Question 1 (20%) excel 6 ex5

The term structure of interest rates is modeled by the following 8 key rates (APR, semi-annually compounded):

Order	Term (year)	Key rate
1	0.5	4.061%
2	1	4.190%
3	2	3.686%
4	3	3.355%
5	5	3.073%
6	7	2.986%
7	10	2.873%
8	30	2.655%

a) (9%) Calculate the first 5 key rate durations (with maturities ranging from 0.5 to 5 years) for each one of the following 6 Treasury bonds:

	Coupon (%)	Maturity (years)
1	5.50%	0.8
2	4.50%	1.7
3	5.25%	2.25
4	4.75%	3.5
5	4.25%	4.75
6	5.75%	5.25

	tvec	rvec
Order	Term (year)	Key rate
1	0.5	4.061%
2	1	4.190%
3	2	3.686%
4	3	3.355%
5	5	3.073%
6	7	2.986%

Calculate bond price: Bond_ZeroRates(par, mat, coupon, freq, tvec, rvec)

Bond	Coupon rate	Maturity (years)	Par	Frequency	Price
1	5.50%	0.80	$ 100.00	2	$ 102.15
2	4.50%	1.70	$ 100.00	2	$ 102.41
3	5.25%	2.25	$ 100.00	2	$ 104.80
4	4.75%	3.50	$ 100.00	2	$ 104.72
5	4.25%	4.75	$ 100.00	2	$ 105.93
6	5.75%	5.25	$ 100.00	2	$ 114.16

Calculate key rate durations (by numerical differentiation):

\delta k	0.0001	-0.0001

New rate

Term(Yrs)	Key Rate #	1		2		3		4		5		6	
		r+	r-	r+	r-	r+	r-	r+	r-	r+	r-	r+	r-
0.5	1	4.0710%	4.0510%	4.0610%	4.0610%	4.0610%	4.0610%	4.0610%	4.0610%	4.0610%	4.0610%	4.0610%	4.0610%
1	2	4.1900%	4.1900%	4.2000%	4.1800%	4.1900%	4.1900%	4.1900%	4.1900%	4.1900%	4.1900%	4.1900%	4.1900%
2	3	3.6860%	3.6860%	3.6860%	3.6860%	3.6960%	3.6760%	3.6860%	3.6860%	3.6860%	3.6860%	3.6860%	3.6860%
3	4	3.3550%	3.3550%	3.3550%	3.3550%	3.3550%	3.3550%	3.3650%	3.3450%	3.3550%	3.3550%	3.3550%	3.3550%
5	5	3.0730%	3.0730%	3.0730%	3.0730%	3.0730%	3.0730%	3.0730%	3.0730%	3.0830%	3.0630%	3.0730%	3.0730%
7	6	2.9860%	2.9860%	2.9860%	2.9860%	2.9860%	2.9860%	2.9860%	2.9860%	2.9860%	2.9860%	2.9960%	2.9760%

Bond price

	P+	P-	P+	P-	P+	P-	P+	P-	P+	P-	P+	P-
1	$ 102.15	$ 102.16	$ 102.15	$ 102.16	$ 102.15	$ 102.15	$ 102.15	$ 102.15	$ 102.15	$ 102.15	$ 102.15	$ 102.15
2	$ 102.41	$ 102.41	$ 102.41	$ 102.42	$ 102.40	$ 102.43	$ 102.41	$ 102.41	$ 102.41	$ 102.41	$ 102.41	$ 102.41
3	$ 104.80	$ 104.80	$ 104.80	$ 104.80	$ 104.78	$ 104.82	$ 104.79	$ 104.80	$ 104.80	$ 104.80	$ 104.80	$ 104.80
4	$ 104.72	$ 104.72	$ 104.72	$ 104.72	$ 104.72	$ 104.72	$ 104.70	$ 104.75	$ 104.71	$ 104.73	$ 104.72	$ 104.72
5	$ 105.93	$ 105.93	$ 105.93	$ 105.93	$ 105.93	$ 105.93	$ 105.92	$ 105.94	$ 105.89	$ 105.97	$ 105.93	$ 105.93
6	$ 114.16	$ 114.16	$ 114.16	$ 114.16	$ 114.16	$ 114.16	$ 114.16	$ 114.17	$ 114.12	$ 114.21	$ 114.16	$ 114.17

Key Dur

1	**0.312994597**	**0.457761317**	**0**	**0**	**0**
2	**0.013059306**	**0.493883228**	**1.097707817**	**0**	**0**
3	**0.015001477**	**0.040877217**	**1.535406745**	**0.499304657**	**0**
4	**0.010892629**	**0.037044263**	**0.082663419**	**2.338709879**	**0.750873767**
5	**0.012014603**	**0.032738346**	**0.07305911**	**0.653061503**	**3.48564593**
6	**0.015082568**	**0.04109818**	**0.091714969**	**0.221367521**	**3.662395933**

Key rate duration summary:

	Key rate duration				
Order	1	2	3	4	5
Term (year)	0.5	1	2	3	5
	Bond #				
1	0.312994597	0.457761317	0	0	0
2	0.013059306	0.493883228	1.097707817	0	0
3	0.015001477	0.040877217	1.535406745	0.499304657	0
4	0.010892629	0.037044263	0.082663419	2.338709879	0.750873767
5	0.012014603	0.032738346	0.07305911	0.653061503	3.48564593
6	0.015082568	0.04109818	0.091714969	0.221367521	3.662395933

b) (4%) Consider the funding operation of the following annual liabilities:

Year	Liability ($000)
1	1000
2	1250
3	950
4	900
5	800

For each year, the annual liability is paid twice a year, the first half at the midpoint of the year and the second half at yearend (for example, half of the first year's $1 million is paid in 6 months and the other half in one year). What are the first 5 key rate durations of the portfolio of the liabilities?

Calculate spot rate given the key rate

			1		2		3		4		5	
Term	**Spot Rate**		**Z+**	**Z-**	**Z+**	**Z-**	**Z+**	**Z-**	**Z+**	**Z-**	**Z+**	**Z-**
0.5	4.0610%		2.0355%	2.0255%	4.0610%	4.0610%	4.0610%	4.0610%	4.0610%	4.0610%	4.0610%	4.0610%
1	4.1900%		2.0950%	2.0950%	4.2000%	4.1800%	4.1900%	4.1900%	4.1900%	4.1900%	4.1900%	4.1900%
1.5	3.9380%		1.9690%	1.9690%	3.9430%	3.9330%	3.9430%	3.9330%	3.9380%	3.9380%	3.9380%	3.9380%
2	3.6860%		1.8430%	1.8430%	3.6860%	3.6860%	3.6960%	3.6760%	3.6860%	3.6860%	3.6860%	3.6860%
2.5	3.5205%		1.7603%	1.7603%	3.5205%	3.5205%	3.5255%	3.5155%	3.5255%	3.5155%	3.5205%	3.5205%
3	3.3550%		1.6775%	1.6775%	3.3550%	3.3550%	3.3550%	3.3550%	3.3650%	3.3450%	3.3550%	3.3550%
3.5	3.2845%		1.6423%	1.6423%	3.2845%	3.2845%	3.2845%	3.2845%	3.2920%	3.2770%	3.2870%	3.2820%
4	3.2140%		1.6070%	1.6070%	3.2140%	3.2140%	3.2140%	3.2140%	3.2190%	3.2090%	3.2190%	3.2090%
4.5	3.1435%		1.5718%	1.5718%	3.1435%	3.1435%	3.1435%	3.1435%	3.1460%	3.1410%	3.1510%	3.1360%
5	3.0730%		1.5365%	1.5365%	3.0730%	3.0730%	3.0730%	3.0730%	3.0730%	3.0730%	3.0830%	3.0630%

semi-annual payment of liabilities

n	**t**	**CF**	**PV(0)**	**PV(+)**	**PV(-)**	**PV(+)**	**PV(-)**	**PV(+)**	**PV(-)**	**PV(+)**	**PV(-)**	**PV(+)**	**PV(-)**
1	0.5	$ 500.00	$ 490.05	$ 490.03	$ 490.07	$ 490.05	$ 490.05	$ 490.05	$ 490.05	$ 490.05	$ 490.05	$ 490.05	$ 490.05
2	1	$ 500.00	$ 479.69	$ 479.69	$ 479.69	$ 479.64	$ 479.74	$ 479.69	$ 479.69	$ 479.69	$ 479.69	$ 479.69	$ 479.69
3	1.5	$ 625.00	$ 589.49	$ 589.49	$ 589.49	$ 589.45	$ 589.53	$ 589.45	$ 589.53	$ 589.49	$ 589.49	$ 589.49	$ 589.49
4	2	$ 625.00	$ 580.97	$ 580.97	$ 580.97	$ 580.97	$ 580.97	$ 580.86	$ 581.09	$ 580.97	$ 580.97	$ 580.97	$ 580.97
5	2.5	$ 475.00	$ 435.31	$ 435.31	$ 435.31	$ 435.31	$ 435.31	$ 435.26	$ 435.37	$ 435.26	$ 435.37	$ 435.31	$ 435.31
6	3	$ 475.00	$ 429.88	$ 429.88	$ 429.88	$ 429.88	$ 429.88	$ 429.88	$ 429.88	$ 429.75	$ 430.00	$ 429.88	$ 429.88
7	3.5	$ 450.00	$ 401.51	$ 401.51	$ 401.51	$ 401.51	$ 401.51	$ 401.51	$ 401.51	$ 401.40	$ 401.61	$ 401.47	$ 401.54
8	4	$ 450.00	$ 396.12	$ 396.12	$ 396.12	$ 396.12	$ 396.12	$ 396.12	$ 396.12	$ 396.04	$ 396.19	$ 396.04	$ 396.19
9	4.5	$ 400.00	$ 347.62	$ 347.62	$ 347.62	$ 347.62	$ 347.62	$ 347.62	$ 347.62	$ 347.58	$ 347.66	$ 347.50	$ 347.73
10	5	$ 400.00	$ 343.43	$ 343.43	$ 343.43	$ 343.43	$ 343.43	$ 343.43	$ 343.43	$ 343.43	$ 343.43	$ 343.26	$ 343.60
			$ 4,494.06	**$ 4,494.04**	**$ 4,494.09**	**$ 4,493.97**	**$ 4,494.16**	**$ 4,493.85**	**$ 4,494.28**	**$ 4,493.66**	**$ 4,494.47**	**$ 4,493.67**	**$ 4,494.46**
			Key Rate Duration	**0.053436826**		**0.201026596**		**0.469336058**		**0.891116451**		**0.883736848**	

Key Rate Duration Summary		
Order	**Term (year)**	**Key Rate Duration**
1	0.5	**0.053436826**
2	1	**0.201026596**
3	2	**0.469336058**

4	3	**0.891116451**
5	5	**0.883736848**

c) (7%) Find a portfolio of the 6 bonds in a) above that match the first 5 key rate durations of the liabilities given in b) above. Assume that the present value of the bond portfolio is 10% higher than the present value of the liabilities. Both shorting (negative weight) and leverage (> 100% weight) are allowed for any of the 6 bonds.

Target: 10%

		Key rate duration					
Order		1	2	3	4	5	
Term (year)		0.5	1	2	3	5	**Weight**
Bond #							
	1	0.312994597	0.457761317	0	0	0	1
	2	0.013059306	0.493883228	1.097707817	0	0	1
	3	0.015001477	0.040877217	1.535406745	0.499304657	0	1
	4	0.010892629	0.037044263	0.082663419	2.338709879	0.750873767	1
	5	0.012014603	0.032738346	0.07305911	0.653061503	3.48564593	1
	6	0.015082568	0.04109818	0.091714969	0.221367521	3.662395933	1
Key Rate Duration		**0.053436826**	**0.201026596**	**0.469336058**	**0.891116451**	**0.883736848**	
Target		0.048578933	0.182751451	0.426669143	0.810105864	0.803397135	1

Portfolio construction:

PV fund	$	4,494.06
Value of target Portfolio	$	4,943.47

Bond	**Weight**	**Holding**	**Price**	**# of Bond**
1	10.81%	$ 534.17	$ 102.15	5.23
2	19.39%	$ 958.69	$ 102.41	9.36
3	9.01%	$ 445.29	$ 104.80	4.25
4	56.98%	$ 2,816.59	$ 104.72	26.90
5	-133.39%	-$ 6,593.93	$ 105.93	-62.25
6	137.20%	$ 6,782.65	$ 114.16	59.41
Sum	1	$ 4,943.47		