Programming:

You are given a five-by-five array of cells, some of which contain letters and the others are empty. You need to fill the empty squares with letters so that there is a path moving only horizontally or vertically (**not** diagonally) at each step that visits all cells with consecutive letters. The letters are lower case 'a' through 'y', and 'a' is present in one of the starting cells.

For example, a solution to

	u	t	a	
	v		r	
	w	x	y	

is

g	f	e	d	c
h	u	t	a	b
i	v	s	r	q
j	w	x	y	p
k	l	m	n	o

Write a program to solve this problem. The input consists of five lines, each containing a string of five lower case letters from 'a' to 'z' with no spaces. An 'a' appears exactly once in the five lines. A 'z' indicates an empty cell. The output is five lines, each containing five characters (without spaces), and the five lines together contain each of the letters 'a' through 'y' according to the rules given above.

For example, if the input is

```
zzzzm
zzzzz
zfzzz
zzzaz
zzzzu
```

then a correct output is

```
ijklm
hgpon
efqrs
dcbat
yxwvu
```

Another way to describe the correct output is that if you place the cursor over the 'a', then using only the up, down, left, and right arrows you can move the cursor over the letters 'a' through 'y' in order.

You may assume that at least one solution exists for any of the test cases. If more than one solution exists, print out just one.

Names your program AtoY.

Exercises (not to turn in):

(1) (Goodrich & Tamassia) An evil king has n bottles of wine, and a spy has just poisoned one of them. Unfortunately, the king does not know *which* bottle was poisoned. The poison is very deadly but slow-acting; a single drop in a bottle of wine will kill anyone tasting it, but only after one month.

By assigning n tasters, one to each bottle, the king can know which bottle is poisoned after one month. Design a scheme which allows him to determine the poisoned bottle within one month but with many fewer tasters, and characterize the number of tasters needed as a function of n.

With your scheme, what are the smallest, largest, and average number of tasters who will die on the job?

(2) Use induction to prove that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

for $n \geq 1$.