

Spot rates in six-month increments are provided in the following table:

Term(Year)	Spot Rate
0.5	4.061%
1	4.172%
1.5	3.950%
2	3.634%
2.5	3.427%
3	3.267%
3.5	3.162%
4	3.059%
4.5	2.985%
5	2.915%

All rates are stated as APR with semi-annual compounding. Use the Black's model to answer the following questions:

- a) (10%) Consider a 5-year cap on the 6-month spot rate with a principal of \$100,000 and semi-annual payments. If the cap rate is 4.25% and the flat volatility for the cap is 17%, how much is the cap worth today?
b) (6%) Consider a 5-year floor on the 6-month spot rate with a principal of \$100,000 and semi-annual payments. If the floor rate is 2.5% and the flat volatility for the floor is 17%, how much is the floor worth today?

Principle	\$ 100,000.00	a)	Cap Rate	4.25%	b)	Floor Rate	2.50%
Maturity	5		Volatility	17%		Volatility	17%
Compounding Frequency	2						

Period	Term(Year)	Spot Rate	Forward Rate	Maturity(yr)	Settlement	Caplet valuation						Floorlet valuation					
						d1	d2	N(d1)	N(d2)	Caplet Payoff	Caplet PV	d1	d2	N(-d1)	N(-d2)	Floorlet Payoff	Floorlet PV
1	0.5	4.061%															
2	1	4.172%	4.283%	0.5	1	0.1246	0.0044	0.5496	0.5017	110.719	106.240	4.5388	4.4186	0.0000	0.0000	0.000	0.000
3	1.5	3.950%	3.507%	1	1.5	-1.0458	-1.2158	0.1478	0.1120	21.129	19.925	2.0755	1.9055	0.0190	0.0284	2.185	2.061
4	2	3.634%	2.689%	1.5	2	-2.0945	-2.3028	0.0181	0.0106	1.719	1.599	0.4540	0.2458	0.3249	0.4029	66.815	62.171
5	2.5	3.427%	2.601%	2	2.5	-1.9220	-2.1624	0.0273	0.0153	3.011	2.766	0.2851	0.0447	0.3878	0.4822	98.392	90.379
6	3	3.267%	2.469%	2.5	3	-1.8863	-2.1551	0.0296	0.0156	3.472	3.150	0.0878	-0.1810	0.4650	0.5718	140.729	127.692
7	3.5	3.162%	2.533%	3	3.5	-1.6102	-1.9046	0.0537	0.0284	7.608	6.817	0.1919	-0.1025	0.4239	0.5408	139.138	124.669
8	4	3.059%	2.339%	3.5	4	-1.7181	-2.0361	0.0429	0.0209	5.824	5.158	-0.0497	-0.3677	0.5198	0.6435	196.285	173.840
9	4.5	2.985%	2.394%	4	4.5	-1.5181	-1.8581	0.0645	0.0316	10.097	8.837	0.0425	-0.2975	0.4830	0.6169	192.993	168.903
10	5	2.915%	2.286%	4.5	5	-1.5391	-1.8998	0.0619	0.0287	9.682	8.377	-0.0677	-0.4284	0.5270	0.6658	229.871	198.904
										Sum	162.870					Sum	948.618

- c) (4%) If a cap is added to the floor in b) above such that the resulting collar has zero value today, what should the cap rate be? Continue to assume that the flat volatility is 17%.

Principle	\$ 100,000.00																
Maturity	5																
Compounding Frequency	2																
Cap Rate	3.21969%																
Floor Rate	2.50%																
Volatility	17%																

Period	Term(Year)	Spot Rate	Forward Rate	Maturity(yr)	Settlement	Caplet valuation						Floorlet valuation					
						d1	d2	N(d1)	N(d2)	Caplet Payoff	Caplet PV	d1	d2	N(-d1)	N(-d2)	Floorlet Payoff	Floorlet PV
1	0.5	4.061%	0	0	0												
2	1	4.172%	4.283%	0.5	1	2.4342	2.3140	0.9925	0.9897	532.338	510.805	4.5388	4.4186	0.0000	0.0000	0.000	0.000
3	1.5	3.950%	3.507%	1	1.5	0.5873	0.4173	0.7215	0.6618	199.699	188.319	2.0755	1.9055	0.0190	0.0284	2.185	2.061
4	2	3.634%	2.689%	1.5	2	-0.7611	-0.9693	0.2233	0.1662	32.667	30.397	0.4540	0.2458	0.3249	0.4029	66.815	62.171
5	2.5	3.427%	2.601%	2	2.5	-0.7672	-1.0076	0.2215	0.1568	35.593	32.694	0.2851	0.0447	0.3878	0.4822	98.392	90.379
6	3	3.267%	2.469%	2.5	3	-0.8534	-1.1222	0.1967	0.1309	32.125	29.149	0.0878	-0.1810	0.4650	0.5718	140.729	127.692
7	3.5	3.162%	2.533%	3	3.5	-0.6673	-0.9617	0.2523	0.1681	48.947	43.857	0.1919	-0.1025	0.4239	0.5408	139.138	124.669
8	4	3.059%	2.339%	3.5	4	-0.8451	-1.1632	0.1990	0.1224	35.785	31.693	-0.0497	-0.3677	0.5198	0.6435	196.285	173.840
9	4.5	2.985%	2.394%	4	4.5	-0.7016	-1.0416	0.2415	0.1488	49.486	43.309	0.0425	-0.2975	0.4830	0.6169	192.993	168.903
10	5	2.915%	2.286%	4.5	5	-0.7693	-1.1299	0.2209	0.1293	44.370	38.393	-0.0677	-0.4284	0.5270	0.6658	229.871	198.904
											948.617						948.618

Error 0.00

Solve Cap Rate by setting error=0