A METHOD TO DERIVE DRIVE EQUATIONS FOR ANY FRC WHEELBASE

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INTRODUCTION

Every robot in the FIRST Robotics Competition must somehow translate from driver input (such as “go forward”) into motor drive speeds (such as “motor 0 full power forward, motor 1 full power reverse”). For many standard wheel configurations, these equations are hidden within the libraries FIRST provides, and the programmer does not need to know them. At the time of writing, these libraries are the WPILib, and the equations reside within the RobotDrive class.

However, teams using unusual drive bases must write their own drive equations because the FIRST-provided libraries cannot possibly cover every combination of wheels. This paper provides a method by which a team can create a set of drive equations to govern an arbitrary combination of:

- Simple wheels, defined as traditional wheels with no rollers
- Omni wheels, defined as compound wheels featuring rollers whose axles are parallel to the direction of the main wheel’s motion
- Mecanum wheels, defined as compound wheels featuring rollers whose axles are 45° inclined to the direction of the main wheel’s motion
- Swerve wheels, defined as powered wheels that can be steered in the yaw axis

Simple wheel | Omni wheel | Mecanum wheel | Swerve wheel

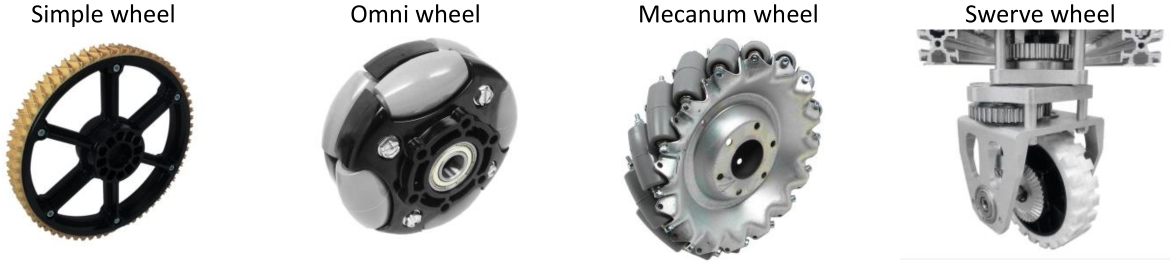

Figure 1 – Supported wheel types - Images courtesy AndyMark®

The equations provided by this method take as input (independent variables):

- The desired robot translational velocity, a 2D vector, $\langle x_R, y_R \rangle$ or $s_R \angle \theta_R$
- The desired robot yaw rate, a scalar, hereafter referred to as angular velocity, ω_R

They provide as output:

- For each wheel, the roll speed required to accomplish the desired maneuver

ASSUMPTIONS AND LIMITATIONS

This paper is written for a coach or an advanced student. Understanding this paper may require familiarity with vectors, a good grasp of basic algebra, and an understanding of basic geometry. Readers who are not familiar with these topics are encouraged to read on, and seek out supporting information as needed.

This paper will not tell you:

- How to move a robot from one point to another on the field
- How to make a wheel roll at the desired speed in spite of resistance and slippage
- How to convert joystick values to desired translational velocity and angular velocity

Drag forces from non-driven wheels are not accounted for by this method.

This method does not account for keeping the wheel roll velocities within the capabilities of each wheel and its drivetrain. Certain combinations of inputs may result in the drive equations commanding more than 100% power to a motor, which will result in the motor running slower than the other equations expect, leading to the robot performing an incorrect maneuver. You can solve this by shifting inputs to the nearest value that does not saturate the drivetrain, to keep your robot performing the correct maneuver.

DEFINITIONS AND SYMBOLS

MATHEMATICAL NOTATION CONVENTIONS

$m\angle\theta$: A vector in polar notation – magnitude at angle.

$\langle x, y\rangle$: A vector in Cartesian notation.

$\vec{A}$: A vector variable

a: A scalar variable

$\vec{A}\ \overrightarrow{proj}\ \vec{B}$: The vector projection of $\vec{A}$ onto $\vec{B}$.

$\vec{A}\ proj\ \vec{B}$: The scalar projection of $\vec{A}$ onto $\vec{B}$.

ROBOT TERMS

Drive base: The wheels, their drivetrains, and the part of the chassis that holds them together.

Drive equations: The set of equations that determine the roll speed of each wheel to accomplish a given robot translational velocity and angular velocity.

ROBOT VARIABLES

Robot translational velocity, $\overrightarrow{S_R} = \langle x_R, y_R \rangle = s_R \angle \theta_R$: The speed and direction, relative to the field, in which the robot is moving across the floor.

Robot angular velocity, ω_R: The rate at which the robot is turning around its center relative to the field. In this paper we draw positive $\omega_R$as counter-clockwise.

Wheel roll velocity, $\vec{S} = \langle x, y \rangle = s\angle\theta_s$, a 2D vector: The speed and direction, relative to the field, at which a wheel would roll across the floor if it was a simple wheel. That is, if its speed and direction were unaffected by the rollers found on compound wheels. This is related to the wheel's angular velocity by the equation $|\vec{S}| = 2\pi R\omega$, where R is the radius of the wheel and ω is the wheel's angular velocity. The angle of $\vec{S}$ is simply the angle at which the wheel is mounted to the robot, where $\vec{S}$ is pointed in the direction the wheel would roll (perpendicular to its axle). This is the output of drive equations regarding swerve wheels.

Wheel roll speed, s: The magnitude of $\vec{S}$, that is, $s = |\vec{S}|$. This is the output of drive equations regarding non-swerve wheels, since they remain at a fixed angle relative to the drive base.

Roller vector, $\vec{r}$: For compound wheels, the unit vector, relative to the field, along which the rollers can push against the floor. This is parallel to the roller axles, as opposed to the direction along which the rollers spin freely. (See Figure 14)

Roller angle, θ_{Sr}: For compound wheels, the angle between the roller vector $\vec{r}$ and the roll velocity $\vec{S}$. For omni wheels, this angle is 0, since the rollers are parallel to the wheel's roll direction. For Mecanum wheels, this is typically$\pm\frac{\pi}{4}$. (See Figure 14)

Tangent vector, $\vec{T}$: When the robot is turning about its center, this is a unit vector tangent to the circle along which the wheel moves. It points in the direction of positive robot angular velocity, and is relative to the field. (See Figure 14)

VECTOR PROJECTION REVIEW

The vector projection of a vector $\vec{a}$ onto another vector $\vec{b}$, which is written in this paper as $\vec{a}\,\overrightarrow{proj}\,\vec{b}$, is the "shadow" that $\vec{a}$ "casts" along $\vec{b}$. Note that $\vec{a}\,\overrightarrow{proj}\,\vec{b}$ can be longer than $\vec{b}$, as seen in the second figure.

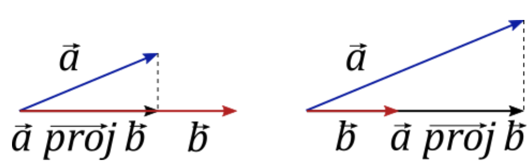

Figure 2 – Vector projection

The resulting vector can be computed with this equation (the dot indicates a dot product):

$$\vec{a}\,\overrightarrow{proj}\,\vec{b} = \left(\vec{a}\cdot\frac{\vec{b}}{|\vec{b}|}\right)\frac{\vec{b}}{|\vec{b}|}$$

In the specific case that $\vec{b}$ is a unit vector, this equation becomes much simpler, because $|\vec{b}| = 1$:

$$\vec{a}\,\overrightarrow{proj}\,\vec{b} = (\vec{a}\cdot\vec{b})\vec{b}$$

The length of this vector, called the scalar projection, is simply:

$$\vec{a}\;proj\;\vec{b} = \vec{a}\cdot\vec{b}$$

Since the dot product of $\vec{a}$ and $\vec{b}$ is $\vec{a}\cdot\vec{b}$=$|\vec{a}\,||\vec{b}|\cos\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$, we can also write:

(1) $$\vec{a}\,\overrightarrow{proj}\,\vec{b} = (|\vec{a}|\cos\theta\,)\vec{b}$$

and

(2) $$\vec{a}\;proj\;\vec{b} = |\vec{a}|\cos\theta$$

when $|\vec{b}| = 1$.

RELATIONSHIP BETWEEN WHEEL MOVEMENT AND WHEEL ROLL

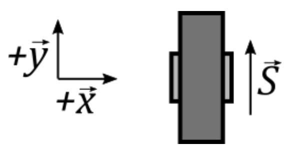

Figure 3 – Constraints on a simple wheel

A simple wheel can only move in the direction of its roll (unless it skids). Thus, the robot's movement is constrained to match the wheel's roll along the vector $\vec{S}$. The wheel holds the robot still in the direction perpendicular to $\vec{S}$. In Figure 3, this means the part of the robot attached to the wheel can only move along the y axis, at the speed s (which is $|\vec{S}|$). It cannot move along the x axis.

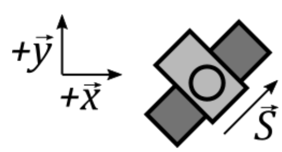

Figure 4 – Constraints on a swerve wheel

A swerve wheel creates the same relationship as a simple wheel, but the direction of $\vec{S}$ can be controlled by the software.

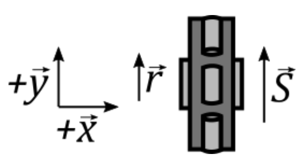

Figure 5 – Constraints on an omni wheel

An omni wheel exerts force along the direction of its roll, but slides freely in the direction parallel to its axle. Like the simple wheel, the robot's movement must match the rolling of the wheel along $\vec{S}$. However, the wheel does not constrain the robot's movement at all parallel to the axle. In Figure 5, the robot is free to move along the $\vec{S}$ axis unconstrained, and will move along the y axis at speed s (which is $|\vec{S}|$).

Movement along the rollers rolling direction is dictated by the other wheels' constraints on the robot's movement. For instance, consider the simple, not-very-useful robot in Figure 6:

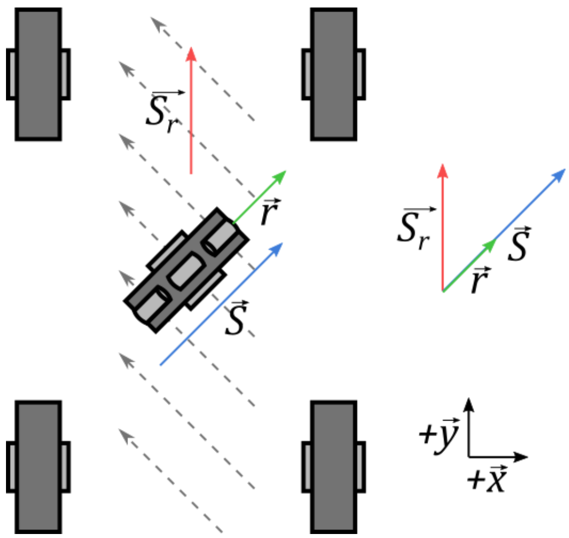

Figure 6 – Robot with one omni wheel and four undriven simple wheels

This robot has one omni wheel, and four simple, undriven wheels that spin freely. Thus, the robot is free to move along the positive y axis, but cannot move along the x axis. It will probably not win any competitions this way. Logically, the driven omni wheel in the center can push the robot along the y axis only. Its rollers roll along the dotted lines, in a movement somewhat analogous to turning screw threads. The robot's velocity $\overrightarrow{S_R}$ projected along $\vec{r}$ is the same as $\vec{S}$. Also $\vec{r}$ and $\vec{S}$ are parallel, so $\vec{S}\ \overrightarrow{proj}\ \vec{r} = \vec{S}$.

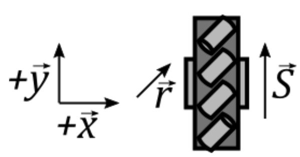

Figure 7 – Constraints on a Mecanum wheel

A Mecanum wheel's relationship with the floor is slightly less obvious. Like with an omni wheel, the floor may move freely under the wheel, but only in the direction perpendicular to the roller axle. So, like the omni wheel, we the movement of the wheel is constrained to match the movement of the robot, but only along the direction parallel to the roller axle. For example, see the simple useless robot in Figure 8:

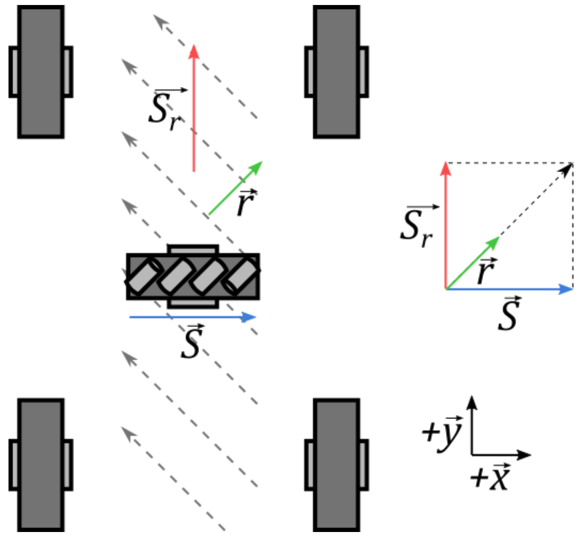

Figure 8 – Robot with one Mecanum wheel and four undriven simple wheels

Like the robot in Figure 6, this robot can only move in the y axis. When the wheel turns, the rollers move along the dotted lines. The rollers, acting analogous to screw threads, push the robot along $\overrightarrow{S_R}$. As long as the wheels continue to grip the floor, the robot's movement will match the wheel's movement, but only in the direction of $\vec{r}$. Another way to phrase this is $\vec{S}\ \overrightarrow{proj}\ \vec{r} = \overrightarrow{S_R}\ \overrightarrow{proj}\ \vec{r}$, and consequently $\vec{S}\ proj\ \vec{r} = \overrightarrow{S_R}\ proj\ \vec{r}$.

METHOD

This method consists of these steps:

For each wheel:

1. Determine the roll velocity that results in translation but no rotation
2. Determine the roll velocity that results in rotation but no translation
3. Compute the average of the above roll velocities

STEP 1: DETERMINE $\vec{S}$ FOR EACH WHEEL WHEN $\omega_R = 0$

For the robot to translate without rotating, each wheel must roll at the same velocity as the robot's movement. Non-compound wheels must be oriented along the robot's movement.

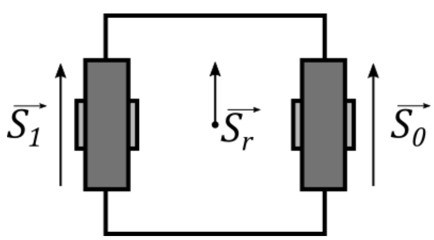

Figure 9 – Wheel roll velocities for two simple wheels

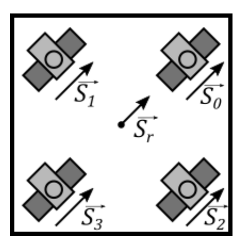

Figure 10 – Wheel roll velocities for four swerve wheels

So for a simple or swerve wheel,

(3) $$\vec{S} = \overrightarrow{S_R}$$

This completes the first half of this wheel's drive equation. Note that a fixed simple wheel will only support $\vec{S}$ and therefore $\overrightarrow{S_R}$ in one direction. That is, without rotating, the robot in Figure 9 can only move along the y axis.

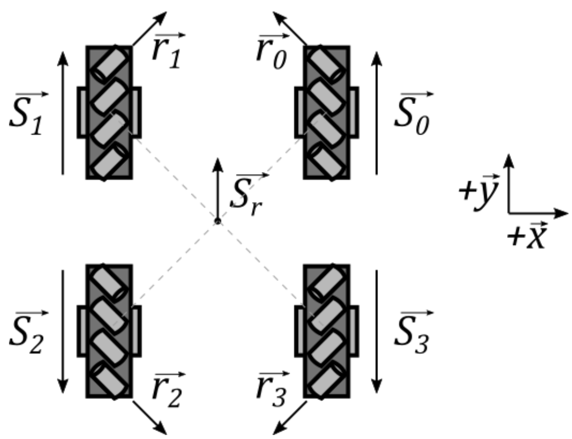

Figure 11 – Wheel roll velocities for four Mecanum wheels

For an omni or Mecanum wheel, the wheel's rolling velocity is constrained to match the robot's velocity, but only in the direction parallel to the roller axles. Therefore, $\vec{S}\ proj\ \vec{r} = \overrightarrow{S_R}\ proj\ \vec{r}$, where sp denotes scalar projection. Since $\vec{r}$ is a unit vector, $|\vec{r}| = 1$, and the scalar projection can be simplified to $\vec{S} \cdot \vec{r} = \overrightarrow{S_R} \cdot \vec{r}$. For compound wheels, the dependent variable we need to find is $s = |\vec{S}|$. We can rewrite the last equation as $\vec{S} \cdot \vec{r} = |\vec{S}| \cos \theta_{Sr} = \overrightarrow{S_R} \cdot \vec{r}$, and therefore

(4) $$s = \frac{\overrightarrow{S_R} \cdot \vec{r}}{\cos \theta_{Sr}}$$

For an omni wheel $\theta_{Sr} = 0$, so

(5) $$s = \overrightarrow{S_R} \cdot \vec{r}$$

For a typical Mecanum wheel, $\theta_{Sr} = \pm\frac{\pi}{4}$, and $\cos \pm\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, so:

(6) $$s = \sqrt{2}\overrightarrow{S_R} \cdot \vec{r}$$

STEP 2: DETERMINE $\vec{S}$ FOR EACH WHEEL WHEN $\overrightarrow{S_R} = 0$

For the robot to rotate without translating, every wheel must follow a circular track centered on the robot's center:

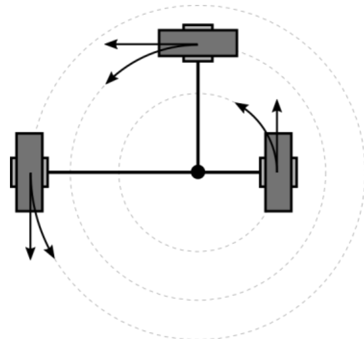

Figure 12 – Rotating robot

Simple wheels must have their axles pointed through the center of the robot, or the robot will be incapable of rotating around its center without the wheels scrubbing along the floor.

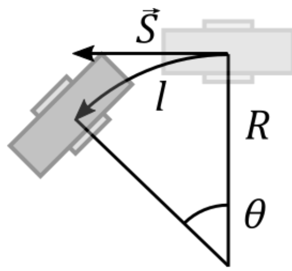

Figure 13 – Wheel sweeping out a sector

Over a small slice of time, dt, the simple wheel will roll the distance dl along the circle, and the robot's angle relative to the world will change by $d\theta$. When θ is in radians, the sector length l is given by $l = R\theta$, and $\frac{dl}{dt} = R\frac{d\theta}{dt}$. By definition, $\omega_R = \frac{d\theta}{dt}$. For very small values of dt, the sector of the circle approaches a straight line segment, so we can approximate $\frac{dl}{dt}$ as $\vec{S}$, and therefore write:

(7) $$\vec{S} = R\omega_R$$

when $\vec{S}$ points along positive ω_R. When $\vec{S}$ points against ω_R, $\frac{dl}{dt}$ is $-\vec{S}$, and so in that case:

(8) $$\vec{S} = -R\omega_R$$

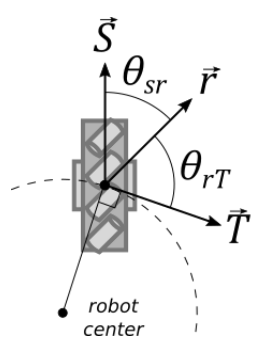

Figure 14 – Roller vector, Roller angle, Tangent vector

For compound wheels, the wheel's rolling velocity is constrained to match a vector tangent to the circle swept out by the wheel's contact patch, but only in the direction parallel to the roller axles. Referring to this tangent vector as $\vec{T}$,a unit vector, this can be written as:

$$R\omega_R = \left(\vec{S}\,\overrightarrow{proj}\,\vec{r}\right) proj\,\vec{T}$$

Since $\vec{r}$ and $\vec{T}$ are unit vectors, we can use (1) and (2) rewrite this as:

$$R\omega_R = \left|\vec{S}\,\overrightarrow{proj}\,\vec{r}\right| \cos\theta_{rT} = \left(\vec{S}\,proj\,\vec{r}\right)\cos\theta_{rT} = \left|\vec{S}\right|\cos\theta_{Sr}\ \cos\theta_{rT} = s\cos\theta_{Sr}\ \cos\theta_{rT}$$

Solving for s results in:

(9) $$s = \frac{R\omega_R}{\cos\theta_{Sr}\ \cos\theta_{rT}}$$

For an omni wheel, $\theta_{Sr} = 0$, so:

(10) $$s = \frac{R\omega_R}{\cos\theta_{rT}}$$

For a typical Mecanum wheel, $\theta_{Sr} = \pm\frac{\pi}{4}$, and $\cos\pm\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, so:

(11) $$s = \frac{\sqrt{2}R\omega_R}{\cos\theta_{rT}}$$

Worth noting is that (7), (8), (10), and (11) are all merely (9) with specific values supplied for $\theta_{Sr}$and θ_{rT}. An omni wheel is just a Mecanum wheel with $\theta_{Sr} = 0$, a simple wheel is just an omni wheel with the rollers locked in place, and an swerve wheel is just a simple wheel where the angle of $\vec{S}$ is actuated.

SUMMARY OF CONSTRAINT EQUATIONS

Wheel type		**When $\omega_R = 0$**		**When $\overrightarrow{S_R} = 0$**
Simple	(3)	$\vec{S} = \overrightarrow{S_R}$	(7)	$\vec{S} = R\omega_R\angle\theta_s$ or (8) $\vec{S} = -R\omega_R\angle\theta_s$
Omni	(5)	$s = \overrightarrow{S_R}\cdot\vec{r}$	(10)	$s = \frac{R\omega_R}{\cos\theta_{rT}}$
Mecanum (45° rollers)	(6)	$s = \sqrt{2}\cdot\overrightarrow{S_R}\cdot\vec{r}$	(11)	$s = \frac{\sqrt{2}R\omega_R}{\cos\theta_{rT}}$
Mecanum (θ_{Sr} rollers)	(4)	$s = \frac{\overrightarrow{S_R}\cdot\vec{r}}{\cos\theta_{Sr}}$	(9)	$s = \frac{R\omega_R}{\cos\theta_{Sr}\ \cos\theta_{rT}}$
Swerve	(3)	$\vec{S} = \overrightarrow{S_R}$	(7)	$\vec{S} = R\omega_R\angle\theta_T$

EXAMPLES

STANDARD ARCADE DRIVE

A common drive base is one with two wheels, mounted parallel on the outside of the chassis.

STEP 1: $\omega_R = 0$

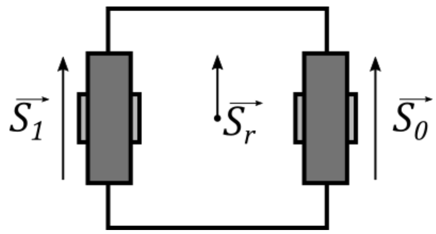

Figure 15 – A standard drive base with two simple wheels, showing translation

Consulting (3) tells us that $\vec{S}_0 = \vec{S}_1 = \vec{S_R}$. Given the wheels' orientation, we can also note that $\vec{S}_0 = \langle 0, s_0 \rangle$ and $\vec{S}_1 = \langle 0, s_1 \rangle$. So, $\langle 0, s_0 \rangle = \langle 0, s_1 \rangle = \langle 0, y_R \rangle$. Therefore, we can say that when $\omega_R = 0$:

(12) $$s_0 = y_R$$
(13) $$s_1 = y_R$$

STEP 2: $\vec{S_R} = 0$

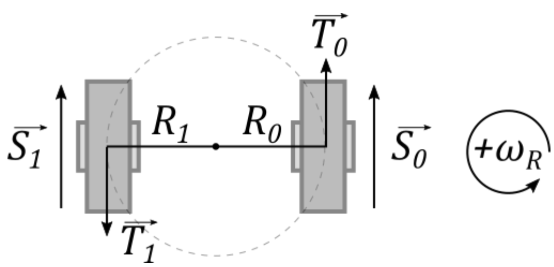

Figure 16 – A standard drive base with two simple wheels, showing rotation

In this case, $\vec{S}_0$ is oriented against ω_R, and thus should use equation (8). However, $\vec{S}_1$ is oriented along ω_R, and thus should use (7). Therefore,

$$\vec{S}_0 = -R_0 \omega_R \angle \theta_s \text{ and}$$

$$\vec{S}_1 = R_1 \omega_R \angle \theta_s$$

Since $\vec{S}_0 = \langle 0, s_0 \rangle$ and $\vec{S}_1 = \langle 0, s_1 \rangle$,

$$\langle 0, s_0 \rangle = -R_0 \omega_R \angle \theta_s \text{ and}$$

$$\langle 0, s_1 \rangle = R_1 \omega_R \angle \theta_s$$

In both cases, $\theta_s = \frac{\pi}{2}$ – that is, $\vec{S}_0$and $\vec{S}_1$ are both pointed "upward", along the positive direction of the y-axis. So:

(14) $$s_0 = -R_0 \omega_R$$
(15) $$s_1 = R_1 \omega_R$$

STEP 3: TAKE THE MEAN

Equations (12) and (14) average to create (16), while (13) and (15) average to create (17).

(16) $$\mathrm{s}_0 = \frac{\mathrm{y_R} - \mathrm{R}_0\omega_\mathrm{R}}{2}$$

(17) $$\mathrm{s}_1 = \frac{\mathrm{y_R} + \mathrm{R}_1\omega_\mathrm{R}}{2}$$

PRACTICAL NOTE

While equations (16) and (17) are the final drive equations, it would be unusual to see them represented exactly this way in software.

A highly skilled FRC veteran could, in theory, compute the maximum desirable robot speed, along with precisely the desired mapping between user inputs and robot speed. In reality, the first time a robot is driven, the controls typically feel either "twitchy" (the robot responds strongly to small movements of the sticks) or feels "slow" (the stick is full-forward and the robot merely crawls). Consequently, most teams wind up inserting multipliers at several parts of the drive equation. Rather than (16) and (17), you are more likely to see robot software for this drive base that looks something like this, where $\mathrm{y_R}$ comes from the y axis of a joystick and ω_R comes from the x axis of the same stick:

(18) $$\mathrm{y}'_\mathrm{R} = C_y \mathrm{y_R}$$
(19) $$\omega'_\mathrm{R} = C_\omega \omega_\mathrm{R}$$
(20) $$\mathrm{s}'_1 = y_R + \omega_R$$
(21) $$\mathrm{s}'_0 = y_R - \omega_R$$
(22) $$\mathrm{s}_0 = C_0 \mathrm{s}'_0$$
(23) $$\mathrm{s}_1 = C_1 \mathrm{s}'_1$$

This accomplishes the same thing as (16) and (17), but the values $\frac{1}{2}$, R_0, and R_1 have been subsumed into C_y, C_ω, C_0, and C_1. It is also common to find (20) and (21) acting as standalone drive equations – without the coefficients, the robot still performs the same maneuvers. Note that in a symmetrical robot, such as the one in this example, $\mathrm{R}_0 = \mathrm{R}_1$, and C_0 will very likely equal C_1.

Many teams also employ features such as deadbanding and exponential scaling of joystick inputs. These are useful features, which would replace (18) and (19) with more advanced functions.

FOUR-MECANUM SQUARE

This is the simplest drive base that features 4 Mecanum wheels. The contact patches of the wheels sit at the corners of a square, and the rollers are mounted at a 45° angle to each axle.

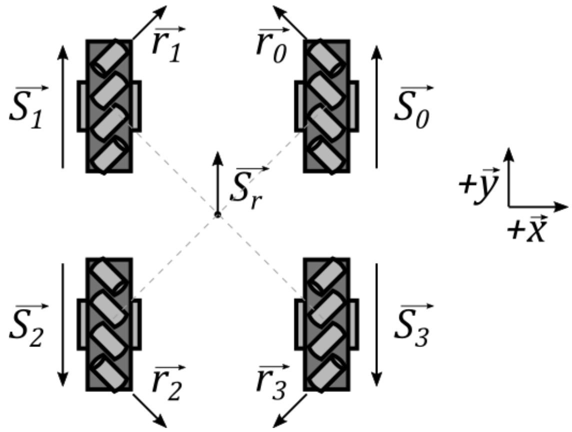

Figure 17 – Wheel roll velocities for four Mecanum wheels

STEP 1: $\omega_R = 0$

Since these are Mecanum wheels with 45° rollers, (6) describes the wheel speed for each wheel. The roller vectors in the field's reference frame are:

(24) $$\vec{r_0} = \langle -\sqrt{2}, +\sqrt{2} \rangle$$
(25) $$\vec{r_1} = \langle +\sqrt{2}, +\sqrt{2} \rangle$$
(26) $$\vec{r_2} = \langle +\sqrt{2}, -\sqrt{2} \rangle$$
(27) $$\vec{r_3} = \langle -\sqrt{2}, -\sqrt{2} \rangle$$

Substituting these into (6) for each wheel yields:

(28) $$s_0 = \sqrt{2} \cdot \vec{S_R} \cdot \langle -\sqrt{2}, +\sqrt{2} \rangle$$
(29) $$s_1 = \sqrt{2} \cdot \vec{S_R} \cdot \langle +\sqrt{2}, +\sqrt{2} \rangle$$
(30) $$s_2 = \sqrt{2} \cdot \vec{S_R} \cdot \langle +\sqrt{2}, -\sqrt{2} \rangle$$
(31) $$s_3 = \sqrt{2} \cdot \vec{S_R} \cdot \langle -\sqrt{2}, -\sqrt{2} \rangle$$

In (32) through (33), $\vec{r_n}$ is expressed in Cartesian format. It would have been equally valid to express $\vec{r_n}$ in polar format. In this case, let's assume the robot's velocity ($\overrightarrow{S_R}$) is also expressed in Cartesian format. This would be the case if the robot's desired translation came from a single joystick's x and y axes.

Substituting $S_R = \langle x, y \rangle$ and computing the dot product results in:

(34) $$s_0 = -2x + 2y$$
(35) $$s_1 = +2x + 2y$$
(36) $$s_2 = +2x - 2y$$
(37) $$s_3 = -2x - 2y$$

STEP 2: $\vec{S_R} = 0$

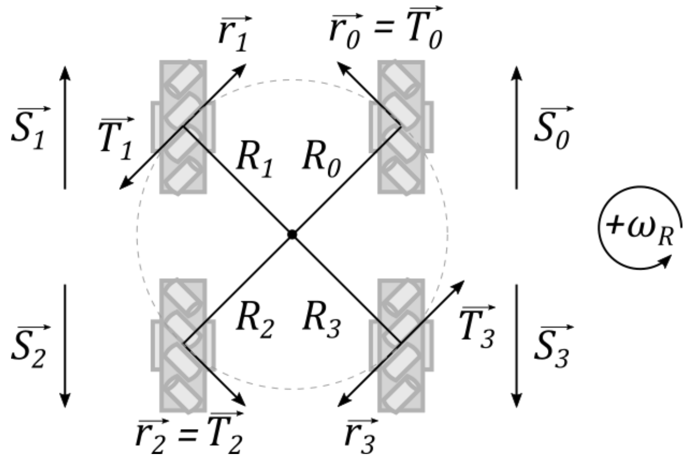

Figure 18 – Roller and tangent vectors for four Mecanum wheels

For this step, we require (11), and consequently we also need the angle between the roller vector and the tangent vector for each wheel. This can be seen by inspecting Figure 18, and can be computed for any robot. In this case, since the robot is a perfect square, the roller vectors happen to align perfectly parallel to the tangent vectors – either along or against them. So for wheels 0 and 2, $\theta_{rT} = 0$ and $\cos\theta_{rT} = 1$. For wheels 1 and 3, $\theta_{rT} = \pi$ and $\cos\theta_{rT} = -1$. Substituting this into (11) results in:

(38) $$s_0 = +\sqrt{2}R\omega_R$$
(39) $$s_1 = -\sqrt{2}R\omega_R$$
(40) $$s_2 = +\sqrt{2}R\omega_R$$
(41) $$s_3 = -\sqrt{2}R\omega_R$$

STEP 3: TAKE THE MEAN

Averaging (42) through (43) with (44) through (45) results in:

(46) $$s_0 = -x + y + \frac{\sqrt{2}}{2}R\omega_R$$
(47) $$s_1 = +x + y - \frac{\sqrt{2}}{2}R\omega_R$$
(48) $$s_2 = +x - y + \frac{\sqrt{2}}{2}R\omega_R$$
(49) $$s_3 = -x - y - \frac{\sqrt{2}}{2}R\omega_R$$

Note that R, the distance from each wheel's contact patch to the center of the robot, is constant. In some robots, R can be different for different wheels. In the robot in this example, R is the same for all four wheels, so it is shown here without subscript.

PRACTICAL NOTE

Like in the previous example, the input values (x, y, and ω_R) will probably need to be scaled so that the robot's speed feels right (in stricter terms, so that the robot's velocity maps well to the operator's expectations). Therefore, the constant term $\frac{\sqrt{2}}{2}R$ will be subsumed into C_ω, and the actual drive equations would likely appear as:

(50) $$\mathrm{x}_{\mathrm{R}}' = C_x \mathrm{x}_{\mathrm{R}}$$
(51) $$\mathrm{y}_{\mathrm{R}}' = C_y \mathrm{y}_{\mathrm{R}}$$
(52) $$\omega_{\mathrm{R}}' = C_\omega \omega_{\mathrm{R}}$$
(53) $$s_0' = -x + y + \omega_R$$
(54) $$s_1' = +x + y - \omega_R$$
(55) $$s_2' = +x - y + \omega_R$$
(56) $$s_3' = -x - y - \omega_R$$
(57) $$s_0 = C_0 s_0'$$
(58) $$s_1 = C_1 s_1'$$
(59) $$s_2 = C_2 s_2'$$
(60) $$s_3 = C_3 s_3'$$