Enhanced Scenario Analysis

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KEY FINDINGS

- The authors use a multivariate measure of statistical distance to estimate probabilities of prospective scenarios.
- They construct portfolios that maximize utility for investors with different risk preferences.
- The authors introduce a procedure for minimally modifying scenarios to render them consistent with one's prespecified views about their probabilities of occurrence.

ABSTRACT: *Investors have long relied on scenario analysis as an alternative to mean–variance analysis to help them construct portfolios. Even though mean–variance analysis accounts for all potential scenarios, many investors find it difficult to implement because it requires them to specify statistical features of asset classes that are often unintuitive and difficult to estimate. Scenario analysis, by contrast, requires only that investors specify a small set of potential outcomes as projections of economic variables and assign probabilities to their occurrence. It is, therefore, more intuitive than mean–variance analysis, but it is highly subjective. In this article, the authors propose to replace the subjective elements of scenario analysis with a robust statistical process. They use a multivariate measure of statistical distance to estimate probabilities of prospective scenarios. Next, they construct portfolios that maximize utility for investors with different risk preferences. Lastly, the authors introduce a procedure for minimally modifying scenarios to render them consistent with prespecified views about their probabilities of occurrence.*

TOPICS: ***Portfolio management/multi-asset allocation, risk management, quantitative methods****

*All articles are now categorized by topics and subtopics. **View at PM-Research.com.**

Mean–variance analysis and scenario analysis are the two most common techniques for constructing asset class portfolios. Though they are implemented quite differently and appeal to different types of investors, ultimately they are variations on one other. Mean–variance analysis requires investors to specify the expected returns, standard deviations, and correlations of asset classes, and in turn it yields efficient portfolios along with their expected returns and standard deviations. If we accept that returns are approximately elliptically distributed,[1] then each portfolio's expected return and standard

[1] In two dimensions (two asset classes), an elliptical distribution features observations that are evenly distributed along the boundaries of ellipses that are centered on the mean observation of the scatter plot. It therefore has no skewness, but it may have non-normal kurtosis. This concept extends to distributions with more than two dimensions, though it cannot be visualized beyond three. Mean–variance optimization assumes either that returns are elliptically distributed or that investor preferences are well approximated by mean and variance.

deviation account for all possible relevant scenarios[2] weighted by their likelihood of occurrence.

Nonetheless, many investors are uncomfortable with mean–variance analysis because it requires them to specify statistical features of asset classes that they find unintuitive and difficult to estimate. As an alternative to mean–variance analysis, some investors prefer scenario analysis, in which they define potential economic scenarios and assign probabilities to their occurrence. Scenario analysis is thus more intuitive, but this intuition often comes at the expense of quantitative rigor. Investors rely on judgment to assign probabilities to the prospective scenarios. As an alternative to subjectively determined probabilities, we introduce a statistical procedure for determining the relative likelihood of prospective scenarios based on a measure called the Mahalanobis distance.

Previous literature has applied the Mahalanobis distance to evaluate stress test scenarios built from market variables in comparison to stress events that have occurred historically (Golub, Greenberg, and Ratcliffe 2018). We also use the Mahalanobis distance to evaluate alternative scenarios, but we focus explicitly on the goal of portfolio choice. We present a collection of techniques for this purpose and illustrate their application in a case study. First, we define scenarios in terms of economic variables that are both intuitive and relevant to asset performance. Second, we define unusualness relative to a chosen anchor, which may be, for example, current economic conditions, normal economic conditions, or a blend that represents an investor's views on persistence versus mean reversion. Third, we use investor utility functions to account for risk when comparing portfolio performance across scenarios. Finally, we specify an algorithm to modify scenario definitions as efficiently as possible to align with an investor's views on their relative probabilities of occurrence.

STATISTICAL PROBABILITIES OF PROSPECTIVE SCENARIOS

Investors typically rely on judgment to assign probabilities to prospective economic scenarios. They might, for example, consider historical patterns of economic variables, their recent trajectories, and prevailing government policies, but ultimately the probabilities they assign are expressions of their subjective views. We instead propose a statistical method for determining the relative likelihood of prospective scenarios based on a measure called the Mahalanobis distance.

The origin of this measure dates back to 1927 when Mahalanobis, an Indian statistician, used characteristics of the human skull to analyze distances and resemblances among various castes in India.[3] The skull characteristics used by Mahalanobis differed by scale and variability. That is, Mahalanobis might have considered a half-inch difference in nasal length between two groups of skulls a significant difference, whereas he considered the same difference in head length to be insignificant. Mahalanobis normalized differences in each characteristic by the characteristic's standard deviation and then squared and summed the normalized differences, thus generating one composite distance measure that was invariant to the variability of each dimension. In a subsequent article, he proposed a more generalized statistical measure of distance that takes into account not only the standard deviations of individual dimensions but also the correlations between dimensions.[4]

This measure has since been applied to detect financial turbulence in the securities markets by measuring the statistical unusualness of a current set of asset returns relative to their historical means and covariances.[5] In this article, we first adapt the Mahalanobis distance to estimate the statistical unusualness of prospective economic scenarios given the current values of the economic variables used to characterize the scenarios, along with their historical covariation. Later we show how to adjust this formula to account for historical norms and prevailing economic conditions. Within this context, the Mahalanobis distance d is computed as shown in Equation 1.[6]

$$d = (x - \gamma)' \Sigma^{-1} (x - \gamma) \tag{1}$$

[2] By *relevant* we mean combinations of asset class returns that are reasonably feasible given the assumed distribution.

[3] See Mahalanobis (1927).

[4] See Mahalanobis (1936).

[5] See, for example, Chow et al. (1999) and Kritzman and Li (2010).

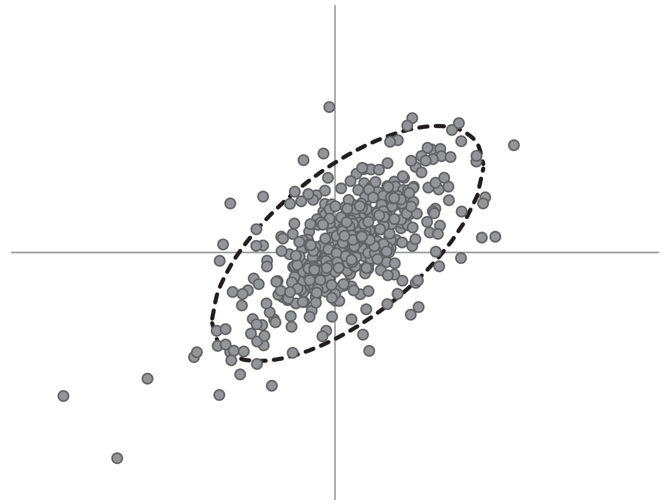
[6] The Mahalanobis distance is often multiplied by $\frac{1}{N}$ so that the average distance score across the dataset equals 1. This is just a scaling factor that we exclude for purposes of our analysis. It is sometimes shown as the square root of this quantity, which is another form of scaling.

In Equation 1, d equals the Mahalanobis distance, x equals the values of a set of economic variables used to characterize a future scenario, γ equals the prevailing values of the economic variables, Σ is the historical covariance matrix of changes in values for those variables, and $'$ indicates a vector transpose. We express all vectors as column vectors.

The term $(x-\gamma)$ captures how unusual each projected economic variable is in isolation given its prevailing value. By multiplying this term by the inverse of the covariance matrix, we capture the unusualness of the co-occurrence of the projected variable changes, and we render the measure scale independent as well. Consider, for example, the scatter plot of two hypothetical economic variables shown in Exhibit 1.

Exhibit 1

Scatter Plot of Two Hypothetical Economic Variables



In Exhibit 1, each dot represents the joint values of the two economic variables for a given observation. The center of the ellipse represents the prevailing values of the two variables. The observations within the ellipse represent reasonably plausible combinations because the observations are not particularly distant from the prevailing values. The observations outside the ellipse are statistically unusual and therefore likely to characterize much less plausible combinations. Notice that combinations just outside the narrow part of the ellipse are closer to the ellipse's center than some combinations within the ellipse at either end. This illustrates the notion that some observations qualify as unusual not because one or more of the values was unusually distant from the prevailing value, but instead because the values moved in the opposite direction despite the fact that these two economic variables are positively correlated, as evidenced by the positive slope of the scatter plot.

This measure of statistical unusualness is scale independent in the following sense: Observations that lie on a particular ellipse all have the same Mahalanobis distance from the center of the scatter plot, even though they have different Euclidean distances. This feature is particularly useful when we deal with economic variables that are measured according to different scales. For example, inflation is measured as a percentage change, whereas the unemployment rate is measured as a level. By multiplying $(x-\gamma)$ by the inverse of the covariance matrix, we convert all of the variables into common units.

Up to now, we have considered deviations in economic conditions relative to their current values. This method implicitly assumes that current economic conditions are likely to persist. However, there may be circumstances in which mean reversion is more likely than persistence. This could be the case when an economic trend has persisted for an unusually long period of time or when economic conditions have deviated far from their normal range. Mean reversion may also be a more reasonable prediction if we are forecasting scenarios far into the future. To reflect these circumstances, we replace deviations from current conditions, $(x-\gamma)$, with deviations from normal conditions, $(x-\mu)$. More generally, investors may anchor probabilities to any chosen blend of current conditions and normal conditions by introducing a variable, θ, which is a weighted average of current and normal conditions.

$$d = (x-\theta)' \Sigma^{-1} (x-\theta) \tag{2}$$

$$\theta = \omega\gamma + (1-\omega)\mu \tag{3}$$

In Equations 2 and 3, ω is the weight applied to current conditions, and $(1-\omega)$ is the weight applied to normal conditions. The covariance matrix, Σ, is estimated from the average of squared historical deviations, and these historical deviations must align with the choice of θ. One may also use θ to express a view that economic conditions will tend toward a value other than the historical norm (e.g., a set of conditions associated with good or bad stock market performance). Equation 4 shows the definition of the deviations we use to calculate the covariance matrix. Note that we use the economic conditions that occurred prior to each historical observation, x_{t-1}, rather than the current conditions, γ, which occur at the time we perform the analysis.

$$Deviation_t = x_t - (\omega x_{t-1} + (1-\omega)\mu) \quad (4)$$

The Mahalanobis distance is related to a scenario's probability of occurrence in the following sense: An observation with a high Mahalanobis distance will tend to occur less frequently than one with a low Mahalanobis distance. If we assume that the economic variables follow a multivariate normal distribution, we can measure the relative likelihood of scenarios precisely. The likelihood of an observation decays as the Mahalanobis distance increases; it decays according to an exponential function, which gives rise to the normal distribution. We measure the likelihood that we would observe a scenario given the chosen blend of prevailing and normal values of the economic variables used to define the scenario, together with their historical covariation, as shown in Equation 5.

$$Likelihood \propto e^{-d/2} \quad (5)$$

In Equation 5, d equals the Mahalanobis distance, e is the base of the exponential function, and $\propto$ denotes a proportionality relationship.[7]

We now have but one more step to derive the relative probabilities of the scenarios. Unlike mean–variance analysis, which implicitly subsumes all possible future scenarios, scenario analysis considers only a finite set of potential scenarios, which is hardly exhaustive. We therefore rescale the likelihoods so they sum to one. These rescaled values represent the relative probabilities of the scenarios.

To summarize, we determine the relative probabilities of prospective scenarios as follows:

1. We define a finite set of prospective scenarios as combinations of economic variables.
2. We use Equation 2 to calculate the Mahalanobis distances of the prospective scenarios from the prevailing and normal values of the variables used to define the scenarios.
3. We use Equation 5 to convert each prospective scenario's Mahalanobis distance into a measure of relative likelihood.
4. We rescale the likelihoods to sum to one, which we interpret as probabilities.

One might be tempted to think that this process yields probabilities based on the historical prevalence of the projected scenarios. This is not the case. The probability of a prospective scenario is determined by its difference from the current or normal economic setting and by how the economic variables co-varied historically, even if the scenario's defining values never actually occurred historically. Next, we present a case study in which we apply this approach to recent data to select a portfolio.

CASE STUDY (DECEMBER 2018)

The first step in scenario analysis is to define the prospective scenarios. We use the following economic variables for the US economy to characterize the prospective scenarios[8]:

- Economic growth (year-over-year percentage change in real gross national product, seasonally adjusted)
- Unemployment rate (civilian unemployment rate, seasonally adjusted)
- Inflation (year-over-year percentage change in the Consumer Price Index, seasonally adjusted)
- Interest rates (three-month Treasury rate)
- Yield curve slope (10-year Treasury rate minus three-month Treasury rate)
- Credit spreads (Moody's BAA rate minus 10-year Treasury constant maturity rate)

In Exhibit 2, we show the current values for each economic variable, and we define three prospective scenarios pertaining to economic growth: normal, weak, and robust. All variables pertain to annual horizons. We characterize normal growth in the context of the past 10 years by eliminating all six-month periods with economic growth in the top 25% or bottom 25% of the distribution and taking the median of each variable over the remaining periods. This definition of normal growth serves as a prospective scenario and as a potential

[7] The probability density function for the multivariate normal distribution has a similar form but includes a constant term that ensures the cumulative probability of all possible outcomes equals one. The scaling is irrelevant to our analysis because we are interested in the relative probabilities of a discrete set of scenarios, which we rescale to sum to one.

[8] All economic data are sourced from the Federal Reserve of St. Louis Economic Data online library.

EXHIBIT 2
Current and Prospective Scenarios

Economic Variables	Current	Normal	Weak	Robust
Economic Growth	3.0%	2.6%	0.9%	4.2%
Unemployment Rate	3.7%	5.3%	6.9%	3.7%
Inflation	2.2%	2.0%	0.6%	3.3%
Interest Rates	2.4%	0.1%	0.0%	2.5%
Yield Curve Slope	0.5%	2.0%	3.0%	0.5%
Credit Spreads	2.3%	2.7%	3.4%	1.9%

anchor (μ) for the scenario probabilities. We define the weak growth scenario as a collection of one-standard-deviation tilts away from normal.[9] In particular, we assume that a weak environment coincides with lower inflation, lower interest rates (bounded at zero), higher yield curve slope, higher credit spreads, lower economic growth, and a higher unemployment rate. The robust growth scenario applies the same size tilts from normal, but this time in a favorable direction. These scenario definitions are admittedly quite simple because they are intended merely for illustration.

The next step is to estimate the relative probability of each scenario. We apply Equations 2 and 5 to derive the scenario probabilities shown in Exhibit 3. It is interesting to note that the robust growth scenario is highly probable when we anchor to current economic conditions but not when we anchor to normal conditions.

We next estimate the asset class returns associated with each of the scenarios. We consider five domestic asset classes:

- Equities (S&P 500 Index)
- Treasury Bonds (Bloomberg US Treasuries Index)
- Corporate Bonds (Bloomberg US Credit Index)
- Commodities (Dow Jones Commodities Index)
- Cash Equivalents (JP Morgan Three-Month Cash Index)

We regress the historical returns of the asset classes on the contemporaneous historical changes in the economic variables used to define the scenarios; we thus derive the equations we then use to predict the asset classes' returns for a given scenario. The dependent variable in each regression is the past year's return in excess

[9] We use standard deviations of the variables (relative to normal conditions) over the past 30 years.

EXHIBIT 3
Scenario Probabilities

	Normal	Weak	Robust
Probability Based on Persistence	16%	0%	84%
Probability Based on Mean Reversion	75%	6%	19%
Probability Based on 50/50 Blend	58%	1%	41%

of cash for the relevant asset class as of each quarter end. The independent variables are year-over-year changes in economic variables for the same periods. We use overlapping annual returns to align with our year-ahead prediction horizon and because the measurement and publication timing of economic data may differ slightly from the time those data are reflected in asset prices. Overlapping time windows do not impose any bias on the coefficient estimates or the R^2. The p-values we report are adjusted to account for the autocorrelation induced by overlapping windows.[10] Exhibit 4 shows the regression results based on data beginning January 1989 and ending December 2018.

The R^2 values for these regressions may seem unusually high because it is commonly known that economic variables are poor predictors of asset class returns. The R^2 values are high because these regressions measure the contemporaneous relation between the economic variables and the asset class returns. When we regress the asset class returns on the prior year's values for these variables, the average R^2 value across the regressions is substantially smaller, as shown in Exhibit 5. Therefore, to profit from the contemporaneous relation of asset class returns and economic variables, we need to predict the future value of these variables, which is essentially what we do when we apply the Mahalanobis distance to estimate the relative likelihood of prospective economic scenarios.

One could argue that we should directly forecast asset class returns, but returns are noisier and more difficult to predict than are economic variables. This is why we first apply the Mahalanobis distance to predict the relative likelihood of prospective economic variables

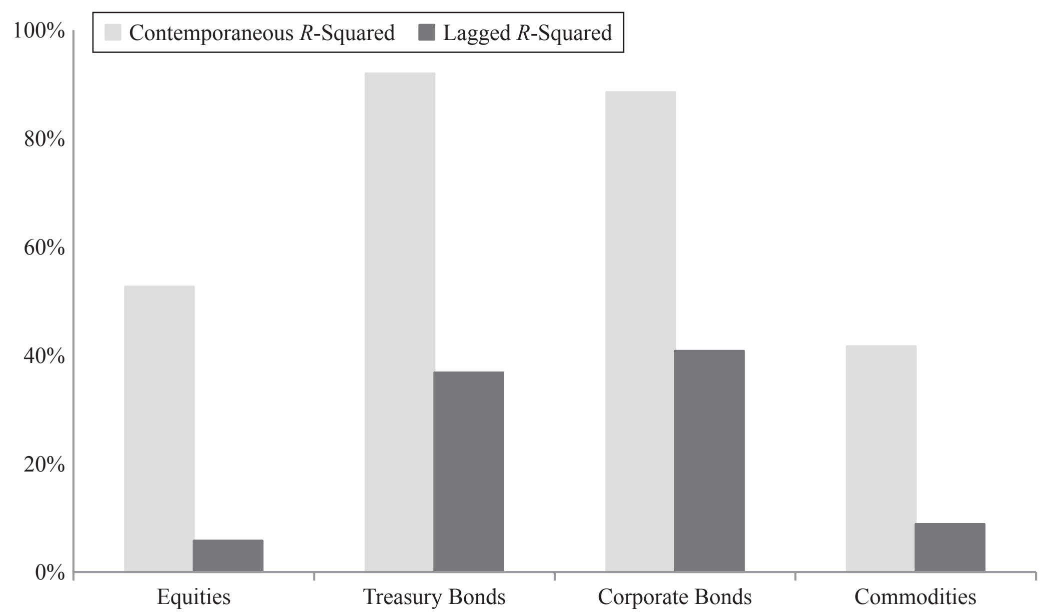
[10] We adjust t-statistics to account for the observed autocorrelation up to four quarterly lags. We use a conservative estimate of degrees of freedom to compute p-values. In particular, we use the number of non-overlapping annual returns, which equals 30 in this case.

EXHIBIT 4
Asset Class Regression Results

	Intercept	Economic Growth	Unemployment Rate	Inflation	Interest Rates	Yield Curve Slope	Credit Spreads	R^2
Equities	**7.4%**	0.7	**–7.7**	–1.3	–4.7	–3.6	**–13.3**	53%
p-Value	**0.00**	0.23	**0.00**	0.06	0.06	0.10	**0.00**	
Treasury Bonds	**2.0%**	**0.2**	**0.4**	–0.1	**–5.2**	**–5.1**	–0.1	92%
p-Value	**0.00**	**0.02**	**0.05**	0.07	**0.00**	**0.00**	0.42	
Corporate Bonds	**2.6%**	0.2	0.4	**–0.5**	**–7.3**	**–6.9**	**–6.6**	89%
p-Value	**0.00**	0.07	0.16	**0.00**	**0.00**	**0.00**	**0.00**	
Commodities	4.0%	0.5	3.3	**9.9**	1.6	1.1	–1.4	42%
p-Value	0.13	0.37	0.24	**0.00**	0.31	0.34	0.34	

Note: Bold font denotes coefficients that are significant at the 5% level.

EXHIBIT 5
R^2 Values for Lagged versus Contemporaneous Regressions



and then use the contemporaneous regression equations to map these economic variables onto asset class returns.

Next, for each scenario, we multiply the prospective change in each economic variable by the corresponding regression coefficients in Exhibit 4 to derive scenario-dependent asset class returns (Exhibit 6).[11]

[11] In the case of cash, we assume that the current interest rate will prevail over the next year. For all other assets, we add cash to the excess returns estimated from the regressions to estimate expected total returns.

The final step of scenario analysis is to select the portfolio that is best suited to an investor's preferences. We consider three portfolios, as shown in Exhibit 7: conservative, moderate, and aggressive.

We multiply the asset class returns in each scenario by the portfolio weights to arrive at the return associated with each portfolio in each scenario (Exhibit 8).

Although we do not calculate risk explicitly, it is reflected in the range of returns that each portfolio experiences across the three scenarios. Most investors are

EXHIBIT 6
Scenario-Dependent Asset Class Returns

	Normal	Weak	Robust
Equities	–2.5%	–27.3%	13.5%
Treasury Bonds	8.9%	4.4%	3.7%
Corporate Bonds	9.1%	–1.8%	6.1%
Commodities	6.7%	–2.2%	18.5%
Cash Equivalents	2.4%	2.4%	2.4%

EXHIBIT 7
Portfolio Weights

	Conservative	Moderate	Aggressive
Equities	20%	40%	60%
Treasury Bonds	30%	15%	0%
Corporate Bonds	10%	15%	20%
Commodities	10%	15%	20%
Cash Equivalents	30%	15%	0%

EXHIBIT 8
Scenario-Dependent Portfolio Returns

	Conservative	Moderate	Aggressive
Normal	4.4%	3.0%	1.6%
Weak	–3.8%	–10.5%	–17.2%
Robust	7.0%	10.0%	13.0%

risk averse, which means they dislike a given size loss more than they like an equal size gain. These preferences are typically expressed by defining utility as a concave function of portfolio return. In this illustration, we use a power utility function as shown in Equation 6, though this approach is valid for any chosen utility function.

$$U_{power}(R) = \frac{(1+R)^{1-\varphi} - 1}{1-\varphi} \tag{6}$$

In Equation 6, U equals utility, R equals return, and φ is a risk aversion coefficient. Power utility increases risk aversion by increasing the degree of curvature in the utility function. Exhibit 9 depicts this utility function for returns ranging from −10% to 30%. For power utility, we set the risk aversion coefficient, φ, equal to 5.

Exhibit 10 shows the scenario-dependent utility for each portfolio. These values reflect the fact that losses are penalized more than gains are rewarded based on the utility function we specified.

Lastly, we multiply these scenario-dependent utilities by the relative probabilities of the prospective scenarios (from Exhibit 3) to arrive at the probability-weighted expected utility of each portfolio. As is the case for mean–variance analysis, the investor should select the portfolio with the highest expected utility. Exhibit 11 shows that anchoring to the current environment (which assumes persistence) suggests that the aggressive portfolio offers the highest expected utility, whereas anchoring to normal conditions (which assumes mean reversion) suggests that the conservative portfolio is optimal. In both of these cases, the portfolio with the highest expected utility also has the highest expected return. However, this result does not always hold. When we anchor to a 50/50 blend of persistence and mean reversion, the aggressive portfolio has the highest expected return, but it does not have the highest expected utility owing to its higher level of risk. An investor with these views is better off holding the moderate portfolio.

SCENARIO MODIFICATION

Suppose that an investor believes that the weak scenario is more likely to occur—say, 10% likely instead of 1%. We could just assign a 10% probability to its occurrence and adjust the other scenarios' probabilities accordingly. Alternatively, we could change one or more of the values that define the scenarios so that the probability of the weak scenario rises to 10%.

Because we rescale the probabilities of the prospective scenarios to sum to one, we may need to modify one or more of the other scenarios to achieve this outcome most efficiently. In fact, the most efficient solution could involve making the largest changes to definitions of scenarios other than the one we are targeting. In any event, it is generally preferable to discover what changes would be called for to achieve our target probability rather than to change the probabilities arbitrarily. If only minor and, upon reflection, reasonable modifications are called for, this approach would seem preferable to an arbitrary revision of the scenario probabilities.

We cannot solve this problem analytically. Instead, we must resort to an iterative procedure to find the solution. We do so by taking the partial derivatives of the targeted probability with respect to the economic

Exhibit 9

Power Utility Function

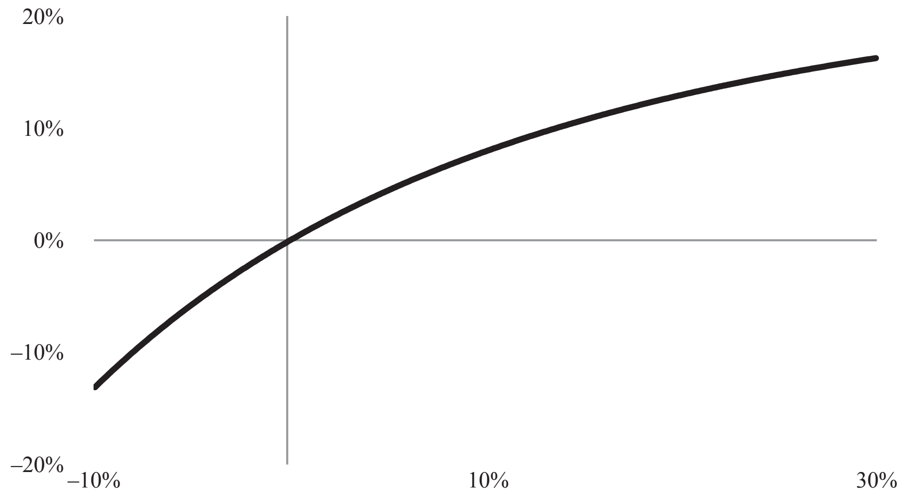


Exhibit 10

Scenario-Dependent Portfolio Utility

	Conservative	Moderate	Aggressive
Normal	4.0%	2.8%	1.6%
Weak	–4.2%	–14.0%	–28.2%
Robust	5.9%	7.9%	9.7%

Exhibit 11

Expected Portfolio Returns and Utility

	Conservative	Moderate	Aggressive
Expected Return—Persistence	6.6%	8.9%	11.2%
Expected Return—Mean Reversion	4.4%	3.5%	2.7%
Expected Return—50/50 Blend	5.4%	5.7%	6.0%
Expected Utility—Persistence	5.6%	7.1%	8.4%
Expected Utility—Mean Reversion	3.9%	2.8%	1.3%
Expected Utility—50/50 Blend	4.7%	4.7%	4.5%

variables across all the scenarios and iteratively change the values of the economic variables by arbitrarily small amounts that are proportional to the direction and size of the partial derivatives. We repeat this process until the rescaled probability of the targeted scenario equals our desired value. After doing so, we should consider how the other scenario probabilities changed to ensure that we are comfortable with all of the new probabilities.

To determine these derivatives, we first compute the gradient of the Mahalanobis distance, d_m, with respect to each of vectors of economic conditions, x_m, for a given scenario, m, to arrive at the collective set of changes that will have the largest impact on the Mahalanobis distance for that scenario. The gradient of the Mahalanobis distance is simply the collection of derivatives for each scenario.

$$\nabla d_m(x_m) = 2\Sigma^{-1}(x_m - \theta) \tag{7}$$

In this expression, ∇d_m is the gradient of the Mahalanobis distance as a function of its vector of inputs, Σ^{-1} is the inverse of the covariance matrix of historical deviations, and θ is the vector of economic variables based on the chosen blend of current conditions and normal conditions. The gradient tells us the directions and proportions in which to adjust each element of x_m to have the greatest impact for a given scenario, m. We repeat this calculation for every scenario.

Equation 7 gives the sensitivity of the Mahalanobis distance to the economic variables, but this is only an intermediate step. Our goal is to determine the sensitivity of the rescaled probabilities to the economic variables. These probabilities are a function of the raw probabilities from Equation 5, which themselves are a function of the Mahalanobis distances from Equation 2. We therefore have a set of nested functions, which requires us to apply the chain rule of calculus to determine the ultimate derivatives of interest (i.e., the derivatives of the rescaled probabilities with respect to the economic variables). These derivatives are given by Equation 8.

EXHIBIT 12
Algorithm to Adjust Scenario Probabilities

Panel A: Baseline Scenarios

Economic Variables	Normal	Weak	Robust
Economic Growth	2.6%	0.9%	4.2%
Unemployment Rate	5.3%	6.9%	3.7%
Inflation	2.0%	0.6%	3.3%
Interest Rates	0.1%	0.0%	2.5%
Yield Curve Slope	2.0%	3.0%	0.5%
Credit Spreads	2.7%	3.4%	1.9%
Mahalanobis Distance	0.90	8.57	1.63
Probability Based on 50/50 Blend	58%	1%	41%

Panel B: Adjusted Scenarios—Interim

	Normal		Weak		Robust	
Economic Variables	**Adjustment***	**Value**	**Adjustment***	**Value**	**Adjustment***	**Value**
Economic Growth	0.003%	2.6%	–0.010%	0.9%	0.005%	4.3%
Unemployment Rate	0.015%	5.3%	–0.082%	6.8%	0.000%	3.7%
Inflation	0.006%	2.0%	0.030%	0.7%	0.009%	3.3%
Interest Rates	–0.010%	0.1%	–0.010%	0.0%	0.000%	2.5%
Yield Curve Slope	0.007%	2.0%	–0.109%	2.9%	–0.012%	0.4%
Credit Spreads	0.002%	2.7%	–0.131%	3.3%	–0.011%	1.9%
Mahalanobis Distance	0.03	0.92	–1.30	7.26	0.03	1.66
Probability Based on 50/50 Blend	–1%	58%	1%	2%	–1%	40%

Panel C: Adjusted Scenarios—Final

	Normal		Weak		Robust	
Economic Variables	**Adjustment***	**Value**	**Adjustment***	**Value**	**Adjustment***	**Value**
Economic Growth	0.016%	2.6%	0.001%	0.9%	0.026%	4.3%
Unemployment Rate	0.086%	5.4%	–0.404%	6.4%	0.001%	3.7%
Inflation	0.035%	2.0%	0.145%	0.8%	0.051%	3.3%
Interest Rates	–0.056%	0.1%	–0.010%	0.0%	–0.006%	2.5%
Yield Curve Slope	0.038%	2.0%	–0.444%	2.6%	–0.075%	0.4%
Credit Spreads	0.010%	2.7%	–0.412%	3.0%	–0.076%	1.9%
Mahalanobis Distance	0.16	1.06	–4.13	4.44	0.26	1.89
Probability Based on 50/50 Blend	–4%	54%	9%	10%	–5%	36%

**Adjustments relative to baseline.*

$$\nabla p_{target}(x_m) = \frac{M}{\sqrt{det(2\pi\Sigma)}}\left(\frac{\delta_{target}(m)}{\sum_{k=1}^{M}\xi_k} - \frac{\xi_m}{\left(\sum_{k=1}^{M}\xi_k\right)^2}\right)e^{-d_m/2}\Sigma^{-1}(x_m-\theta) \quad (8)$$

In this expression, ∇p_{target} is the gradient of the probability as a function of the vector of inputs, the term $\delta_{target}(m)$ is equal to one if scenario m is the same as the scenario for which we are targeting a probability and equal to zero otherwise, M is the number of economic scenarios, N is the number of economic variables, d_m is the Mahalanobis distance of x_m, ξ_m is the raw probability density of d_m, and $det(\,)$ represents the determinant of a matrix.

Though the expression is algebraically complicated, it is straightforward to evaluate. We identify the scenario that has the largest directional derivative, and we adjust the vector of that scenario's economic conditions by a small increment in this direction. We proceed iteratively until the probability equals our desired target. This method is generally called *gradient descent*, and it is used commonly to minimize functions. In this application, we simply stop the algorithm as soon as the desired probability has been reached, rather than proceeding to minimize the function. Exhibit 12 provides a sample illustration of the algorithm applied to the case study in which we increase the probability of the weak scenario to 10% (based on the 50/50 anchor).

CONCLUSION

Investors often use scenario analysis as an alternative to mean–variance analysis to construct portfolios. Investors who prefer scenario analysis find it more intuitive than mean–variance analysis. However, this intuition often comes at the expense of quantitative rigor because investors who use scenario analysis rely on subjective judgment to define prospective scenarios and to assign probabilities to their occurrence. We introduce a new procedure that removes much of the subjectivity of scenario analysis. We first introduce a robust statistical procedure for determining the relative probabilities of prospective scenarios, and we show how this procedure can be applied flexibly to reflect the duration, unusualness, or imminence of current economic conditions. We illustrate in a detailed case study how an investor can use this approach to select a portfolio to maximize expected utility, taking into account both expected return and risk across scenarios. We then introduce an additional procedure for modifying scenarios to render them consistent with prespecified views about their probabilities of occurrence, which we illustrate with an example.

This statistically enhanced approach to scenario analysis is no doubt mathematically complex, which would seem to defeat the appeal of scenario analysis. But this is not so. The complexity arises in the construction of the statistical process. Once this process is in place, its implementation and the assessment of its output is as intuitive as any subjective approach to scenario analysis. It differs only to the extent that it rests on a sound scientific foundation.

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inputs to imply plausible shocks to a global set of risk factors. These factor shocks are finally put into a portfolio valuation engine, yielding hypothetical fund profit and loss (P&L) that can be decomposed into its underlying drivers. Key to the effectiveness of this approach is the cross-functional involvement of investors, risk managers, and economists. In conjunction, the authors define potential geopolitical or other macro events, specify potential economic outcomes, and translate them into shocks to key policy risk variables and risk model factors. The process is completed by applying the shocks to portfolios and evaluating whether P&L outcomes are consistent with fund mandates and whether positioning is deliberate, diversified, and scaled.

Sources of Excess Return and Implications for Active Fixed-Income Portfolio Construction

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ABSTRACT: *The continued development of indexes, factor frameworks, and attribution tools in fixed-income markets provides a deeper understanding of manager performance and optimal portfolio construction techniques. In this article, the authors use quarterly holdings data for a broad sample of fixed-income mutual funds to attribute active returns to (1) the returns to static factor exposures, such as a structural tilt to credit spreads; (2) time-varying factor exposures, such as varying duration over the cycle; and (3) individual bond security selection. They find that, although bond funds in aggregate demonstrated positive alpha, a nontrivial amount of their performance was driven by exposure to static factors as opposed to dynamic timing or security selection. The authors illustrate how active portfolio managers can employ index products to more efficiently express factor views and help capture more excess return through reduced costs and frictions.*