Week 10: Set Theory and the Continuum Hypothesis

Mathematical Logic Course

April 24, 2023

Introduction

- Welcome to Week 10 of our Mathematical Logic Course!
- This week, we'll explore Set Theory and the Continuum Hypothesis.
- We'll cover the following topics:
  - Introduction to set theory: Zermelo-Fraenkel axioms
  - The Continuum Hypothesis: statement and independence

Set Theory: Zermelo-Fraenkel Axioms

- What is set theory?
- Zermelo-Fraenkel axioms: foundation of modern set theory
- Understanding axiomatic set theory and its role in mathematics
- Examples of sets, operations, and relations

Infinite Sets: Countable and Uncountable

- Introduction to infinite sets
- Distinction between countable and uncountable sets
- Examples of countable and uncountable sets
- Cardinality of infinite sets: aleph numbers

The Continuum Hypothesis

- Statement of the Continuum Hypothesis (CH)
- Understanding the significance of the CH in set theory
- Exploring the concept of the cardinality of the continuum

Independence of the Continuum Hypothesis

- Gödel and Cohen's work on the independence of the CH
- Understanding the concept of independence in mathematical logic
- Implications for set theory and mathematical reasoning

Summary and Conclusion

- Recap of the topics covered in this lecture
- Set theory and its foundational role in mathematics
- The Continuum Hypothesis and its independence
- Next week, we'll explore Model Theory and Nonstandard Models

Questions and Discussion

- Do you have any questions about today's lecture?
- Let's discuss the material and explore any questions you may have

Coding Exercises

- Implementing set operations and exploring infinite sets in Python
- Understanding the Continuum Hypothesis through coding exercises