Project 2

This project is based on topics covered in Chapter 9.

Information:

Chapter 9 presents different frameworks for estimating a parameters with one value i.e. point estimation. The three main methods are MME, MLE and Bayes.
In this project you will use a simulation study to see of you can recover the parameter values (when your point estimate is close to the true value) and also use some real data to estimate parameter values or functional forms thereof.

Question 1

Summarize the three estimation methods (MME, MLE and Bayes) on one page. Use an algorithmic style of writing to explain to someone the exact steps they need to take to perform the inference from these three methods.

Question 2

Suppose we have a sample y from the model

$$f(y;\lambda,\theta) = \frac{\lambda}{\theta}\left(\frac{y}{\theta}\right)^{\lambda-1} e^{-\left(\frac{y}{\theta}\right)^{\lambda}} \quad \text{for} \quad y > 0.$$

Assume $\theta = 1$ is fixed and use λ_0 as the true value of λ (Choose it yourself). Use a simulation study to calculate the point estimate of λ. You must use MME, MLE and Bayes with an Exponential prior for λ such that a priori, $E(\lambda) = \lambda_0$. Use the mode of the posterior density as the Bayes point estimate.

Show graphically the maximum of the likelihood on the likelihood function as well as the mode of the posterior.

Question 3

Suppose we have a sample y from the model

$$f(y;\lambda,\theta) = \frac{\lambda}{\theta}\left(\frac{y}{\theta}\right)^{\lambda-1} e^{-\left(\frac{y}{\theta}\right)^{\lambda}} \quad \text{for} \quad y > 0.$$

This model is very well-used in the real world to model wind speed and many scientists now use it to predict wind speed to build wind farms in order to generate eco-energy[1].

Assume both θ and λ are unknown and use the data "Proj2_2021.csv" to calculate the point estimates of θ and λ. You must use MME, MLE and Bayes with an Exponential prior for λ such that a priori, $E(\lambda) = 2$ and an Exponential prior over θ with $E(\theta) = 1$. Use the mode of the posterior density as the Bayes point estimates.

Show graphically the maximum of the likelihood on the likelihood function as well as the mode of the posterior.
HINT: Here you have to maximize over two dimensions, so you should produce a 3d graph of the likelihood function and the joint posterior.

[1] Kadhem, A.A., Wahab, N.I.A., Aris, I., Jasni, J. and Abdalla, A.N., 2017. Advanced wind speed prediction model based on a combination of weibull distribution and an artificial neural network. Energies, 10(11), p.1744.
Parajuli, A., 2016. A statistical analysis of wind speed and power density based on Weibull and Rayleigh models of Jumla, Nepal. Energy and Power Engineering, 8(7), pp.271-282.
Chiodo, E., Mazzanti, G. and Karimian, M., 2015, June. Bayes estimation of Inverse Weibull distribution for extreme wind speed prediction. In 2015 International Conference on Clean Electrical Power (ICCEP) (pp. 639-646). IEEE.