Stress hedging in portfolio construction

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May 2015

Abstract

Scenario stress testing is a useful and increasingly popular approach to assess portfolio performance under different market conditions. In this paper we focus on how to incorporate stress scenario information directly in portfolio construction as additional constraints to control for potential losses and risks. To broaden the applicability of stress testing we propose robust constrained optimization approach for handling uncertainty in scenario parameters. An example of the "Oil-Crisis" event is used as a numerical illustration.

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Introduction

Recent event-driven fluctuations in financial markets made investors increasingly concerned with finding ways to assess portfolio performance under different economic conditions. A growing number of financial practitioners and academics have begun to explore alternative approaches to portfolio construction that go beyond conventional mean-variance analysis [Laubsch 1999]. Scenario-based portfolio stress testing is one of such methodologies. It attempts to estimate the impact and design protection against extreme events [BIS Report 2009, Berkowitz 2000]. In applications of stress testing one starts with identifying a relevant scenario, e.g. “Oil Crisis” or “SP500 Drop”. The next step is modeling that event and valuing expected losses for individual securities and for portfolio as a whole. As a result, this framework enables to analyze portfolio performance subject to a concrete market event which makes it increasingly widespread [Ionescu and Yermo 2014].

Most commonly, stress-testing is used to *estimate* the impact of adverse events on investment portfolio [Kupiec 1999]. In this paper we focus on how to *control* potential event-driven losses and risks by using constraint-based framework within conventional mean-variance portfolio construction process. In addition, we show how to address practical situations when stress event cannot be fully specified and only a range of possible values can be assigned to model parameters. This additional complexity has not been previously considered and we introduce robust constrained optimization methodology to solve for an optimal stress protection in the presence of uncertainty. As a result, optimal stress hedging can now be applied more broadly to inform portfolio construction decisions.[1]

The organization of this paper is as follows. In the next section we describe how to integrate optimal hedging in portfolio construction both when stress-test scenario is fully defined and in presence of uncertainty. Proposed methodology is illustrated in the following section using “Oil Crisis” scenario as an example.

Portfolio construction with stress exposure control

Control of stress test exposure can be accommodated in portfolio construction by formulating it as a constrained portfolio optimization. Consider first the case when scenario is described by potential losses that each asset can exhibit. In order to control aggregated portfolio loss we extend conventional mean variance framework by adding a linear constraint on total or active exposure to the selected stress scenario (see e.g. [Ruban and

[1] AU wants to thank Prof. Steven Boyd for pointing on his paper describing worst case portfolio risk analysis in presence of model uncertainty.

Melas 2010]). In that case, resulting portfolio construction methodology has the following form:

$$\underset{\text{Given all constraints}}{\text{Maximize}}\,[\alpha^T \bullet (\text{h-h}_b) - \gamma \bullet (\text{h-h}_b)^T V\ (\text{h-h}_b)] \qquad (1)$$

$$h^T L \le \varepsilon_1$$

$$(h - h_b)^T L \le \varepsilon_2,$$

where

α − Asset expected returns
V − Covariance matrix
h − Portfolio holdings
h_b − Benchmark holdings
L − Stress test loss estimates, defined to have positive values
ε_1 − Stress test loss threshold, defined to be positive
ε_2 − Active stress test loss threshold, defined to be positive

Alternatively investor can decide to control both stress P&L and stress risk simultaneously which results in the following optimization formulation:

$$\underset{\text{Given all constraints}}{\text{Maximize}}\,[\alpha^T \bullet (\text{h-h}_b) - \gamma \bullet (\text{h-h}_b)^T V\ (\text{h-h}_b)] \qquad (2)$$

$$h^T L \le \varepsilon_1$$

$$(h - h_b)^T L \le \varepsilon_2$$

$$h^T V_S h \le \delta_1$$

$$(h - h_b)^T V_S (h - h_b) \le \delta_2\ ,$$

where

V_s − Stress test covariance matrix
δ_1 − Stress test risk threshold.
δ_2 − Active stress test risk threshold

The benefit of controlling stress exposure does not come for free. As always, introduction of additional constraints will come at a cost, so one should consider a cost-benefit analysis to assess when hedging stress exposure is indeed beneficial. Multiple metrics can be utilized. For example, one can measure the cost of hedging by the amount of reduction in alpha exposure (see e.g. [Ruban and Melas 2010]). In this paper we adopt a different metric. It is based on risk-scaled deviation to the current holdings. In practical terms, this approach would help to assess if sufficient reduction in stress P&L and/or risk can be achieved without significant change in the current investor holdings. Under this metric portfolio construction problem with loss and risk control under stress scenario can be written as

$$\underset{\text{Given all constraints}}{\text{Minimize}}\,[(\text{h-h}_p)^T V\,(\text{h-h}_p)] \qquad (3)$$

$$h^T L \leq \varepsilon$$

$$h^T V_S h \leq \delta.$$

where

h_p - Current portfolio holdings
ε - Stress test loss threshold
δ - Stress test risk threshold

Advantage of this measure is that it can be employed when there is no explicit information on manager's alpha views. As a result, that approach can also be implemented for non-optimized portfolios. In addition, methodology described by Eq. 3 is quite flexible – it can accommodate controls on alpha exposure as well as incorporate bounds on total and/or active exposures to stress test scenarios.

However, there is one important limitation for all stress-test hedging solutions discussed so far in Eq. 1-3. These methods can be put to work only when all model parameters describing scenario are available. That may not always be the case. What if investor is not certain that he/she can accurately estimate asset losses or a risk model describing stress test? One reason for that can be simply the rarity of such events. Another one is that investor tries to avoid replicating history and just assumes some of the information about stress scenario. As a result, the only available information may be the ranges of available values or, perhaps, just the sign for some parameters, like correlations.

Presence of any uncertainty in scenario parameters creates a problem that has not being addressed before. In this paper we tackle this complexity by using robust optimization approach. Proposed technique enables to control the worst-case outcome under stress conditions and provides a practical portfolio construction solution.

Before proceeding with proposed robust optimization setup and solution, let's consider different types of input uncertainty that were previously discussed in the context of portfolio construction. Box uncertainty assumes that parameter lies within a range around the point estimate for each component [Tutuncu and Koenig 2004]. Ellipsoid uncertainty sets define a surface around the point estimate (center) with axes parameters determining the size of the set [Goldfarb and Iyengar 2003]. Both approaches can be applied to handle incomplete information in linear terms, like stress P&L. A more complicated case is how to handle uncertainty in stress-scenario risk model. In particular what should be done when user can specify only the sign for correlations? While seemingly more complex, the last example can be explicitly formulated as a special case of box-type uncertainty [see e.g. Lobo and Boyd 1999]. In that paper uncertainty was present in the contemporaneous risk model used in portfolio construction. Here we allowed for uncertainty in description of the stress scenario, while we assume that risk model corresponding to current market conditions is fully specified.

In this paper we aim to consider a rather general case of input uncertainty. We assume that both stress test covariance matrix (i.e. the covariance matrix which is associated with stress scenario) and stress test loss estimates are not explicitly known. The only available information is that they are subject to a box type uncertainty with lower and upper bounds ($\underline{C}, \overline{C}$) and ($\underline{L}, \overline{L}$), respectively. In that case portfolio construction with stress exposure hedging can be formulated as a worst-case scenario optimization:

$$\underset{\text{Given all constraints}}{\text{Minimize}_h} \left[\text{Maximize}_{V_S} [(\text{h-h}_p)^T V \, (\text{h-h}_p) + \theta \bullet h^T V_S h]\right] \qquad (4)$$

$$h^T L < \varepsilon$$

$$\underline{C} \le V_S \le \overline{C}$$

$$\underline{L} \le L \le \overline{L}.$$

where θ controls the aversion to uncertain stress scenario risk. In practice, bounds on losses can be explicitly selected based on scenario expectations, while bounds on covariance matrix can be set by making assumptions about volatilities and correlations which will translate into bounds on covariance. For example, when investor wants to account for positive correlations between assets without specifying an explicit value – the bound on that correlation is particularly simple – it is between zero and one.

Using robust optimization techniques outlined in [Lobo and Boyd 1999], this mini-max problem can be cast into a convex optimization form which can be solved using any numerical package for semi-definite convex programming (Appendix A). We use CVX in this study [Grant and Boyd 1999].

The following section illustrates application of proposed portfolio construction framework for stress scenario hedging both in the case of fully defined scenario and in the case of uncertainty.

2. Portfolio construction example with "Oil Crisis" scenario

To illustrate proposed portfolio construction methodology for scenario exposure hedging we use an "Oil Crisis" stress test example.[2] This "crisis" is defined by realization of a ~12% monthly loss in the value of Crude Oil futures (NYMEX). Based on historical events, the probability of oil prices plunging more than this scenario is ~1%.

Also for illustration let's assume a long-short, dollar neutral multi-strategy fund that has established positions in 10 different strategies. Table 1 contains strategy correlations under normal market conditions (see upper triangle) and current fund holdings. Portfolio allocation decision is made given the current correlation/volatility structure in the market.

[2] Any of the approaches described by Eq. 1-3 can be further extended to simultaneously control exposure to multiple stress events.

Alpha expectations and other portfolio manager conditions result in ex-ante portfolio risk of 4.4% per year.

Strategy Name	Allocation (%)	Volatility (%)	Oil Crisis Volatility (%) *	Oil Crisis P&L (bps)	Asset Correlations (Current & Oil Crisis)**									
Strategy -1	-5%	6.5%	7.3%	-25	1.00	0.76	0.14	0.02	0.16	0.07	0.25	-0.01	-0.11	-0.10
Strategy -2	15%	5.6%	5.5%	-25	0.76	1.00	0.05	0.18	-0.02	0.10	0.19	0.07	-0.10	-0.16
Strategy -3	-25%	3.4%	4.7%	-20	0.27	0.24	1.00	0.45	0.55	0.18	0.03	0.10	0.04	0.61
Strategy -4	-5%	3.1%	3.9%	-400	0.33	0.36	0.61	1.00	0.02	0.22	-0.23	0.17	0.15	0.55
Strategy -5	-5%	10.1%	10.8%	50	0.23	0.19	0.29	0.09	1.00	0.28	0.23	-0.02	-0.06	0.21
Strategy -6	5%	3.5%	5.4%	-150	0.29	0.33	0.34	0.62	0.15	1.00	0.22	0.11	-0.10	0.16
Strategy -7	10%	13.0%	14.7%	-100	0.47	0.53	0.14	0.23	0.32	0.32	1.00	-0.03	-0.08	-0.02
Strategy -8	-10%	3.4%	3.3%	-200	0.01	0.07	0.06	0.11	0.04	0.11	0.14	1.00	-0.17	0.24
Strategy -9	10%	5.0%	5.3%	50	0.16	0.20	0.29	0.38	0.06	0.34	0.09	-0.04	1.00	0.13
Strategy -10	10%	3.8%	5.9%	-300	0.19	0.19	0.82	0.56	0.00	0.36	0.10	0.08	0.28	1.00

* Lowest volatility under "Oil Crisis" stress test conditions is set to be 90% of stress test risk volatility level, while highest stress test volatility is set to be 150% of stress test risk volatility for each strategy

** Upper triangle shows the strategy correlations with current risk model, while lower triangle shows the scenario-weighted correlations under "Oil Crisis" stress test conditions.

** Lower bounds on correlation uncertainty under "Oil Crisis" is calculated by substracting 10% of correlation difference between "Oil Crisis" scenario and current correlation matrices from "Oil Crisis" scenario correlations for each entry

** Upper bounds on correlation uncertainty under "Oil Crisis" is calculated by adding 50% of correlation difference between "Oil Crisis" scenario and current correlation matrices to "Oil Crisis" scenario correlations for each entry

** Upper (lower) bounds on covariance uncertainty under "Oil Crisis" is calculated by the maximum (minimum) of 4 different values implied by upper and lower bounds on volatility and correlation values for each strategy pairs

Table 1: The inputs for Oil Crisis stress test scenario

In the case of deterministic stress-test scenario models we can estimate correlations and P&L for each strategy in a crisis mode e.g. by a scenario-weighting approach similar to described in [Silva and Ural 2010, Ruban and Melas 2010]. Results are provided in Table 1 where the lower triangle of correlation matrix contains "stressed" values.

Consider first the deterministic case where we only hedge against stress P&L. The resulting portfolio construction approach is described by Eq. 3 with only linear constraint providing an upper bound for possible scenario total loss. With fully defined stress scenario parameters we can easily compute portfolio P&L and show that with current holdings the fund loss will be ~120 bps if the "Oil Crisis" scenario materializes. Chart 1 shows how portfolio risk changes when potential stress test loss is reduced from 120 bps to 70 bps. Interestingly, our results indicate that stress-protection results in a lower total risk of portfolio. As shown on the green line, the risk of Portfolio A decreases 44 bps if the impact of "Oil Crisis" scenario is hedged to 80 bps. The purple line shows that the cost of hedging, as measured by portfolio tracking error with respect to current holdings, increases by 1% to achieve the hedged Portfolio A. Portfolio manager needs to balance the cost of portfolio adjustments against the benefit of hedging stress test scenario to select the appropriate reallocation solution.

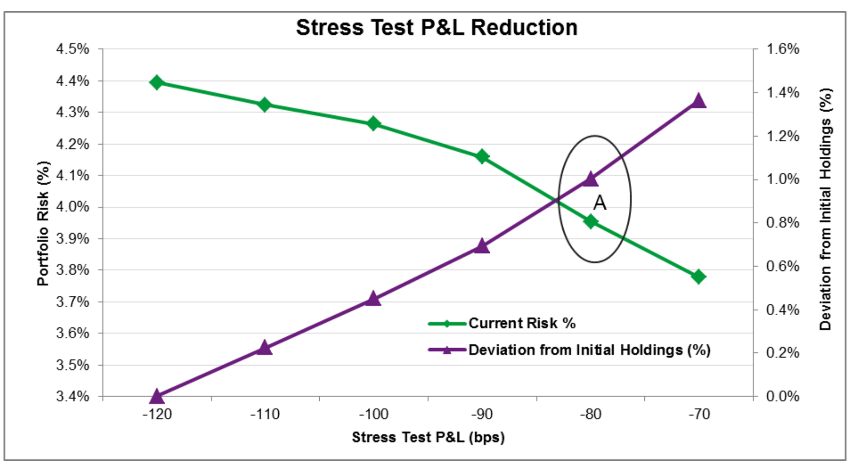

Chart 1: The current risk and deviation from initial holdings across various stress test P&L thresholds for Stress Test P&L Reduction model.

Alternatively investor can control simultaneously for stress P&L and stress risk (see Eq. 3.). The constraint on stress test risk can be equivalently modelled as a penalty term in the objective function. Optimizations at different level of "stress-risk" aversion parameter θ will result in efficient frontiers as shown on Chart 2. On this plot, we present risk profiles of hedged portfolios against the deviation from initial holdings for varying levels of stress test risk.

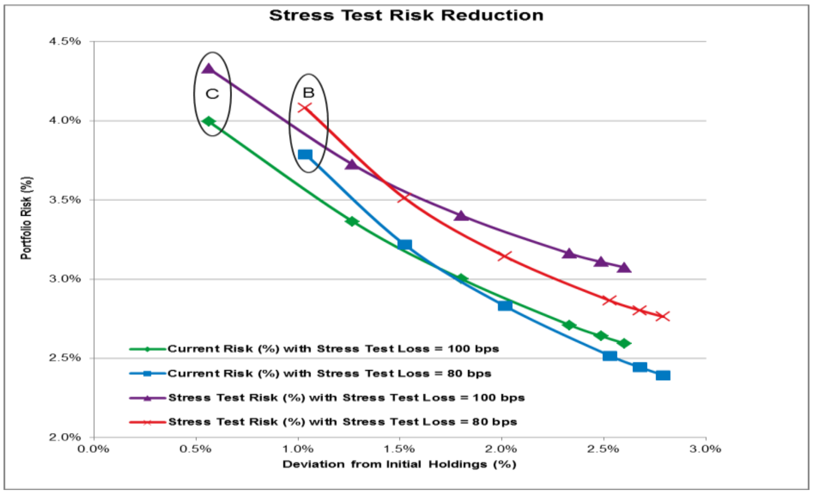

Chart 2: The chart shows the current risk and stress test risk when stress test risk is controlled with a constraint in Stress Test Risk Reduction model, achieving stress test P&L of -100 bps and -80 bps

Purple and red lines show the stress test risks of different hedged solutions that achieve 100 bps and 80 bps loss under "Oil Crisis" scenario respectively. Compared to the green and blue lines, scenario-weighted stress test risk is 13% more than the current risk on average. Portfolio B shows a hedged solution which reduces "Oil Crisis" risk by 70 bps and "Oil Crisis" loss by 40 bps with 1% deviation from the current holdings. Compared to Portfolio C, Portfolio B achieves 20 bps more reduction in "Oil Crisis" with 25 bps less stress test risk, 21 bps less current risk, and 47 bps more deviation from initial holdings at the same level of stress test risk aversion.

While actual numbers depend on model details, the overall shape of these efficient frontiers can be explained qualitatively. Indeed, simultaneous improvement in stress P&L and stress risk exhibited by Portfolio B vs. Portfolio C is in line with the expectation that stress-test risk and stress-test loss reduction are positively correlated. The hedging opportunities increase with higher loss reduction level (blue line vs green line), leading to higher benefit of risk reduction per one unit of deviation from initial holdings. However, there is a limit to risk reduction for a given level of stress test loss. The curvature of blue line shows that stress test reduces at diminishing rates with further deviation from initial holdings. In practice, computing these "efficient hedging frontiers" will inform investor on the desired level of hedging.

To illustrate our approach for a scenario with uncertainty, we decided to use point estimates for stress P&L while allowing box-type constraints for elements of stress covariance matrix. Here we use the following rules to specify the range for correlations. The upper bound is defined as stress-based estimated value increased by 50% of the difference between stress and normal regime correlation. The lower bound is defined as estimated stressed value reduced by 10% of the same difference. This asymmetry reflects the expectation that correlations tend to increase in the crisis periods. In addition, for each strategy we allowed volatility to be within 90% to 150% of the "Oil Crisis" values. The resulting risk model (see Table 2) combines uncertainty coming from both sources.

Table 2: The upper and lower bounds on volatility and correlations for Oil Crisis scenario

Strategy Name	Lower Bound (%)	Upper Bound (%)	Asset Correlations (Upper & Lower Bound for Oil Crisis)***									
Strategy -1	6.5%	10.9%	1.00	0.64	0.23	0.01	0.21	0.00	0.35	0.15	0.26	0.05
Strategy -2	4.9%	8.2%	0.61	1.00	0.27	0.11	0.20	0.13	0.28	0.24	0.27	0.04
Strategy -3	4.2%	7.1%	0.17	0.20	1.00	0.45	0.41	0.19	0.35	0.05	0.52	0.04
Strategy -4	3.5%	5.9%	-0.29	-0.09	0.20	1.00	0.02	0.39	0.14	0.18	0.04	0.36
Strategy -5	9.7%	16.2%	0.17	0.19	0.40	0.00	1.00	0.10	0.35	0.15	0.33	-0.04
Strategy -6	4.8%	8.0%	-0.17	-0.01	0.13	0.32	0.08	1.00	0.18	0.14	0.11	0.13
Strategy -7	13.2%	22.0%	0.18	0.20	0.25	0.03	0.17	0.16	1.00	-0.04	0.51	-0.08
Strategy -8	3.0%	5.0%	-0.01	0.07	-0.17	0.07	0.12	0.09	-0.16	1.00	-0.09	0.20
Strategy -9	4.7%	7.9%	0.04	0.11	0.29	-0.01	0.15	0.02	0.28	-0.09	1.00	-0.16
Strategy -10	5.3%	8.8%	-0.08	-0.03	-0.20	0.28	-0.07	0.13	-0.23	0.19	-0.21	1.00

*** Upper triangle shows the upper bound on strategy correlations under "Oil Crisis" scenario, while lower triangle shows the lower bound on correlations for "Oil Crisis" scenario.

Effect of robust optimization was measured by changing stress risk aversion parameter to determine the impact of worst-case scenario on portfolio allocations. That is similar to our analysis for fully defined stress risk model and allowed us to compare both approaches.

Efficient stress hedging frontiers with box-type uncertainty in stress-test risk model are depicted in Chart 3. It shows how the current risk (green line), the worst-case risk (blue line) and the deviation from initial portfolio (purple line) change with increasing impact of stress test risk uncertainty (horizontal axis) while expected stress test scenario loss was kept constant at 80 bps. As expected, worst-case risk values can be significantly higher than current portfolio risk estimate. In fact in this example worst-case volatility associated with crisis scenario is almost twice the value based on regular model.

To illustrate efficiency of robust optimization in reducing worst-case scenario risk we can compare two portfolios that have the same level of loss reduction. Portfolio A was based on loss hedging in a deterministic scenario (see Chart 1) and Portfolio D achieves the same level of loss protection and is a solution for robust optimization for scenario risk control in presence of uncertainty. We can estimate the worst-case risk for both portfolios and, also as expected, including risk control results in a lower value of worst-case volatility.

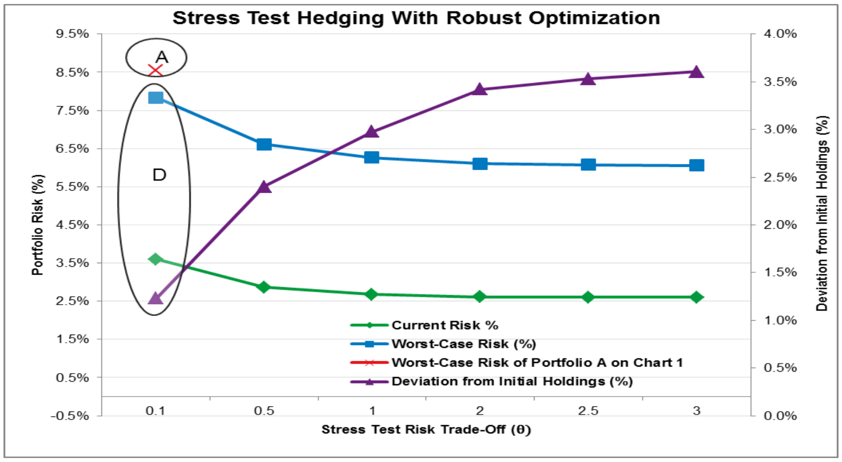

Chart 3: The chart highlights how portfolio risk and deviation from initial holdings change when stress test risk trade-off is varied for Stress Test Hedging with Robust Optimization model for a fixed stress test loss threshold of -80 bps.

Finally, let's compare risk profiles for portfolios that achieve the same level of loss reduction while employing no stress scenario risk hedging (Portfolio A), stress hedging using deterministic stress risk model (Portfolio B) and stress risk hedging under uncertainty (Portfolio D). Results are provided in Table 3. As expected, Portfolio B achieves less stress test risk than Portfolio A. In addition, stress risk control also helps reducing the worst-case risk profile of Portfolio B, although it is not directly controlled in the optimization model. Not surprisingly, worst-case risk exposure is further reduced in Portfolio D since it provides the least risky portfolio in terms of this metric.

	Current Risk (%)	**Deviation from Initial Holdings (%)**	**Stress Test Loss (bps)**	**Stress Test Risk (%)**	**Worst-Case Risk (%)**
Current Portfolio	4.4%	0.0%	-120	4.8%	9.1%
Portfolio A (Stress Test P&L Reduction)	4.0%	1.0%	-80	4.3%	8.5%
Portfolio B (Stress Test Risk Reduction)	3.8%	1.0%	-80	4.1%	8.3%
Portfolio D (Stress Test Hedging with Robust Optimization)	3.6%	1.2%	-80	3.9%	7.8%

Table 3. Current risk, deviation from initial holdings, stress test loss, stress test risk, and worst-case risk of all portfolios.

The strategy allocation differences among portfolios depend on the level of scenario loss, stress test risk covariance and uncertainty in the volatility and correlation structure of stress test risk. While details depend on multiple scenario parameters, qualitatively, as expected, portfolio construction models reduce allocation to strategies with high stress scenario loss. Stress risk control enables further reduction in risk by identifying assets with increased volatility and, finally, robust optimization provides more prudent solutions for investors, finding better worst-case risk hedged portfolios than deterministic models for a fixed level of risk aversion.

3. Summary

Stress test analysis provides an efficient framework to identify potential impact of a market event on investment portfolio. In this paper, we describe how to control that negative impact by constraining loss and/or risk associated with the stress scenario in portfolio construction. Furthermore we propose a novel robust constrained optimization methodology that directly addresses challenges posed by uncertainty in modeling stress test scenarios. Using computational example of "Oil Crisis" we illustrate application of proposed methodologies and effectiveness of the resulting solutions.

Finally, presented stress test hedging framework is quite flexible. It can be extended to multiple scenarios by simultaneously controlling different drivers of risks. In addition, proposed robust optimization approach can handle uncertainty not only in asset-by-asset risk models but also can be applied to partially defined factor-based structural risk models.

ACKNOWLEDMENTS

The authors thank Prof. Stephen Boyd, Dr. Ronald Ratcliffe, and Vivek Gupta for their valuable input and comments.

Appendix: Robust Optimization Approach for Stress Test Hedging

In order to highlight the difference between robust optimization approach and deterministic stress test risk control lets start with deterministic case described in Eq.3 as:

$$\min_h (h - h_p)^T V \left(h - h_p\right)$$

$$h^T L \leq \varepsilon$$
$$h^T V_s \, h \leq \delta$$

Here, again, V is the current risk model, V_s represents the stress test risk and L represents the scenario loss. ε is the upper bound on scenario loss, and δ is the upper bound on stress test risk.

The upper bound on stress test risk can be equivalently applied as a penalty in the objective function and controlled by the risk aversion to stress test risk - parameter θ:

$$\min_h [(h - h_p)^T V \left(h - h_p\right) + \theta \, h^T V_s \, h]$$

$$h^T L \leq \varepsilon$$

Now, let's assume that expected return and stress test covariance matrix (i.e. the covariance matrix associated with the stress test scenario) are subject to box type uncertainty with lower and upper bounds $(\underline{L}, \overline{L})$ and $(\underline{V}, \overline{V})$, respectively. Portfolio construction with stress test hedging will result in a robust optimization problem which can be formulated as:

$$\min_h \{max_{V_s}(h - h_p)^T V \left(h - h_p\right) + \theta \, h^T V_s \, h\}$$

$$max_h h^T L \leq \varepsilon$$

The uncertainty in stress covariances can be driven by e.g. uncertainty in correlations while stress test loss is represented with the following set:

$$L = \{L: L_0 + \gamma, |\gamma_i| \leq \mu_i, i = 1, \dots, n\}$$

where L_0 shows the point estimate for scenario loss and γ is the uncertainty around each scenario loss estimate.

After using the techniques outlined in [Lobo and Boyd 1999, Goldfarb and Iyengar 2003], this problem can be cast into a convex robust optimization model as follows:

$$\min_{h,h^+,h^-,\underline{Q},\overline{Q}} \; (h - h_p)^T V \left(h - h_p\right) + \theta(< \overline{Q}, \overline{V} > - < \underline{Q}, \underline{V} >)$$
$$h^T L_0 + \mu^T (h^+ + h^-) \leq \varepsilon$$
$$h = h^+ - h^-, h^+ \geq 0, h^- \geq 0$$
$$\underline{C} \geq 0, \overline{C} \geq 0$$
$$\begin{bmatrix} \overline{Q} - \underline{Q} & h \\ h^T & 1 \end{bmatrix} \geq 0$$

The robust model includes auxiliary variables $(h^+, h^-, \underline{Q}, \overline{Q})$ in order to choose a worst-case risk model from the covariance uncertainty set. Here $< A, B >$ represents the trace of the matrix product AB. θ controls the trade-off between the deviation from the initial holdings and the worst-case stress test risk. The inequality constraints (≥) on matrices $\underline{V}, \overline{V}, \underline{Q}, \overline{Q}$ define the positive semi-definite structure of these matrices. For illustration of this approach we include only uncertainty in stress scenario risk model in section 2. Resulting convex semi-definite optimization problem was solved using CVX [Grant and Boyd 1999].

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