

Developing

① ID-GNN with Commonsense Reasoning

ID-GNN의 preliminaries

$$G = (V, E), \quad V = \{v_1, \dots, v_n\} \text{ (node set)}$$

$$E \subseteq V \times V \text{ (edge set)}$$

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$$\text{Node feature } X = \{x_v | v \in V\}$$

h_v : node embedding

$$\text{edge feature } f = \{f_{uv} | v \in N(u)\}$$

$$h_v^{(0)} = x_v$$

k -th iteration (k -th layer)

$m_v^{(k)}$ = message embedding

$$m_u^{(k)} = \text{MSG}^{(k)}(h_u^{(k-1)})$$

MSG(\cdot), AGG(\cdot) 은 다양하다

$$h_v^{(k)} = \text{AGG}^{(k)}(\{m_u^{(k)}, u \in N(v)\}, h_v^{(k-1)})$$

이제 ID-GNN !!

ID-GNN is built with **two important components**

(1) inductive identity coloring where identity information is injected to each node

(2) heterogeneous message passing where the identity information is utilized in message passing

① Inductive identity coloring

- first extract the k -hop ego network $G_v^{(k)}$ of v center node

- then assign a unique coloring to the central node of the ego network $G_v^{(k)}$

- coloring & non coloring 노드 구분 짓기

② Heterogeneous message passing

k rounds of message passing are then applied to all the extracted ego networks.

$$m_s^{(k)} = MSG_{1[S=v]}^{(k)}(h_s^{(k-1)})$$

$$h_u^{(k)} = AGG^{(k)}(\{m_s^{(k)}, s \in N(u)\}, h_u^{(k-1)})$$

$MSG_1^{(k)}(\cdot)$: applied to nodes with Identity coloring.

$MSG_0^{(k)}(\cdot)$: // " without coloring

$$1[S=v] = \begin{cases} 1 & S=v \\ 0 & \text{otherwise} \end{cases}$$

This way, the inductive identity coloring's encoded into the ID-GNN computational graph

이 방법은 모든 MPNN에 적용 가능!!

edge attributes f_{su} 를 추가하면?

$$m_{su}^{(k)} = MSG_{1[S=v]}^{(k)}(h_s^{(k-1)}, f_{su})$$

$$h_u^{(k)} = AGG^{(k)}(\{m_{su}^{(k)}, s \in N(u)\}, h_u^{(k-1)})$$

Algorithm: ID-GNN embedding computation

Input: Graph $G(V, E)$,

input node feature $\Rightarrow \{x_v, \forall v \in V\}$

number of layers k

trainable functions $\Rightarrow MSG_1^{(k)}, MSG_0^{(k)}$

$EGO(v, k)$: extracts the k-hop ego network centered at node v

Output

Node embedding h_v for all $v \in V$

for $v \in V$ do

$G_v^{(k)} \leftarrow EGO(v, k), h_u^{(0)} \leftarrow x_u, \forall u \in G_v^{(k)}$

for $k=1, \dots, K$ do

for $u \in G_v^{(k)}$ do

$h_u^{(k)} \leftarrow AGG^{(k)}(\{MSG_{1[S=v]}^{(k)}(h_s^{(k-1)}, f_{su}), s \in N(u)\}, h_u^{(k-1)})$

$h_v \leftarrow h_v^{(K)}$

ID-GNNs can count cycles.

$h_v^{(k)}[i] \Rightarrow$ equals the number of length i cycles starting and ending at node v
For $i=1, \dots, k$

they prove this by showing that ID-GNNs can count paths from any node u to the identity node v

이론에서 Cycle count가 Node-level (predict clustering coefficient) 이
도용이 된 것 같다 \Rightarrow Reasoning 이 어떻게 사용 할까

A_{uv}^{ℓ} 은 #walks of length ℓ between u and v

ID-GNN-Fast : Injecting Identity via Augmented Node features

$$X_v^+[\ell] = \text{Diag}(A^{\ell})[v]$$

$$X_v = \text{Concat}(x_v, X_v^+)$$

그런다면 Commonsense Reasoning에서 도용이 된 것 같은 것들을 생각해? node-level?, edge-level?
Graph-level?

Cycle specific 한 Commonsense Reasoning

\Rightarrow Cycle를 카운트 해서