# Model Selection : MAP, MLE

## Introduction

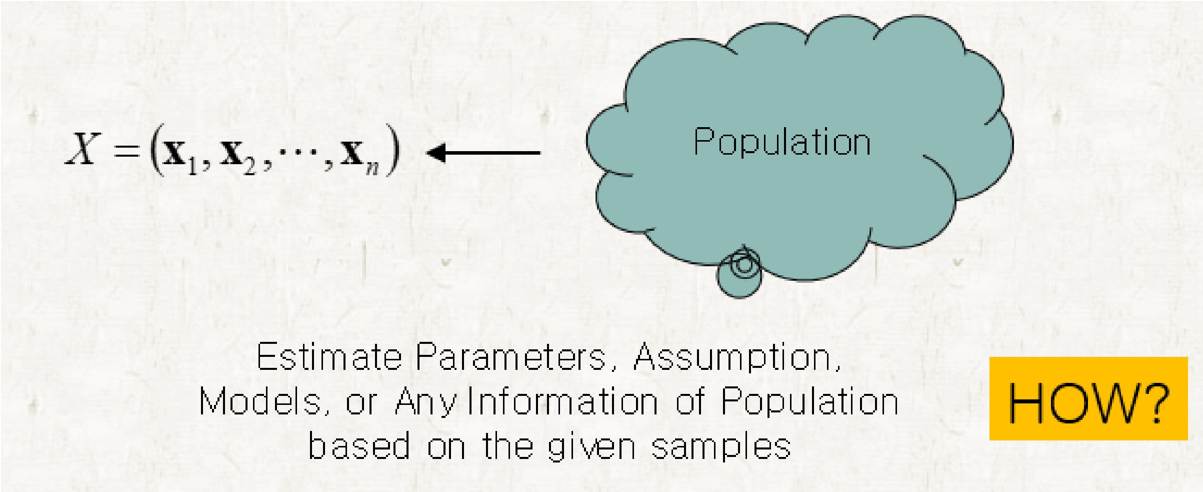
### How to find a model for given data

- Based on **Error Function**
    - Define an error function
    - Find a model which **minizes the error function**
        - solving linear equation
        - Gradient descent method
- Based on Probability
    - Evaluate the probability of given data generated
    - Find a model which **maximize the probability**
        - Solving linear equation
        - Gradient descent method
        - EM(Expectation-Maximization)
        - MCMC(Makov Chain Monte Carlo)

## Model Selection

### You have sample data x=(x1,x2,,....xn)

- Do not know about population.
- want to estimate some hidden parameters of population>>model
- sample data가 있는데 이건 어떤 모델에서 나올까??




## Let's assume that we have

- a set of sample data

$$D = \{d_1, d_2, \cdots, d_n\}$$

- a set of candidate models, parameters or assumptions

$$M = \{m_1, m_2, \cdots, m_k\}$$

D를 가장 잘 설명하는 모델m을 골라야한다.>>p(m|D)

## Usually

- P(D|mi) is easy to evaluate, but P(mi|D) is hard

## Questions

- Choose the most probable model given the training data

## Choose a model by Maximum A Poesterior(MAP)

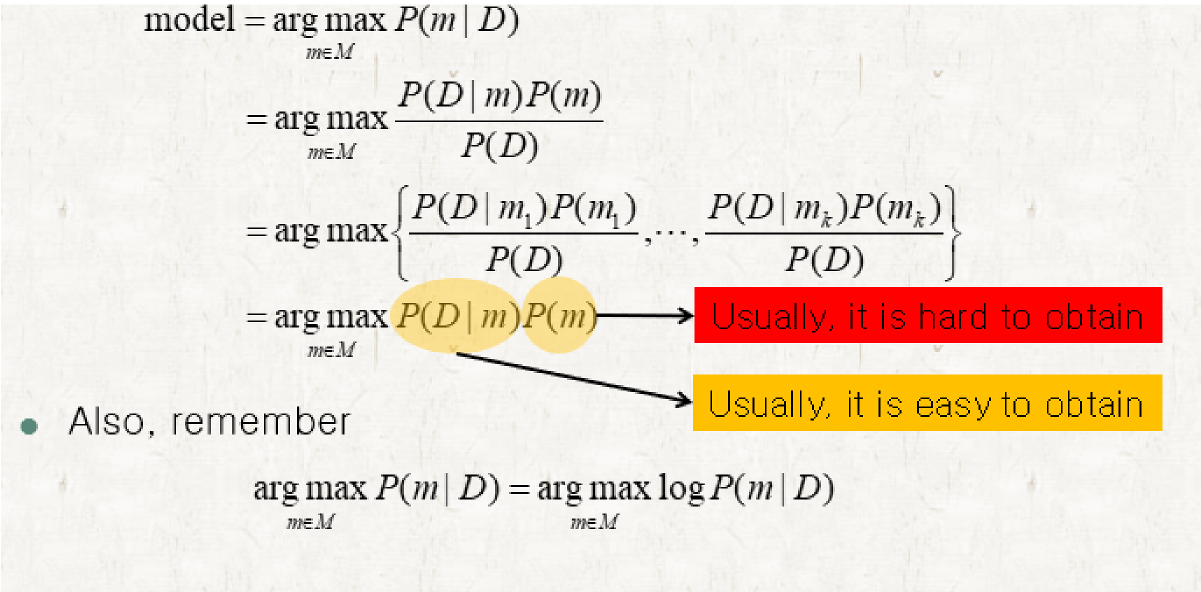
(Choose the most probable model given the training data)

- Evaluate the conditional probability of each m given D

$$P(m_1 \mid D), P(m_2 \mid D), \cdots, P(m_k \mid D)$$

- Choose the model which has the maximum probability
- In short

$$\text{model} = \underset{m \in M}{\arg\max}\, P(m \mid D)$$

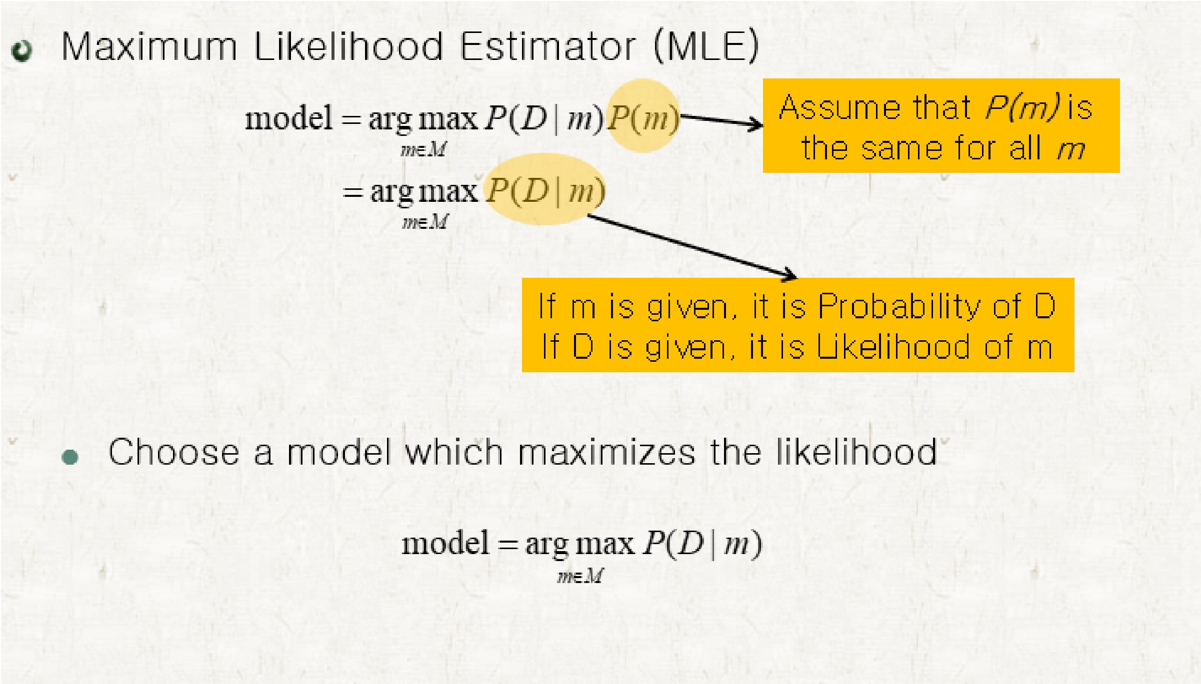



그런데 왜 p(m)은구하기 어려운가?

예를들어 모델이 10개가 있다치면 1/10이아닌가?

>>단정지을수 없다.

## Maximum Likelihood Estimator(MLE)




MAP에서 P(m)을 다 동일하다 가정 즉 모델이 k개 있으면 p(m)=1/k

그러면 p(m)은 상수가 되니까

argmax P(D|m)만구하면된다<< 이것이 MLE

보통 MLE를 구하는것이 MAP보다 쉽다.

☞ **likelihood**: p(z|x), 어떤 모델에서 해당 데이터(관측값)이 나올 확률

☞ **사전확률(prior probability)**: p(x), 관측자가 관측을 하기 전에 시스템 또는 모델에 대해 가지고 있는 선험적 확률. 예를 들어, 남여의 구성비를 나타내는 p(남자), p(여자) 등이 사전 확률에 해당한다.

☞ **사후확률(posterior probability)**: p(x|z), 사건이 발생한 후(관측이 진행된 후) 그 사건이 특정 모델에서 발생했을 확률

- Evaluate the conditional probability of D given each of m

$$P(D|m_1), P(D|m_2), \cdots, P(D|m_k)$$

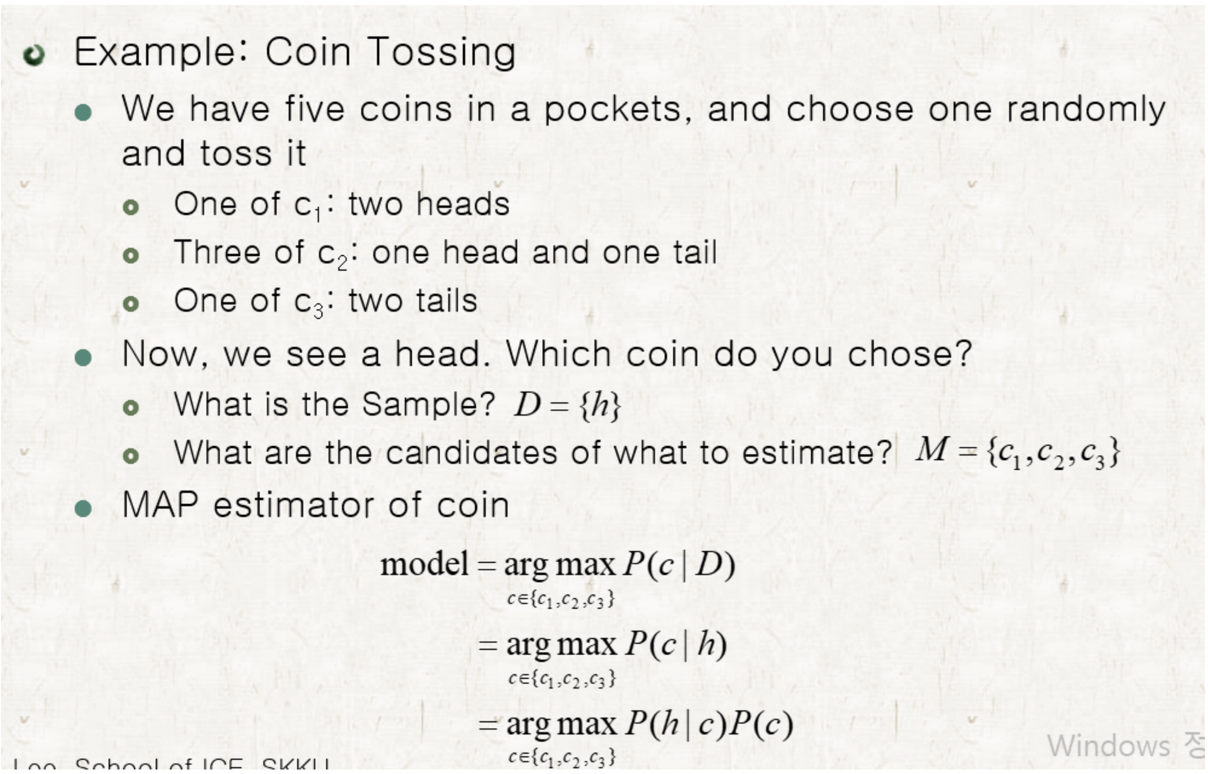
- Choose the model which has the maximum probability

- In short

$$\text{model} = \arg\max_{m \in M} P(D \mid m)$$

$$P(h \mid c_1) = 1.0, P(h \mid c_2) = 0.5, P(h \mid c_3) = 0.0,$$

## Example - Coin Tossing




$$P(c_1) = 0.2, P(c_2) = 0.6, P(c_3) = 0.2$$

$$P(h|c_1)=1.0, P(h|c_2)=0.5, P(h|c_3)=0.0,$$

$$\left.\begin{aligned} P(h|c_1)P(c_1)&=0.2 \\ P(h|c_2)P(c_2)&=0.3 \\ P(h|c_3)P(c_3)&=0.0 \end{aligned}\right\} \xrightarrow{\text{max}} P(h|c_2)P(c_2)=0.3$$

MAP estimator is $c_3$