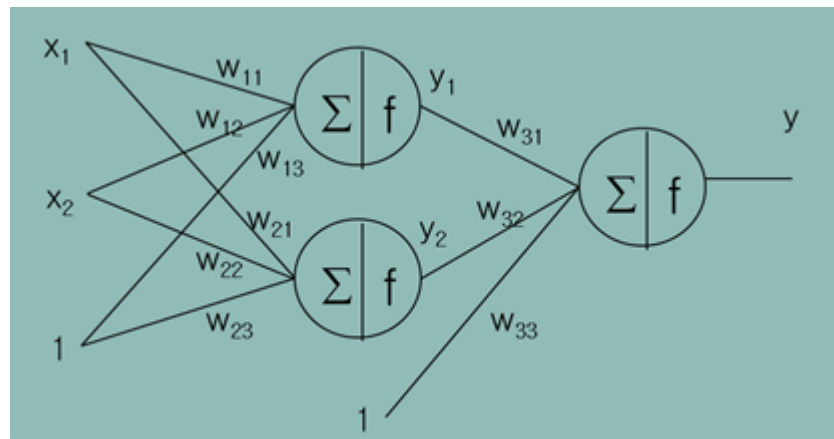


Neural Networks-Configuration

Regression

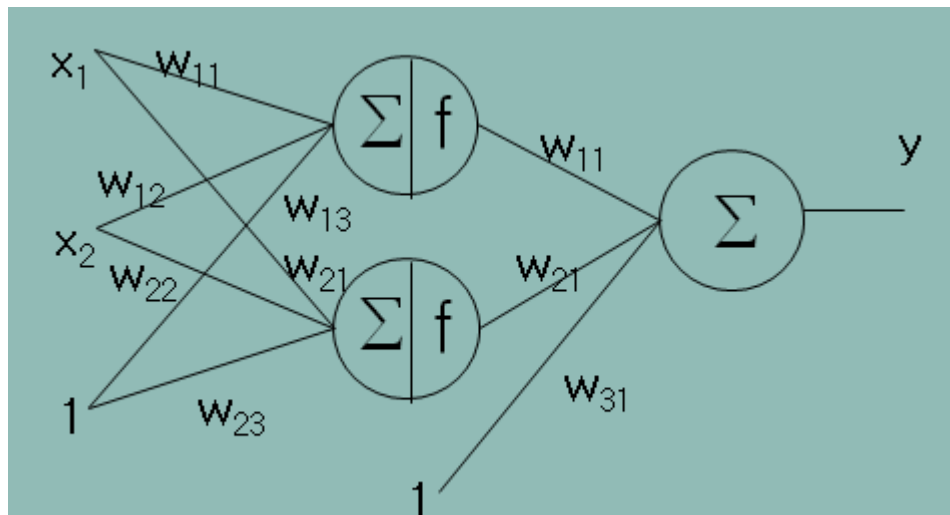


The activation functions produce a value between $[0,1]$

regression문제는 임의의 real value를 출력해야한다

Solution

- Normalize the outputs into $[0,1]$
- Or, Use a linear output node(=보통 항등함수)

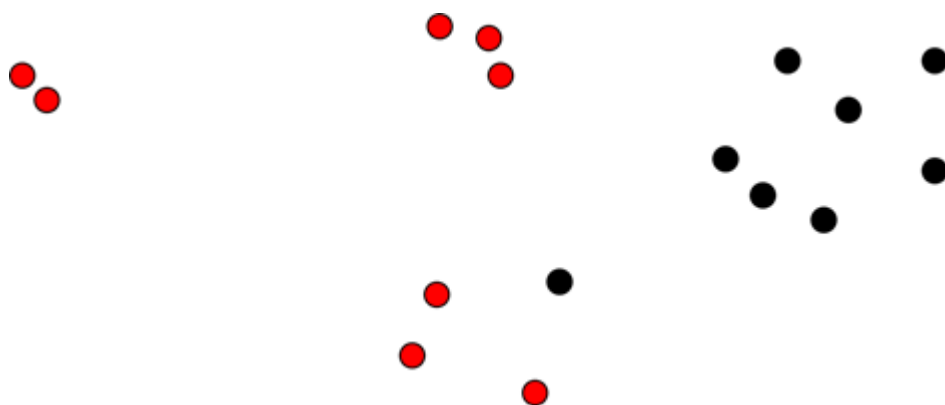


activation func : Non-linear transformation으로 해야 deep 하게 쌓는 의미가 있는것이다

Binary-Class Classification

You have Two Problems

$(x_{11}, x_{12}, Red), (x_{21}, x_{22}, Red), (x_{31}, x_{32}, Black), (x_{41}, x_{42}, Red), (x_{51}, x_{52}, Black), \dots$



output 0 | Two class

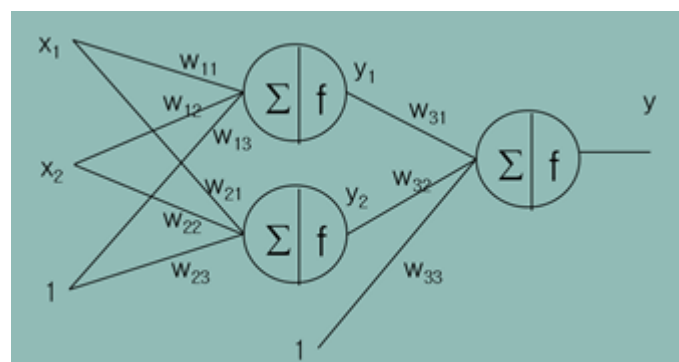
- P1 : NN cannot produces nominal values
- P2 : Error Function for training—>XMSE

P1 : Handling Nominal Values

- Use 0 and 1 for class labels

$(x_{11}, x_{12}, 1), (x_{21}, x_{22}, 1), (x_{31}, x_{32}, 0), (x_{41}, x_{42}, 1), (x_{51}, x_{52}, 0), \dots$

- Use sigmoid



P2 : Error Function for Training

NNs will not be trained in some special cases!!>>MSE는 잘안됨 ,그래서 cross entropy사용

Cross Entropy

$$H_p(q) = - \sum_{i=1}^n q(x_i) \log p(x_i)$$

크로스 엔트로피는 실제 분포 q 에 대하여 알지 못하는 상태에서, 모델링을 통하여 구한 분포 p 를 통하여 q 를 예측하는 것이다.

q 와 p 가 모두 들어가서 크로스 엔트로피라고 한다.

머신러닝을 하는 경우에 실제 환경의 값과 q 를, 예측값(관찰값) p 를 모두 알고 있는 경우가 있다. **머신러닝의 모델은 몇%의 확률로 예측했는데, 실제 확률은 몇%야!**라는 사실을 알고 있을 때 사용한다.

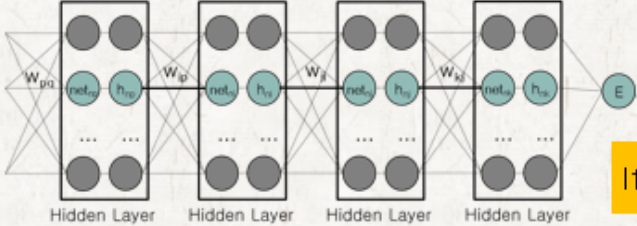
—>크로스 엔트로피는 출력을 확률로 가정하고 만들어진 error func

크로스 엔트로피에서는 실제값과 예측값이 맞는 경우에는 0으로 수렴하고, 값이 틀릴 경우에는 값이 커지기 때문에, **실제 값과 예측 값의 차이를 줄이기 위한 엔트로피**라고 보시면 될 것 같습니다.

$$E = - \sum_{n=1}^N (t_n \log(y_n) + (1 - t_n) \log(1 - y_n))$$

그럼 왜 Error func로 MSE를 안쓰고 cross entropy로 쓸까??

예를 들어, target값=1이고, 출력값(y)=0일 경우

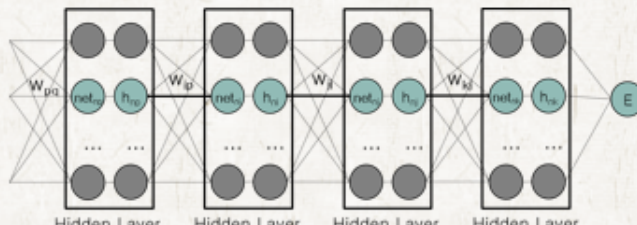


if $h = \text{Sigmoid}(\text{net})$

$$\frac{\partial E}{\partial w_{kj}} = \frac{\partial E}{\partial \text{net}_{nk}} \frac{\partial \text{net}_{nk}}{\partial w_{kj}} = \delta_{nk} h_{nj} \quad \delta_{nk} = \frac{\partial E}{\partial h_{nk}} \frac{\partial h_{nk}}{\partial \text{net}_{nk}} = -(t_n - h_{nk}) h_{nk}(1 - h_{nk})$$

$$\frac{\partial E}{\partial w_{ji}} = \frac{\partial E}{\partial \text{net}_{nj}} \frac{\partial \text{net}_{nj}}{\partial w_{ji}} = \delta_{nj} h_{ni} \quad \delta_{nj} = \left(\sum_{k=1}^K \delta_{nk} w_{kj} \right) \frac{\partial h_{nj}}{\partial \text{net}_{nj}}$$

$$\frac{\partial E}{\partial w_{ip}} = \frac{\partial E}{\partial \text{net}_{ni}} \frac{\partial \text{net}_{ni}}{\partial w_{ip}} = \delta_{ni} h_{np} \quad \delta_{ni} = \left(\sum_{j=1}^J \delta_{nj} w_{ji} \right) \frac{\partial h_{ni}}{\partial \text{net}_{ni}}$$



if $h = \text{Sigmoid}(\text{net})$

$$\frac{\partial E}{\partial w_{kj}} = \frac{\partial E}{\partial \text{net}_{nk}} \frac{\partial \text{net}_{nk}}{\partial w_{kj}} = -(t_n - h_{nk}) h_{nk}(1 - h_{nk}) h_{nj}$$

$$\frac{\partial E}{\partial w_{ji}} = \frac{\partial E}{\partial \text{net}_{nj}} \frac{\partial \text{net}_{nj}}{\partial w_{ji}} = \left(\sum_{k=1}^K -(t_n - h_{nk}) h_{nk}(1 - h_{nk}) w_{kj} \right) \frac{\partial \text{net}_{nj}}{\partial w_{ji}}$$

$$\frac{\partial E}{\partial w_{ip}} = \frac{\partial E}{\partial \text{net}_{ni}} \frac{\partial \text{net}_{ni}}{\partial w_{ip}} = \left(\sum_{j=1}^J \left(\sum_{k=1}^K -(t_n - h_{nk}) h_{nk}(1 - h_{nk}) w_{kj} \right) \frac{\partial \text{net}_{nj}}{\partial w_{ji}} \right) \frac{\partial \text{net}_{ni}}{\partial w_{ip}}$$

If h_{nk} is close to 1 or 0, all gradients for n -th training data is 0

what if h_{nk} is close to 1 or 0 but it's wrong?

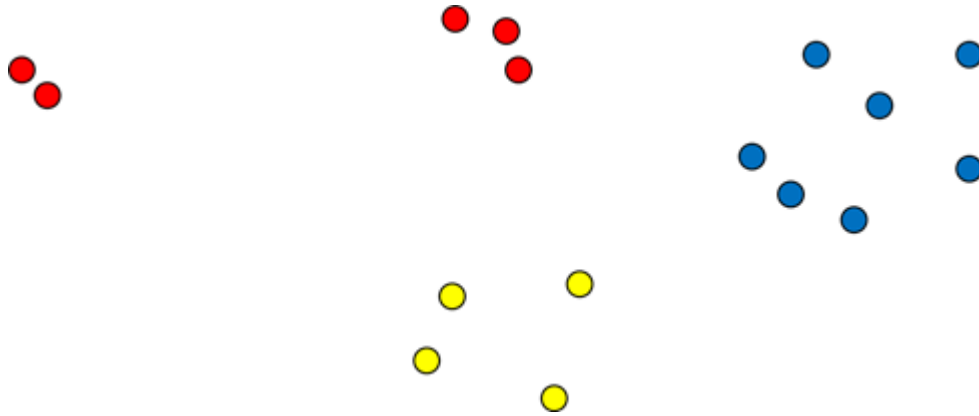
요약: Binary -class classification은 class output가 2개로 주어지며, output activation func는 sigmoid

Error_func는 cross entropy를 사용한다!!

Multi-Class Classification

Problem

$(\mathbf{x}_1, Red), (\mathbf{x}_2, Yellow), (\mathbf{x}_3, Blue), (\mathbf{x}_4, Red), (\mathbf{x}_5, Blue), \dots$



Nominal value를 다루는데 예를들어, Red:1,Yello:0.5,Blue:0으로 하면될까??

결론만 따지면 별로다.

왜냐?

Red	→	1.0
Yellow	→	0.5
Blue	→	0.0

Red가 1인데 0.5를 선택하면 틀린거다. 0을 선택해도 마찬가지다.

그러나 0.5일때 MSE가 더작다...

They are just names. We cannot say that Red>yellow>Blue

그러면 어떻게 해야하나?

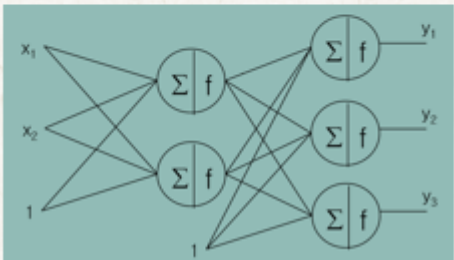
Create virtual outputs!!

- Create virtual outputs

$(x_{11}, x_{12}, Red),$
 $(x_{21}, x_{22}, Yellow),$
 $(x_{31}, x_{32}, Blue),$
 $(x_{41}, x_{42}, Red),$
 $(x_{51}, x_{52}, Blue),$
 ...

➔

$(x_{11}, x_{12}, 1, 0, 0),$
 $(x_{21}, x_{22}, 0, 1, 0),$
 $(x_{31}, x_{32}, 0, 0, 1),$
 $(x_{41}, x_{42}, 1, 0, 0),$
 $(x_{51}, x_{52}, 0, 0, 1),$
 ...
- Place nodes at the output layer as many as virtual outputs



$$E = \sum_{n=1}^{Data} \sum_{k=1}^{Class} -t_{nk} \log(y_{nk})$$

Hmm?? $-(t_n \log(y_n) + (1 - t_n) \log(1 - y_n))$

원핫인코딩이란(one-hot encoding)?

0으로 이루어진 벡터에 단 한개의 1의 값으로 해당 데이터의 값을 구별하는 것이 원핫 인코딩이다.

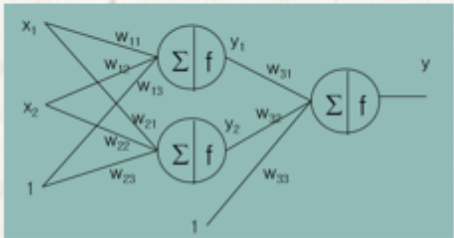
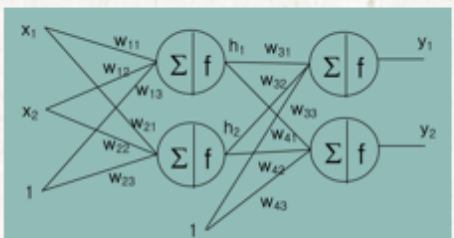
Binary Class Cross Entropy vs Multi Class Cross Entropy

- Binary Class Cross Entropy vs Multi-Class Cross Entropy

(x_{11}, x_{12}, Red)
 (x_{21}, x_{22}, Red)
 $(x_{31}, x_{32}, Black)$
 (x_{41}, x_{42}, Red)
 $(x_{51}, x_{52}, Black)$

$(x_{11}, x_{12}, 1)$
 $(x_{21}, x_{22}, 1)$
 $(x_{31}, x_{32}, 0)$
 $(x_{41}, x_{42}, 1)$
 $(x_{51}, x_{52}, 0)$

$(x_{11}, x_{12}, 1, 0)$
 $(x_{21}, x_{22}, 1, 0)$
 $(x_{31}, x_{32}, 0, 1)$
 $(x_{41}, x_{42}, 1, 0)$
 $(x_{51}, x_{52}, 0, 1)$

$$-(t_n \log(y_n) + (1 - t_n) \log(1 - y_n)) \quad -(t_{n1} \log(y_{n1}) + t_{n2} \log(y_{n2})) = - \sum_{k=1}^{Class} t_{nk} \log(y_{nk})$$

binary나 multi나 cross entropy가 다르다 생각할 수 있지만, 같다!

그렇다면 Activation function??

- Outputs of Multi-Class Classification satisfies

$$1 = \sum_{k=1}^{Class} t_{nk} \quad \text{for } n\text{-th training data}$$

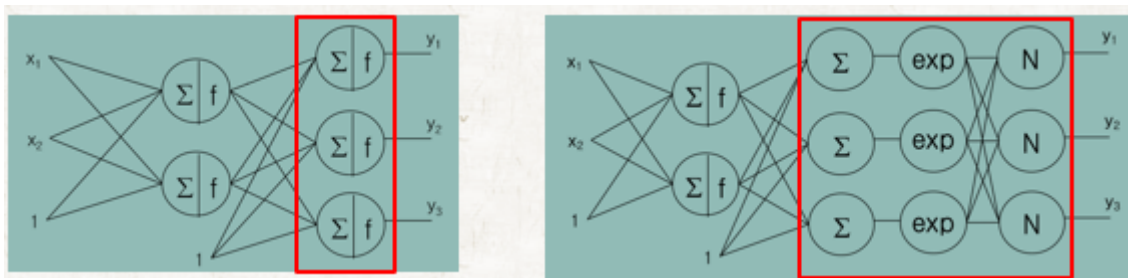
- But, Sigmoid does not satisfy this

$$1 \neq \sum_{k=1}^{Class} \text{Sigmoid}(\text{net}_{nk}) \quad \text{for } n\text{-th training data}$$

- So, We use Softmax layer

$$y_{nk} = \frac{\exp(\text{net}_{nk})}{\sum_{i=1}^{Class} \exp(\text{net}_{ni})}$$

Softmax layer



$$y_{n2} = \frac{\exp(\text{net}_{n2})}{\exp(\text{net}_{n1}) + \exp(\text{net}_{n2}) + \exp(\text{net}_{n3})}$$

How about Error func?

- Softmax layer + Cross Entropy

$$E = \sum_{n=1}^{Data\ Class} \sum_{k=1} CE(t_{nk}, y_{nk})$$

Multi-Label Classification

Multi-Label:Multi-output(with binary or multi values)

$(x_{11}, x_{12}, Yes, Yes, Male)$
 $(x_{21}, x_{22}, No, Yes, Male)$
 $(x_{31}, x_{32}, Yes, No, Female)$
 $(x_{41}, x_{42}, No, No, Female)$
 $(x_{51}, x_{52}, No, Yes, Male)$
 ...

위의그림에서

$(x_{11}, x_{12}, 1, 1, 1)$

$(x_{21}, x_{22}, 0, 1, 1)$ 이런식으로 원핫 인코딩을 적용하면 된다.

output들의 각 label은 독립적이다.

$$E = \sum_{n=1}^{Data\ Label} \sum_{k=1} CE(t_{nk}, y_{nk})$$

$$CE(t_{nk}, y_{nk}) = -(t_{nk} \log(y_{nk}) + (1 - t_{nk}) \log(1 - y_{nk}))$$

Nominal Inputs

Two inputs and one output

$$\begin{aligned} x_1 &\in \mathbb{R} \\ x_2 &\in \{\text{Red}, \text{Yellow}, \text{Blue}\} \\ y &\in \{0, 1\} \end{aligned}$$

(0.1, Red, 0)	→	(0.1, 1, 0, 0, 0)
(0.2, Blue, 1)		(0.2, 0, 0, 1, 1)
(0.3, Yellow, 0)		(0.3, 0, 1, 0, 0)
(0.4, Red, 1)		(0.4, 1, 0, 0, 1)

Summary

Problem	Activation Function		Loss function
	Hidden Layer	Output Layer	
Regression	ReLU	Linear	MSE
2-class Classification	ReLU	Sigmoid	CE
Multi-class Classification	ReLU	Softmax layer	CE
Multi-label Classification	ReLU	Sigmoid	CE