FRE6711 Quantitative Portfolio Management
Memo for the Final Project

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1 Overview

This project aims to build a factor-based model Long/Short Global Macro Strategy with a Beta Target and to evaluate its sensitivity to variations of Beta.

Concretely, we want to build an investment strategy that maximizes the return of the portfolio subject to a constraint of target Beta, where Beta is the usual single factor Market risk measure. The performance and the risk profiles of such a strategy may be quite different depending of the target Beta and the market environment. A low Beta meaning a strategy aiming to be de-correlated to the global market represented by the S&P 500, and conversely a high Beta meaning that, having a big appetite for risk, we are aiming to ride or scale up the market risk. In addition to that, such a strategy is likely to to be quite sensitive to the length of the estimators used for the input covariance matrix (Risk Model) and the expected returns (Alpha Model), so it is important to understand the impact of those estimators on the Portfolio's characteristics: performance, volatility, skewness, VAR/CVAR and risk to performance ratios.

For practical considerations, we will assume that our universe of investment is a set of ETFs large enough to represent the World global economy and that our factor model is the French-Fama 3-factor-model for which free data are available for downolad at the website www.quandl.com.

As an example - just to fix ideas, if you assume that you have simple Trend estimators for the risk and the performance, those estimators can be based on long or short term historical data, which in their simplest form are giving by the sample covariance and the sample mean with a long or a short look-back period. A similar remark can be made if the returns are estimated using a 3-factor model as the estimated coefficients of the model are then computed using a regression on the factors using a large or small set of historical data. In that case, the central question is assess the impact of the length of those regression-based estimators on the realized performance and risk indicators of the optimized portfolio.

In summary, the behavior of the optimal portfolio built from an estimator (long term -LT, midterm - MT or short term - ST) may change with the Market environment and the target Beta. So the goal of this project is to understand the behavior of the strategies built using combinations of return/risk estimators and Target Beta during several historical periods : before the subprime (2008) crisis, during that crisis and after the crisis. Two benchmark portfolios will be considered.

1. The Market Portfolio (S&P 500)

2. The Mean-variance Long/Short Portfolio with yearly return target of 15%, and holdings between -2 and +2.

2 Investment Strategy

We will consider an portfolio optimization problem of the form:

$$\begin{cases} \max_{\omega \in \mathbb{R}^n} \rho^T \omega - \lambda(\omega - \omega_p)^T Q(\omega - \omega_p) \\ \sum_{i=1}^{n} \beta_i^m \omega_i = \beta_T^m \\ \sum_{i=1}^{n} \omega_i = 1, \ -2 \le \omega_i \le 2, \end{cases} \tag{1}$$

where

- Q is the Identity matrix (with diagonal elements equal to 1), ω_p is the composition of a reference Portfolio (the previous Portfolio for backtesting, otherwise ω_p has all its components equal to $1/n$) and λ is a small regularization parameter to limit the turnover (alternative: $Q = \Sigma$, the covariance matrix);
- $\beta_i^m = \dfrac{cov(r_i, r_M)}{\sigma^2(r_M)}$ is the Beta of security S_i as defined in the CAPM Model so that $\beta_P^m = \sum_{i=1}^{n} \beta_i^m \omega_i$ is the Beta of the Portfolio;
- β_T^m is the Portfolio's Target Beta, for example $\beta_T^m = 0.5$, $\beta_T^m = 1$, $\beta_T^m = 1.5$.

The French Fama 3-factor model is well documented in the litterature but we remind it for reference. Under that factor model, the return of a security is given by the formula

$$r_i = r_f + \beta_i^3 (r_M - r_f) + b_i^s r_{SMB} + b_i^v r_{HML} + \alpha_i + \varepsilon_i \tag{2}$$

with $\mathbb{E}(\varepsilon_i) = 0$ in such a way that we have in terms of Expected Returns

$$\rho_i = r_f + \beta_i^3 (\rho_M - r_f) + b_i^s \rho_{SMB} + b_i^v \rho_{HML} + \alpha_i. \tag{3}$$

In equation (2), the 3 coefficients β_i^3 , b_i^s and b_i^v and estimated by making a linear regression of the time series $y_i = \rho_i - r_f$ against the time series $\rho_M - r_f$, $x_i^2 = r_{SMB}$ and ρ_{HML}[1] (historical time series available from quandl.com). Note that in general $\beta_i^m \neq \beta_i^3$ and needs to be estimated by a separated regression or computed directly.

[1] ρ_M for Market, ρ_{SMB} for Small minus Big and ρ_{HML} for High minus Low

If you use a long series of historical data, say 200 dates to estimate the 3 coefficients of the factor model, then we will say that you have a long terrm model; likewise you could use between 45 and 60 dates to have a short-term model , or between 90 and 120 dates for a midterm estimate....

3 Investment Universe

We will consider the following set of ETFs that you can download for the period 01/01/2007 to 9/30/2020

1. CurrencyShares Euro Trust (FXE)
2. iShares MSCI Japan Index (EWJ)
3. SPDR GOLD Trust (GLD)
4. Powershares NASDAQ-100 Trust (QQQQ)
5. SPDR S&P 500 (SPY)
6. iShares Lehman Short Treasury Bond (SHV)
7. PowerShares DB Agriculture Fund (DBA)
8. United States Oil Fund LP (USO)
9. SPDR S&P Biotech (XBI)
10. iShares S&P Latin America 40 Index (ILF)
11. SPDR S&P Emerging Middle Est & Africa (GAF)
12. iShares MSCI Pacific ex-Japan Index Fund (EPP)
13. SPDR DJ Euro Stoxx 50 (FEZ)

4 Practical aspects

For estimation of the parameters of the factor model, you can use a cross sectional regression model by gathering all the individual securities model in a single "big" factor model. If you assume, that you have 3 factors, then the model at time t for each asset is given by

$$r_{it}^e = \alpha_i + \epsilon_t \beta_i^3 (r_{Mt} - r_{ft}) + b_i^s r_{SMBt} + b_i^v r_{HMLt} + \alpha_i + \varepsilon_{it} \tag{4}$$

with $r^e_{it} = r_{it} - r_{ft}$, and moreover the ε_{it} are independant of the factors and satisfy

$$cov(\varepsilon_{it}, \varepsilon_{js}) = \begin{cases} \sigma_i^2 \text{ when } i = j, \text{ and } t = s \\ 0 \text{ otherwise.} \end{cases}$$

4.1 Time Series Model for a given Security

If we consider T observations of the excess return of Security S_i stacked in a column vector $R_i = \begin{bmatrix} r^e_{i1} \\ r^e_{i2} \\ \cdots \\ r^e_{iT} \end{bmatrix}$, we have the time series regression model for Security i:

$$R_i = \mathbf{1}_T \alpha_i + F\beta_i + \varepsilon_i \text{ for } i = 1, 2, \ldots n \tag{5}$$

where

- $\beta_i = \begin{bmatrix} \beta^3_i \\ b^s_i \\ b^v_i \end{bmatrix}$ is the (3 by 1) vector of Betas
- $\mathbf{F} = \begin{bmatrix} \mathbf{f}'_1 \\ \vdots \\ \mathbf{f}'_T \end{bmatrix} = \begin{bmatrix} r_{M1} - r_{f1} & r_{SMB1} & r_{HML1} \\ \vdots & \ddots & \vdots \\ r_{MT} - r_{fT} & r_{SMBT} & r_{HMLT} \end{bmatrix}$ is the is the (T x 3) matrix of observations of the factors.
- the residual term ε_i is a (T by 1) vector satisfying $\mathbb{E}(\varepsilon_i \varepsilon'_i) = \sigma_i^2 \mathbb{I}_T$

The previous model is well-suited for a regression to estimate the coefficients of the model using data for the securities and the factors.

4.2 Cross Sectional Model

Alternatively, we can use a cross sectional formulation that can be useful for risk analysis including the derivation of the covariance matrix of the returns. If we define $R_t = \begin{bmatrix} r^e_{1t} \\ r^e_{2t} \\ \cdots \\ r^e_{nt} \end{bmatrix}$, the vector of all Securities excess returns at time t, then we can write

$$R_t = \alpha + \mathbf{B}\mathbf{f}_t + \varepsilon_t \text{ for } t = 1, 2, \ldots T, \tag{6}$$

where

- $B \begin{bmatrix} \beta_1' \\ \beta_2' \\ \cdots \\ \beta_n' \end{bmatrix} = \begin{bmatrix} \beta_1^3 & b_1^s & b_1^v \\ \vdots & \ddots & \vdots \\ \beta_n^3 & b_n^s & b_n^v \end{bmatrix}$ is a N by 3 matrix,
- $\mathbf{f}_t = \begin{bmatrix} r_{Mt} - r_{f1} \\ r_{SMBt} \\ r_{HMLt} \end{bmatrix}$ is the vector of factor returns at time t.
- $\mathbb{E}(\varepsilon_t \varepsilon_t' | \mathbf{f}_t) = D = diag(\sigma_1^2, \dots, \sigma_n^2)$

The cross sectional model implies that if Ω_f is the covariance of the factors, then

$$cov(R_t) = \mathbf{B}\Omega_f\mathbf{B}' + D \tag{7}$$

which implies that

$$cov(R_{it}) = \beta_i\Omega_f\beta_i + \sigma_I^2 \tag{8}$$

and

$$cov(R_{it}, R_{jt}) = \beta_i\Omega_f\beta_j \tag{9}$$

4.3 Min Variance with 15% Target Return

As a consequence of the previous section, the benchmark problem designed as min-variance with a 15% annual return target is to be formulated as follows:

$$\begin{cases} \min_{\omega \in \mathbb{R}^n} \omega^T \Sigma \omega + \lambda(\omega - \omega_p)^T Q \omega - \omega_p) \\ \rho^T \omega = 15\% \\ \sum_{i=1}^{n} \omega_i = 1 \\ -2 \le \omega_i \le 2, \end{cases} \tag{10}$$

where $\rho_i = r_f + \beta_i^3(\rho_M - r_f) + b_i^s \rho_{SMB} + b_i^v \rho_{HML} + \alpha_i$, and $\Sigma = \mathbf{B}\Omega_f\mathbf{B}' + D$. The inputs λ, ω_p, and Q are defined as in equation (1).

5 Performance and Risk Reporting for comparing Strategies

Use the **PerformanceAnalytics Module of R** for the Risk and Performance Reporting.
Below is the list of Key Indicators to report your Optimal Strategies. All daily indicators will be annualized assuming that each year has 250 business days.

For example, the Reporting for a Strategy over a given period[2] can be provided by:

1. A summarizing Table with the following lines for comparison with the underlyings
   - Cumulated PnL or Return
   - Daily Mean Geometric Return
   - Daily Min Return
   - Max 10 days Drawdown
   - Volatility
   - Sharpe Ratio
   - Skewness
   - Kurtosis
   - Modified VaR
   - CVaR

	Security 1	Security 2	...	Security n	Optimal Portfolio
Daily Mean Return			...	5	12
Daily Min Return			...	9	9
⋮			...	8	7
Max drawdown			⋮	8	7

For comparison with between the 2 built strategies and the S&P, a summary table will look like:

	MaxRet	MaxRet	MaxRet	MinVol	S&P 500
	Target$\beta^m = 0.5$	$\beta^m = 1$	$\beta^m = 1.5$	Target $\rho = 15\%$	
Mean Return			5	12	
⋮			8	7	
Max DD			8	7	

2. The evolution of the daily performance illustrated by the graph of cumulated PnLs, assuming that you invest 100 at the first allocation date.

3. The distribution of daily Returns.

4. Note that to benchmark your strategies against the Market, the SPY is representing the S&P500 Index.

[2] please, indicate clearly the period report date, example from 03/01/1997 to 12/30/2008

6 Submision of the Final Report

Your final report can be a Word, Latex File or PPT slides presenting your findings and conclusions about the impact of the estimators on the behavior of your strategy, and also what kind of estimators would recommend to use, when and why (before, during ands after the crisis). Please submit also the code developed for the project (Python, R, or Matlab) and all supporting graphs, tables and simulation results.