

Project 3

Information:

This project is based on the confidence intervals covered in chapter 11 as well as bootstrap confidence intervals. Bootstrapping is a popular statistical technique that allows the estimation of the sampling distribution of almost any statistics using random sampling methods.

Question 1

Explain the use of the pivotal quantity and bootstrap methods for obtaining confidence intervals for a population parameter θ .

Include the following ideas:

1. What does a confidence interval with a confidence level of γ represent?
2. Which requirements should be met to determine a confidence interval for θ with the pivotal quantity method?
3. Which steps should be followed to determine a confidence interval for θ with the pivotal quantity method?
4. Why do we use bootstrapping in statistics?
5. How do we obtain bootstrap samples?
6. Which steps should be followed to obtain a bootstrap confidence interval using the percentile method?
7. What are the arguments for and against the bootstrap percentile method? Hint: refer to the sample size and symmetry of the statistic of interest.

Question 2

Consider a random sample, X_1, \dots, X_n , from a population which is normally distributed with expected value μ and variance σ^2 where both population parameters are unknown. Design a simulation study to empirically show that the confidence intervals for μ and σ^2 given by Theorems 11.3.1b and 11.3.1c are indeed a $100(1 - \alpha)\%$ confidence regions. Consider a small and large sample size. You can choose the population parameters. Include the theoretical methodology and design, the code and results as well as a discussion of the results.

Question 3

Consider a random sample, X_1, \dots, X_n , from a population which is normally distributed with expected value μ and variance σ^2 where both population parameters are unknown. Design a simulation study to empirically show the behavior of the percentile bootstrap confidence intervals for μ and σ^2 with respect to small vs. large sample size as well as the symmetry of the sampling distribution. You can choose the population parameters. Include the theoretical methodology and design, the code and results as well as a discussion of the results.