Solutions to Homework 1

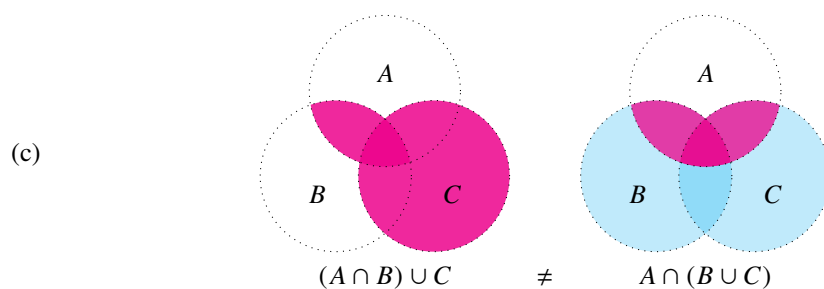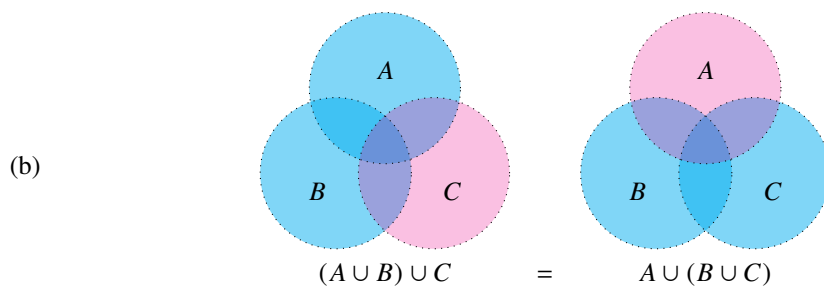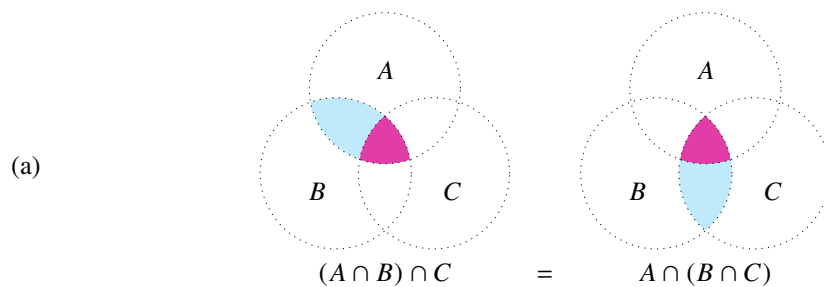## A. Sets

1. Evaluate the following claims:
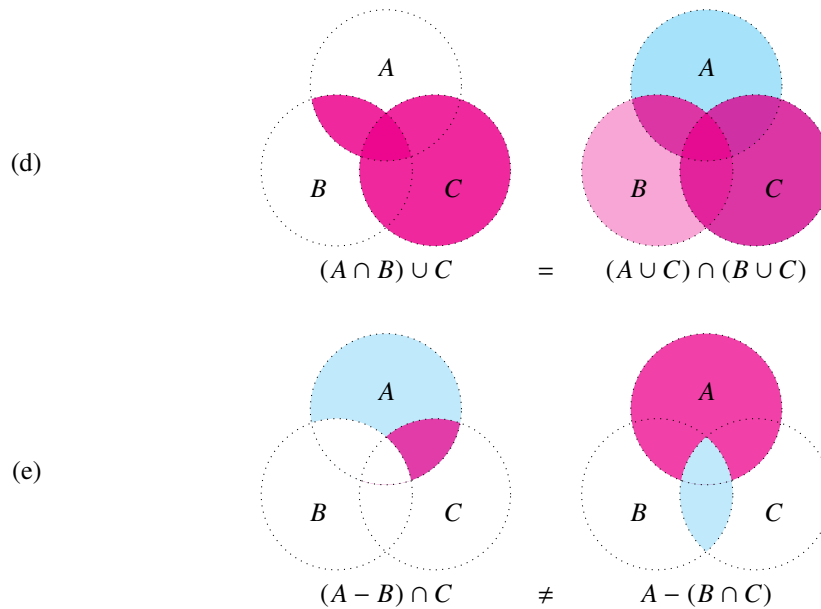
    (a) $\emptyset \in \{\emptyset\}$: True!

    (b) $\emptyset \subset \{\emptyset\}$: **Also** True! First, $\emptyset \subseteq \{\emptyset\}$ since everything in $\emptyset$ is in $\{\emptyset\}$. Second, $\{\emptyset\} \nsubseteq \emptyset$.

2. $A \cup Z = A$ whenever $Z \subseteq A$.

3. $A \cap Z = A$ whenever $A \subseteq Z$.

4. Venn diagrams (where there is a dark magenta, take that to be the final set):

    (a)

    $(A \cap B) \cap C$ $\quad = \quad$ $A \cap (B \cap C)$

    (b)

    $(A \cup B) \cup C$ $\quad = \quad$ $A \cup (B \cup C)$

    (c)

    $(A \cap B) \cup C$ $\quad \neq \quad$ $A \cap (B \cup C)$

(d)

$(A \cap B) \cup C \quad = \quad (A \cup C) \cap (B \cup C)$

(e)

$(A - B) \cap C \quad \neq \quad A - (B \cap C)$

B. Relations and functions

1. (a) The at-least-as-old-as relation is not an equivalence relation since it isn't symmetric. E.g., I'm (probably) as old as you, but you're not (probably) as old as me.

   (b) The same-age-as relation is an equivalence relation. It's reflexive since everyone's the same age as themself. It's symmetric since if x is the same age as y, y is the same age as y. And it's transitive since if x and y have the same age, and so do y and z, we can conclude that x and z have the same age.

2. (a) If $f(x) = x \times 5$, $f^{-1}(x) = \frac{x}{5}$.

   (b) The inverse of the identity function is the identity function.

   (c) f^{-1} fails to be a function whenever $f(x_1) = f(x_2)$, for distinct choices of x_1 and x_2.

   (d) $f^{-1}(f(x)) = x$

3. (a) $(f \circ g)(x)$ = the father of (the mother of x) = x's maternal grandfather.

   (b) $(f^{-1} \circ f)(x) = f^{-1}(f(x)) = x$

C. Problem 4

Gricean reasoning doesn't apply to sentences of the form $\neg(p \vee q)$ in the same way it applies to sentences of the form $p \vee q$. In the latter case, the existence of a **strictly stronger** utterance $p \wedge q$ that the speaker could have uttered leads (by the Maxim of Quantity) to the conclusion that the speaker doesn't believe $p \wedge q$. The reason this fails for $\neg(p \vee q)$ is that $\neg(p \wedge q)$ **isn't** strictly stronger; in fact, it's strictly weaker.

By the way, this reasoning would lead you to expect that when a speaker utters something of the form $\neg(p \wedge q)$, she must believe that the stronger possible utterance $\neg(p \vee q)$ is false — in other words, she believes $p \vee q$. This seems to be correct: "Emma didn't read P&P and GE" does seem to implicate that she read at least one of them.