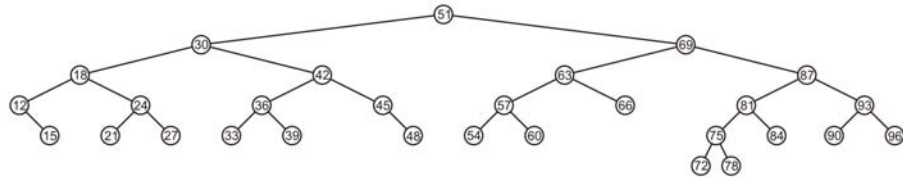


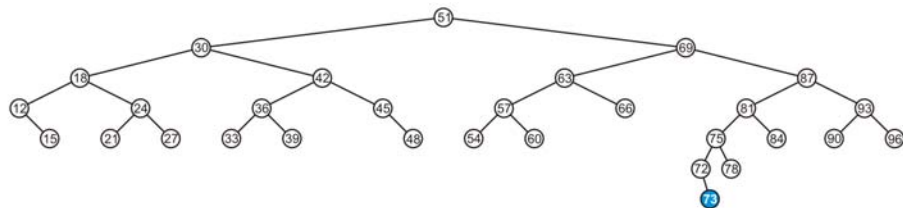
More examples : Insertion

Consider this AVL tree



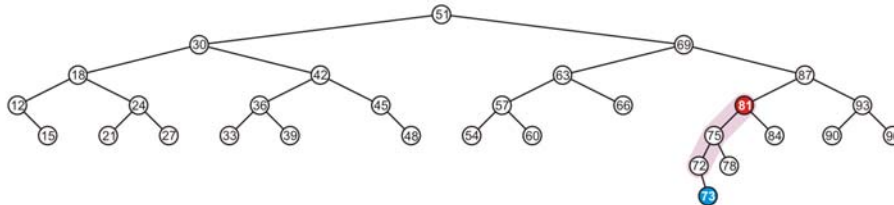
More examples : Insertion

Insert 73



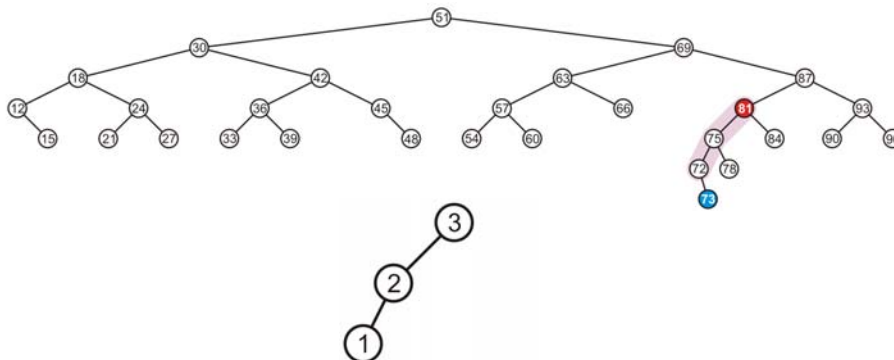
More examples : Insertion

The node 81 is unbalanced
– A left-left imbalance



More examples : Insertion

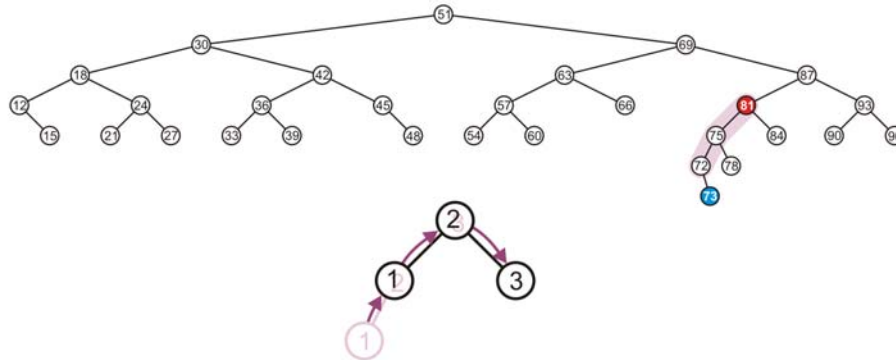
The node 81 is unbalanced
– A left-left imbalance



More examples : Insertion

The node 81 is unbalanced

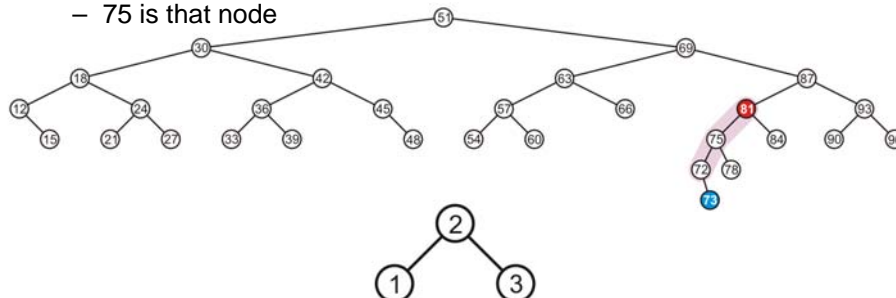
- A left-left imbalance
- Promote the intermediate node to the imbalanced node



More examples : Insertion

The node 81 is unbalanced

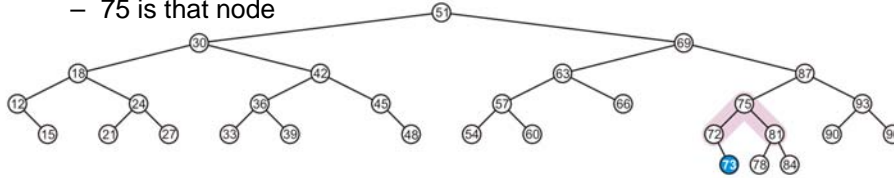
- A left-left imbalance
- Promote the intermediate node to the imbalanced node
- 75 is that node



More examples : Insertion

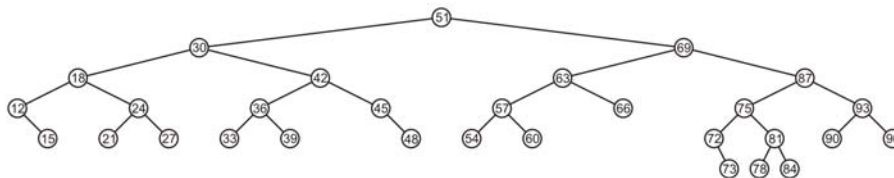
The node 81 is unbalanced

- A left-left imbalance
- Promote the intermediate node to the imbalanced node
- 75 is that node



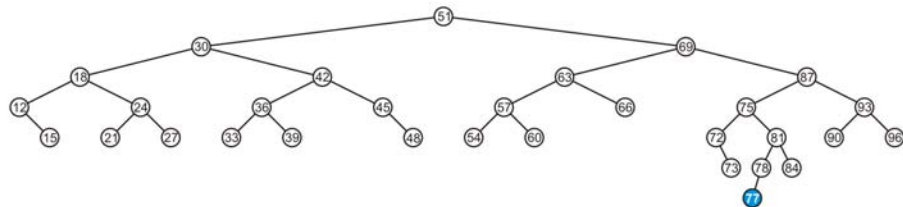
More examples : Insertion

The tree is AVL balanced



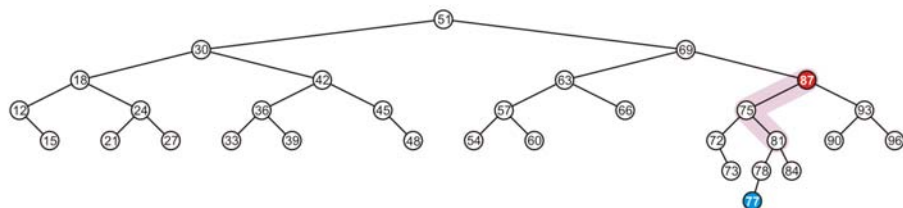
More examples : Insertion

Insert 77



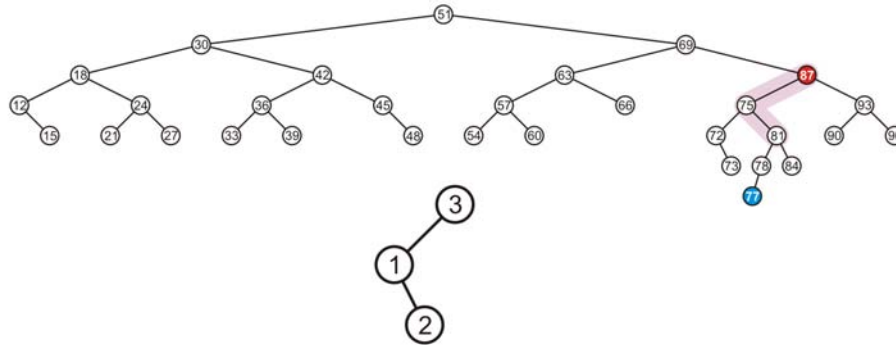
More examples : Insertion

The node 87 is unbalanced
– A left-right imbalance



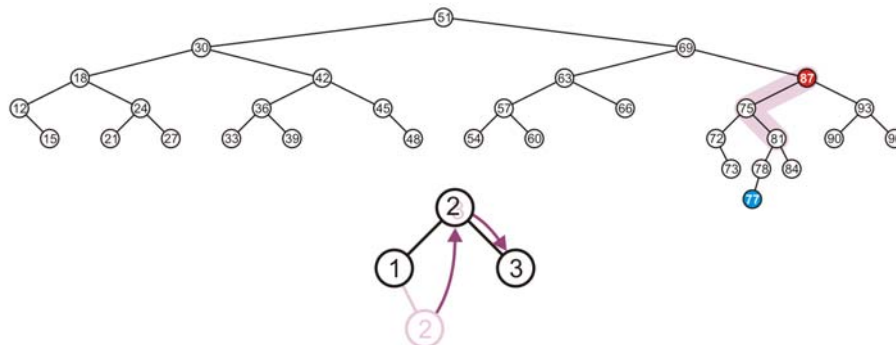
More examples : Insertion

- The node 87 is unbalanced
- A left-right imbalance



More examples : Insertion

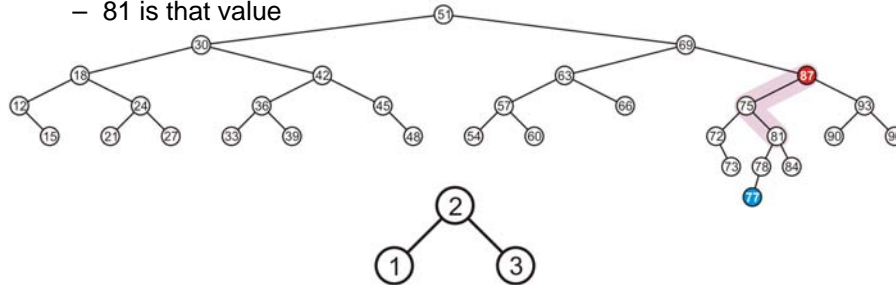
- The node 87 is unbalanced
- A left-right imbalance
 - Promote the intermediate node to the imbalanced node



More examples : Insertion

The node 87 is unbalanced

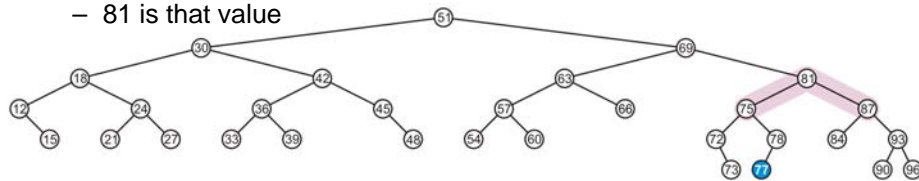
- A left-right imbalance
- Promote the intermediate node to the imbalanced node
- 81 is that value



More examples : Insertion

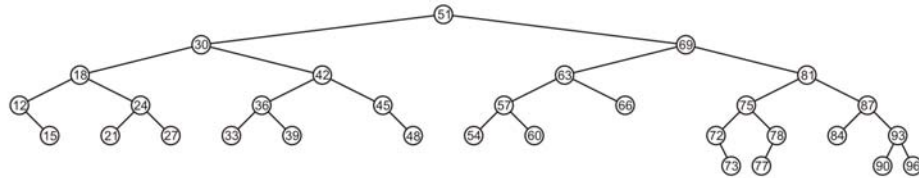
The node 87 is unbalanced

- A left-right imbalance
- Promote the intermediate node to the imbalanced node
- 81 is that value



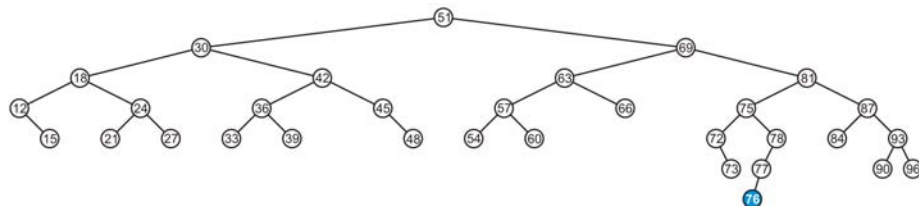
More examples : Insertion

The tree is balanced



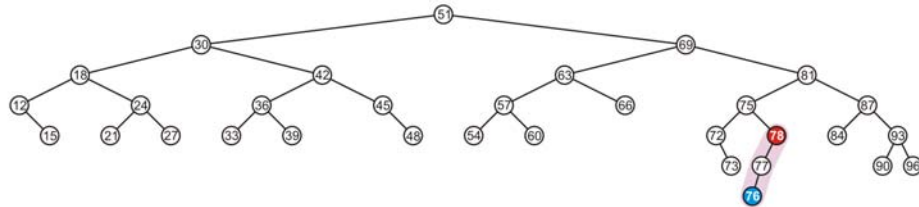
More examples : Insertion

Insert 76



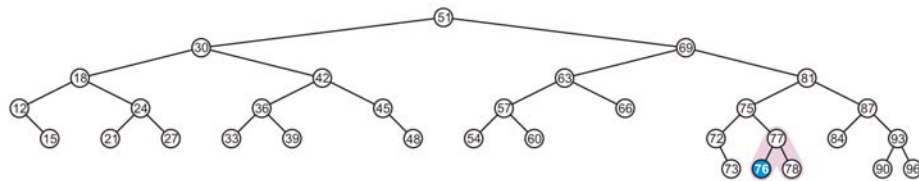
More examples : Insertion

The node 78 is unbalanced
– A left-left imbalance



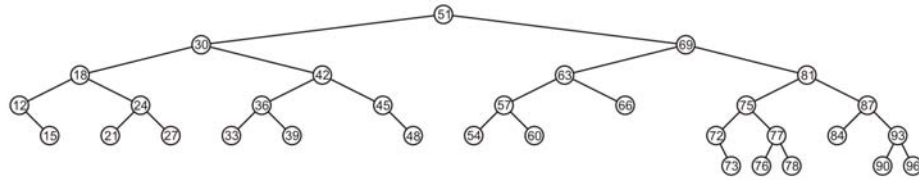
More examples : Insertion

The node 78 is unbalanced
– Promote 77



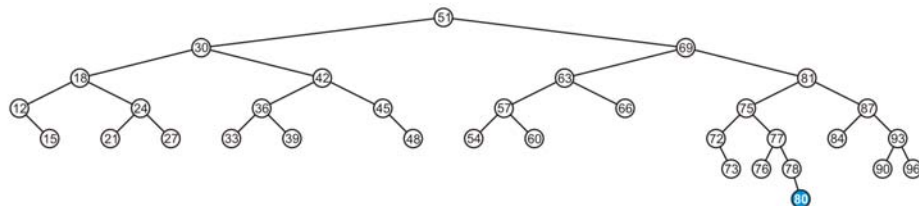
More examples : Insertion

Again, balanced



More examples : Insertion

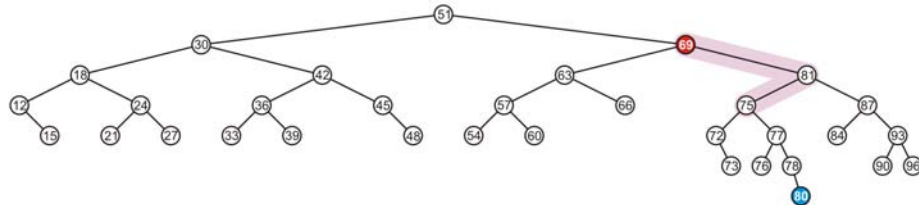
Insert 80



More examples : Insertion

The node 69 is unbalanced

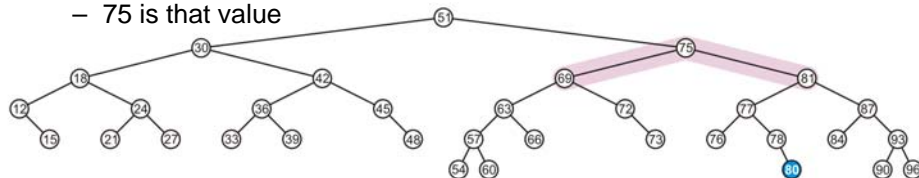
- A right-left imbalance
- Promote the intermediate node to the imbalanced node



More examples : Insertion

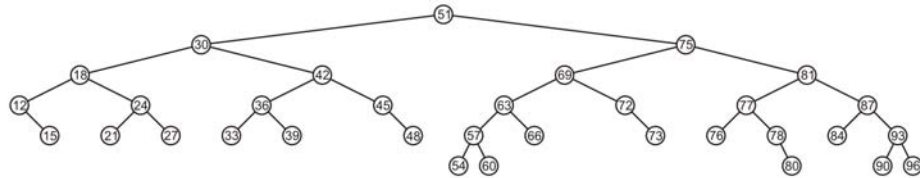
The node 69 is unbalanced

- A left-right imbalance
- Promote the intermediate node to the imbalanced node
- 75 is that value



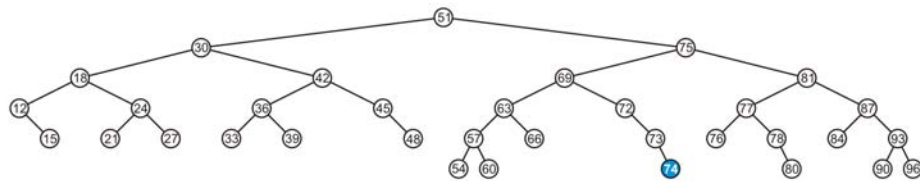
More examples : Insertion

Again, balanced



More examples : Insertion

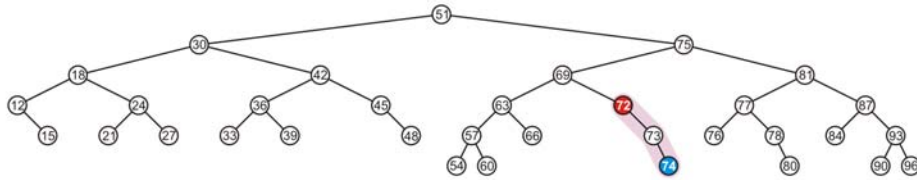
Insert 74



More examples : Insertion

The node 72 is unbalanced

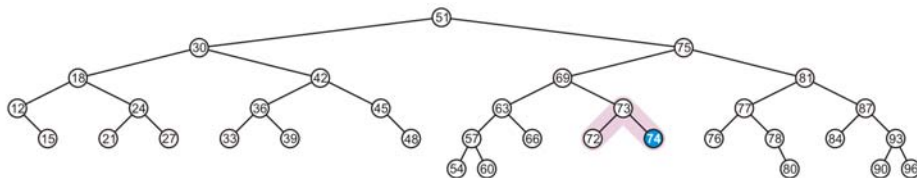
- A right-right imbalance
- Promote the intermediate node to the imbalanced node



More examples : Insertion

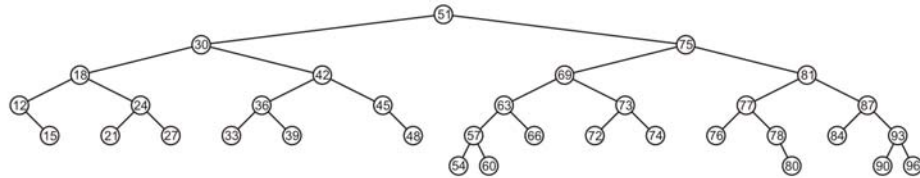
The node 72 is unbalanced

- A right-right imbalance
- Promote the intermediate node to the imbalanced node



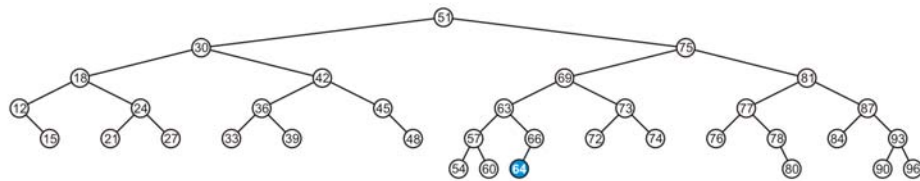
More examples : Insertion

Again, balanced



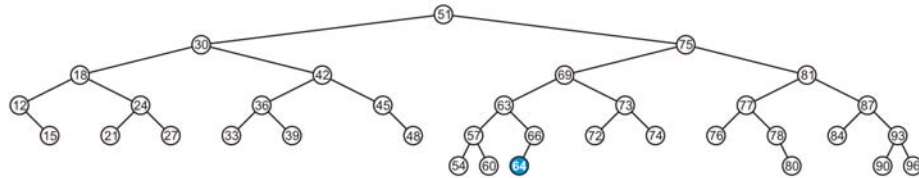
More examples : Insertion

Insert 64



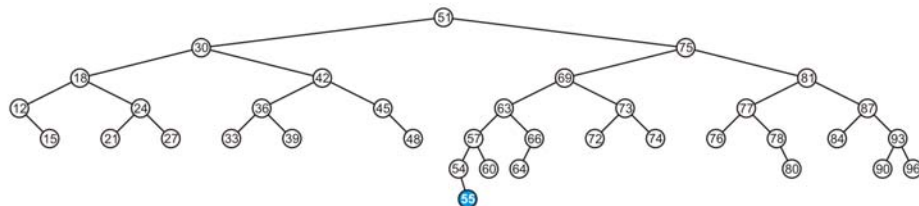
More examples : Insertion

This causes no imbalances



More examples : Insertion

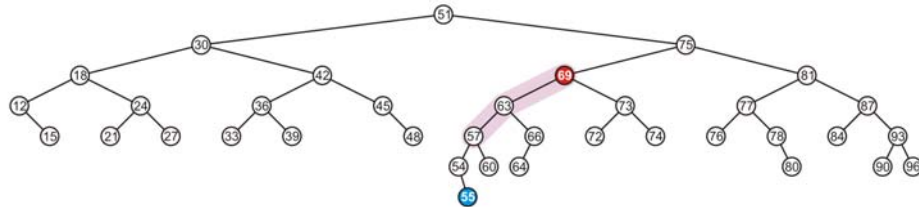
Insert 55



More examples : Insertion

The node 69 is imbalanced

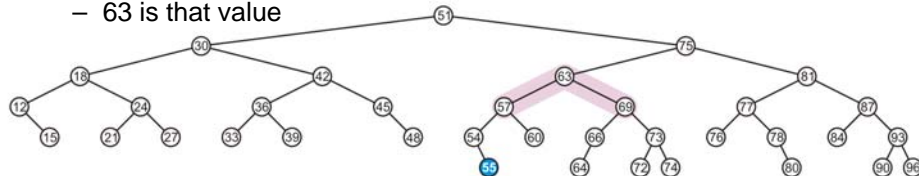
- A left-left imbalance
- Promote the intermediate node to the imbalanced node



More examples : Insertion

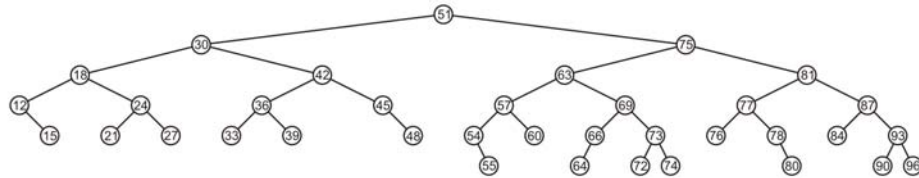
The node 69 is imbalanced

- A left-left imbalance
- Promote the intermediate node to the imbalanced node
- 63 is that value



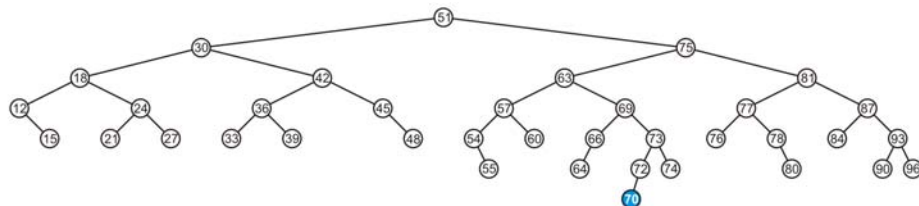
More examples : Insertion

The tree is now balanced



More examples : Insertion

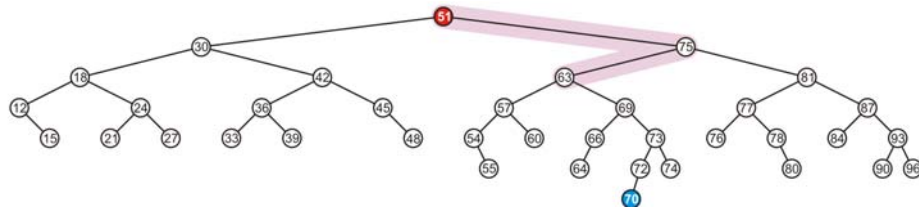
Insert 70



More examples : Insertion

The root node is now imbalanced

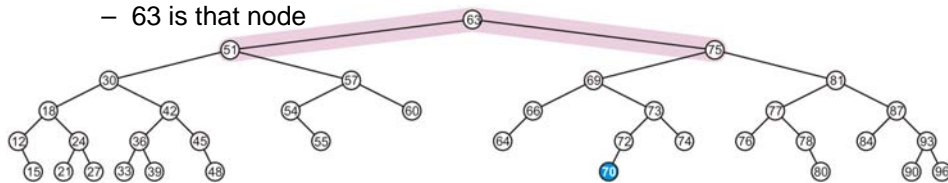
- A right-left imbalance
- Promote the intermediate node to the root



More examples : Insertion

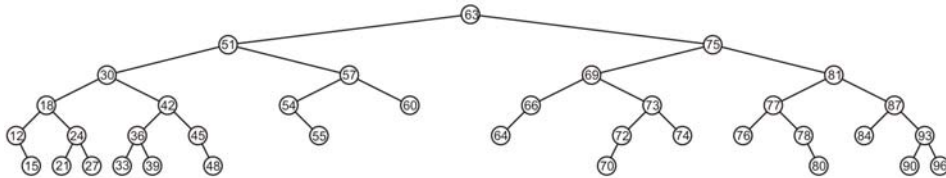
The root node is imbalanced

- A right-left imbalance
- Promote the intermediate node to the root
- 63 is that node



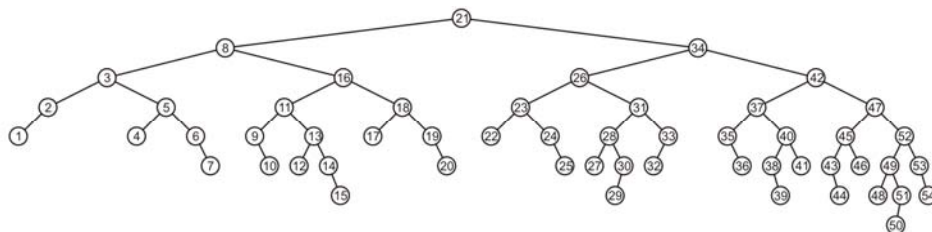
More examples : Insertion

The result is AVL balanced



More examples : Removal

Consider the following AVL tree



More examples : Removal

Suppose we erase the front node: 1

```
graph TD; 21((21)) --- 8((8)); 21 --- 34((34)); 8 --- 3((3)); 8 --- 16((16)); 3 --- 2((2)); 3 --- 5((5)); 5 --- 4((4)); 5 --- 6((6)); 6 --- 7((7)); 16 --- 11((11)); 16 --- 18((18)); 11 --- 9((9)); 11 --- 13((13)); 13 --- 10((10)); 13 --- 12((12)); 12 --- 14((14)); 14 --- 15((15)); 18 --- 17((17)); 18 --- 19((19)); 19 --- 20((20)); 34 --- 26((26)); 34 --- 42((42)); 26 --- 23((23)); 26 --- 31((31)); 23 --- 22((22)); 23 --- 24((24)); 24 --- 25((25)); 31 --- 28((28)); 31 --- 33((33)); 28 --- 27((27)); 28 --- 30((30)); 30 --- 32((32)); 42 --- 37((37)); 42 --- 47((47)); 37 --- 35((35)); 37 --- 40((40)); 40 --- 36((36)); 40 --- 38((38)); 38 --- 39((39)); 47 --- 45((45)); 47 --- 52((52)); 45 --- 43((43)); 45 --- 46((46)); 46 --- 44((44)); 52 --- 49((49)); 52 --- 53((53)); 49 --- 48((48)); 49 --- 51((51)); 51 --- 50((50)); 50 --- 54((54)); 1((1))
```

More examples : Removal

While its previous parent, 2, is not unbalanced, its grandparent 3 is

- The imbalance is in the right-right subtree

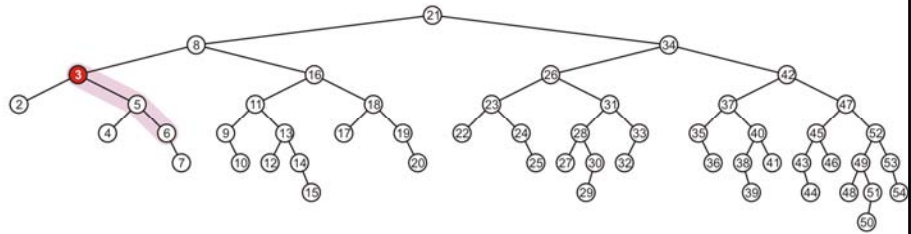
```
graph TD; 21((21)) --- 8((8)); 21 --- 34((34)); 8 --- 2((2)); 8 --- 16((16)); 2 --- 1((1)); 2 --- 5((5)); 16 --- 11((11)); 16 --- 18((18)); 11 --- 9((9)); 11 --- 13((13)); 18 --- 17((17)); 18 --- 19((19)); 13 --- 10((10)); 13 --- 14((14)); 14 --- 12((12)); 14 --- 15((15)); 34 --- 26((26)); 34 --- 42((42)); 26 --- 23((23)); 26 --- 31((31)); 23 --- 22((22)); 23 --- 24((24)); 31 --- 25((25)); 31 --- 33((33)); 25 --- 27((27)); 25 --- 30((30)); 30 --- 28((28)); 30 --- 32((32)); 42 --- 37((37)); 42 --- 47((47)); 37 --- 35((35)); 37 --- 40((40)); 40 --- 36((36)); 40 --- 41((41)); 47 --- 45((45)); 47 --- 52((52)); 45 --- 43((43)); 45 --- 46((46)); 52 --- 48((48)); 52 --- 53((53)); 48 --- 44((44)); 48 --- 49((49)); 49 --- 50((50)); 49 --- 51((51)); 51 --- 54((54)); 51 --- 55((55)); style 1 fill:#ff0000; linkStyle 1 stroke:#ff0000,stroke-width:2px;
```

More examples : Removal

While its previous parent, 2, is not unbalanced, its grandparent 3 is

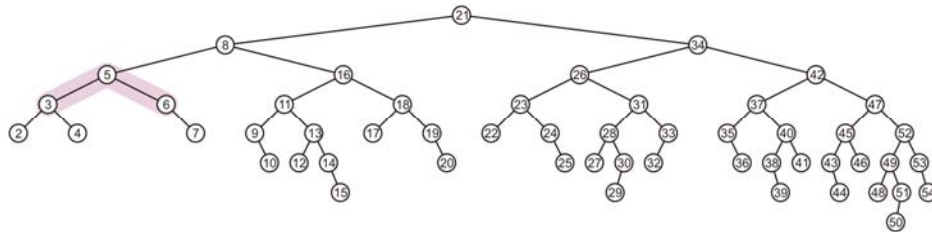
- The imbalance is in the right-right subtree

```
graph TD; 21((21)) --- 8((8)); 21 --- 34((34)); 8 --- 2((2)); 8 --- 16((16)); 2 --- 1((1)); 2 --- 5((5)); 16 --- 11((11)); 16 --- 18((18)); 11 --- 9((9)); 11 --- 13((13)); 18 --- 17((17)); 18 --- 19((19)); 13 --- 10((10)); 13 --- 14((14)); 14 --- 12((12)); 14 --- 15((15)); 34 --- 26((26)); 34 --- 42((42)); 26 --- 23((23)); 26 --- 31((31)); 23 --- 22((22)); 23 --- 24((24)); 31 --- 25((25)); 31 --- 33((33)); 25 --- 27((27)); 25 --- 30((30)); 30 --- 28((28)); 30 --- 32((32)); 42 --- 37((37)); 42 --- 47((47)); 37 --- 35((35)); 37 --- 40((40)); 40 --- 36((36)); 40 --- 41((41)); 47 --- 45((45)); 47 --- 52((52)); 45 --- 43((43)); 45 --- 46((46)); 52 --- 48((48)); 52 --- 53((53)); 48 --- 44((44)); 48 --- 49((49)); 49 --- 50((50)); 49 --- 51((51)); 51 --- 54((54)); 51 --- 55((55)); style 1 fill:#ff0000; linkStyle 1 stroke:#ff0000,stroke-width:2px;
```



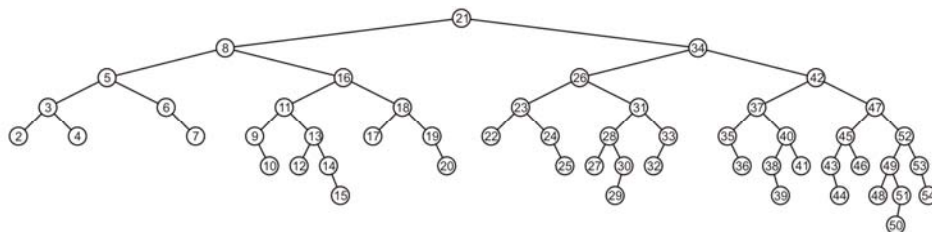
More examples : Removal

We can correct this with a simple balance



More examples : Removal

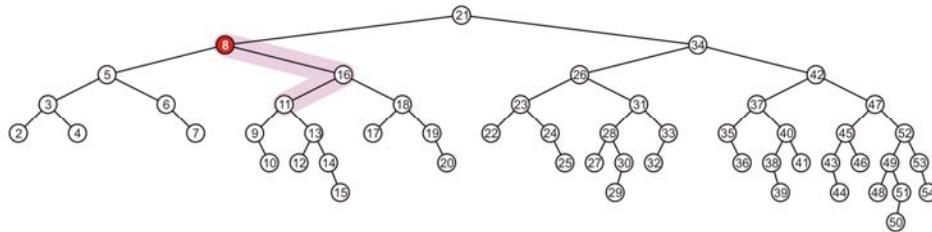
The node of that subtree, 5, is now balanced



More examples : Removal

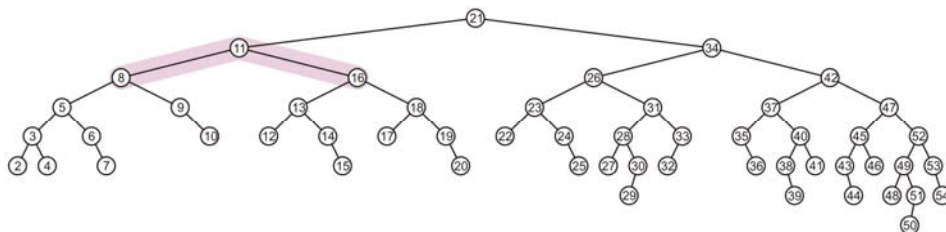
Recurring to the root, however, 8 is also unbalanced

- This is a right-left imbalance



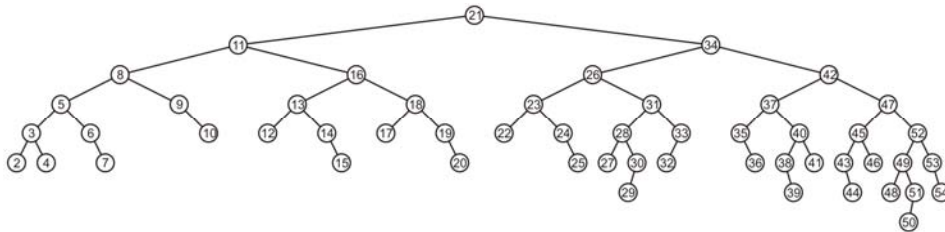
More examples : Removal

Promoting 11 to the root corrects the imbalance



More examples : Removal

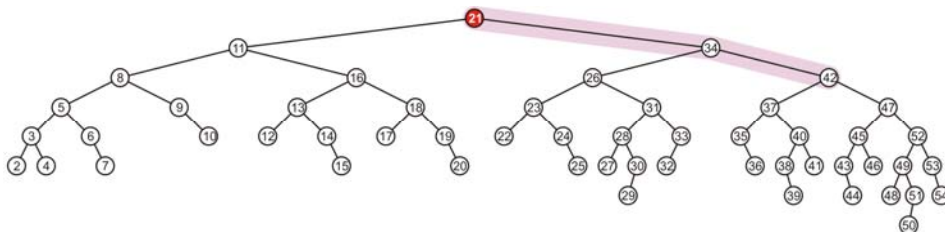
At this point, the node 11 is balanced



More examples : Removal

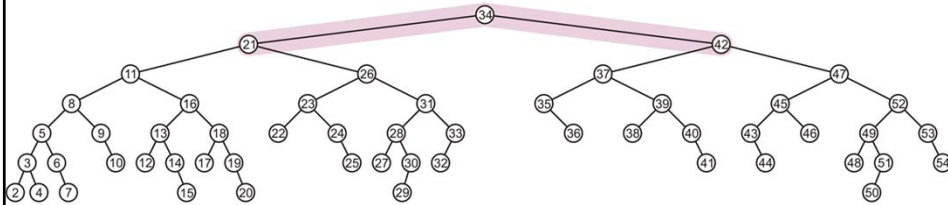
Still, the root node is unbalanced

- This is a right-right imbalance



More examples : Removal

Again, a simple balance fixes the imbalance



More examples : Removal

The resulting tree is now AVL balanced

- Note, few erases will require one balance, even fewer will require more than one

