

# Week 2: First-Order Logic and Formal Systems

Mathematical Logic Course

April 24, 2023

# Overview

- ▶ Introduction to first-order logic
- ▶ Syntax and semantics of first-order logic
- ▶ Formal systems: axioms, rules of inference, proofs
- ▶ Formalizing mathematical theories (e.g., Peano arithmetic)
- ▶ Coding exercises

# Introduction to First-Order Logic

- ▶ Extension of propositional logic with quantifiers
- ▶ Allows for reasoning about objects and their properties
- ▶ Quantifiers: universal ( $\forall$ ) and existential ( $\exists$ )
- ▶ Example:  $\forall x, P(x)$

# Syntax and Semantics

- ▶ Syntax: rules for constructing well-formed formulas
- ▶ Semantics: rules for interpreting formulas
- ▶ Terms, predicates, and quantifiers
- ▶ Example:  $\forall x, P(x) \Rightarrow Q(x)$

# Formal Systems

- ▶ Axioms: foundational statements assumed to be true
- ▶ Rules of inference: rules for deriving new statements from existing ones
- ▶ Proofs: sequences of steps leading to a conclusion

# Formalizing Mathematical Theories

- ▶ Example: Peano arithmetic
- ▶ Axioms for the natural numbers
- ▶ Defining addition, multiplication, and other operations

# Coding Exercises

- ▶ Building a simple theorem prover for propositional logic in Python
- ▶ Representing first-order formulas and evaluating them

# Summary and Next Steps

- ▶ We learned the basics of first-order logic and formal systems
- ▶ Next topic: Computability and Turing machines
- ▶ Coding exercises to reinforce concepts