

```
In [1]: import yfinance as yf
import datetime
import pandas as pd
import numpy as np
from scipy.stats import norm
import statsmodels.api as sm

# Define the stock symbols
symbols = ["RY.TO", "SHOP.TO", "CNQ.TO", "VCN.TO"]

# Define the start and end dates
start_date = datetime.datetime(2018, 5, 1)
end_date = datetime.datetime(2023, 4, 29)

# Download the stock data
data = yf.download(symbols, start=start_date, end=end_date)["Adj Close"]

# Print the downloaded data
print(data)
```

[\*\*\*\*\*100%\*\*\*\*\*] 4 of 4 completed

	CNQ.TO	RY.TO	SHOP.TO	VCN.TO
Date				
2018-05-01	33.902885	79.370392	16.350000	27.247820
2018-05-02	34.234322	79.402901	16.099001	27.264994
2018-05-03	33.490417	79.362267	17.349001	27.239231
2018-05-04	33.645088	79.500404	17.495001	27.453918
2018-05-07	33.365208	80.256187	17.855000	27.574144
...	...	...	...	...
2023-04-24	80.814072	132.105591	65.019997	41.602375
2023-04-25	79.579285	130.413055	63.340000	41.106163
2023-04-26	79.292816	130.878265	63.279999	40.967224
2023-04-27	79.905273	132.422302	64.669998	41.314575
2023-04-28	81.554939	133.134949	65.639999	41.532906

[1254 rows x 4 columns]

```
In [2]: # Calculate the percentage change
pct_returns = data.pct_change()

# Print the percentage change
print(pct_returns)
```

```

# Define the initial investment for each stock
initial_investment = 25000.00

# Create an empty DataFrame to store the stock values
stock_values = pd.DataFrame(index=data.index)

# Iterate over the stock symbols and calculate the stock values
for symbol in symbols:
    stock_values[symbol] = (pct_returns[symbol] + 1).cumprod() * initial_investment
    stock_values.loc[start_date, symbol] = initial_investment

# Print the stock values
print(stock_values)

```

	CNQ.TO	RY.TO	SHOP.TO	VCN.TO
Date				
2018-05-01	NaN	NaN	NaN	NaN
2018-05-02	0.009776	0.000410	-0.015352	0.000630
2018-05-03	-0.021730	-0.000512	0.077645	-0.000945
2018-05-04	0.004618	0.001741	0.008415	0.007882
2018-05-07	-0.008319	0.009507	0.020577	0.004379
...	...	...	...	...
2023-04-24	0.010125	-0.003807	-0.018862	0.000000
2023-04-25	-0.015279	-0.012812	-0.025838	-0.011927
2023-04-26	-0.003600	0.003567	-0.000947	-0.003380
2023-04-27	0.007724	0.011798	0.021966	0.008479
2023-04-28	0.020645	0.005382	0.014999	0.005285

[1254 rows x 4 columns]

	RY.TO	SHOP.TO	CNQ.TO	VCN.TO
Date				
2018-05-01	25000.000000	25000.000000	25000.000000	25000.000000
2018-05-02	25010.239602	24616.208800	25244.401142	25015.757010
2018-05-03	24997.440700	26527.523740	24695.845979	24992.119745
2018-05-04	25040.951200	26750.765185	24809.900221	25189.096373
2018-05-07	25279.006936	27301.221905	24603.516221	25299.404195
...	...	...	...	...
2023-04-24	41610.475817	99418.952792	59592.325707	38170.370311
2023-04-25	41077.362851	96850.150879	58681.793336	37715.093515
2023-04-26	41223.894181	96758.405662	58470.551356	37587.616432
2023-04-27	41710.233239	98883.786942	58922.177572	37906.312639
2023-04-28	41934.701856	100366.969202	60138.641755	38106.631769

[1254 rows x 4 columns]

# Value-at-Risk Calculations for Individual Securities

Historical Value-at-Risk at 95% and 99% Confidence Levels

```
In [3]: # Define the confidence levels
confidence_levels = [0.95, 0.99]

# Define the initial investment amount in each security
investment_amount = 10000

# Calculate the HVaR
for symbol in symbols:
    print("Security:", symbol)
    returns = pct_returns[symbol].dropna()
    for confidence in confidence_levels:
        var = np.percentile(returns, (1 - confidence) * 100)
        hvar = -var * investment_amount
        print("1-Day HVaR ({}%):".format(confidence * 100), hvar)
    print()
```

```
Security: RY.TO
1-Day HVaR (95.0%): 158.45065509060126
1-Day HVaR (99.0%): 344.8108077840831
```

```
Security: SHOP.TO
1-Day HVaR (95.0%): 601.8191151678225
1-Day HVaR (99.0%): 1091.710540384408
```

```
Security: CNQ.TO
1-Day HVaR (95.0%): 356.39910605849576
1-Day HVaR (99.0%): 640.0848652525268
```

```
Security: VCN.TO
1-Day HVaR (95.0%): 137.63982696858247
1-Day HVaR (99.0%): 275.1044096269951
```

. Parametric Value-at-Risk at 95% and 99% Confidence Levels

```
In [4]: # Define the confidence levels
confidence_levels = [0.95, 0.99]
```

```

# Calculate the PVaR
for symbol in symbols:
    print("Security:", symbol)
    returns = pct_returns[symbol].dropna()
    std = returns.std()
    for confidence in confidence_levels:
        z_score = norm.ppf(1 - confidence)
        pvar = -z_score * std * investment_amount
        print("Parametric VaR ({}%):".format(confidence * 100), pvar)
    print()

```

Security: RY.TO  
 Parametric VaR (95.0%): 210.4067358659806  
 Parametric VaR (99.0%): 297.5822618167457

Security: SHOP.TO  
 Parametric VaR (95.0%): 649.8579713071781  
 Parametric VaR (99.0%): 919.1065303366046

Security: CNQ.TO  
 Parametric VaR (95.0%): 477.443579792514  
 Parametric VaR (99.0%): 675.2575661600388

Security: VCN.TO  
 Parametric VaR (95.0%): 185.38641831584874  
 Parametric VaR (99.0%): 262.195549232202

## Value-at-Risk Calculations for the Portfolio

```

In [5]: # Calculate the portfolio value by summing the values of each stock
portfolio_values = stock_values.sum(axis=1)

# Print the portfolio values
print(portfolio_values)

# Calculate the percentage change
portfolio_returns = portfolio_values.pct_change()

# Print the portfolio return
print(portfolio_returns)

# Calculate the weight of each stock in the portfolio

```

```
stock_weights = stock_values.div(portfolio_values, axis=0)

# Print the stock weights
print(stock_weights)
```

```

Date
2018-05-01    100000.000000
2018-05-02     99886.606555
2018-05-03    101212.930165
2018-05-04    101790.712979
2018-05-07    102483.149257
...
2023-04-24    238792.124628
2023-04-25    234324.400581
2023-04-26    234040.467630
2023-04-27    237422.510393
2023-04-28    240546.944582
Length: 1254, dtype: float64
Date
2018-05-01         NaN
2018-05-02    -0.001134
2018-05-03     0.013278
2018-05-04     0.005709
2018-05-07     0.006803
...
2023-04-24    -0.006131
2023-04-25    -0.018710
2023-04-26    -0.001212
2023-04-27     0.014451
2023-04-28     0.013160
Length: 1254, dtype: float64
      RY.TO  SHOP.TO  CNQ.TO  VCN.TO
Date
2018-05-01  0.250000  0.250000  0.250000  0.250000
2018-05-02  0.250386  0.246442  0.252731  0.250442
2018-05-03  0.246979  0.262096  0.243999  0.246926
2018-05-04  0.246004  0.262802  0.243734  0.247460
2018-05-07  0.246665  0.266397  0.240074  0.246864
...
2023-04-24  0.174254  0.416341  0.249557  0.159848
2023-04-25  0.175301  0.413317  0.250430  0.160952
2023-04-26  0.176140  0.413426  0.249831  0.160603
2023-04-27  0.175679  0.416489  0.248174  0.159658
2023-04-28  0.174331  0.417245  0.250008  0.158417

[1254 rows x 4 columns]

```

```

In [6]: # Define the initial investment for the portfolio
portfolio_initial_investment = 100000.00

```

```

# Get the weight of the last day (2023-04-28)
last_day_weights = stock_weights.loc['2023-04-28']

# Calculate the weighted return for each stock
stock_weighted_returns = pct_returns * last_day_weights

# Calculate the weighted return for the portfolio
portfolio_weighted_return = stock_weighted_returns.sum(axis=1)

# Create a new DataFrame for portfolio values
portfolio_adjusted_values = pd.DataFrame(index=pct_returns.index, columns=['Value'])

# Calculate the portfolio values for each day
for i in range(len(portfolio_adjusted_values)):
    if i == 0:
        # Set the initial value for the first day
        portfolio_adjusted_values.iloc[i] = portfolio_initial_investment
    else:
        portfolio_adjusted_values.iloc[i] = portfolio_adjusted_values.iloc[i-1] * (1 + portfolio_weighted_return.iloc[i])

# Print the portfolio values
print(portfolio_adjusted_values)

```

Date	Value
2018-05-01	100000.0
2018-05-02	99620.994304
2018-05-03	102283.393194
2018-05-04	102919.385513
2018-05-07	103830.948923
...	...
2023-04-24	347209.892152
2023-04-25	340708.793103
2023-04-26	340296.943331
2023-04-27	345229.902146
2023-04-28	349785.274871

[1254 rows x 1 columns]

Historical Value-at-Risk of the Portfolio at 95% and 99% Confidence Levels

```

In [7]: # Define the confidence levels
confidence_levels = [0.95, 0.99]

# Define the initial investment amount

```

```

portfolio_investment = 10000

# Calculate the HVaR
for confidence in confidence_levels:
    portfolio_returns = portfolio_returns.dropna()
    var = np.percentile(portfolio_returns, (1 - confidence) * 100)
    hvar = -var * portfolio_investment
    print("1-Day HVaR ({}%):".format(confidence * 100), hvar)
print()

```

1-Day HVaR (95.0%): 340.99458498769536

1-Day HVaR (99.0%): 606.2085179651153

Parametric Value-at-Risk of the Portfolio at 95% and 99% Confidence Levels

```

In [8]: # Define the confidence levels
confidence_levels = [0.95, 0.99]

# Define the initial investment amount
portfolio_investment = 10000

# Calculate the PVaR
for confidence in confidence_levels:
    portfolio_returns = portfolio_returns.dropna()
    std = portfolio_returns.std()
    z_score = norm.ppf(1 - confidence)
    pvar = -z_score * std * portfolio_investment
    print("1-Day PVaR ({}%):".format(confidence * 100), pvar)
print()

```

1-Day PVaR (95.0%): 379.47870258766466

1-Day PVaR (99.0%): 536.7039708236819

Adjusted Portfolio Return (HVar & PVar)

```

In [9]: # Define the confidence levels
confidence_levels = [0.95, 0.99]

# Define the initial investment amount
portfolio_investment = 10000

# Calculate the HVaR
for confidence in confidence_levels:

```



```

var = np.percentile(portfolio_weighted_return, (1 - confidence) * 100)
hvar = -var * portfolio_investment
print("1-Day HVaR for adjusted portfolio ({})%:".format(confidence * 100), hvar)
print()

```

1-Day HVaR for adjusted portfolio (95.0%): 301.7430821380544  
1-Day HVaR for adjusted portfolio (99.0%): 562.6241837124171

```

In [10]: # Define the confidence levels
confidence_levels = [0.95, 0.99]

# Define the initial investment amount
portfolio_investment = 10000

# Calculate the PVaR
for confidence in confidence_levels:
    adjusted_std = portfolio_weighted_return.std()
    z_score = norm.ppf(1 - confidence)
    pvar = -z_score * adjusted_std * portfolio_investment
    print("1-Day PVaR for adjusted portfolio ({})%:".format(confidence * 100), pvar)
print()

```

1-Day PVaR for adjusted portfolio (95.0%): 344.9642165005178  
1-Day PVaR for adjusted portfolio (99.0%): 487.8894744959699

## Parametric Value-at-Risk with the S&P/TSX Composite Index as the Risk Factor

Calculating Parametric VaR at 95% and 99% Confidence Levels with S&P/TSX Composite Index

```

In [11]: # Define the stock symbols
symbols_five= ["RY.TO", "SHOP.TO", "CNQ.TO", "VCN.TO", "^GSPTSE"]

# Define the start and end dates
start_date = datetime.datetime(2018, 5, 1)
end_date = datetime.datetime(2023, 4, 29)

# Download the stock data
data_with_SP = yf.download(symbols_five, start=start_date, end=end_date)["Adj Close"]

```

```
# Print the downloaded data
```

```
print(data_with_SP)
```

```
[*****100%*****] 5 of 5 completed
```

	CNQ.TO	RY.TO	SHOP.TO	VCN.TO	^GSPTSE
Date					
2018-05-01	33.902874	79.370399	16.350000	27.247822	15618.900391
2018-05-02	34.234322	79.402901	16.099001	27.264996	15627.900391
2018-05-03	33.490417	79.362274	17.349001	27.239233	15621.500000
2018-05-04	33.645088	79.500404	17.495001	27.453918	15729.400391
2018-05-07	33.365200	80.256157	17.855000	27.574141	15808.599609
...	...	...	...	...	...
2023-04-24	80.814072	132.105591	65.019997	41.602375	20676.699219
2023-04-25	79.579285	130.413055	63.340000	41.106163	20439.900391
2023-04-26	79.292816	130.878265	63.279999	40.967224	20366.699219
2023-04-27	79.905273	132.422302	64.669998	41.314575	20522.599609
2023-04-28	81.554939	133.134949	65.639999	41.532906	20636.500000

```
[1254 rows x 5 columns]
```

In [12]:

```
# Calculate the returns
```

```
returns = data_with_SP.pct_change().dropna()
```

```
# Split the data into x and y variables
```

```
x_variable = returns["^GSPTSE"]
```

```
y_variables = returns[["RY.TO", "SHOP.TO", "CNQ.TO", "VCN.TO"]]
```

```
# Add a constant column to the x_variables
```

```
x_variables = sm.add_constant(x_variable)
```

```
# Perform separate regressions and extract coefficients
```

```
betas = {}
```

```
for column in y_variables:
```

```
    model = sm.OLS(y_variables[column], x_variables)
```

```
    results = model.fit()
```

```
    beta = results.params[1] # Extract the coefficient for the independent variable (x)
```

```
    betas[column] = beta
```

```
# Print the beta coefficients
```

```
for symbol, beta in betas.items():
```

```
    print(f"Beta for {symbol}: {beta}")
```

Beta for RY.TO: 0.944065042615045  
Beta for SHOP.TO: 1.51054031825421  
Beta for CNQ.TO: 1.6904186073090786  
Beta for VCN.TO: 0.9874635326790003

In [13]: *# Calculate the square of each beta and multiply by the variance of ^GSPTSE*

```
sd_betas = {}  
variance = x_variable.var()  
for symbol, beta in betas.items():  
    sd_beta = np.sqrt(beta ** 2 * variance)  
    sd_betas[symbol] = sd_beta
```

*# Print the adjusted beta values*

```
for symbol, sd_beta in sd_betas.items():  
    print(f"SD for {symbol}: {sd_beta}")
```

SD for RY.TO: 0.010744458254609002  
SD for SHOP.TO: 0.017191545771496307  
SD for CNQ.TO: 0.01923875086904657  
SD for VCN.TO: 0.011238378952608378

In [14]: *# Define the confidence levels*

```
confidence_levels = [0.95, 0.99]
```

*# Define the initial investment amount*

```
investment_amount = 10000
```

*# Calculate the PVaR*

```
for symbol, sd_beta in sd_betas.items():  
    print("Security:", symbol)  
    for confidence in confidence_levels:  
        z_score = norm.ppf(1 - confidence)  
        pvar = -z_score * sd_beta * investment_amount  
        print("Parametric VaR ({}%):".format(confidence * 100), pvar)  
    print()
```

Security: RY.TO  
Parametric VaR (95.0%): 176.73061129722302  
Parametric VaR (99.0%): 249.95347618330214

Security: SHOP.TO  
Parametric VaR (95.0%): 282.77576415147945  
Parametric VaR (99.0%): 399.9351595699624

Security: CNQ.TO  
Parametric VaR (95.0%): 316.4492914496704  
Parametric VaR (99.0%): 447.5602718340786

Security: VCN.TO  
Parametric VaR (95.0%): 184.85488381252978  
Parametric VaR (99.0%): 261.4437898406583

```
In [15]: weighted_beta_variance = {}

for date in returns.index:
    weighted_beta_var = 0

    for symbol, beta in betas.items():
        weight = stock_weights.loc[date, symbol]
        variance = x_variable.var()
        weighted_beta_var += beta ** 2 * weight ** 2 * variance

    weighted_beta_variance[date] = weighted_beta_var

# Calculate the average weighted beta variance
average_weighted_beta_variance = np.mean(list(weighted_beta_variance.values()))

weighted_beta_sd = np.sqrt(average_weighted_beta_variance)

print("portfolio SD:", weighted_beta_sd)

portfolio SD: 0.010265698315931836
```

```
In [16]: # Define the confidence levels
confidence_levels = [0.95, 0.99]

# Define the initial investment amount
portfolio_investment = 10000

# Calculate the PVaR
```

```

for confidence in confidence_levels:
    z_score = norm.ppf(1 - confidence)
    pvar = -z_score * weighted_beta_sd * portfolio_investment
    print("1-Day PVaR ({}%):".format(confidence * 100), pvar)
print()

```

1-Day PVaR (95.0%): 168.85571108150103

1-Day PVaR (99.0%): 238.81585452812666

```

In [17]: # Calculate the squared weights
squared_adjusted_weights = np.square(last_day_weights)

# Calculate the squared betas
squared_adjusted_betas = {symbol: beta ** 2 for symbol, beta in betas.items()}

# Calculate the adjusted portfolio variance
weighted_adjusted_betas = squared_adjusted_weights * np.array(list(squared_adjusted_betas.values())) * variance

adjusted_portfolio_variance = np.sum(weighted_adjusted_betas)

# Caculate SD
adjusted_portfolio_sd = np.sqrt(adjusted_portfolio_variance)

print(f"Adjusted Portfolio SD: {adjusted_portfolio_sd}")

```

Adjusted Portfolio SD: 0.009014753551404539

Calculating Portfolio Parametric VaR with S&P/TSX Composite Index as the Risk Factor

```

In [18]: # Define the confidence levels
confidence_levels = [0.95, 0.99]

# Define the initial investment amount
portfolio_investment = 10000

# Calculate the PVaR
for confidence in confidence_levels:
    z_score = norm.ppf(1 - confidence)
    pvar = -z_score * adjusted_portfolio_sd * portfolio_investment
    print("1-Day adjusted PVaR ({}%):".format(confidence * 100), pvar)
print()

```

1-Day adjusted PVaR (95.0%): 148.27950075101418

1-Day adjusted PVaR (99.0%): 209.71452759312066

## Parametric VaR Using Exponentially Weighted Moving Average (EWMA)

Calculating EWMA Parametric VaR at 95% and 99% Confidence Levels for each Security

```
In [19]: # Set the Lambda ( $\lambda$ ) value
lambda_ = 0.94

# Calculate the EWMA volatility for each security
ewma_volatility = pct_returns.ewm(alpha=1 - lambda_).std()

# Drop NaN values from the EWMA volatility DataFrame
ewma_volatility = ewma_volatility.dropna()

# Define the initial investment amount
investment_amount = 10000

# Calculate the PVaR
confidence_levels = [0.95, 0.99]

for security in ewma_volatility.columns:
    print("Security:", security)
    ewma_std = ewma_volatility[security].values[-1] # Get the latest EWMA volatility
    for confidence in confidence_levels:
        z_score = norm.ppf(1 - confidence)
        ewma_pvar = -ewma_std * z_score * investment_amount
        print("EWMA PVaR ({}%):".format(confidence * 100), ewma_pvar)
    print()
```

Security: CNQ.TO  
EWMA PVaR (95.0%): 267.793347068433  
EWMA PVaR (99.0%): 378.74524117355384

Security: RY.TO  
EWMA PVaR (95.0%): 129.25660111053975  
EWMA PVaR (99.0%): 182.81007760948958

Security: SHOP.TO  
EWMA PVaR (95.0%): 418.6481488590663  
EWMA PVaR (99.0%): 592.1020661725761

Security: VCN.TO  
EWMA PVaR (95.0%): 99.1750002041381  
EWMA PVaR (99.0%): 140.2650953875444

```
In [20]: # Set the lambda ( $\lambda$ ) value
lambda_ = 0.94

# Calculate the EWMA volatility for the portfolio
ewma_volatility = portfolio_returns.ewm(alpha=1 - lambda_).std()

# Drop NaN values from the EWMA volatility Series
ewma_volatility = ewma_volatility.dropna()

# Define the initial investment amount
investment_amount = 10000

# Calculate the PVaR for the portfolio
confidence_levels = [0.95, 0.99]
portfolio_ewma_pvar = {}

for confidence in confidence_levels:
    z_score = norm.ppf(1 - confidence)
    portfolio_pvar_ewma = -z_score * ewma_volatility[-1] * investment_amount
    portfolio_ewma_pvar[confidence] = portfolio_pvar_ewma

# Print the results
for confidence, pvar in portfolio_ewma_pvar.items():
    print("Portfolio EWMA PVaR ({}%):".format(confidence * 100), pvar)
```

Portfolio EWMA PVaR (95.0%): 219.43384957876177  
Portfolio EWMA PVaR (99.0%): 310.34948100899356

## Calculating EWMA Parametric VaR for the Portfolio at 95% and 99% Confidence Levels

```
In [21]: # Set the Lambda ( $\lambda$ ) value
lambda_ = 0.94

# Calculate the EWMA volatility for the portfolio
ewma_volatility = portfolio_weighted_return.ewm(alpha=1 - lambda_).std()

# Drop NaN values from the EWMA volatility Series
ewma_volatility = ewma_volatility.dropna()

# Define the initial investment amount
investment_amount = 10000

# Calculate the PVaR for the portfolio
confidence_levels = [0.95, 0.99]
portfolio_ewma_pvar = {}

for confidence in confidence_levels:
    z_score = norm.ppf(1 - confidence)
    portfolio_pvar_adjusted_ewma = -z_score * ewma_volatility[-1] * investment_amount
    portfolio_ewma_pvar[confidence] = portfolio_pvar_adjusted_ewma

# Print the results
for confidence, pvar in portfolio_ewma_pvar.items():
    print("Portfolio EWMA PVaR ({}%):".format(confidence * 100), pvar)
```

Portfolio EWMA PVaR (95.0%): 220.5736342958308

Portfolio EWMA PVaR (99.0%): 311.9615002853422