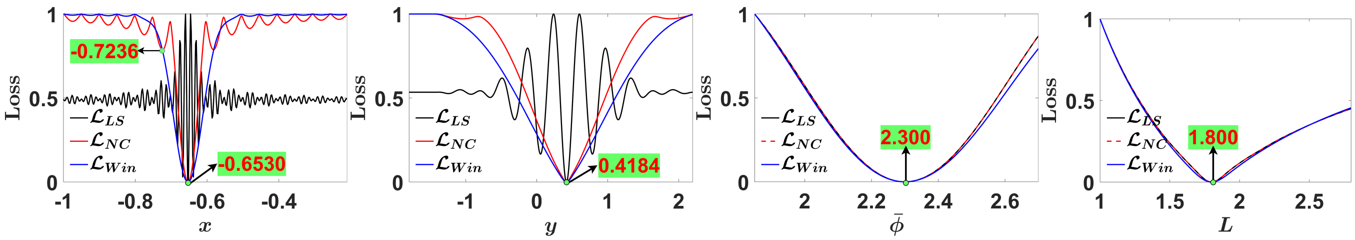

Fig. 1. Comparison of convergence domains of the three loss functions. Using a scattering center with ground truth (GT) parameters $[x = -0.6530, y = 0.4184, \bar{\phi} = 2.300, L = 1.800]$ as an example, each curve was generated using a control variable method by sweeping a single parameter, calculating the loss between the reconstructed and reference echoes, and normalizing the loss value to the [0, 1] range.

azimuth-dependent factor of the ASC, γ_k is a minimum value, which is directly 0 in the estimation. n represents the noise.

ASCs can be divided into two main categories: distributed SCs ($L > 0$) and localized SCs ($L = 0$). Different combinations of α, L correspond to the scattering geometry of ASC, and the target characteristics can be retrieved by ASC extraction. The idea behind extracting ASCs from complex targets is to iteratively estimate θ of each ASC. After each iteration, the residual echo is updated using the clean strategy, and then the next ASC will be estimated. Ultimately, a set of parameters $[\theta_1, \theta_2, ..., \theta_N]$ for N ASCs is obtained to reconstruct the target's echo. In each estimation, the ASC parameters are optimized by minimizing the difference between the reconstructed echo and the current residual echo. The least squares loss function is defined as follows

$$\mathcal{L}_{LS}(\theta) = \|E_v^{res} - \hat{E}_v(\theta)\|_2^2 \tag{2}$$

where, E_v^{res} represents the vectorized residual echo, and $\hat{E}_v$ represents the vectorized reconstructed echo by estimated θ.

Due to the non-linearity of the $sinc$ term in the ASC model, parameter estimation based on the ASC model becomes a non-linear, non-convex optimization problem [13]. By visualizing the loss function, as shown in Fig. 1, it can be observed from the loss values of $\mathcal{L}_{LS}$ that the estimation of the position parameters x, y is a highly non-convex optimization problem with a narrow convergence domain, posing significant challenges for gradient-based algorithms. Global optimization GA relies on randomness and is generally less sensitive to parameter initialization. In contrast, gradient-based methods face two main issues that need to be addressed:

(1) **Design a More Convex-like Loss Function**: Since $\mathcal{L}_{LS}$ has many local minima, it is easy to get trapped in local optima. Thus, a loss function with a wider convergence domain needs to be designed to reduce the difficulty of gradient-based optimization.

(2) **Design More Accurate Parameter Initialization**: Ensuring that the parameters fall within the convergence domain after initialization can improve optimization efficiency and accelerate convergence speed, avoiding local optima caused by inappropriate initial values.

III. PROPOSED METHOD

A. Design of Loss function

Due to the tendency of $\mathcal{L}_{LS}$ to easily fall into local optima, $\mathcal{L}_{LS}$ is expanded and derived a non-coherent loss function $\mathcal{L}_{NC}$ through an approximation approach [12].

$$\begin{aligned}\mathcal{L}_{LS}(\theta) &= \left[E_v^{res} - \hat{E}_v(\theta)\right]^H \left[E_v^{res} - \hat{E}_v(\theta)\right] \\ &= E_v^{res\,H} E_v^{res} - 2\Re\left(E_v^{res\,H}\hat{E}_v(\theta)\right) + \hat{E}_v(\theta)^H \hat{E}_v(\theta) \\ &\approx \underbrace{E_v^{res\,H} E_v^{res} - 2\left|E_v^{res\,H}\hat{E}_v(\theta)\right| + \hat{E}_v(\theta)^H \hat{E}_v(\theta)}_{\mathcal{L}_{NC}(\theta)}\end{aligned} \tag{3}$$

where, $\Re(\cdot)$ represents the real part.

From Fig. 1, it can be observed that for the position parameters x, y, the convergence domain of $\mathcal{L}_{NC}$ is wider compared to $\mathcal{L}_{LS}$, but it is still prone to falling into local extremum regions. For x, y, $\mathcal{L}_{NC}$ resembles the $sinc$ function in signal processing, where its convergence domain is analogous to the main lobe of the $sinc$ function, and the local extremum regions are similar to the side lobes of the $sinc$ function. To further obtain a more convex-like loss function $\mathcal{L}_{Win}$, a windowing operation can be applied. By leveraging the property of window functions to suppress side lobes, the possibility of parameters falling into local optima can be effectively reduced, thereby $\mathcal{L}_{Win}$ is more convex-like than $\mathcal{L}_{NC}$. $\mathcal{L}_{Win}$ is as follows

$$\mathcal{L}_{Win}(\theta) = W \cdot \mathcal{L}_{NC}(\theta) \tag{4}$$

where W denotes a -35dB Taylor window, whose length depends on the SAR image size.

B. Image-frequency domain joint initialization

In order to apply gradient-based optimization strategies to non-convex ASC extraction, it is necessary not only to design a more convex-like loss function but also to perform accurate parameter initialization. x, y represent the position information of the ASC, which can be roughly estimated by imaging the echo and selecting the strongest scattering points in the image domain. The corresponding pixel values in the imaging results (without normalization) serve as the relative amplitudes of different ASCs. The subsequent clean strategy removes the strongest point at each step, ensuring accuracy and accelerating convergence. The initial values for x, y, denoted as x^0, y^0, can be calculated as

$$x^0 = (X_{max} - X_{center})\, dr, y^0 = (Y_{max} - Y_{center})\, da \tag{5}$$

where X_{max}, Y_{max} are the indices of the strongest scattering point in the image domain, X_{center}, Y_{center} are the indices of the image center, dr/da are the range/azimuth resolution.

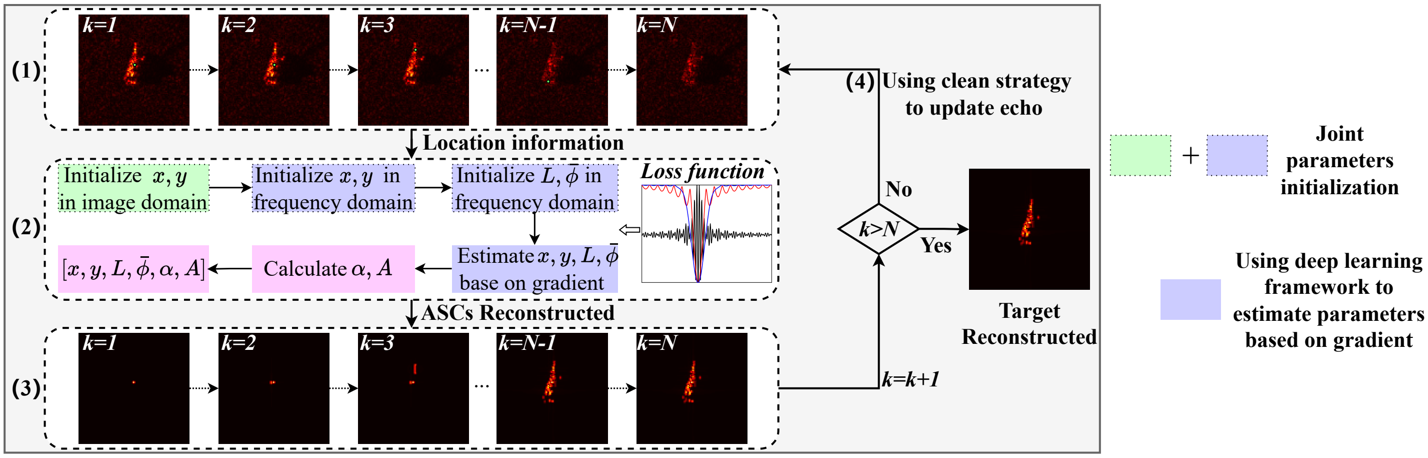

Fig. 2. The process of the proposed method: (1) Extract the strongest point's location information from the SAR image. (2) Perform the k-th ASC estimation. (3) ASCs reconstruction. (4) Update the echo using the clean strategy. Repeat (1)-(4) to proceed to the $(k+1)$-th iteration.

For localized SCs, coarse estimation based on the image domain typically yields satisfactory results. However, for distributed SCs, as shown in Fig. 1, due to the length of the SCs, estimating a single strong point solely from the image domain can introduce errors, leading the optimization process to fall into local optima. Therefore, more accurate parameter initialization is also required from the frequency domain.

ASC estimation essentially minimises the difference between the frequency-domain echo and the reconstructed echo. Frequency domain initialization belongs to the signal-level estimation, which can effectively compensate for the shortcomings of image domain initialization. The parameters of ASC model are coupled and interact with each other. Directly performing joint estimation of all parameters can be computationally expensive. To address this issue, [11] adopted a dictionary scaling approach, first performing a rough estimation of the position parameters x, y, then a rough estimation of the distributed parameters $\bar{\phi}, L$, and finally performing joint optimization of the four parameters. The proposed frequency domain initialization follows this strategy, performing frequency domain initialization for position and distributed parameters separately based on gradient optimization.

Although $\mathcal{L}_{NC}$ and $\mathcal{L}_{Win}$ can reduce the difficulty of parameter estimation, ASC extraction remains a non-convex optimization problem. To address this, we choose the optimization algorithm that integrates gradient and momentum from deep learning frameworks for frequency domain parameter initialization, as momentum helps improve convergence stability and mitigate the impact of local minima. The parameter update function is as follows

$$\theta^* = \mathcal{O}\left(\theta, \mathcal{L}, \nabla\mathcal{L}, m\right) \tag{6}$$

where θ^* represents the updated parameters, $\nabla\mathcal{L}$ represents the gradient, and m represents the momentum.

From Fig. 1, it can be seen that $\mathcal{L}_{Win}$ effectively reduces the difficulty of estimating the x, y. Therefore, $\mathcal{L}_{Win}$is chosen to further initialize the position parameters x, y in the frequency domain. The updated $x^1, y^1$are calculated as follows

$$\left[x^1, y^1\right] = \mathcal{O}\left(\left[x^0, y^0\right], \mathcal{L}_{win}, \nabla\mathcal{L}, m\right) \tag{7}$$

$\bar{\phi}$ and L are difficult to estimate from the image domain, so their initial values are set to a small value ε for estimation. The updated $\bar{\phi}^1$ and L^1 are calculated as follows

$$\left[\bar{\phi}^1, L^1\right] = \mathcal{O}\left(\left[\varepsilon, \varepsilon\right], \mathcal{L}_{NC}, \nabla\mathcal{L}, m\right) \tag{8}$$

C. Complete ASC Extraction Process

The efficient ASC extraction for complex targets adopts an iterative and clean strategy, with the processing flow illustrated in Fig. 2. Taking the estimation of the k-th ASC parameter as an example, the process is described as follows

1. Perform image-frequency domain joint initialization to obtain $\left[x_k^1, y_k^1, \bar{\phi}_k^1, L_k^1\right]$.

2. Perform joint estimation of the four parameters to obtain accurate results $\left[x_k, y_k, \bar{\phi}_k, L_k\right]$. Meanwhile, use the least squares method to estimate the complex-valued A_k.

$$\left[x_k, y_k, \bar{\phi}_k, L_k\right] = \mathcal{O}\left(\left[x_k^1, y_k^1, \bar{\phi}_k^1, L_k^1\right], \mathcal{L}_{NC}, \nabla\mathcal{L}, m\right) \tag{9}$$

$$A_k = \left(\hat{E}_{v,k}^H \hat{E}_{v,k}\right)^{-1} \hat{E}_{v,k}^H E_v^{res} \tag{10}$$

where, $\hat{E}_{v,k}$ represents the echo vector reconstructed for the k-th ASC, and $A_k = 1$.

3. Use the enumeration method to estimate α_k.

4. Use the least squares method again to estimate complex-valued A_k.

5. Based on the clean strategy, subtract the echo of the k-th ASC from the current residual echo E_v^{res}, and then, the estimation of the $(k+1)$-th ASC is performed, and this process continues until the estimation of all N ASCs is completed, reconstructing the complete target echo.

IV. Experimental Results

In this section, we design relevant experiments to verify the effectiveness of the proposed method.

A. Comparison of parameter initialization methods

We define the residual energy reconstruction ratio η to evaluate target reconstruction accuracy. A smaller η indicates

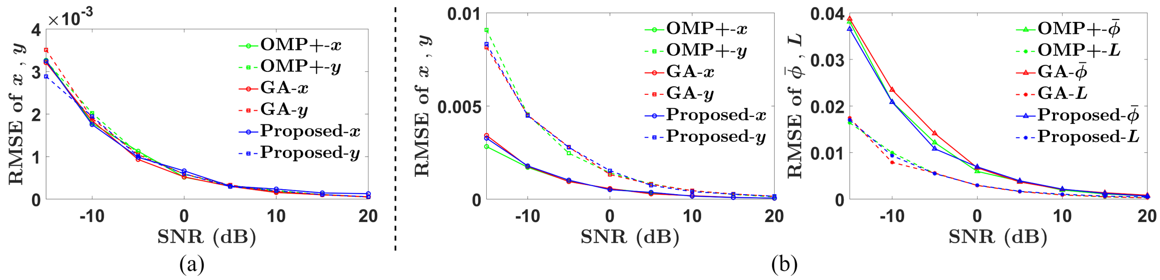

Fig. 3. Estimation of localized and distributed SCs under different SNR conditions. (a)/(b) represents the result of localized/distributed SCs, respectively.

TABLE I
COMPARISON OF PARAMETER ESTIMATION RESULTS USING DIFFERENT INITIALIZATION METHODS.

Initialization	x (m)	y (m)	α	$\bar{\phi}$ (°)	L (m)	$\lvert A \rvert$	η
I †	-0.7238	0.4170	1.0	2.300	1.7990	3.5158	0.9528
I+F † ($\mathcal{L}_{NC}$)	-0.7236	0.4169	1.0	2.299	1.7996	3.5163	0.9527
I+F † ($\mathcal{L}_{Win}$)	-0.6530	0.4184	1.0	2.299	1.7997	16.1838	9.4484e-7
GT	-0.6530	0.4184	1.0	2.300	1.8000	16.1852	-

† I and F represent image and frequency domain initialization, respectively.

better target extraction and more accurate parameter estimation. It is computed as follows

$$\eta = (E_v^{res\,H} E_v^{res})/(E_v^H E_v) \tag{11}$$

where, E_v represents the complete echo vector.

The proposed method primarily achieves accurate and efficient gradient-based estimation by designing a more convex-like loss function and joint image-frequency domain parameter initialization. To validate the effectiveness of initialization, we designed three sets of comparative experiments. As shown in Table I, the GT displays the information of the distributed SC. Initialization based on the image domain (I row), or further using $\mathcal{L}_{NC}$ for frequency-domain initialization (I+F ($\mathcal{L}_{NC}$) row) of the position parameters x, y, easily causes the estimation of the position parameters x, y to fall into local optima. As demonstrated by the convergence domain visualization in Fig. 1, the estimation of x falls near a local minimum of -0.7236 due to $\mathcal{L}_{NC}$. The designed $\mathcal{L}_{Win}$ achieves optimal estimation of the position parameters by expanding the convergence domain. The fact that the η tends to 0 also verifies this, proving the effectiveness of the proposed joint image-frequency domain initialization and $\mathcal{L}_{Win}$.

B. Analysis of parameter estimation performance

To further validate the parameter estimation performance of the proposed method, 100 Monte Carlo experiments were conducted under different SNRs to evaluate the parameter estimation performance for both localized and distributed SCs. The OMP+ [15] and GA [12] method are selected for comparison. OMP+ uses dictionary scaling [11] for parameter coarse estimation, and then Broyden-Fletcher-Goldfarb-Shanno (BFGS) is applied for local refinement. For the evaluation of continuous parameters $[x, y, \bar{\phi}, L]$, the root mean square error (RMSE) is calculated. Fig. 3 shows the estimation results under different SNRs. As all three methods work in the frequency domain, they remain effective even at low SNRs.

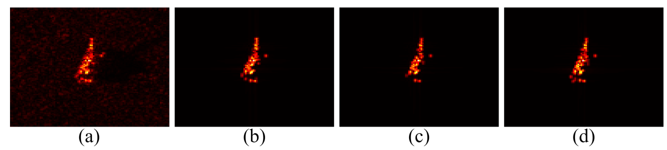

Fig. 4. ASCs extraction results of T72. (a) shows T72. (b), (c) and (d) show the target reconstruction results using OMP+, GA and the proposed method.

TABLE II
COMPARISON OF DIFFERENT METHODS FOR EXTRACTING T72 ASCS.

Methods	η	Time for extract 40 ASCs (s) †
OMP+	0.4109	52.41
GA	0.4160	3472.40
The proposed	0.4131	**11.85**

† Record on NVIDIA GeForce RTX 3090 GPU, an Intel(R) Core(TM) i9-9900K CPU, and the PyTorch framework.

C. Experiment on measured SAR data

We validate the proposed method using the measured T72 tank (MSTAR [14] dataset, ID: HB03335.015). After preprocessing the image, we extract 40 ASCs in the frequency domain using OMP+, GA and the proposed method for comparison. As shown in Fig. 4, OMP+, GA, and the proposed method can reconstruct a relatively complete target, with the proposed method achieving a more complete extraction of ASCs at the tank's rear. As shown in Table II, the three methods show negligible differences in their η values. OMP+ combines dictionary scaling with gradient-based refinement via BFGS, which improves the efficiency. Since the GA method requires global optimization of parameters, which is more challenging and necessitates multiple repeated estimations using the relax strategy, the extraction process is extremely time-consuming. In contrast, the proposed method, through simple initialization and fully gradient-based estimation, completes the extraction in just $11.85s$ while ensuring extraction accuracy.

V. CONCLUSION

This letter proposes a fully gradient-based efficient ASC extraction method. By designing a more convex-like loss function and a joint image-frequency domain initialization strategy, it mitigates the time-consuming issue of global optimization methods, achieving second-level ASC extraction of tank targets while maintaining accuracy. This improvement benefits downstream tasks such as SAR target characteristic analysis and recognition.

REFERENCES

[1] M. Gerry, L. Potter, I. Gupta, and A. Van Der Merwe, "A parametric model for synthetic aperture radar measurements," *IEEE Transactions on Antennas and Propagation*, vol. 47, no. 7, pp. 1179–1188, 1999.

[2] F.-Y. Zhu, S.-R. Chai, Y.-F. Zou, Z.-X. He, and L.-X. Guo, "An efficient and accurate rcs reconstruction technique using adaptive tls-esprit algorithm," *IEEE Antennas and Wireless Propagation Letters*, vol. 23, no. 1, pp. 49–53, 2024.

[3] G. Peng, X. Zhang, X. Zhao, Z. Lin, Y. Zhang, and S. Zuo, "An efficient attributed scattering center extraction method accelerated by dictionary reuse," *IEEE Antennas and Wireless Propagation Letters*, vol. 23, no. 12, pp. 4847–4851, 2024.

[4] L. Potter and R. Moses, "Attributed scattering centers for sar atr," *IEEE Transactions on Image Processing*, vol. 6, no. 1, pp. 79–91, 1997.

[5] J. Zhou, S. Feng, H. Sun, L. Zhang, and G. Kuang, "Attributed scattering center guided adversarial attack for dcnn sar target recognition," *IEEE Geoscience and Remote Sensing Letters*, vol. 20, pp. 1–5, 2023.

[6] J. Luan, J. Ding, and Y. Zhang, "Unsupervised cross-domain radar target recognition using multilevel alignment," *IEEE Transactions on Radar Systems*, vol. 3, pp. 630–644, 2025.

[7] J. Zhang, M. Xing, and Y. Xie, "Fec: A feature fusion framework for sar target recognition based on electromagnetic scattering features and deep cnn features," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 59, no. 3, pp. 2174–2187, 2021.

[8] J. Zhang, K. Ji, X. Xing *et al.*, "Feature extraction and analysis of attributed scattering centers on sar targets," *Radar Science and Technology*, vol. 9, no. 3, pp. 207–212, 2011.

[9] J. Chen, X. Zhang, H. Wang, and F. Xu, "A reinforcement learning framework for scattering feature extraction and sar image interpretation," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 62, pp. 1–14, 2024.

[10] X. Zhang, "Noise-robust target recognition of sar images based on attribute scattering center matching," *Remote Sensing Letters*, vol. 10, pp. 186 – 194, 2018.

[11] H. Liu, B. Jiu, F. Li, and Y. Wang, "Attributed scattering center extraction algorithm based on sparse representation with dictionary refinement," *IEEE Transactions on Antennas and Propagation*, vol. 65, no. 5, pp. 2604–2614, 2017.

[12] M. Jing and G. Zhang, "Attributed scattering center extraction with genetic algorithm," *IEEE Transactions on Antennas and Propagation*, vol. 69, no. 5, pp. 2810–2819, 2021.

[13] X. Shen, Z. Zhuang, H. Wang, and F. Shu, "An effective method for attributed scattering center extraction based on an improved esprit algorithm," *IEEE Transactions on Antennas and Propagation*, pp. 1–1, 2024.

[14] E. R. Keydel, S. W. Lee, and J. T. Moore, "Mstar extended operating conditions: A tutorial," *Algorithms for Synthetic Aperture Radar Imagery III*, vol. 2757, pp. 228–242, 1996.

[15] S. Xiao, "A study on electromagnetic scattering parametric model extraction based on radar image," Master's thesis, Xidian University, Xi'an, China, 2024, [Online]. Available: OMP+.