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INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

MA 374: Financial Engineering Lab

Lab 01

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Check for the *no-arbitrage condition*

We first need to check for the no-arbitrage condition.

We know that,

$$E[S(t_1)|S(t_0)] = S(t_0)\exp(r\delta t)$$

$$\implies puS(t_0) + (1-p)dS(t_0) = S(t_0)\exp(r\delta t)$$

$$\implies p = \frac{\exp(r\delta t) - d}{u-d}$$

Hence, we will calculate this probability and see if $p \epsilon [0, 1]$, and if itdoes not, we know that the ***no-arbitrage principle is getting violated.***

Question 1

Page

$d < R < u$

$$e^{r\frac{T}{m}} - e^{-\sigma\sqrt{\delta t} + (r - \frac{1}{2}\sigma^2)\delta t} > 0$$

so

$$e^{r\frac{T}{m}} > e^{-\sigma\sqrt{\frac{T}{m}} + (r - \frac{1}{2}\sigma^2)\frac{T}{m}}$$

so

so

$$r\frac{T}{m} > -\sigma\sqrt{\frac{T}{m}} + (r - \frac{1}{2}\sigma^2)\frac{T}{m}$$

so

Take $\sqrt{\frac{T}{m}} = k$ so

$$rk^2 > -\sigma k + (r - \frac{1}{2}\sigma^2)k^2$$

$k \neq 0$ so

$$r \cdot k > -\sigma + (r - \frac{1}{2}\sigma^2)k$$

so

$$\frac{1}{2}\sigma^2 k > -\sigma$$

This is alway true

Now

$$R < u$$

so

similarly as earlier

$$r\frac{T}{m} < \sigma\sqrt{\frac{T}{m}} + \left(r - \frac{1}{2}\sigma^2\right)\frac{T}{m}$$

$$\sqrt{\frac{T}{m}} = k$$

so

$$\frac{1}{2}\sigma^2 k < \sigma$$

$$\frac{1}{2}\sigma k < 1$$

$$\frac{1}{2}\sigma\sqrt{\frac{T}{m}} < 1$$

so

$$\frac{1}{2}\sigma\sqrt{T} < \sqrt{m}$$

so

$$0.1125 < m$$

So m can be infinitely large there is no upper bound on it

There is no upperbound on M.

1. we calculate the price of the option using the respective payoff function for both the call and put option, i.e,

$$C_n^M = \max(S_n^M - K, 0), \qquad 0 <= n <= M$$
$$P_n^M = \max(K - S_n^M, 0), \qquad 0 <= n <= M$$

where, C_n^M is the nth possible price of the call option for the Mth interval, and P_n^M is the nth possible price of the put option for the Mth interval

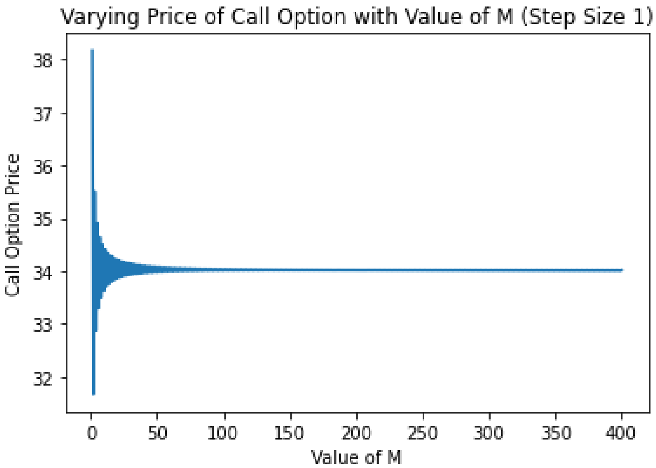
2. Now, we continuously apply **Backward Induction** to find out the option price at t = 0by using following relation:

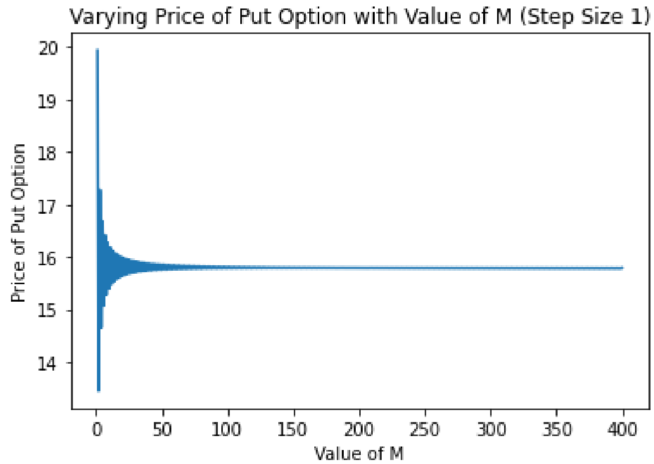
$$C_n^i = (1 - p) . C_{n+1}^{i+1} + p . C_n^{i+1} \quad , 0 <= n <= i \ \& \ 0 <= i <= M - 1$$
$$P_n^i = (1 - p) . C_{n+1}^{i+1} + p . P_n^{i+1} \quad , 0 <= n <= i \ \& \ 0 <= i <= M - 1$$

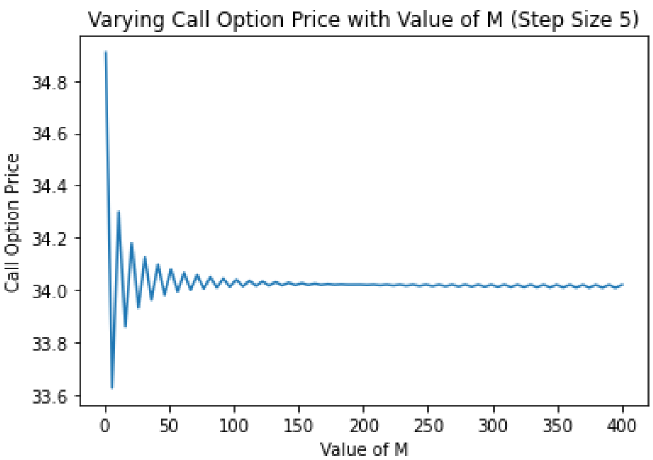
3. $C0^0$ and $P0^0$ are the required values, i.e, initial option prices.

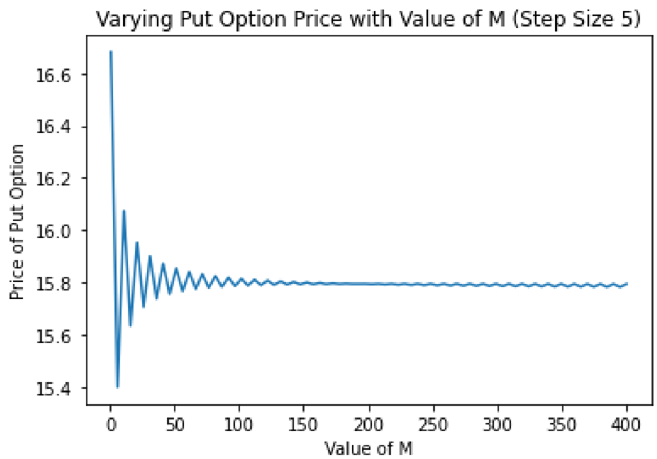
	Step Size	Call Option Price	Put Option Price
0	1	38.167635	19.941717
1	5	34.906533	16.680615
2	10	33.625022	15.399104
3	20	33.859449	15.633532
4	50	33.981184	15.755267
5	100	34.011161	15.785243
6	200	34.019579	15.793661
7	400	34.019132	15.793214

Question 2


Varying Price of Call Option with Value of M (Step Size 1)
Call Option Price
Value of M


Varying Price of Put Option with Value of M (Step Size 1)
Price of Put Option
Value of M


Varying Call Option Price with Value of M (Step Size 5)
Call Option Price
Value of M


Varying Put Option Price with Value of M (Step Size 5)
Price of Put Option
Value of M

Observations:

1. We observe that the value of the **European call option** converges to **34.0** while the **European put option** converges to**15.7**.

2. The convergence of the plot is faster when step value is 5. The deviations from the convergence value is higher when the value of M is less.

3. This convergence is however not perfect. There is still somedeviation(values tend to oscillate around specified values as in first point).

4. On increasing the value of M, the prices of both the call and theput options slowly converge and the deviation between consecutive values reduces. However, on further increasing M, the error increases. This pattern repeats and the option value converges slowly.

Question 3.

The values of the options at **t = 0, 0.50, 1, 1.50, 3, 4.5** for the case **M = 20** are tabulated below:

We have (i+1) different values of option as there are (i+1) possible values for the underlying asset for the ith step.

For call and put option Respectively

```
PS D:\sem-6\6th\Semester 6\MA 374 (Financial Engg. Lab)\Lab 1> python -u "d:\sem-6\6th\Semester 6\MA 374 (Financial Engg. Lab)\Lab 1\200123026_q3.py"
              d0           d1          d2          d3          d4          d5          d6         d7         d8        d9  d10  d11  d12  d13  d14  d15  d16  d17  d18
0.0    33.736359          NaN         NaN         NaN         NaN         NaN         NaN        NaN        NaN       NaN  NaN  NaN  NaN  NaN  NaN  NaN  NaN  NaN  NaN
0.5    59.783741    31.784024   15.035018         NaN         NaN         NaN         NaN        NaN        NaN       NaN  NaN  NaN  NaN  NaN  NaN  NaN  NaN  NaN  NaN
1.0   100.431372    57.544383   29.708395   13.418246    5.131482         NaN         NaN        NaN        NaN       NaN  NaN  NaN  NaN  NaN  NaN  NaN  NaN  NaN  NaN
1.5   160.325055    98.235182   55.159042   27.491091   11.725109    4.103586    1.119238        NaN        NaN       NaN  NaN  NaN  NaN  NaN  NaN  NaN  NaN  NaN  NaN
3.0   518.687766   359.629024  241.804154  154.674922   91.073467   46.896969   19.681692   6.130906   1.231475  0.117798  0.0  0.0  0.0  NaN  NaN  NaN  NaN  NaN  NaN
4.5  1419.169249  1024.804270  732.651507  516.219417  355.882581  237.102132  149.107411  83.919318  36.229016  8.139997  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
            d0         d1         d2         d3         d4         d5         d6  ...        d12        d13        d14        d15        d16        d17        d18
0.0  15.678049        NaN        NaN        NaN        NaN        NaN        NaN  ...        NaN        NaN        NaN        NaN        NaN        NaN        NaN
0.5   8.508324  15.529151  24.724001        NaN        NaN        NaN        NaN  ...        NaN        NaN        NaN        NaN        NaN        NaN        NaN
1.0   3.518466   8.030547  15.308652  25.031664  36.015923        NaN        NaN  ...        NaN        NaN        NaN        NaN        NaN        NaN        NaN
1.5   0.946912   3.010006   7.459553  14.999497  25.316165  37.017144  48.347257  ...        NaN        NaN        NaN        NaN        NaN        NaN        NaN
3.0   0.000000   0.000000   0.008745   0.172801   1.239805   4.970879  13.246405  ...  78.239478        NaN        NaN        NaN        NaN        NaN        NaN
4.5   0.000000   0.000000   0.000000   0.000000   0.000000   0.000000   0.000000  ...  60.832399  71.607918  79.590619  85.504349  89.885349  93.130873  95.535216
```

Thank you